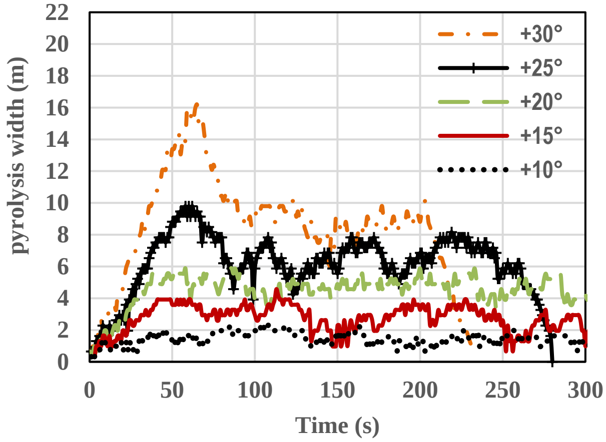
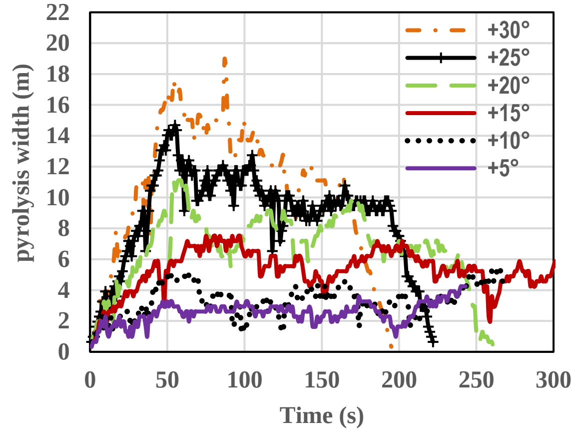
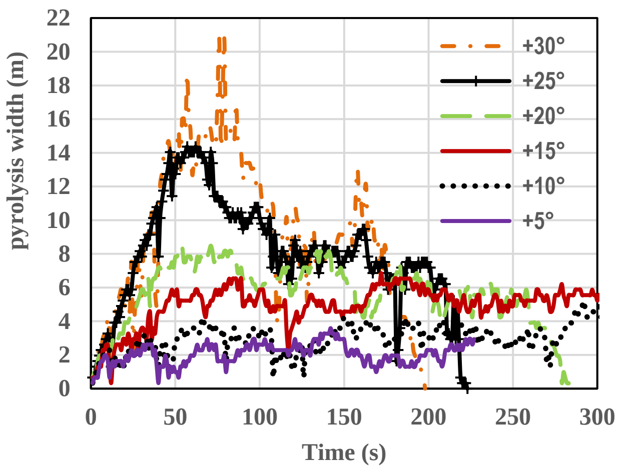
Annexure A: Supplementary document



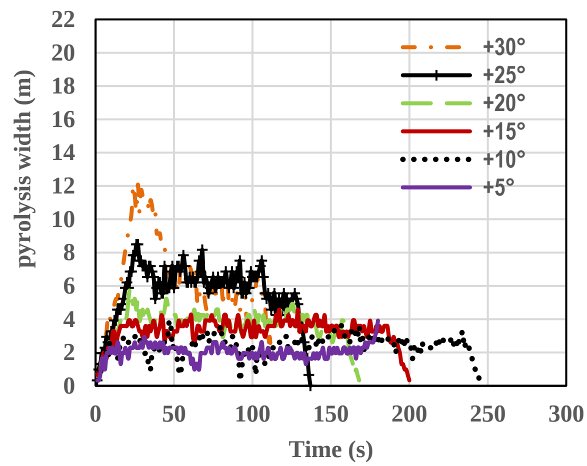
(a) Set 1, 0.1 m/s



(b) Set 2, 1 m/s

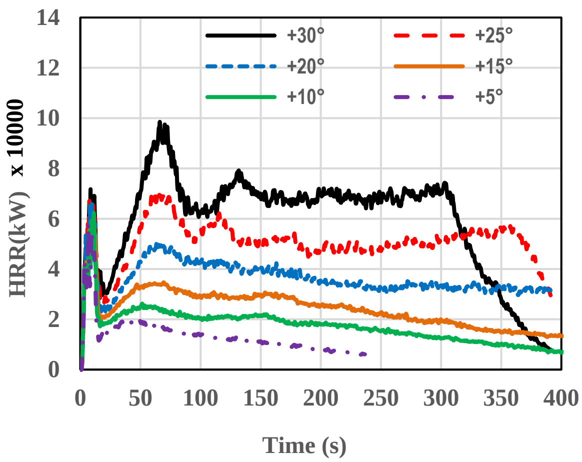


(c) Set 3, 1 m/s

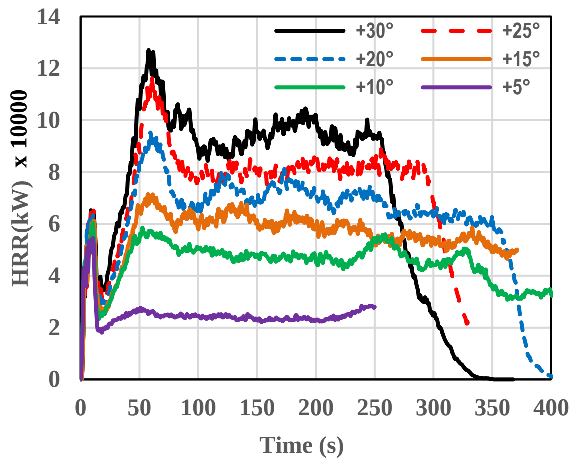


(d) Set 4, 1 m/s

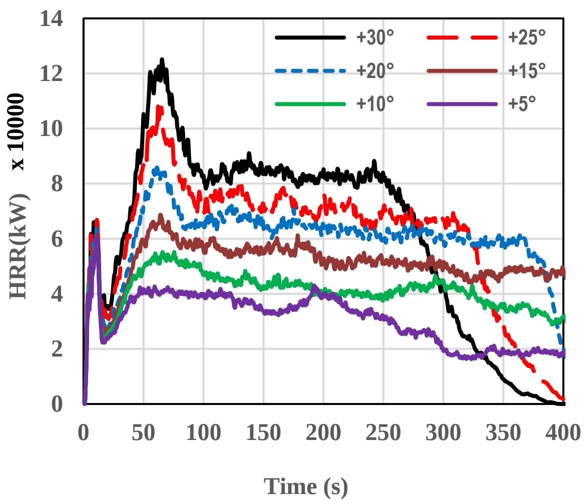
Figure S1. Pyrolysis width vs time: (a) Set 1, original domain at 0.1 m/s; (b) Set 2, original domain at 1 m/s ; (c) Set 3, larger domain with original fuel, at 1 m/s ; and (d) Set 4, larger domain with changed ('lighter& drier') fuel, at 1 m/s



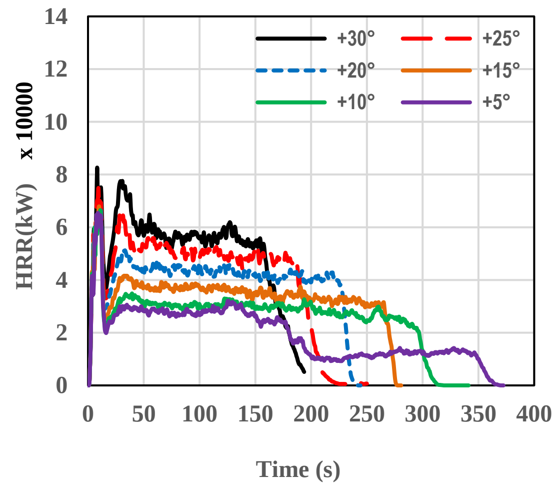
(a) Set 1, 0.1 m/s



(b) Set 2, 1 m/s

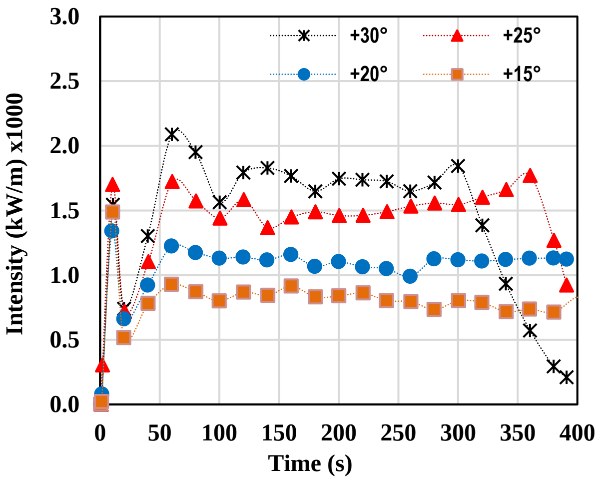


(c) Set 3, 1 m/s

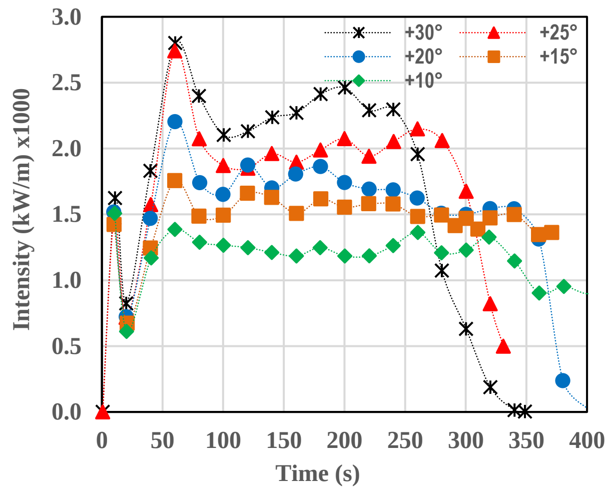


(d) Set 4, 1 m/s

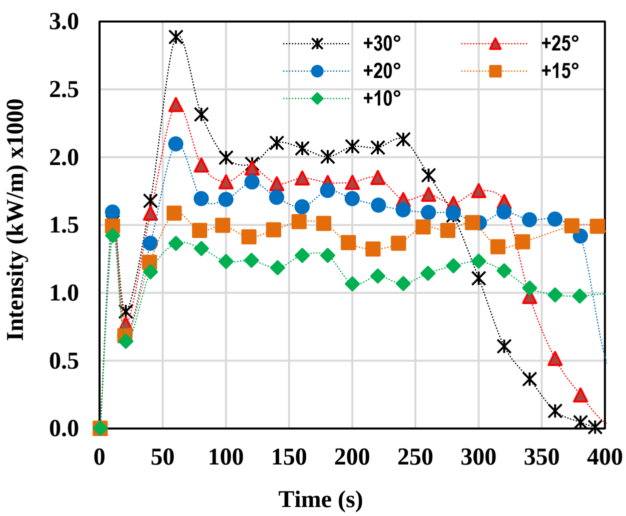
Figure S2. HRR vs time: (a) Set 1, original domain at 0.1 m/s; (b) Set 2, original domain at 1 m/s; (c) Set 3, larger domain with original fuel, at 1 m/s; (d) Set 4, larger domain with changed ('lighter & drier') fuel parameters at 1 m/s.



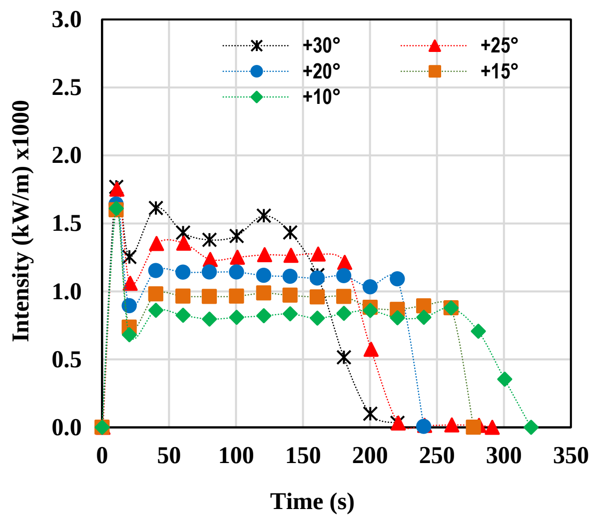
(a) Intensity vs time – Set 1, 0.1 m/s



(b) Intensity vs time – Set 2, 1 m/s

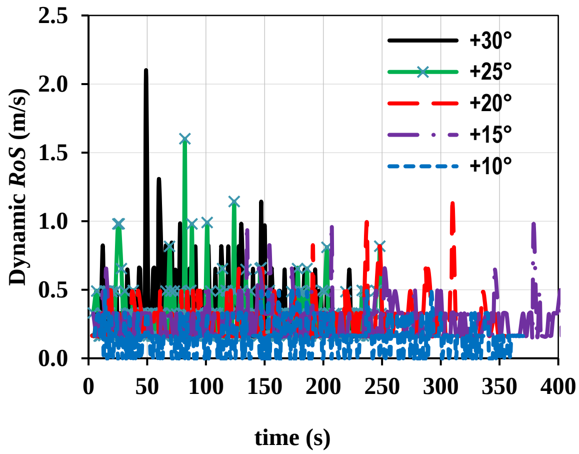


(c) Intensity vs time – Set 3, 1 m/s

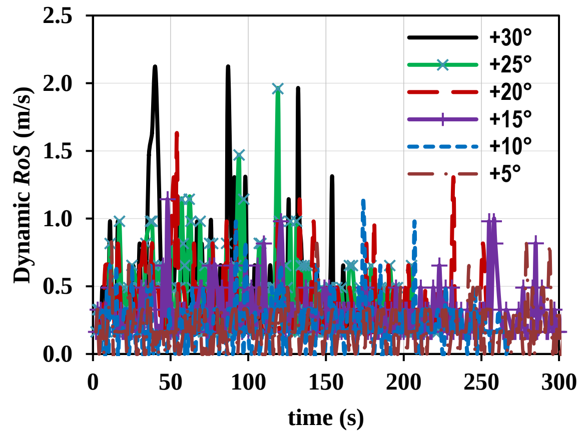


(d) Intensity vs time – Set 4, 1 m/s

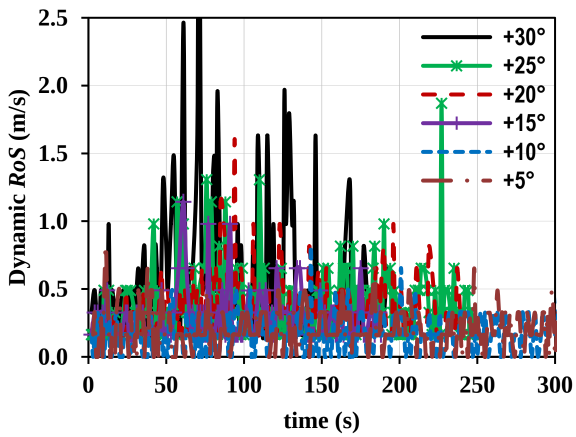
Figure S3. Fireline intensity vs time: (a) Set 1, original domain at 0.1 m/s; (b) Set 2, original domain at 1 m/s; (c) Set 3, larger domain with original fuel , at 1 m/s; (d) Set 4, larger domain with changed ('lighter & drier') fuel at 1 m/s;



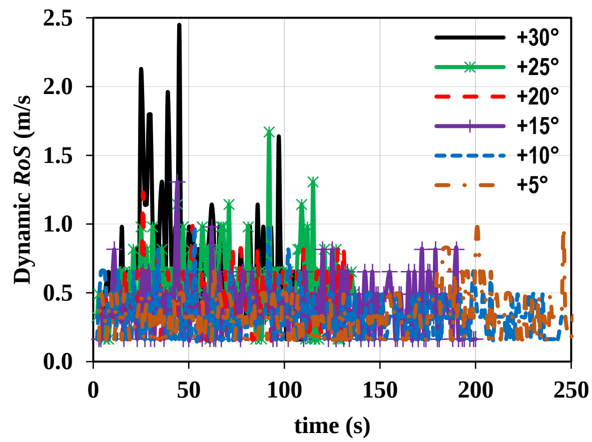
(a) Set 1, 0.1 m/s



(b) Set 2, 1 m/s

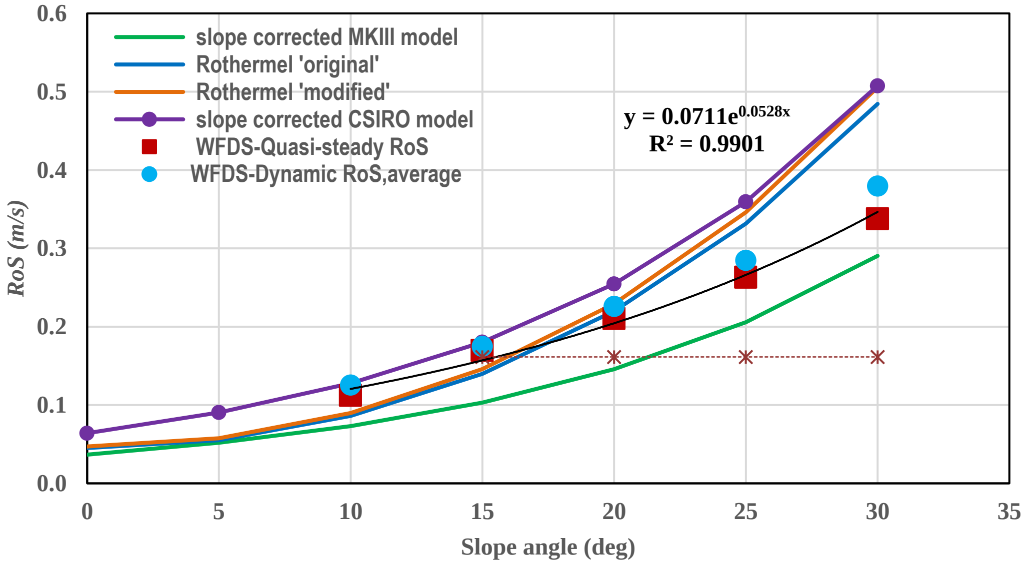


(c) Set 3, 1 m/s

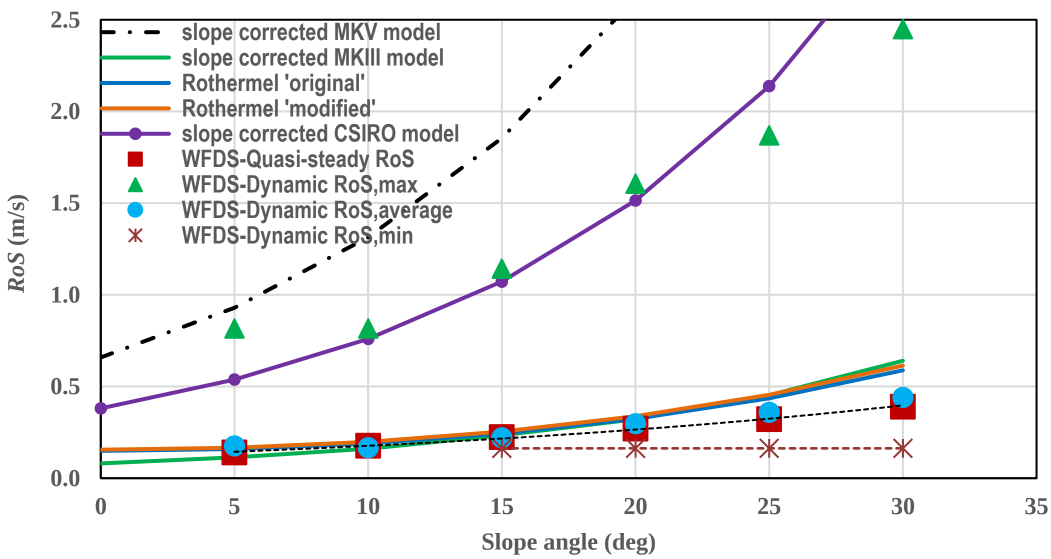


(d) Set 4, 1 m/s

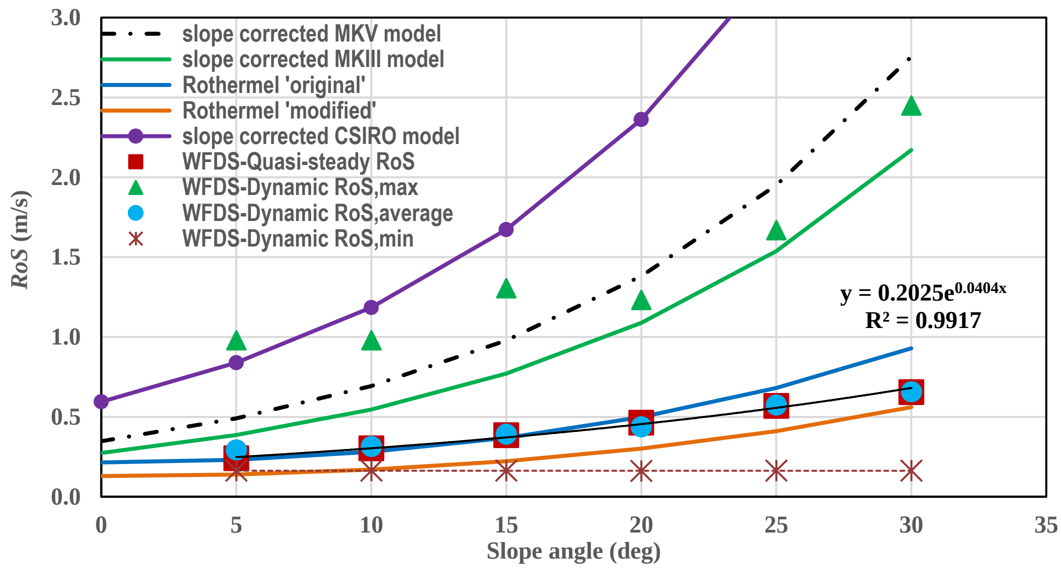
Figure S4. Dynamic *RoS* : (a) Set 1, at 0.1 m/s; (b) Set 2, at 1 m/s; (c) Set 3, large domain with original fuel, at 1 m/s; (d) Set 4, large domain with changed ('lighter & drier') fuel at 1 m/s.



(a) RoS vs slope angle, 0.1 m/s (Set 1). Dynamic maximum values are not shown as these are too large.



(b) RoS vs slope angle, 1 m/s (Set 3)

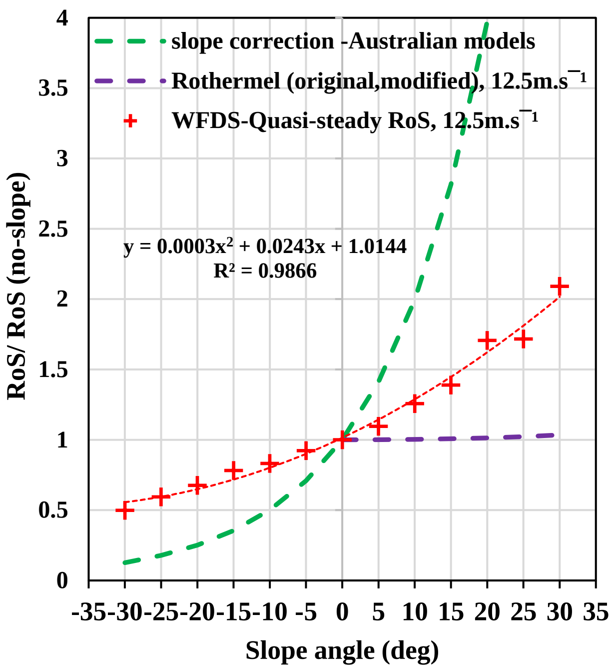


(c) *RoS* vs slope angle, 1 m/s (Set 4)

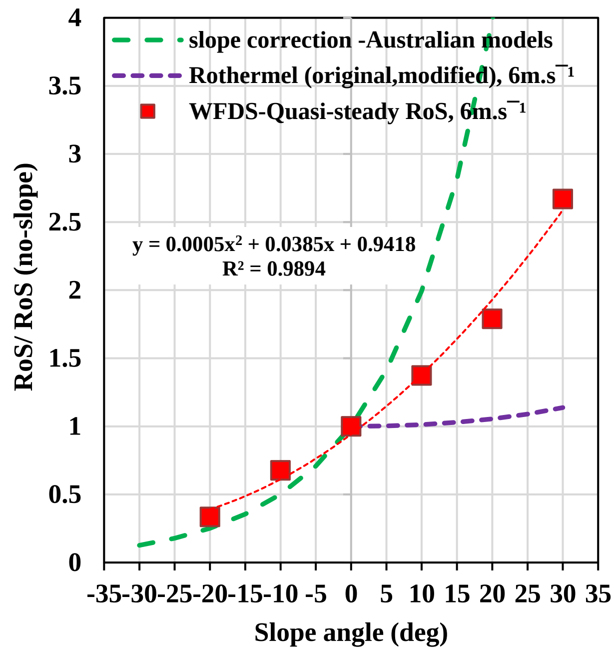
Figure S5. – *RoS* correlations, *RoS* vs slope angle: (a) at 0.1 m/s (Set 1); (b) at 1 m/s (Set 3), original fuel; and (c) at 1 m/s (Set 4), changed ('lighter & drier') fuel parameters.

(The slope values obtained from the linear fit equations to the fire front plots in Fig. 5(a) for each case are plotted against slope angles in Fig. S5(a) for 0.1 m/s wind velocity. Since we did not observe any notable variations in the fire isochrones pattern and fire front locations between these two sets, only set 3 simulation results are presented in the *RoS* calculations for 1m/s cases. *RoS* results for 1 m/s cases are shown in Fig. S5(b), obtained from linear fit equations to the fire front locations plotted for set 3 in Fig. 5(c). In Fig. S5(c), the *RoS* results of changed ('lighter & drier') fuel parameter cases are presented, obtained from the fire front locations plotted in Fig. 5(d) for changed fuel (set 4) simulations.

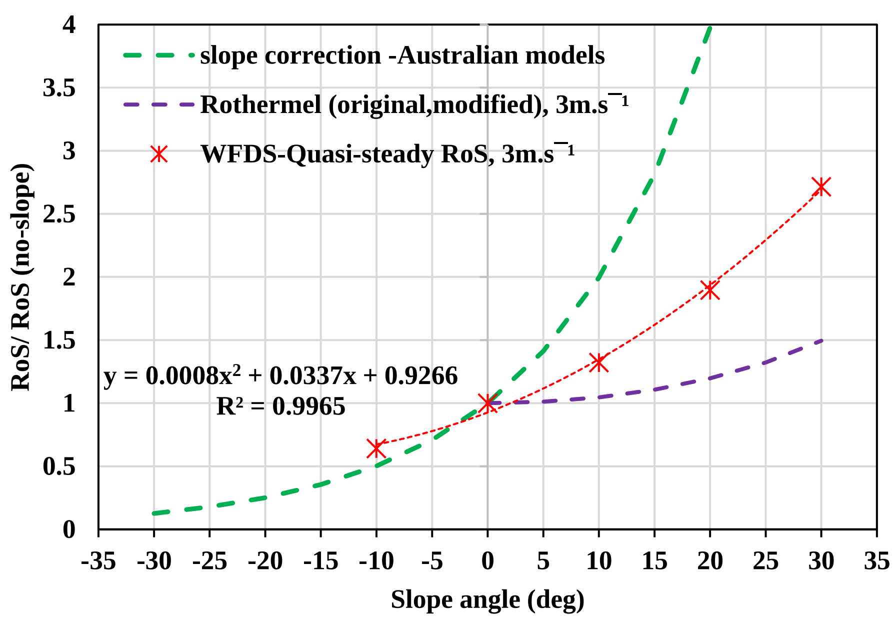
The dynamic *RoS* values are also plotted along with the quasi-steady *RoS* values in Fig.S5(a) – (c). The dynamic *RoS* values (with maximum and minimum bounds and average values) are extracted from Fig.S4(a), (c) and (d). Averaged dynamic *RoS* values are extracted from approximately 40–250 s from ignition for all cases. Note that dynamic *RoS* values are instantaneous values obtained by differentiating the fire front location data along the centreline of the burnable grass plot at each simulation output time, whereas the quasi-steady value is obtained from a linear fit to the fire front location. The two methodologies used to determine *RoS* may not be consistent in some scenarios, especially for lower slope angles at such a low wind velocity (nearly zero), where the fire front moves very slowly).



(a) Relative *RoS*: 12.5 m/s



(b) Relative RoS: 6 m/s



(c) Relative RoS: 3 m/s

Figure S6. Comparison of the effect of slope at different driving wind velocities: RoS/RoS (at no-slope) between WFDS quasi-steady results and empirical models for higher wind velocities, (a) at 12.5 m/s; (b) at 6 m/s; (and (c) at 3 m/s.

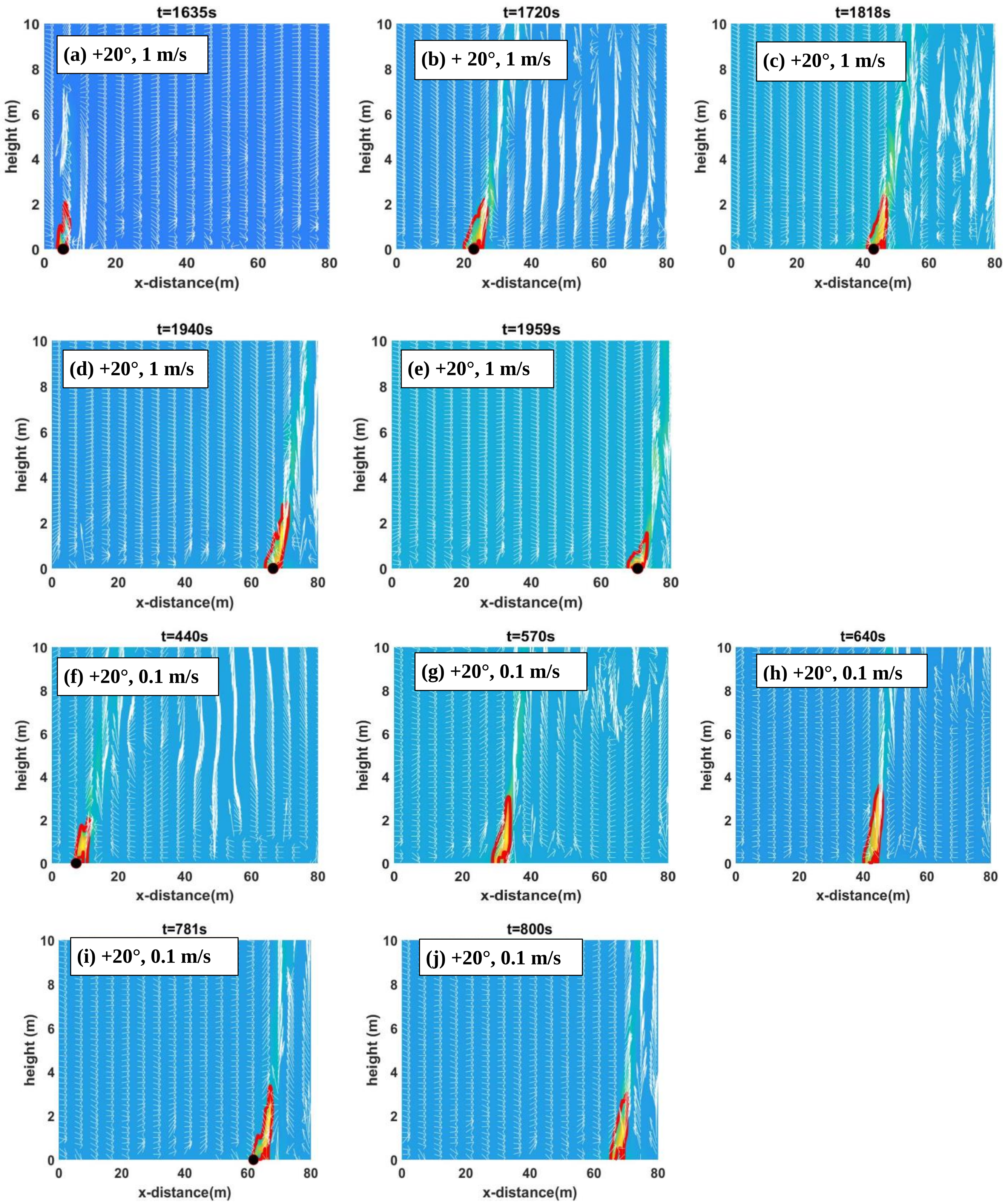
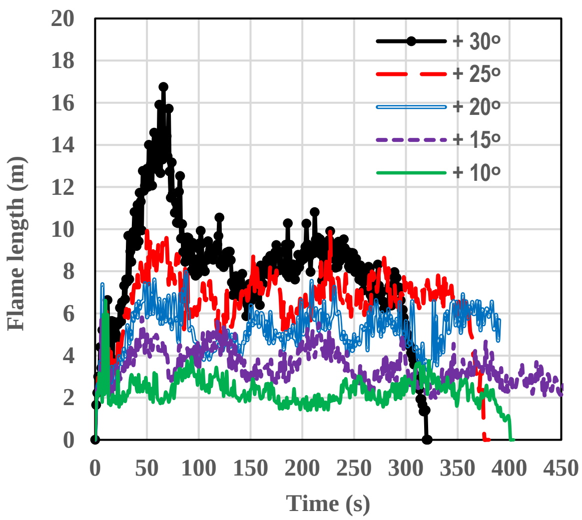
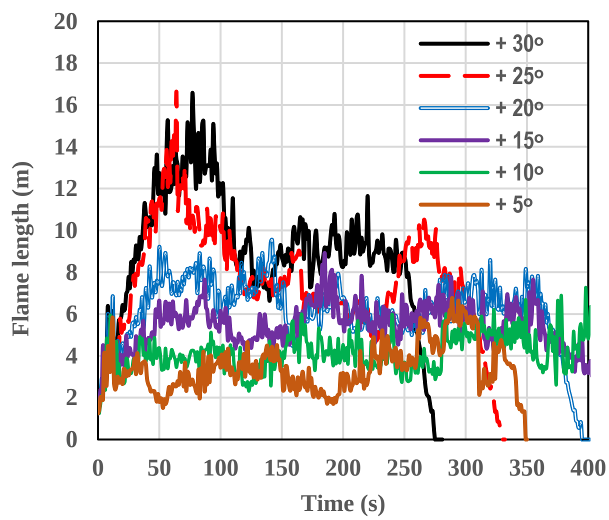


Figure S7. Flame contour (red) with temperature contour shaded (in yellow) in the background along with detachment location (black dot) and wind vector plots (white arrows), at various times: Frames (a–e): +20° at 1 m/s (Set 3); Frames (f–j): +20° at 0.1 m/s (Set 1)

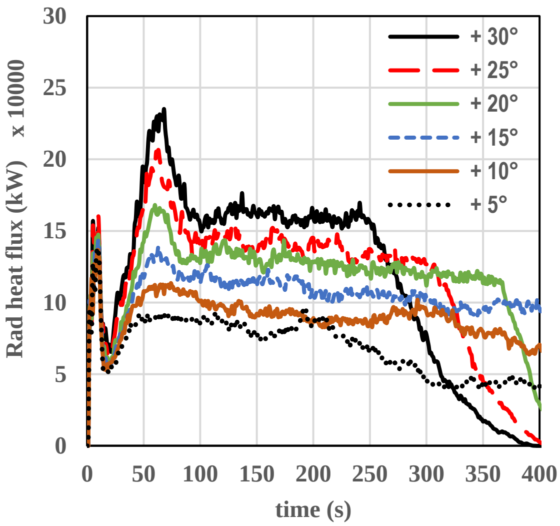


(a) Flame length vs time – Set 1, 0.1 m.s^{-1}

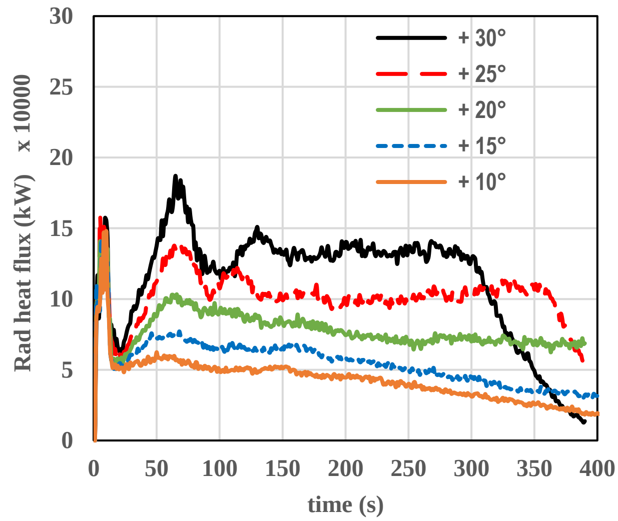


(b) Flame length vs time – Set 3, 1 m.s^{-1}

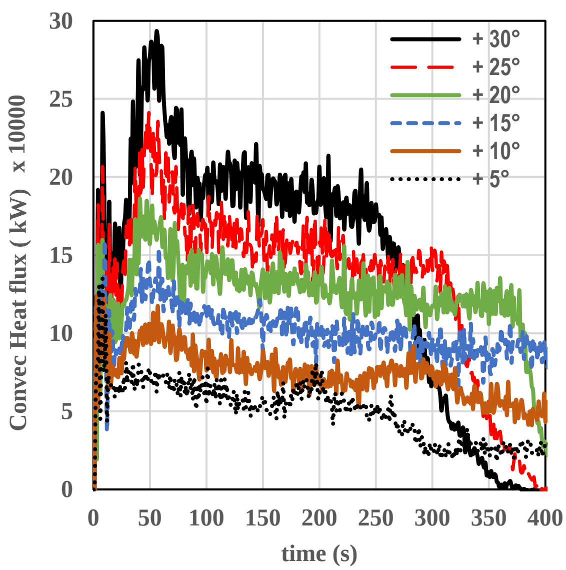
Figure S8. Flame length vs time: (a) at 0.1 m/s (Set 1); (b) at 1 m/s (Set 3); (c) Quasi-steady flame length L vs slope with empirically derived values for at 0.1 m/s (Set 1) and at 1 m/s (Set 3)



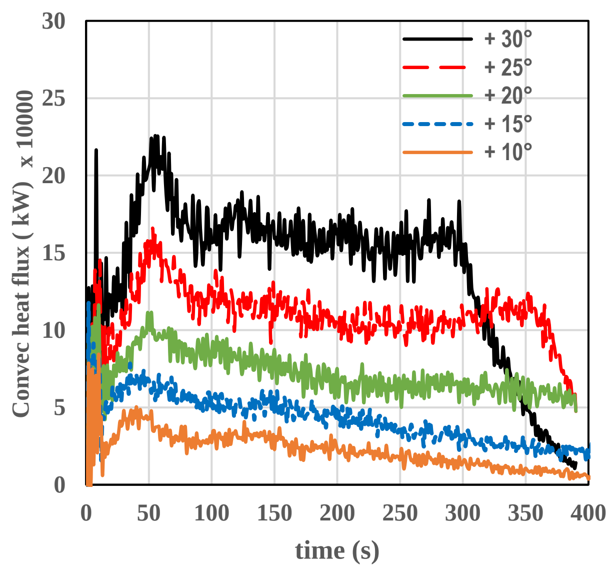
(a) Radiative flux vs time- 1 m/s (Set 3)



(b) Radiative flux vs time- 0.1 m/s (Set 1)



(c) Convec flux vs time-1 m/s (Set 3)



(d) Convec flux vs time-0.1 m/s (Set 1)

Figure S9. Total Heat fluxes as a function of time: (a) Radiative heat flux vs time at velocity 1 m/s; (b) Radiative heat flux vs time at velocity 0.1 m/s; (c) Convective heat flux vs time at 1 m/s; (d) Convective heat flux vs time at 0.1 m/s.

Rothermel models equations and correlations – SI units [1-4]

'Original' Rothermel model equation to determine RoS

$$RoS = f(W_o, \delta, \sigma, M_f, M_x, U, \Phi_s, \Phi_w, h, \rho_p, S_T, S_e) \quad (S1)$$

w_o Ovendry fuel loading (kg/m2)

δ fuel depth (m)

σ Surface area-to-volume ratio, cm^{-1}

M_f Fuel moisture content, dimensionless fraction

M_x Fuel moisture extinction, dimensionless fraction

U Windspeed at midflame height, m/min

Φ_s slope factor

Φ_w wind factor

$\tan \Phi$ slope, dimensionless fraction

h fuel heat content (fuel's heat of combustion), kJ/kg

ρ_p ovendry particle density, kg/m^3

S_T Fuel total mineral content, dimensionless fraction

S_e Fuel effective mineral content, dimensionless fraction

Rate of spread RoS (m/sec):

$$RoS = I_R * \xi \frac{1 + \Phi w + \Phi s}{\rho b * \varepsilon * Qig} \quad (S2)$$

Reaction Intensity I_R (kW/m^2):

$$I_R = \frac{1}{60}(\Gamma' * wn * h * \acute{\eta} M * \acute{\eta} s) \quad (S3)$$

Optimum reaction velocity (min^{-1})

$$\Gamma' = \Gamma' \max \left[\frac{\beta}{\beta op} \exp\left(1 - \frac{\beta}{\beta op}\right)\right]^A \quad (S4)$$

Maximum reaction velocity (min^{-1})

$$\Gamma' max = (0.0591 + 2.926\sigma \wedge - 1.5) \wedge - 1 \quad \text{(S5)}$$

Packing ratio β

$$\beta = \frac{\rho b}{\rho p} \quad \text{(S6)}$$

Ovendry bulk density ϱb (kg/m^3)

$$\rho b = \frac{w_o}{\delta} \quad \text{(S7)}$$

Optimum packing ratio *βop*

$$\beta op = 0.20395\sigma^{-0.8189} \quad \text{(S8)}$$

Dimensionless parameter A

$$A = 8.9033\sigma^{-0.7913} \quad \text{(S9)}$$

Net fuel loading Wn (kg/m^2)

$$Wn = w_o(1 - ST) \quad \text{(S10)}$$

Moisture damping coefficient (dimensionless parameter), *ήM*

$$ήM = 1 - \left(2.59 * \frac{Mf}{Mx}\right) + 5.11\left(\frac{Mf}{Mx}\right)^2 - 3.52\left(\frac{Mf}{Mx}\right)^3 \quad \text{(S11)}$$

Mineral damping coefficient (dimensionless parameter), *ής*

$$ήS = 0.174 s_e^{-0.19} \quad \text{(S12)}$$

Propagating flux ratio (dimensionless parameter), ξ

$$\xi = (192 + 7.9095\sigma)^{-1} \exp[(0.792 + 3.7597\sigma^{0.5})(\beta + 0.1)] \quad \text{(S13)}$$

Wind coefficient (dimensionless parameter), Φ_w

$$\Phi w = C(3.281U)^B \left(\frac{\beta}{\beta op}\right)^{-E} \quad \text{(S14)}$$

Dimensionless parameter C

$$C = 7.47 \exp(-0.8711\sigma^{0.55}) \quad \text{(S15)}$$

Dimensionless parameter B

$$B = 0.15988\sigma^{0.54} \quad \text{(S16)}$$

Dimensionless parameter E

$$E = 0.715 \exp(-0.01094\sigma) \quad \text{(S17)}$$

Slope factor (dimensionless parameter), Φ_s

$$\Phi s = 5.275\beta^{-0.3}(\tan \Phi)^2 \quad \text{(S18)}$$

Effective heating number (dimensionless parameter), ε

$$\varepsilon = \exp\left(-\frac{4.528}{\sigma}\right) \quad \text{(S19)}$$

Heat of pre-ignition (kJ/kg), Qig

$$Qig = 581 + 2594Mf \quad \text{(S20)}$$

'Modified' Rothermel model equations to determine RoS [4]

w_o	Ovendry fuel loading (kg/m2)
δ	fuel depth(m)
σ	Surface area-to-volume ratio, cm^{-1}
M_f	Fuel moisture content, dimensionless fraction
M_x or M_c	Fuel moisture extinction, dimensionless fraction
U	Windspeed at midflame height, m/min
Φ_s	slope factor
Φ_w	wind factor
tan Φ	slope, dimensionless fraction
hv	heat of combustion of volatile gas, kJ/kg
ρ_p	ovendry particle density, kg/m^3

T	Temperature ambient ºC
Q_f	Heat of Pyrolysis kJ/kg

Rate of spread *RoS* (m/sec):

$$RoS = \frac{(\xi * hv * w_o * Ʈ * ή M)(1 + \Phi w + \Phi s)}{[\rho b * \varepsilon * (Qf + Mf * Qw)]} \quad (S21)$$

Where;

Propagating flux ratio (dimensionless parameter), ξ

$$\xi = 1 - exp(-0.17\sigma\beta) \quad (S22)$$

Packing ratio β

$$\beta = \rho b / \rho p \quad (S23)$$

Ovendry bulk density (kg/m³), ρb

$$\rho b = \frac{w_o}{\delta} \quad (S24)$$

Optimum packing ratio βop

$$\beta op = 0.20395\sigma^{-0.8189} \quad (S25)$$

Maximum reaction velocity Ʈ (min^{-1})

$$Ʈ = 0.34\sigma\ (\sigma * \beta * \delta)^{-0.5} \exp\left(-\sigma * \frac{\beta}{3}\right) \mathrm{P}f(nx) \quad (S26)$$

Extinction index *Pf(nx)*

$$\mathrm{P}f(nx) = \frac{\ln\left(\sigma * \beta * \delta * \frac{hv}{Qw}\right)}{Mf + \frac{Qf}{Qw}} \quad (S27)$$

Heat of vaporization *Qw,* kJ/kg

$$Qw = 4.18(100 - T + 540) \tag{S28}$$

Moisture damping coefficient (dimensionless parameter), *ήM*

$$\acute{\eta}M = \exp\left(-\frac{mf}{mc}\right) \tag{S29}$$

Wind coefficient (dimensionless parameter), Φ_w

$$\Phi w = C(3.281U)^B \left(\frac{\beta}{\beta op}\right)^{-E} \tag{S30}$$

Dimensionless parameter C

$$C = 7.47 \exp(-0.8711\sigma^{0.55}) \tag{S31}$$

Dimensionless parameter B

$$B = 0.15988\sigma^{0.54} \tag{S32}$$

Dimensionless parameter E

$$E = 0.715 \exp(-0.01094\sigma) \tag{S33}$$

Slope factor (dimensionless parameter), Φ_s

$$\Phi s = 5.275\beta^{-0.3}(\tan \Phi)^2 \tag{S34}$$

Effective heating number (dimensionless parameter), ε

$$\varepsilon = \exp\left(-\frac{4.528}{\sigma}\right) \tag{S35}$$

References:

1. Andrews, P.L. *The Rothermel Surface Fire Spread Model and Associated Developments: A Comprehensive Explanation*; General Techncial Report RMRS-GTR-371; United States Department of Agriculture, Forest Service: Washington, DC, USA, 2018.
2. Rothermel, R.C. *A Mathematical Model for Predicting Fire Spread in Wildland Fuels*; General Technical Report INT-115; Intermountain Forest and Range, USDA Forest Service: Ogden, UT, USA, 1972.
3. Weise, D.; Biging, G. A Qualitative Comparison of Fire spread model incorporating wind and slope effects. *For. Sci.* **1997**, *43*, 170–180.
4. Wilson, R. *Reexamination of Rothermels Fire Spread Equations in No-Wind and No-Slope Conditions*; Research Paper INT-434; United States Department of Agriculture, Forest Service: Washington, DC, USA, 1990.