

Review

Image of the Electron Suggested by Nonlinear Electrodynamics Coupled to Gravity

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Abstract: We present a systematic review of the basic features that were adopted for different electron models and show, in a brief overview, that, for electromagnetic spinning solitons in nonlinear electrodynamics minimally coupled to gravity (NED-GR), all of these features follow directly from NED-GR dynamical equations as model-independent generic features. Regular spherically symmetric solutions of NED-GR equations that describe electrically charged objects have obligatory de Sitter center due to the algebraic structure of stress–energy tensors for electromagnetic fields. By the Gürses–Gürsey formalism, which includes the Newman–Janis algorithm, they are transformed to axially symmetric solutions that describe regular spinning objects asymptotically Kerr–Newman for a distant observer, with the gyromagnetic ratio $g = 2$. Their masses are determined by the electromagnetic density, related to the interior de Sitter vacuum and to the breaking of spacetime symmetry from the de Sitter group. De Sitter center transforms to the de Sitter vacuum disk, which has properties of a perfect conductor and ideal diamagnetic. The ring singularity of the Kerr–Newman geometry is replaced with the superconducting current, which serves as the non-dissipative source for exterior fields and source of the intrinsic magnetic momentum for any electrically charged spinning NED-GR object. Electromagnetic spinning soliton with the electron parameters can shed some light on appearance of a minimal length scale in the annihilation reaction $e^+e^- \rightarrow \gamma\gamma(\gamma)$.

Keywords: electron; nonlinear electrodynamics; de Sitter vacuum



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1. Introduction. Electron Story

“The electron is inexhaustible” [1]

In the first models of the electron, as proposed by Abraham [2] and Lorentz [3,4] soon after its discovery by Sir Joseph John Thomson in 1897, the electron was visualized as an extended spherical electrically charged object with the finite total field energy. The models that were based on assumptions about the distribution of a charge density were plagued by the problem of preventing the electron from flying apart under the Coulomb repulsion, which required introducing *cohesive* forces of non-electromagnetic origin (the Poincaré stress) testifying for impossibility to construct an electron model within electrodynamics. Later analyzing extended electron models, Dirac noted the most attractive idea of the Lorentz model [3] concerning the electromagnetic origin of the electron mass. At the same time, he did not find any physical reason for assumptions concerning the character of additional non-electromagnetic forces [5].

In quantum electrodynamics, electron is considered to be a point and the question of its structure is not addressed. The classical models of point-like spinning particles encounter the problem of divergent self-energy for a point charge and approach this problem in the frame of various generalizations of the classical lagrangian $(-mc\sqrt{\dot{x}\dot{x}})$ by introducing terms with higher derivatives or extra variables [6–15], and then restricting undesirable effects by applying geometrical [16,17] or symmetry [18,19] constraints.

Another type of point-like models goes back to the Schrödinger suggestion [20] relating the electron spin to its Zitterbewegung motion—trembling motion due to the rapid oscillation

of a spinning particle about its classical worldline. The approach based on the concept of Zitterbewegung was motivated by attempts to understand the intrinsic nature of the electron spin and it involved studying the fundamental questions of quantum mechanics [21–25]. In models developed in the frame of this approach, the electron was associated with the mean motion of a point-like constituent, whose trajectory is a cylindrical helix [26,27]. The effects of spacetime curvature on Zitterbewegung of spin-1/2 particles have been studied in the paper [28], applying the approach introduced for Zitterbewegung in the local rest frame [29]. It was shown that coupling of Zitterbewegung frequency terms to the Ricci curvature tensor would lead to the appearance of non-trivial contributions to the relative position and momentum operators, which suggested a formal violation of the weak equivalence principle [28].

The concept of a fundamental rotator as a dynamical system described by position, a single null direction, and two additional parameters, mass and length, has been developed in [30] in the frame of the hamiltonian dynamics represented in such a way that the Casimir invariants are not constants of motion, but model parameters [30]. In [31], the fundamental rotator was considered as the relativistic model for a point-like relativistic spinning particle, including its interaction with an external electromagnetic field [31].

The development of point models of spinning particles gradually involved the appearance of tools that are needed for a description of an extended particle and revealed some of its typical features. In order to overcome difficulties with point charges, Dirac developed nonlinear electrodynamics, which allows for describing electric currents starting from a theory without charges, exploiting possibilities of a vector potential A as extra variables entering the electromagnetic theory due to its gauge invariance. He imposed a nonlinear gauge in which the potential 1-form A is time-like ($A^2 = k^2 = \text{positive constant}$), so that A can be regarded as proportional to a velocity field, and identified it with a current, obtaining the motion of a continuous stream of electricity rather than the motion of point charges [32]. In 1962, Dirac proposed the model of the electron as a charged conducting surface endowed with a non-Maxwellian surface tension; outside the surface, the Maxwell equations hold, while, inside it, there is no field; thus, the electron was pictured as a bubble in the electromagnetic field [33]. In 1982, Righi and Venturi have shown that the field equations of the Dirac nonlinear electrodynamics admit an extended-type, spherically symmetric, static solution that can be considered to be a charged particle [34].

Later on it appeared, in the frame implying a point-like image of a particle, that there exists an alternative approach to the investigation of the structure of charged particles [35], based on generalization of the Dirac nonlinear electrodynamics [32]. In the generalized Maxwell equations of a new formulation of the Dirac nonlinear electrodynamics, the first vector potential A is orthogonal to the second pseudo-vector potential B . A solution of the field equations is constituted by the electric field of a conducting sphere and by the magnetic field of a magnetized sphere, while the angular momentum that is stored in the total electromagnetic field is equal to $\hbar/2$. The field equations admit a soliton-like solution that can represent a charged particle, endowed with a Coulomb field and the field of a magnetic dipole. The soliton mass is finite, and the angular momentum that is produced by its electromagnetic field can be identified—for the suitable choice of the parameters—with the spin of the charged particle. This approach gives the model of a charged spinning particle as a sphere with the radius r_e introduced for the dimensional reason, while the magnetic momentum μ_e (as well as the related electric charge e) comes as a constant of integration [35]; they are given by $r_e = \frac{e^2}{2mc^2} \left[1 + \frac{3}{4} \left(\frac{\hbar c}{e^2} \right)^2 \right]$ and $\mu_e = \frac{3}{8} \frac{e\hbar}{mc} \left[1 + \frac{3}{4} \left(\frac{\hbar c}{e^2} \right)^2 \right]$. The magnetic momentum vanishes when $\hbar \rightarrow 0$, in such a case r_e is equal to the classical radius of the electron without spin. For the particle with the spin ($\hbar \neq 0$), the radius r_e takes a high value of the order of the Bohr radius. The interesting feature is the complete accessibility of a sphere r_e interior to any other particle (except for possible electromagnetic repulsions) [35].

In the course of independent development, Boyer presented the extended models for rotating fluid masses in the frame of general relativity [36,37]. In the paper [36], the general question has been addressed whether a perfect-fluid interior can be matched to a

given exterior field. For an isolated, axially symmetric, uniformly rotating perfect-fluid mass in a steady state, Boyer has given conditions for the class of all possible boundaries, which an interior surface would have to satisfy for a given exterior field. In the paper [37], the properties of a stationary, isolated, axially symmetric, rotating body were specialized to the case of constant specific entropy and constant angular velocity as well as for the case of differential rotation. Cohen [38] and by Trümper [39] have presented similar models.

A new road for searches on the electron structure has been opened in 1965 by the discovery of the Kerr–Newman solution to the Einstein–Maxwell equations [40]

$$ds^2 = -dt^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \frac{(2mr - e^2)}{\Sigma} (dt - a \sin^2 \theta d\phi)^2 + (r^2 + a^2) \sin^2 \theta d\phi^2; \quad A_i = -\frac{er}{\Sigma} [1; 0, 0, -a \sin^2 \theta] \quad (1)$$

where A_i is associated electromagnetic potential, and

$$\Sigma = r^2 + a^2 \cos^2 \theta; \quad \Delta = r^2 - 2mr + a^2 + e^2, \quad (2)$$

which inspired the search for an electromagnetic image of the electron since Carter found that the parameter a directly related to the angular momentum $J = ma$ couples with the charge giving an asymptotic magnetic momentum $\mu = ea$, so that the gyromagnetic ratio e/m , as measured by a distant observer, is the same as required for a spinning particle by the Dirac equation [41]. For the solution parameters corresponding to the electron, in the units $\hbar = c = G = 1$, the mass, angular momentum, and squared charge are given by $m \approx 10^{-22}$; $ma = 1/2$; $e^2 \approx 1/137$, from which one obtains $a = 1/2m \approx 10^{22}$; $e \approx 1/12$, so that the length scale that is determined by a is the Compton radius of the electron [41].

The Kerr–Newman solution belongs to the Kerr family of solutions that can represent the exterior fields of rotating charged or uncharged ($e = 0$ in (1)) bodies. The Kerr–Newman solution is the source-free solution; the only contribution to the stress–energy tensor comes from a source-free electromagnetic field. The question of existence of a physically reasonable interior material source for the exterior fields is the most intriguing question for the Kerr–Newman geometry owing to the Carter result suggesting the classical image of the spinning electron visualized as a massive charged source of the Kerr–Newman fields [41,42].

The Kerr–Newman solution (1) was obtained from the Reissner–Nordström metric by the algebraic trick that was discovered by Newman and Janis [43] for “derivation” of the Kerr solution from the Schwarzschild solution. The Newman–Janis algorithm [43] consists of the coordinate mapping $(r, t) \rightarrow (r, u)$ where $u = t - \int \frac{dr}{g(r)}$ and the complex coordinate transformation $r \rightarrow r + ia \cos \theta$; $u \rightarrow u - ia \cos \theta$ introducing the additional parameter a . Quotation marks in “derivation” are original, accompanied by the remark “There is no clear reason for this operation to yield a new solution”, but the new solution has been obtained [43].

The reason was found by Gürses and Gürsey in 1975 [44]. The point is that both Schwarzschild and Reissner–Nordström metrics have the form

$$ds^2 = g(r)dt^2 - \frac{dr^2}{g(r)} - r^2 d\Omega^2. \quad (3)$$

Metrics (3) belong to the Kerr–Schild class [45], which is the special class of algebraically degenerated solutions to the Einstein equations, which, in this algebraically special case, take the linear form, $\partial_\mu T^\mu_\nu = 0$, and pseudotensor of gravitational energy vanishes, $t_{\mu\nu} = 0$ [44,45].

The axially symmetric metrics of the Kerr–Schild class can be presented as [45]

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2f(r)}{\Sigma} k_\mu k_\nu \quad (4)$$

where $\eta_{\mu\nu}$ is the Minkowski metric and the function $f(r)$ in (4) comes from a spherical solution under transformation. For the Kerr–Newman geometry

$$f(r) = mr - e^2/2. \quad (5)$$

A null vector field k_μ in (4) is tangent to a principal null congruence existing in metrics of the Kerr family, since they belong to the type D in the Petrov–Pirani classification. Two double principal null geodesic congruences (analogous to the null radial geodesics of the spherical geometry) are given by

$$\frac{dt}{d\tau} = \frac{r^2 + a^2}{\Delta} E; \quad \frac{dr}{d\tau} = \pm E; \quad \frac{d\theta}{d\tau} = 0; \quad \frac{d\phi}{d\tau} = \frac{a}{\Delta} E \quad (6)$$

and are used, with choosing the constant $E = 1$, for constructing a null basis for the Newman–Penrose formalism adopted in the type-D space-times ([46] for a systematic description).

The coordinate r is geometrically defined as an affine parameter along either of two principal null congruences, and the surfaces of constant r are the confocal ellipsoids that are given by

$$r^4 - (x^2 + y^2 + z^2 - a^2)r^2 - a^2z^2 = 0 \quad (7)$$

which degenerate, for $r = 0$, to the equatorial disk

$$x^2 + a^2 \leq a^2, \quad z = 0 \quad (8)$$

centered on the symmetry axis and bounded by the ring

$$x^2 + y^2 = a^2, \quad z = 0 \quad (9)$$

which comprises the ring singularity of the Kerr–Newman geometry [46].

The Cartesian coordinates x, y, z are related with the Boyer–Lindquist coordinates r, θ, ϕ by $x + iy = (r + ia)e^{i\phi} \sin \theta$; $z = r \cos \theta$; $x^2 + y^2 = (r^2 + a^2) \sin^2 \theta$.

Newman and Janis found, using the gravitational multipoles analysis [47], that the Kerr metric is compatible with the structure of a rotating ring of mass [43]. The Kerr–Newman solution (1) was interpreted by the authors as representing the gravitational and electromagnetic fields generated by a ring of mass and charge rotating about its axis of symmetry, but a difficulty in this interpretation was notified as conditioned by a multivalued behavior of the metric when a closed loop threads the singular ring [40].

The point clarified by Israel [42] is that r tends to zero through positive values when approaching the disk (8) from above either below, while $\text{grad } r$ (directed outward from the disk) does not vanish. This admits two ways of representation of the Kerr–Newman manifold corresponding to two options of the metric behavior around the disk [42]: (1) the manifold is defined in such a way that $r \geq 0$ everywhere, but there is discontinuity in the normal derivative of the metric across the disk. (2) The metric is smooth everywhere except the singular ring, but an observer crossing the disk (8) from a region with $r > 0$ emerges in a region with $r < 0$. The two sheets, $r > 0$ and $r < 0$, are to be joined on the disk $r = 0$ (for details [46]) which serves as a branch cut, while the singular ring (9) is a branch line of space on two sheets.

The main disaster of the Kerr–Newman geometry is the nontrivial causality violation, just in the case of a particle, when $a^2 + e^2 > m^2$. In this case, there are no Killing horizons, the manifold is geodesically complete, except for geodesics that reach the singularity at $\Sigma = 0$, and the whole space is a single vicious set, i.e. such a set in which any point can be connected to any other point by both a future and past directed timelike curve [41]. The condition of the causality violation [41]

$$g_{\phi\phi} = r^2 + a^2 + \Sigma^{-1}(2mr - e^2)a^2 \sin^2 \theta < 0 \quad (10)$$

is satisfied for small values of r in the vicinity of the disk. In this region, the vector $\partial/\partial\phi$ is timelike, so that the circles $t = \text{const}, r = \text{const}, \theta = \text{const}$, are themselves closed timelike lines. Closed timelike curves are not restricted to the region where the condition (10) is satisfied, but it can extend over the whole space and cannot be removed by taking a covering space [41].

Two lines of research conditioned by two ways of presenting a manifold were to remove or cover the pathological region of the Kerr–Newman geometry, either to adopt it for a construction of a source, including its physical interpretation. The second approach resulted in the string-like models [48–52]. Attempts to introduce a source by truncation the negative- r sheet along the disk (8) break the analyticity and lead to appearance of a singular distribution of a source material on the disk [42]. The choice of truncation is not unique, and various singular models [42,53,54] display this fact.

Assuming a source of the Kerr–Newman metric as a layer of mass and charge distributed over the equatorial disk spanning the ring singularity and applying the well developed theory of surface layers, Israel found that the charged disk must be composed of material having negative proper surface density σ , prevented from flying off by a radial tension $|\sigma|$ (negative pressure), and rotating with the superluminal velocity. The disk material has negative mass, both mass and angular momentum of the disk diverge to $-\infty$ as the intrinsic radial coordinate on the disk tends to $|a|$, the singular ring carries infinite positive charge, and it must carry infinite positive mass and angular momentum in order to yield the finite net values m and ma . As a result, the gyromagnetic ratio is e/m for every annulus of the disk. This allows for accounting for some spin properties of the electron on a purely classical basis by visualizing it as a charged disk with the angular momentum $ma = \hbar/2$ and diameter $2a = \hbar/m$ equal to the Compton wavelength. Because of negative mass of the disk material, electrostatic repulsive forces that are normal to the plane of the disk would provide stability of the disk structure for $|e| > m$ [42].

The Israel approach was essentially different from the existing models, in that the results came from precise analysis of the precise equations, so that information that came from equations reflected typical features of the Kerr–Newman geometry, stated actually of its bad adaptability to the deep interior of an object, but revealed its basic feature—the appearance of a negative pressure as a source of the gravitational repulsion [42].

The repulsive character of the gravitational field close to the disk has been first revealed in the analysis of geodesics: for a particle falling down the axis, the gravitational force becomes repulsive when $r < |a|$ [55,56].

Hamity demonstrated the appearance of a negative pressure for the Kerr geometry in the disk-like model of a neutral spinning particle that is responsible for the Kerr field and visualized as a thin rigidly rotating disk with a regular interior and a singular rim. The interior of the disk represents a material with the positive isotropic stress (negative pressure), which is minimal at the center of the disk and tends to infinity at its rim [57].

Tiomno has used the fact that the electromagnetic field in the Kerr–Newman solution is independent of the gravitational constant, and found two solutions of the Maxwell equations in the flat space for the field of a rotating charged oblate ellipsoid of infinite conductivity and either (a) magnetic susceptibility of vacuum or (b) infinite magnetic susceptibility. It has been shown that, for small $\omega = a/(R^2 + a^2)$ (where R is the semi-minor radius of the ellipsoid), the conductive surface current (case (a)) or the volume magnetization (case (b)) contributes to the total magnetic moment with the needed value of the gyromagnetic ratio [58].

In the Lopez extended model [59], a rigidly rotating charged shell of zero thickness endowed with the surface tension is defined by $r_s^2 = e^2/2m$. At this value of r , all of the gravitational potentials vanish, which allows the KN metric to be matched to the interior flat space-time metric replacing the unphysical region of the Kerr–Newman geometry responsible for the breakdown of causality. The electron is visualized as a bubble of a flat space-time immersed in the Kerr–Newman geometry. Outside the bubble, the metric coefficient $g_{\phi\phi}$ never changes its sign and, hence, there are no closed time-like curves.

The shell of the bubble is the surface of an oblate ellipsoid with a minor axis equal to the classical electron radius and a focal distance of the order of the Compton wavelength λ_e . The magnetic moment is obtained as $\mu = ea$ and the angular momentum as $J = -ma$, so that the gyromagnetic ratio $\mu/|J| = e/m$ takes the proper value. The oblateness of the bubble produces an electric quadrupole moment that is proportional to λ_e^2 [59].

The question of the role of the repulsive gravity in the electron models has been analyzed by Grøn [60], who compared the Lopez model [59] with the earlier Cohen & Cohen model [61], in which the de Sitter equation of state $p = -\rho$ is explicitly introduced for an isotropic vacuum fluid interior matched to the KN metric on a spherical shell. The charged shell confines the interior with the repulsive gravity due to $p = -\rho$, while, in the Lopez model, a bubble interior is flat, but the stress–energy tensor on the shell contains a “gas” with the imposed negative pressure and repulsive gravitation [59].

The question of the origin of mass for a charged particle has been studied by Tiwari et al [62] using the properties of the metric (3) that were applied for a charged perfect-fluid interior. It was shown, by analysis of the relation between the metric coefficients, that the mass-energy density and the pressure of the interior are of electromagnetic origin. A presented particular solution that smoothly matches this interior to the Reissner–Nordström exterior represents a spherically symmetric charged particle whose mass is entirely of electromagnetic origin [62].

For the Kerr geometry, Burinskii constructed the supersymmetric superconducting bag model as a core of the Kerr spinning particle [63] on the basis of an earlier model combining the Kerr spinning particle and superparticle, which describes a neutral rotating black hole [64]. In this model, the Kerr geometry is presented in a complex form, as created by a complex source. A natural supergeneralization of the model results in a complex “supersource” developed by a supershift to the Kerr and Kerr–Sen solutions to metrics of supergravity black holes with a nonlinear realization of the broken supersymmetry [64].

The models for regular interior sources of the Kerr–Newman solution were constructed in [65] on the basis of a smooth deformation of space in the neighborhood of the disk (8), keeping the Kerr–Schild form of the metric. Deformed metric induces a stress–energy tensor in the right hand side of the Einstein equations which corresponds to appearance of a source of the Kerr–Newman geometry in the form of an oblate ellipsoid of revolution with a smooth matter distribution. The singular ring is regularized by introducing an anisotropic matter rotating in the equatorial plane, so that the negative- r sheet is absent. The sources have the form of bags, which can have de Sitter or anti de Sitter interiors and a domain wall boundary at the transition layer r_s at which the Kerr–Newman function $f(r) = mr - e^2/2$ is matched to the de Sitter function $f(r) = r^4/r_0^2$ [65].

Burinskii applied a later similar approach in the lepton bag model [66,67], in the supersymmetric domain-wall bubble model [68], and in the supersymmetric bag model for the unification of gravity with spinning particles ([69] and references therein), in which the external Kerr–Newman solution is matched to the free of gravity (flat) superconducting interior by the domain wall boundary interpolating between them.

The models that are based on the dominating role of the electron spin have been put forward by Pope and Hofer [70] and by Burinskii ([71] and the references therein).

Among the models with an imposed regular (Anti) de Sitter core matched directly to the external Kerr–Newman metric ([63,65] and references therein), there is the model that involves nonlinear electrodynamics [72]. This model represents the solution of a hybrid type, in which the electrically charged KN space-time is matched to the magnetically charged internal core. The model [72] is obtained by modification of the Ayón–Beato–Garcia black hole solution [73]. Magnetic interior is matched to an electric exterior at the surface $r = r_s$, where the electric susceptibility diverges, so that this surface looks like the ideal conducting surface. The magnetic charges are confined inside the surface $r = r_s$. For the external observer, this solution only exhibits electric charge. The internal magnetic core expels electric charges by construction, so one can speculate that it possesses

superconducting properties, whereas the exterior region that is open to distant observers displays the dual superconductivity, expelling the magnetic charges [72].

The problem of fitting the Kerr–Newman exterior to a rotating material source does not have a unique solution, because of the arbitrariness in the choice of the boundary between the exterior and interior [42], as well as of a freedom in choosing an interior model.

To avoid this uneasy choice and yet learn something about a possible reasonable model of the electron structure, it seems natural to appeal to equations, to ask—What do equations know? In the case of structure related by electromagnetic interaction as well as by gravitational interaction (acting in any structure), the appropriate equations come from nonlinear electrodynamics minimally coupled to gravity (minimal coupling does not require introducing additional assumptions) and give certain model-independent information about generic properties of spinning electromagnetic solitons, as described by regular, causally safe solutions, asymptotically Kerr–Newman, and characterized by the gyromagnetic ratio $g = 2$ for a distant observer.

In Section 2.2, we present basic equations of nonlinear electrodynamics minimally coupled to gravity (NED-GR) and outline the basic features and internal structure of electromagnetic spinning solitons that are predicted by analysis of NED-GR dynamical equations for an arbitrary gauge-invariant electromagnetic lagrangian. In Section 3, we summarize and discuss the results.

2. Electromagnetic Spinning Soliton of NED-GR

Nonlinear electrodynamics has been proposed by Born and Infeld in 1934 on the basis of two principles: to consider electromagnetic field and particles within one physical frame, and to avoid infinite values for physical quantities characterizing particles [74]. In the electron model, the finite value of the electromagnetic energy was ensured by imposing an upper limit on the electric field related to the electron size, but geometry remains singular without physical restrictions on gravity in the center of an object. Later, it has been found that NED theories appear as low-energy effective limits in certain models of string/M-theories [75–77].

Two aims of the Born–Infeld program can be achieved in the nonlinear electrodynamics minimally coupled to gravity, which admits regular solutions describing compact finite-energy objects related by electromagnetic and gravitational interactions. A NED-GR electromagnetic spinning soliton is made of a nonlinear electromagnetic field and defined in the spirit of Coleman lump [78] as non-singular non-dissipative object localized in the confined region and keeping itself together by its own self-interaction.

2.1. Basic Equations and Spacetime Structure

Regular electrically charged spherically symmetric solutions obtained in the frame of the Lagrange dynamics [73,79–82] have been found in the alternative P -form of nonlinear electrodynamics related to the Lagrangian F -form by the Legendre transformation [83]. F - P duality turns into electric-magnetic duality in the Maxwell limit, but, in the general case, it connects different theories [84], which leads to branching of a Lagrangian in the F frame [82,84].

In this case, the dynamical system is described by the non-uniform variational problem [85]

$$\mathcal{I} = \frac{1}{16\pi} \left[\int_{\Omega_{int}} (R - \mathcal{L}_{int}(F)) \sqrt{-g} d^4x + \int_{\Omega_{ext}} (R - \mathcal{L}_{ext}(F)) \sqrt{-g} d^4x \right] \quad (11)$$

where R is the scalar curvature, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the tensor of electromagnetic field, and $F = F_{\mu\nu} F^{\mu\nu}$ is the electromagnetic field invariant. The gauge-invariant electromagnetic Lagrangian $\mathcal{L}_{ext}(F)$ should have the Maxwell limit, $\mathcal{L} \rightarrow F$, $\mathcal{L}_F \rightarrow 1$, where $\mathcal{L}_F = d\mathcal{L}/dF$, in the weak field regime, as $r \rightarrow \infty$.

Each part of the manifold, Ω_{int} and Ω_{ext} , is confined by the initial and final spacelike hypersurfaces t_{in} and t_{fin} , and by the timelike internal common boundary between them, denoted as Σ_c ; the exterior part Ω_{ext} is confined by the timelike three-surface Σ_{ext} extended

to infinity where electromagnetic fields vanish. Dynamical variables in the Lagrangian $\mathcal{L}(F)$ are electromagnetic potentials A_μ . At the surfaces t_{in} and t_{fin} their variations should satisfy $\delta A_\mu = 0$. The boundary surface Σ_c is specified by the condition of continuity of the dynamical variables $A_{int}^\mu = A_{ext}^\mu$, and the standard boundary condition on the surface Σ_c is given by [85]

$$\int_{\Sigma_c} \left(\mathcal{L}_F (int) F_{\mu\nu} (int) - \mathcal{L}_F (ext) F_{\mu\nu} (ext) \right) \sqrt{-g} \delta A^\mu d\sigma^\nu = 0 \quad (12)$$

The stress–energy tensor of a nonlinear electromagnetic field, calculated in the standard way [86] with the electromagnetic lagrangian $\mathcal{L}(F)$

$$T_\nu^\mu = -2\mathcal{L}_F F_{\nu\alpha} F^{\mu\alpha} + \frac{1}{2} \delta_\nu^\mu \mathcal{L} \quad (13)$$

provides the only source of the gravitational field in the Einstein equations $G_\nu^\mu = -8\pi G T_\nu^\mu$.

Basic common features of the electron models, such as the adopted interior de Sitter vacuum and electromagnetic origin of mass, as well as presumed superconducting behavior, for NED-GR objects follow directly from the dynamical equations governing their behavior with the only condition–satisfaction of WEC (the Weak Energy Condition [87], which requires the positivity of density, as measured by any local observer, and ensures the positivity of mass of an object).

The key point is that the stress–energy tensors for electromagnetic field have the algebraic structure such that [82]

$$T_t^t = T_r^r. \quad (14)$$

The metrics obtained with these stress–energy tensors belong to the Kerr–Schild class and have the form (3) with the metric function [88,89]

$$g(r) = 1 - \frac{2G\mathcal{M}(r)}{r}; \quad \mathcal{M}(r) = 4\pi \int_0^r \rho(x) x^2 dx. \quad (15)$$

Regular spherical solutions of this class have an obligatory de Sitter center provided that WEC is satisfied [90]. The mass of objects $m = 4\pi \int_0^\infty \rho(r) r^2 dr$ is generically related with the interior de Sitter vacuum and breaking of spacetime symmetry from the de Sitter group [90,91]. In the NED-GR solutions, the mass function $\mathcal{M}(r)$ in (15) is determined by the electromagnetic field density, $\rho(r) = T_t^t(r)$ in (13). Mass m of electromagnetic origin enters into axially symmetric solutions that are obtained by the Gürses–Gürsey formalism for rotating objects [92].

The Gürses and Gürsey formalism presents the general, model-independent approach for description of the axially symmetric metrics of the Kerr–Schild class based on the complex Trautman–Newman translations (that include the Newman–Janis algorithm). In the Boyer–Lindquist coordinates, the Gürses–Gürsey metric reads [44]

$$ds^2 = \frac{2f - \Sigma}{\Sigma} dt^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 - \frac{4af \sin^2 \theta}{\Sigma} dt d\phi + \left(r^2 + a^2 + \frac{2fa^2 \sin^2 \theta}{\Sigma} \right) \sin^2 \theta d\phi^2 \quad (16)$$

where now

$$\Delta = r^2 + a^2 - 2f(r); \quad f(r) = r\mathcal{M}(r) \quad (17)$$

and the master function $f(r)$ comes from a related spherically symmetric solution of the Kerr–Schild class with the mass function given in (15) specified by the density profile of the spherical solution, denoted from now on as $\tilde{\rho}(r)$, $\mathcal{M}(r) = 4\pi \int_0^r \tilde{\rho}(x) x^2 dx$, for convenience when we shall consider the density of an axially symmetric solution, $\rho(r, \theta)$.

For spherical solutions satisfying WEC, the mass function $\mathcal{M}(r)$ is everywhere positive function of r monotonically growing from $\mathcal{M}(r) = 4\pi \tilde{\rho}(0) r^3 / 3$ as $r \rightarrow 0$ to $m - e^2 / 2r$ as

$r \rightarrow \infty$ [92]. Approaching the disk (8), the function $2f(r)$ in (16) approaches the de Sitter form

$$2f(r) \rightarrow \frac{8\pi G\rho(0)}{3}r^4 = \frac{r^4}{r_0^2}; \quad r_0^2 = \frac{3}{8\pi G\rho(0)} = \frac{3}{\Lambda}. \quad (18)$$

The metric (16) is regular for $r \rightarrow 0$ since $r^2/\Sigma \rightarrow 1$ as $r \rightarrow 0$, and asymptotically de Sitter for $r \rightarrow 0$ and asymptotically Kerr–Newman for $r \rightarrow \infty$ [92]. The function $f(r)$ is an everywhere positive function evolving from the de Sitter value $f(r) = 4\pi\rho(0)r^4/3$ at $r \rightarrow 0$ to the Kerr–Newman value $f(r) = mr - re^2/2$. The condition of causality violation (10) would have to read $g_{\phi\phi} = r^2 + a^2 + \Sigma^{-1}2f(r)a^2 \sin 2\theta < 0$ which never occurs due to non-negativity of $f(r)$. The vector $\partial/\partial\phi$ is spacelike over the whole space, so that the whole manifold is causally safe [92].

In the Kerr–Schild form, the metric (16) reads

$$ds^2 = dx^2 + dy^2 + dz^2 - dt^2 + \frac{2f(r)}{\Sigma} \left(\frac{r(xdx + ydy) - a(xdy - ydx)}{r^2 + a^2} + \frac{zdz}{r} + dt \right)^2. \quad (19)$$

Expressing Σ in the Cartesian coordinates, we get, for the rotating de Sitter vacuum,

$$\frac{2f(r)}{\Sigma} = \frac{r^4}{r_0^2} \frac{r^2}{(r^4 + a^2z^2)}. \quad (20)$$

In the equatorial plane $2f(r)/\Sigma = r^2/r_0^2$, so that formally the disk $r = 0$ is totally (together with the ring) flat. But Λ is non-zero on the disk. The metric (19) originates from spherical metric with the de Sitter center. Rotation transforms the center $r = 0$ into the disk (8). Asymptotic (20) represents a rotating de Sitter vacuum with Λ being spread over the disk. Thus, the de Sitter center becomes the de Sitter vacuum disk [92].

A stress–energy tensor responsible for the metric (16) is given by [44]

$$T_{\mu\nu} = \frac{1}{\Sigma^2} [(2(f'r - f) - f''\Sigma)g_{\mu\nu} + (4(f'r - f) - f''\Sigma)(u_\mu u_\nu - l_\mu l_\nu)] \quad (21)$$

in the orthonormal tetrad with the time-like vector u^μ and three space-like vectors l^μ, n^μ, m^μ such that $u^\mu u_\mu = -1$; $l^\mu l_\mu = 1$; $n^\mu n_\mu = 1$; $m^\mu m_\mu = 1$. They are given by

$$u^\mu = \frac{1}{\Delta\Sigma} [(r^2 + a^2)\delta_0^\mu + a\delta_3^\mu]; l^\mu = \sqrt{\frac{\Delta}{\Sigma}}\delta_1^\mu; n^\mu = \frac{1}{\Sigma}\delta_2^\mu; m^\mu = -\frac{1}{\sqrt{\Sigma}\sin\theta}[a\sin^2\theta\delta_0^\mu + \delta_3^\mu]. \quad (22)$$

Obtained by Gürses and Gürsey field equations with the source term (21) are the same as the field equations for the Kerr–Schild metric in the Lorentz covariant coordinate system [44], so that all the axially symmetric solutions from the Kerr–Schild class are described by the Gürses–Gürsey metric (16) obtained with the stress–energy tensor (21) as a source.

The eigenvalues of the stress–energy tensor (21), its components in the co-rotating reference frame, where each of ellipsoidal layers of constant r rotates with its angular velocity $\omega(r) = u^\phi/u^t = a/(r^2 + a^2)$, are [44]

$$T_{\mu\nu}u^\mu u^\nu = \rho(r, \theta); T_{\mu\nu}l^\mu l^\nu = p_r = -\rho; T_{\mu\nu}n^\mu n^\nu = T_{\mu\nu}m^\mu m^\nu = p_\perp(r, \theta). \quad (23)$$

They are related with the function $f(r)$ by

$$\rho(r, \theta) = \frac{2(f'r - f)}{\Sigma^2}; \quad p_\perp = \frac{2(f'r - f) - f''\Sigma}{\Sigma^2}. \quad (24)$$

In terms of the eigenvalues, the stress–energy tensor takes the form

$$T_{\mu\nu} = (\rho(r, \theta) + p_\perp(r, \theta))(u_\mu u_\nu - l_\mu l_\nu) + p_\perp(r, \theta)g_{\mu\nu} \quad (25)$$

The density and pressures are expressed through the density of a related spherical solution as

$$\rho(r, \theta) = \frac{r^4}{\Sigma^2} \tilde{\rho}(r); \quad p_{\perp} = \left(\frac{r^4}{\Sigma^2} - \frac{2r^2}{\Sigma} \right) \tilde{\rho}(r) - \frac{r^3}{2\Sigma} \tilde{\rho}'(r). \quad (26)$$

The equation of state that relates the radial pressure with density in (23) actually follows from the algebraic structure of a stress–energy tensor (14), while the transversal pressure is related with the density by functional dependence which follows from $T_{\nu,\mu}^{\mu} = 0$. This gives [92]

$$p_r(r, \theta) = -\rho(r, \theta); \quad p_{\perp}(r, \theta) = -\rho - \frac{\Sigma}{2r} \frac{\partial \rho(r, \theta)}{\partial r}. \quad (27)$$

In the limit $z \rightarrow 0$, the expression $r^2/\Sigma \rightarrow 1$ in (26), and we get in the equatorial plane [92]

$$\rho(r, \theta) = \rho(r) = \tilde{\rho}(r); \quad p_{\perp} = -\tilde{\rho} - \frac{r}{2} \tilde{\rho}'. \quad (28)$$

For spherical solutions regularity requires $r\tilde{\rho}'(r) \rightarrow 0$ as $r \rightarrow 0$, and the density $\tilde{\rho}(r)$ achieves its maximal de Sitter value [82]. As a result, the density on the disk (8) takes the de Sitter value

$$\rho(r \rightarrow 0) = \tilde{\rho}(0) = \frac{\Lambda}{8\pi G} \quad (29)$$

while in (28) $p_{\perp} = -\tilde{\rho}$ and, hence, $p_{\perp} = p_r = -\tilde{\rho}$, so that the equation of state on the disk (8) represents the de Sitter vacuum [92]

$$p_r = p_{\perp} = -\rho. \quad (30)$$

We have convinced above that *interior de Sitter vacuum, mass of electromagnetic origin, and causal safety* are generic features of all regular compact objects in NED-GR determined by the space-time structure for regular geometry with the metric from the Kerr–Schild class. In what follows, we shall see that the electromagnetic properties of the disk (8) and superconductive origin of the fields are determined in a general setting by solutions for electromagnetic fields.

2.2. Dynamics of Electromagnetic Fields

The dynamic equations for the electromagnetic field obtained by variation of the action (11) with respect to the electromagnetic potential A_{μ} in each part of the manifold, are given by

$$\nabla_{\mu}(\mathcal{L}_F F^{\mu\nu}) = 0 \quad (31)$$

where $\mathcal{L}_F = d\mathcal{L}/dF$, and the contracted Bianchi identities give

$$\nabla_{\mu} {}^*F^{\mu\nu} = 0. \quad (32)$$

An asterisk denotes the Hodge dual that is defined by [86]

$${}^*F^{\mu\nu} = \frac{1}{2} \eta^{\mu\nu\alpha\beta} F_{\alpha\beta}; \quad {}^*F_{\mu\nu} = \frac{1}{2} \eta_{\mu\nu\alpha\beta} F^{\alpha\beta} \quad (33)$$

and the totally antisymmetric unit tensor is chosen in such a way that $\eta_{0123} = \sqrt{-g}$.

In terms of the field vectors that are defined as ([92] and references therein)

$$\mathbf{E} = \{F_{0k}\}; \quad \mathbf{D} = \{\mathcal{L}_F F^{0k}\}; \quad \mathbf{B} = \{{}^*F^{k0}\}; \quad \mathbf{H} = \{\mathcal{L}_F {}^*F_{0k}\}; \quad k = 1, 2, 3 \quad (34)$$

the field Equations (31) and (32) take the form of the Maxwell equations

$$\nabla \mathbf{D} = 0; \quad \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}; \quad \nabla \mathbf{B} = 0; \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}. \quad (35)$$

Non-zero field components that are compatible with the axial symmetry are $F_{01}, F_{02}, F_{13}, F_{23}$; in geometry with the metric (16), they are related by

$$F_{31} = a \sin^2 \theta F_{10}; \quad F_{23} = \frac{r^2 + a^2}{a} F_{02}. \quad (36)$$

The field intensities are expressed via the field components as

$$E_r = F_{10}; \quad E_\theta = F_{20}; \quad H_r = \mathcal{L}_F \sqrt{-g} F^{23}; \quad H_\theta = \mathcal{L}_F \sqrt{-g} F^{31}; \quad (37)$$

$$D^r = \mathcal{L}_F F^{01}; \quad D^\theta = \mathcal{L}_F F^{02}; \quad \sqrt{-g} B^r = F_{23}; \quad \sqrt{-g} B^\theta = F_{31} \quad (38)$$

where

$$F^{01} = \frac{r^2 + a^2}{\Sigma} F_{10}; \quad F^{31} = \frac{1}{\Sigma \sin^2 \theta} F_{31}; \quad F^{02} = \frac{1}{\Sigma} F_{20}; \quad F^{23} = \frac{1}{\Sigma(r^2 + a^2) \sin^2 \theta} F_{23}. \quad (39)$$

The electric field intensity $\mathbf{E}$ is connected with the electric induction $\mathbf{D}$, as

$$D_\alpha = \epsilon_{\alpha\beta} E_\beta \quad (40)$$

where $\epsilon_{\alpha\beta}$ is the tensor of the dielectric permeability, so that the nonlinear electromagnetic field in geometry (16) behaves like anisotropic dielectric medium. Equipotential surfaces are ellipsoidal layers $r = \text{const}$ that are determined by (7). In geometry (16), the symmetry of an ellipsoid (7) gives two independent eigenvalues of $\epsilon_{\alpha\beta}$ [92]

$$\epsilon_r = \frac{(r^2 + a^2)}{\Delta} \mathcal{L}_F; \quad \epsilon_\theta = \mathcal{L}_F. \quad (41)$$

The magnetic field intensity $\mathbf{H}$ is related with the magnetic induction $\mathbf{B}$ by [92]

$$B_\alpha = \mu_{\alpha\beta} H_\beta \quad (42)$$

where $\mu_{\alpha\beta}$ is the tensor of magnetic permeability whose independent eigenvalues are

$$\mu_r = \frac{(r^2 + a^2)}{\Delta} \frac{1}{\mathcal{L}_F}; \quad \mu_\theta = \frac{1}{\mathcal{L}_F}. \quad (43)$$

Dynamical equations (31) and (32) form the system of four equations for two independent functions (with taking into the relations (36)) [93]

$$\frac{\partial}{\partial r} [(r^2 + a^2) \sin \theta \mathcal{L}_F F_{10}] + \frac{\partial}{\partial \theta} [\sin \theta \mathcal{L}_F F_{20}] = 0; \quad \frac{\partial F_{01}}{\partial \theta} + \frac{\partial F_{20}}{\partial r} = 0; \quad (44)$$

$$\frac{\partial}{\partial r} \left[\frac{1}{\sin \theta} \mathcal{L}_F F_{31} \right] + \frac{\partial}{\partial \theta} \left[\frac{1}{(r^2 + a^2) \sin \theta} \mathcal{L}_F F_{32} \right] = 0; \quad \frac{\partial F_{23}}{\partial r} + \frac{\partial F_{31}}{\partial \theta} = 0. \quad (45)$$

Solutions to this system should satisfy the compatibility condition which is given by [93]

$$\frac{\partial}{\partial r} \left(\frac{1}{L_F} \frac{\partial L_F}{\partial \theta} \right) \frac{\partial}{\partial \theta} \left(\frac{1}{L_F} \frac{\partial L_F}{\partial r} \right) + \frac{4a^2 \sin^2(\theta)}{\Sigma^2} \frac{1}{L_F^2} \left[r \frac{\partial L_F}{\partial r} + \cot(\theta) \frac{\partial L_F}{\partial \theta} \right]^2 = 0 \quad (46)$$

as the necessary and sufficient condition of compatibility of Equations (44) and (45), and as the necessary condition for the existence of solutions [93].

Equations (44) and (45) and compatibility condition (46) are satisfied by the functions [92–94]

$$\Sigma^2(\mathcal{L}_F F_{01}) = -e(r^2 - a^2 \cos^2 \theta); \quad \Sigma^2(\mathcal{L}_F F_{02}) = ea^2 r \sin 2\theta; \quad (47)$$

$$\Sigma^2(\mathcal{L}_F F_{31}) = ae \sin^2 \theta (r^2 - a^2 \cos^2 \theta); \quad \Sigma^2(\mathcal{L}_F F_{23}) = aer(r^2 + a^2) \sin 2\theta. \quad (48)$$

in two limiting cases: in the linear regime $\mathcal{L}_F = 1$, when the solutions (47) and (48) coincide with the solutions to the Maxwell–Einstein equations obtained in the Kerr–Newman geometry [41,58], and in the strongly nonlinear regime when (47) and (48) satisfy the system (44) and (45) as the asymptotic solutions in the limit $\mathcal{L}_F \rightarrow \infty$ [93–95].

The relation connecting density and pressure with the electromagnetic fields directly follows from (13) and reads [92]

$$(p_{\perp} + \rho) = 2\mathcal{L}_F \left(F_{10}^2 + \frac{F_{20}^2}{a^2 \sin^2 \theta} \right). \quad (49)$$

This allows to investigate the behavior of the fields on the disk, since geometry tells us about the behavior of the left-hand side there.

The first conclusion is that the weak energy condition that requires $p_{\perp} + \rho \geq 0$ should be satisfied for electrically charged NED-GR objects, since its violation would lead to $\mathcal{L}_F < 0$ in (49) and to negative values of the electric permeability in (41), which is excluded by the basic requirement of electrodynamics of continued media [96].

Putting the solutions (47) in (49), we obtain

$$(p_{\perp} + \rho) = \frac{2e^2}{\mathcal{L}_F \Sigma^2}. \quad (50)$$

It follows that $\mathcal{L}_F \rightarrow \infty$ exactly on the disk (8), where $p_{\perp} + \rho = 0$ by virtue of (30). On the disk the electromagnetic density achieves its maximum, Equation (29), so that the behavior $\mathcal{L}_F \rightarrow \infty$ corresponds to the realization of the underlying hypothesis about nonlinearity replacing a singularity.

On the de Sitter disk (8), where $\mathcal{L}_F \rightarrow \infty$, Equation (41) yields $\epsilon_r = \epsilon_{\theta} = \mathcal{L}_F \rightarrow \infty$, while Equation (43) gives $\mu_r^r = \mu_{\theta}^{\theta} = \mathcal{L}_F^{-1} = \mu \rightarrow 0$. As a result, the disk displays the properties of a perfect conductor and ideal diamagnetic.

The magnetic induction $\mathbf{B}$ in (42) also zeros out on the disk by virtue of (38) and (47) [92,97]. This suggests the appearance of an uncertainty in the definition of a surface current $\mathbf{j}_s = \frac{(1-\mu)}{4\pi\mu} [\mathbf{n}\mathbf{B}]$, where $\mathbf{n}$ is the normal to the surface, which would testify for a transition to a superconducting state [96].

The surface current on the disk is defined by $4\pi j_k = [e_{(k)}^{\alpha} F_{\alpha\beta} n^{\beta}]$ [42], where $e_{(k)}^{\alpha}$ are the base vectors that are related to the intrinsic coordinates on the disk $t, \phi, 0 \leq \xi \leq \pi/2$; the vector $n_{\alpha} = \delta_{\alpha}^1 (1 + e^2/a^2)^{-1/2} \cos \xi$ is the unit normal to the disk, and the symbol $[\dots]$ denotes a jump across its surface in the direction orthogonal to it [42]. Using the solutions (47) and (48) on the disk and taking into account that $\mu = 1/\mathcal{L}_F$ there, we obtain [97]

$$j_{\phi} = -\frac{e}{2\pi a} \sqrt{1 + e^2/a^2} \sin^2 \xi \frac{\mu}{\cos^3 \xi}. \quad (51)$$

Intrinsic coordinate ξ on the disk changes within $0 \leq \xi \leq \pi/2$. The magnetic permeability $\mu = 0$ over the whole disk. Therefore, the current j_{ϕ} is zero throughout the disk, except the ring $\xi = \pi/2$, where numerator and denominator in the second fraction go to zero independently. As a result, the surface current on the ring can be any and amount to a non-zero total value, hence the general condition for transition to a superconducting state [96] is satisfied.

The electric field vanishes on the disk (8) [92,98], and the superconducting current (51) replacing the ring singularity represents a non-dissipative source of the exterior fields that can, in principle, provide a practically unlimited life time of an object [97,98].

As any circular current, the superconducting current (51) produces a magnetic momentum. NED-GR equations for electromagnetic field (31) are source-free and, hence, this magnetic momentum is *intrinsic* for any regular rotating compact object in NED-GR [99].

The current (51) flows in the region of the perfect conductivity. In addition, geometry on the disk is flat, the metric function $g(r) = 1$ at $r = 0$ for a related spherical solution

in (3) and, consequently, in the axially symmetric metric (16), the function $f(r)$ for $r \rightarrow 0$ approaches $2f(r) \rightarrow r^4/r_0^2 \rightarrow 0$; hence, the disk (8) is intrinsically flat together with the ring [92]. Therefore, the magnetic momentum is simply

$$\mu_{in} = \frac{1}{c} j_\phi S = -\frac{eS}{2\pi a} \sqrt{1 + e^2/a^2} U \quad (52)$$

where S is the disk area and U is an undetermined coefficient for an uncertainty in the fraction $\frac{\mu}{\cos^3 \xi}$ in (51). When the magnetic moment of the spinning object is known, the coefficient U can be restored from (52), which shall give the value of the superconducting ring current powering the regular spinning object. The circular current has the form

$$j_\phi = \frac{ec}{2\pi a} \sqrt{1 + e^2/a^2} U. \quad (53)$$

The magnetic momentum produced by this current is

$$\mu_e = \frac{e}{2\pi a} U \pi a^2. \quad (54)$$

For the electron $e = 1.6022 \times 10^{-19}$ C, and $2a = \lambda_e$ [41] where $\lambda_e = 3.8616 \times 10^{-11}$ cm is its Compton wavelength. In the geometrical units ($c = G = 1$) the electron charge is $e = 1.381 \times 10^{-34}$ cm, and $a = 1.931 \times 10^{-11}$ cm, which gives $e^2/a^2 = 0.5115 \times 10^{-46} \ll 1$ in (53). The experimental value of the electron magnetic moment reads [100]

$$\mu_e = \frac{e\hbar}{2m_e c} (1 + 0.00116) = \frac{e\lambda_e}{2} (1 + 0.00116). \quad (55)$$

A comparison of (54) with (55) gives $U = 2.00232$ and $j_\phi = 79.277$ A [99].

In the observer region $r \gg \lambda_e$, the superconducting current (51) produces the electromagnetic fields, which, in the region of a distant observer, are described by the Kerr–Newman limit of solutions (47) and (48) coinciding with the results presented in [41,58]

$$E_r = -\frac{e}{r^2} \left[1 - \frac{\hbar^2}{m_e^2 c^2} \frac{3 \cos^2 \theta}{4r^2} \right]; E_\theta = \frac{e\hbar^2}{m_e^2 c^2} \frac{\sin 2\theta}{4r^3}; B^r = -\frac{e\hbar}{m_e c} \frac{\cos \theta}{r^3}; B_\theta = -\frac{e\hbar}{2m_e c} \frac{\sin \theta}{r^4}. \quad (56)$$

The Planck constant appears due to discovered by Carter ability of the Kerr–Newman solution to present the electron as seen by a distant observer. In terms of the Coleman lump, the leading term in E_r gives the Coulomb law as the classical limit $\hbar = 0$, and the higher terms represent the quantum corrections.

We see that the behavior of solutions for electromagnetic fields determines typical generic features of regular NED-GR spinning objects: *Interior de Sitter vacuum disk (8) has the properties of a perfect conductor and ideal diamagnetic. The superconducting current j_ϕ flowing over the edge of the disk replaces the ring singularity of the Kerr–Newman geometry, powers the electromagnetic fields of an object, and provides the origin of its intrinsic magnetic momentum. For the electromagnetic soliton with the parameters of the electron $j_\phi = 79.277$ A.*

3. Summary and Discussion

Electromagnetic spinning solitons are described by the regular solutions of the source-free NED-GR equations, the only contribution to stress–energy tensors comes from source-free nonlinear electromagnetic field. The mass of a NED-GR soliton is determined by the electromagnetic density. Mass is positive and related to the interior de Sitter vacuum and to breaking of spacetime symmetry from the de Sitter group for NED-GR objects satisfying the Weak Energy Condition, which requires the positivity of density, as measured by any local observer. Electrically charged NED-GR solitons represent regular charged spinning objects related by electromagnetic and gravitational interaction. Their basic generic properties

are defined in the model-independent way by the spacetime structure and by the typical behavior of solutions for electromagnetic fields.

The regular interior of electromagnetic spinning soliton consists of the equatorial disk of the rotating de Sitter vacuum at which the energy density of the electromagnetic vacuum achieves its maximal de Sitter value $8\pi G\rho_{em} = \Lambda$. At approaching the disk the magnetic permeability $\mu \rightarrow 0$ while the electric susceptibility $\epsilon \rightarrow \infty$, which corresponds to behavior typical for the perfect conductor and ideal diamagnetic. The magnetic induction and magnetic permeability independently vanish on the disk, which testifies for a superconducting behavior. The ring singularity of the Kerr–Newman geometry is replaced with the superconducting ring current which provides the non-dissipative source of electromagnetic fields and the origin of an intrinsic magnetic momentum for any electrically charged regular object described by NED-GR. Generic features of the electromagnetic soliton with the parameters of the electron, $m_e a = \hbar/2, g = 2$, suggest that the intrinsic origin of the electron fields and of its magnetic momentum is the superconducting ring current evaluated as $j_\phi = 79.277$ A. One can say that the basic generic features of NED-GR spinning solitons suggest the existence of superconductive behavior within a single particle with the spin $\hbar/2$.

This picture is verified by the results that were obtained for the annihilation reaction $e^+e^- \rightarrow \gamma\gamma(\gamma)$. The experimental data reveal, with a 5σ significance, the appearance of a minimal length scale $l_e = 1.57 \times 10^{-17}$ cm at the energy $E = 1.253$ TeV [101,102]. It corresponds to the minimum in the χ^2 -fit and it characterizes the region of the closest approach of annihilating particles.

The hypotheses of quantum electrodynamics applied in the χ^2 -test assume a scattering center as a point. For the extended electron, its structure should modify the QED cross-section if the test distances are smaller than its characteristic size. The experimental results indicate decreasing cross section with respect to that predicted by QED and testify for extended particles rather than point-like. The purely electromagnetic reaction $e^+e^- \rightarrow \gamma\gamma(\gamma)$ can be interpreted with using the basic properties of an electromagnetic spinning soliton. For the electron visualized as a NED-GR spinning soliton, its characteristic size $\lambda_e = 3.8616 \times 10^{-11}$ cm is certainly bigger than the characteristic test distance $l_e = 1.57 \times 10^{-17}$ cm. The definite feature of the annihilation process is that at its final stage a region of interaction is neutral and spinless. It can be modeled by a spherical lump with the de Sitter vacuum interior. In the internal structure of any structure with the de Sitter interior, there exists the characteristic surface of zero gravity $r_* \simeq (r_0^2 r_g)^{1/3}$, at which the strong energy condition ($\rho + \sum p_k \geq 0$ [87]) is violated and beyond which the gravitational acceleration becomes repulsive [90,103]. The gravitational radius r_g that is related to the energy $E = 1.253$ TeV, and de Sitter radius r_0 related to the Higgs vacuum expectation value responsible for the electron mass at the scale $E_{EW} = 246$ GeV, give $r_* \simeq 0.86 \times 10^{-16}$ cm. The test scale $l_e = 1.57 \times 10^{-17}$ cm fits inside a region where gravity is repulsive and can be understood as a distance at which electromagnetic attraction is balanced by the gravitational repulsion of the interior de Sitter vacuum [102].

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