

Article

Giant Dipole Multi-Resonances Excited by High-Frequency Laser Pulses

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Abstract: The worldwide advent of new laser facilities makes possible the investigation of the nuclear response to a very strong electromagnetic field. In this paper, we inquire on the excitation of one of the most conspicuous collective excitations, the giant dipole resonance, within the hydrodynamical model for a proton-neutron fluid mixture placed in a Skyrme mean-field and interacting with an external ultra-strong electromagnetic field. The variables of this approach are: proton and neutron displacement (velocity) fields, density fluctuations, and fluctuations of the electric field due to the coupling of the laser electromagnetic field to the dynamical distortions of the baryonic system (electromagneto-hydrodynamical effect). We point out the occurrence of a multiresonance structure of the absorption cross-section.

Keywords: laser-nucleus interaction; ultra-intense laser pulse; giant resonance; plasma oscillations

1. Introduction

As emphasized in a number of recent publications, high-intensity lasers, ranging from the optical to the X-ray domain, may influence various nuclear processes, e.g., nuclear transitions by electronic [1,2] or muonic transitions [3], proton emission from nuclei via the nuclear photoeffect in an intense laser field and accompanied by a γ -ray [4], modification of α - [5–7] and proton decay rates [8], modification of heavy ion elastic scattering differential cross-sections [9,10] or the deuteron-triton fusion probability enhancement [11,12]. However, excepting perhaps the case of elastic scattering of heavy ions, the low photon energies and the insufficiently high power of these types of laser pulses are rather weakly disturbing the quantum states of the atomic nucleus. For this reason, the newly entered-into-operation ELI-NP facility intends to use ultra-intense laser fields with intensities reaching up to 10^{22} – 10^{23} W/cm² to produce intense low-energy gamma beams (<20 MeV) by Compton back-scattering and therefore allow nuclear structure and reactions investigations [13,14]. On the other hand, there are perspectives to achieve intense X-ray fields produced by free-electron lasers. Intense laser pulses are expected to reach power densities of over 10^{20} W/cm² using 9.9 keV photons [15].

Giant resonances are one of the most remarkable examples of collective excitations in the atomic nuclei [16]. The giant dipole resonance (GDR), discovered 75 years ago, can be excited, for example, by photon absorption by a nucleus. The electric component of the incident photon, homogeneous over the nucleus since the wavelength is much larger than the nuclear radius ($\lambda \gg R$), induces a coherent displacement of the proton distribution with respect to the neutron distribution [17]. Goldhaber and Teller proposed a simple description of the dipole mode in spherical nuclei by the reciprocal vibrations of the rigid proton and neutron distributions [18]. Following the original work of Flügge on the eigenvibrations of a one-component nuclear liquid drop [19], Steinwedel and Jensen operated the extension to two fluids (proton-neutron mixture) interpenetrating each other [20]. The hydrodynamical treatment of protons vibrating against neutrons provided one of the most transparent descriptions of this excitation mode in nuclei, as showed by further developments [21–30].



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Below, we lay down a framework aiming to describe the coupling of a laser field to a proton-neutron fluid.

2. Neutron-Proton Mixtures in an External Electromagnetic Field

In what follows, we extend the traditional framework of nuclear hydrodynamics, as presented, for example, in Refs. [17,22], to a proton-mixture interacting via Skyrme-type forces [28,30].

Let us introduce the constituent velocities $\dot{\chi}_{p,n}$ via the definition of the corresponding mass-current densities (we employ the term mass-current density, present in the Skyrme energy density \mathcal{H}_{Sky} , to differentiate from the charge-current density.)

$$j_q = \frac{m}{\hbar} \rho_q \dot{\chi}_q \quad (q = p, n) \tag{1}$$

The kinetic energy of the fluid mixture then reads

$$T = \frac{1}{2} m \int d\mathbf{r} (\rho_p \dot{\chi}_p^2 + \rho_n \dot{\chi}_n^2). \tag{2}$$

The internal energy originating from nuclear interactions is expressed by means of Skyrme parametrization (encoded in the B -coefficients, as seen below) of the energy density

$$\int d\mathbf{r} \mathcal{H}_{\text{Sky}} \tag{3}$$

where \mathcal{H}_{Sky} , neglecting the spin-orbit contribution, justified for spin-saturated nuclei, assumes the compact form in terms of the neutron and proton ($q = n, p$) local ρ_q , kinetic energy τ_q and the mass-current j_q densities [31]

$$\begin{aligned} \mathcal{H}_{\text{Sky}} &= \frac{\hbar^2}{2m} (\tau_p + \tau_n) + B_1 \rho^2 + B_2 (\rho_p^2 + \rho_n^2) \\ &+ B_3 (\rho \tau - j^2) + B_4 (\rho_p \tau_p - j_p^2 + \rho_n \tau_n - j_n^2) \\ &- B_5 (\nabla \rho)^2 - B_6 [(\nabla \rho_p)^2 + (\nabla \rho_n)^2] + [B_7 \rho^2 + B_8 (\rho_p^2 + \rho_n^2)] \rho^\alpha. \end{aligned} \tag{4}$$

The corresponding central one-body potential \mathcal{U}_q is defined as the functional derivative of the energy density with respect to the q -th constituent density

$$\mathcal{U}_q \equiv \frac{\delta \mathcal{H}_{\text{Sky}}}{\delta \rho_q(\mathbf{r})} = \frac{\partial \mathcal{H}_{\text{Sky}}}{\partial \rho_q} - \nabla \cdot \frac{\partial \mathcal{H}_{\text{Sky}}}{\partial \nabla \rho_q} + \Delta \frac{\partial \mathcal{H}_{\text{Sky}}}{\partial \Delta \rho_q} \tag{5}$$

In a finite system to the above form of the internal energy density, the following contribution has to be added:

$$U_C(\mathbf{r}) = \frac{1}{2} e \rho_p(\mathbf{r}) \Phi(\mathbf{r}) = \frac{e^2}{2} \rho_p(\mathbf{r}) \int d\mathbf{r}' \frac{\rho_p(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}. \tag{6}$$

where $\Phi(\mathbf{r})$ is the Coulomb potential.

Thus, the internal energy can be written as

$$U = \int d\mathbf{r} (\mathcal{H}_{\text{Sky}}(\mathbf{r}) + U_C(\mathbf{r})) \tag{7}$$

In this paper, we consider a neutron-proton “plasma” placed in an external electromagnetic field (laser) of frequency ω and electric and magnetic strengths

$$E_0(t) = \hat{\mathbf{e}} \mathcal{E}_0 \cos \omega t, \quad B_0(t) = (\hat{\mathbf{k}} \times \hat{\mathbf{e}}) \frac{\mathcal{E}_0}{c} \sin \omega t, \tag{8}$$

where \hat{e} is the unit vector along the laser polarization, and \hat{k} is the unit vector along the propagation direction such that $\hat{k} \perp \hat{e}$. The interaction of the charged component of the fluid mixture with the external electromagnetic field of strength (E, B) reads [32]

$$W_{em} = e \int d\mathbf{r} \rho_p \chi_p \cdot (E + \dot{\chi}_p \times B), \tag{9}$$

where χ_p is the proton fluid displacement field, trivially related to the above introduced proton fluid velocity field

$$\dot{\chi}_p(\mathbf{r}, t) = \frac{\partial \chi_p(\mathbf{r}, t)}{\partial t} \tag{10}$$

Since the coherent motion tends to be distributed in the nuclear volume mainly by two-body scattering, a damping mechanism occurs. This can be viewed from a microscopic point of view as the friction between the proton and neutron fluids. According to the traditional form of nuclear hydrodynamics [17], the damping of collective modes is described by the friction exerted by one fluid against the other, i.e.,

$$W_{fric} = -m\Gamma \int d\mathbf{r} \rho_{red} (\chi_p - \chi_n)(\dot{\chi}_p - \dot{\chi}_n) \tag{11}$$

where Γ simulates the spreading width of the giant resonance and

$$\rho_{red} = \frac{\rho_p \rho_n}{\rho_p + \rho_n} \tag{12}$$

This functional dependence on the relative proton-neutron velocity is justified in §11 of ref. [33]: due to the motion of the proton fluid relative to the neutron fluid, the neutron fluid acquires an additional momentum, leading thus to the appearance of a reaction force on the proton fluid proportional to $-(\dot{\chi}_p - \dot{\chi}_n)$.

A study [34] of giant resonance damping in heavy nuclei ($A \approx 200$) based on the thermalization processes going via the excitation of $2p - 2h$ states (*doorway states*) yields for the width the range of values: $0.42 \text{ MeV} \leq \hbar\Gamma \leq 2.25 \text{ MeV}$. The description of dissipative processes was extended in ref. [24] to also account for friction forces of the proton fluid with itself and neutron fluid with itself. These authors fitted the experimental widths, $\hbar\Gamma \approx 3 \text{ MeV}$ of the isoscalar quadrupole state, and $\hbar\Gamma \approx 4 \text{ MeV}$ of the isovector 1^- (GDR) in ^{208}Pb . Thus, the viscosity is estimated to be $\eta = 10^{-23} \text{ MeV}\cdot\text{s}\cdot\text{fm}^{-3}$. An estimation for the viscosity, consistent with the giant resonance study mentioned above, was given in ref. [35] in the case of heavy nuclei fission using a viscous liquid drop model.

In order to derive the dynamical equations governing the continuum-mechanical system combining the proton and neutron fluids, we apply the Hamilton principle to the four-fold action integral [36,37]

$$\delta S = \delta \int dt \mathcal{L} = 0, \tag{13}$$

where the Lagrangean reads

$$\mathcal{L} = T + T_{add} - U + W_{em} + W_{fric} \tag{14}$$

Before the variation, a gauge term related to the mass balance in the mixture, is added to the Lagrangean by means of undetermined multipliers $\lambda_{p,n}$ [38],

$$C = \int d\mathbf{r} \sum_q \left[\frac{\partial}{\partial t} (\lambda_q \rho_q) + \nabla \cdot (\lambda_q \rho_q \dot{\chi}_q) - \lambda_q \frac{\partial \rho_q}{\partial t} - \lambda_q \nabla \cdot (\rho_q \dot{\chi}_q) \right]. \tag{15}$$

The particles of the fluid mixture are subjected to a virtual variation with respect to the dynamical variables ρ_q and $\dot{\chi}_q$. The Lagrange equations corresponding to the densities are

$$\frac{\partial \mathcal{L}}{\partial \rho_q} - \frac{\partial}{\partial x_i} \left(\frac{\partial \mathcal{L}}{\partial \rho_{q,i}} \right) - \frac{\partial}{\partial t} \left(\frac{\partial \mathcal{L}}{\partial \rho_{q,t}} \right) = 0 \tag{16}$$

and after some mathematical manipulations we are led to the following two equations,

$$\frac{1}{2} m \dot{\chi}_p^2 - \mathcal{U}_p - e\Phi + e\chi_p \cdot (E + \dot{\chi}_p \times B) - m\Gamma \frac{\rho_n}{\rho} (\chi_p - \chi_n) (\dot{\chi}_p - \dot{\chi}_n) + \frac{\partial}{\partial t} \lambda_p + \dot{\chi}_p \cdot \nabla \lambda_p = 0 \tag{17}$$

$$\frac{1}{2} m \dot{\chi}_n^2 - \mathcal{U}_n - m\Gamma \frac{\rho_p}{\rho} (\chi_p - \chi_n) (\dot{\chi}_p - \dot{\chi}_n) + \frac{\partial}{\partial t} \lambda_n + \dot{\chi}_n \cdot \nabla \lambda_n = 0. \tag{18}$$

The Lagrange equations for the proton and neutron fluid velocities yield

$$m\rho_p \dot{\chi}_p - e\rho_p (\chi_p \times B) - m\Gamma \rho_{\text{red}} (\chi_p - \chi_n) + \rho_p \nabla \lambda_p = 0 \tag{19}$$

$$m\rho_n \dot{\chi}_n + m\Gamma \rho_{\text{red}} (\chi_p - \chi_n) + \rho_n \nabla \lambda_n = 0. \tag{20}$$

Combining these equations and discarding the quadratic terms in the velocities, we get

$$m \dot{\chi}_p = e \left[E + \nabla (\chi_p \cdot E) + \chi_p \times B - \chi_p \times (\nabla \times E) \right] - \nabla \mathcal{U}_p + m\Gamma \frac{\rho_n}{\rho} (\dot{\chi}_p - \dot{\chi}_n) \tag{21}$$

$$m \dot{\chi}_n = -\nabla \mathcal{U}_n - m\Gamma \frac{\rho_p}{\rho} (\dot{\chi}_p - \dot{\chi}_n). \tag{22}$$

The hydrodynamical equations established above are supplemented with the equations relating the electromagnetic fields to the charge and current distributions of the fluid mixture (Maxwell equations):

$$\nabla \cdot E = \frac{e}{\epsilon_0} \rho_p; \quad \nabla \cdot B = 0; \quad \nabla \times E = -\frac{\partial B}{\partial t}; \quad \frac{1}{\mu_0} \nabla \times B = e\rho_p \dot{\chi}_p + \epsilon_0 \frac{\partial E}{\partial t} \tag{23}$$

In the non-perturbed state, the following relations are satisfied:

$$\rho = \rho_p + \rho_n = \rho_0; \quad \dot{\chi}_p = \dot{\chi}_n = 0 \tag{24}$$

At $t = 0$, the $p - n$ “plasma” is perturbed, and thus, the density, mean-field, velocity, electric and magnetic fields fluctuate according to

$$\rho_q \longrightarrow \rho_q^0 + \delta\rho_q, \quad \mathcal{U}_q \longrightarrow \mathcal{U}_q^0 + \delta\mathcal{U}_q, \quad \dot{\chi}_q \longrightarrow v_q, \quad E \longrightarrow E_0 + \delta E, \quad B \longrightarrow B_0 + \delta B; \tag{25}$$

where ρ_q denotes the equilibrium densities, and $\delta\rho_q \ll \rho_q$. In what follows, we neglect the surface effects (e.g., $\nabla \rho_q^0, \nabla \mathcal{U}_q^0 = 0$) as well as second-order terms, since we assumed small perturbations.

Neglecting the nonlocal effects that are generating gradient contributions of the densities, and therefore of the mean-field potentials, we are left with a linear dependence on the proton and neutron density fluctuations,

$$\delta\mathcal{U}_q = G_{qq}^{(1)} \delta\rho_q + G_{qq'}^{(1)} \delta\rho_{q'}, \tag{26}$$

where the G -coefficients in the Skyrme parametrization are given in Section 3 of ref. [30].

Neglecting non-dipole and magnetic terms in Equation (21), the linearized hydrodynamic equations of the $p - n$ “plasma” are

$$\frac{\partial \mathbf{v}_p}{\partial t} = -\frac{1}{m} \nabla \delta \mathcal{U}_p + \frac{\Gamma \rho_n^0}{\rho} (\mathbf{v}_p - \mathbf{v}_n) + \frac{e}{m} (\mathbf{E}_0 + \delta \mathbf{E}) \tag{27}$$

$$\frac{\partial \mathbf{v}_n}{\partial t} = -\frac{1}{m} \nabla \delta \mathcal{U}_n - \frac{\Gamma \rho_p^0}{\rho} (\mathbf{v}_p - \mathbf{v}_n). \tag{28}$$

The above two equations that express the momentum balance, are supplemented with the two equations ensuring the mass balance,

$$\frac{\partial \delta \rho_p}{\partial t} + \rho_p^0 \nabla \cdot \mathbf{v}_p = 0 \tag{29}$$

$$\frac{\partial \delta \rho_n}{\partial t} + \rho_n^0 \nabla \cdot \mathbf{v}_n = 0 \tag{30}$$

On the other hand, the Maxwell equations for the fluctuated fields are obtained upon substitution of the fluctuated fields according to (25) in (23)

$$\begin{aligned} \nabla \cdot \delta \mathbf{E} &= \frac{e}{\epsilon_0} \delta \rho_p, \quad \nabla \cdot \delta \mathbf{B} = 0 \\ \nabla \times \delta \mathbf{E} &= -\frac{\partial}{\partial t} \delta \mathbf{B}, \quad \nabla \times \delta \mathbf{B} = \mu_0 e \rho_p^0 \dot{\chi}_p + \frac{1}{c^2} \frac{\partial \delta \mathbf{E}}{\partial t} \end{aligned} \tag{31}$$

This third set of equations completes the macroscopic description of the $p - n$ “plasma”.

Taking the time-derivative of the set of Equations (29) and (30) and substituting inside them the set (27) and (28), we obtain the coupled wave equations for the density fluctuations. Introducing the *proton “plasma” frequency*

$$\omega_p^2 = \frac{e^2 \rho_p^0}{m \epsilon_0}, \tag{32}$$

and the square of the qq' matrix element of the 2×2 speed of sound matrix

$$c_{qq'}^2 = \frac{\rho_q^0}{m} G_{qq'} \tag{33}$$

we write down these two coupled wave equations

$$\delta \ddot{\rho}_p = c_{pp}^2 \Delta \delta \rho_p + c_{pn}^2 \Delta \delta \rho_n^0 + \frac{\Gamma}{\rho} (\rho_n^0 \delta \dot{\rho}_p - \rho_p^0 \delta \dot{\rho}_n) + \omega_p^2 \delta \rho_p \tag{34}$$

$$\delta \ddot{\rho}_n = c_{np}^2 \Delta \delta \rho_p + c_{nn}^2 \Delta \delta \rho_n - \frac{\Gamma}{\rho} (\rho_n^0 \delta \dot{\rho}_p - \rho_p^0 \delta \dot{\rho}_n) \tag{35}$$

For an incompressible nucleus, the total density, $\rho_p + \rho_n = \rho_0$, is preserved in the perturbed state, and therefore, $\delta \rho_p = -\delta \rho_n$. Under this assumption, the wave equations are decoupled, and thence Equation (34) can be rewritten as

$$\delta \ddot{\rho}_p = (c_{pp}^2 - c_{pn}^2) \Delta \delta \rho_p + \Gamma \delta \dot{\rho}_p + \omega_p^2 \delta \rho_p \tag{36}$$

For a low-frequency laser pulse, the external electric field can be taken as quasi-static, i.e., $|\mathbf{E}(t)| \approx E_0$, and from the standpoint of the laser-nucleus interaction, one speaks of a zero-photon exchange approximation. It is then obvious that the solution does not depend on the frequency of the incoming radiation ω , and one can safely assume a harmonic time dependence on the giant dipole oscillation Ω , i.e.,

$$\delta \rho_p \sim e^{i\Omega t} \tag{37}$$

One then has to solve the scalar Helmholtz equation

$$\Delta\delta\rho_p + k^2\delta\rho_p = 0, \tag{38}$$

where

$$\text{Re}\{k^2\} = \frac{\Omega^2}{c_{pp}^2 - c_{pn}^2} \left(1 + \frac{\omega_p^2}{\Omega^2} \right), \quad \text{Im}\{k^2\} = \frac{\Omega\Gamma}{c_{pp}^2 - c_{pn}^2}. \tag{39}$$

Since the strong polarization induced by the electric field results in the excitation of a pure dipole displacement of the charged fluid, we select the space-dependent solution in the form

$$\delta\rho_p(\mathbf{r}) = Aj_1(kr)Y_{10}(\theta, \phi), \tag{40}$$

where the amplitude A is extracted from the boundary condition that ensures no flow takes place outside the spherical nuclear surface of radius R ,

$$\mathbf{n} \cdot \frac{\partial \mathbf{v}_q}{\partial t} \Big|_{r=R} = 0. \tag{41}$$

Using the Euler equation for the proton component (27) the boundary condition can be turned to

$$\left[\rho_p^0(G_{pp} - G_{pn}) + \rho_n^0(G_{np} - G_{nn}) \right] \mathbf{n} \cdot \nabla \delta\rho_p|_{r=R} = e\rho_p^0 \mathbf{n} \cdot (\mathbf{E}_0 + \delta\mathbf{E})|_{r=R} \tag{42}$$

The case when no electric field is induced in the nucleus upon the dynamic polarization due to the external field was discussed in the literature [17]. This case also applies in the present approximation, as can be deduced from the Ampère law (fourth eq. in (31)); if we neglect the curl of the magnetic effect, which is of secondary importance, we have that $\delta\dot{\mathbf{E}} \sim \mathbf{v}_p$, and therefore,

$$\mathbf{n} \cdot \delta\mathbf{E}|_{r=R} \approx 0$$

Consequently, the amplitude of the proton density vibrations relates to the strength of the external electric field E_0 :

$$A = 2\sqrt{\frac{\pi}{3}} \frac{eE_0R}{G_{pp} - G_{nn}} [kRj_1'(kR)]^{-1} \tag{43}$$

Thence, the eigenmodes of the proton-neutron out-of-phase vibrations are given as the roots of the transcendental equation:

$$j_1'(kR) = 0 \tag{44}$$

In the case of an incoming pulse of high-frequency, the time-dependence is expressed in terms of a Fourier series, such as the one used to solve Hill's equation [39]

$$\delta\rho_p = e^{-i\Omega t} \sum_{n=-\infty}^{+\infty} \mathcal{R}_n(\mathbf{r})e^{-in\omega t} \tag{45}$$

In a quantum approach, the central term ($n = 0$) in the sum above corresponds to the zero-photon exchange channel. Inserting the above infinite sum in Equation (36) and equating each term of the resulting sum to zero, we obtain the wave equation of the density fluctuation for an n -photon exchange:

$$(c_{pp}^2 - c_{pn}^2)\Delta\mathcal{R}_n(\mathbf{r}) + \left[(\Omega + n\omega)^2 + i\Gamma(\Omega + n\omega) + \omega_p^2 \right] \mathcal{R}_n = 0 \tag{46}$$

Introducing the square of the wave-vector associated with the n -th channel,

$$k_n^2 = \frac{(\Omega + n\omega)^2}{c_{pp}^2 - c_{pn}^2} \left(1 + \frac{i\Gamma}{\Omega + n\omega} + \frac{\omega_p^2}{(\Omega + n\omega)^2} \right) \tag{47}$$

we obtain the amplitudes of the Fourier series (45),

$$\mathcal{R}_n(\mathbf{r}) = A_n j_1(k_n r) Y_{10}(\theta, \phi) \tag{48}$$

By making the notation

$$\zeta_n^2 = \frac{k_n^2}{R^2} (c_{pp}^2 - c_{pn}^2) \tag{49}$$

it is then possible to invert Equation (47) and obtain the eigenfrequency for n exchanged photons:

$$\Omega_n = n\omega + \zeta_n \sqrt{1 - \left(\frac{\omega_p}{\zeta_n}\right)^2 - \left(\frac{\Gamma}{2\zeta_n}\right)^2} - \frac{1}{2}i\Gamma \tag{50}$$

In what follows, we assume that the proton velocity field splits in a longitudinal ($\nabla \times \mathbf{V}_n^L = 0$) and transverse ($\nabla \cdot \mathbf{V}_n^T = 0$) component of the same parity (see for details [40,41]). Thus,

$$\mathbf{v}_p(\mathbf{r}) = e^{-i\Omega t} \sum_{n=-\infty}^{+\infty} \left(\mathbf{V}_n^L(\mathbf{r}) + \mathbf{V}_n^T(\mathbf{r}) \right) e^{-i\omega t} \tag{51}$$

where

$$\mathbf{V}_n^L(\mathbf{r}) = -\frac{i}{k_n} B_n^L \mathbf{L}_n, \quad \mathbf{V}_n^T(\mathbf{r}) = -\frac{i}{k_n} B_n^T \mathbf{N}_n \tag{52}$$

These components are expressed in terms of the fundamental solutions of the Helmholtz equation [42],

$$\mathbf{L}_n = \frac{1}{\sqrt{3}} \left(j_0(k_n r) \mathbf{Y}_{10}^0 + \sqrt{2} j_2(k_n r) \mathbf{Y}_{10}^2 \right) \tag{53}$$

$$\mathbf{N}_n = \sqrt{\frac{1}{3}} \left(\sqrt{2} j_0(k_n r) \mathbf{Y}_{10}^0 - j_2(k_n r) \mathbf{Y}_{10}^2 \right) \tag{54}$$

where \mathbf{Y}_{JM}^L are vector spherical harmonics [43]. Due to the relation set by the continuity equation, the amplitude of the longitudinal component can be expressed in terms of the amplitude of the density fluctuation in each channel, i.e.,

$$B_n^L = \frac{1}{\rho_p^0} (\Omega + n\omega) A_n \tag{55}$$

Let us return to the fluctuations of the electromagnetic field. If we differentiate the Ampère law (last equations in (31)) with respect to time and take the curl of the Maxwell–Faraday law (third equations in (31)), thus eliminating $\nabla \times \partial \delta \mathbf{B} / \partial t$, we end up with the non-homogeneous wave-equation for the electric field fluctuations:

$$\Delta \delta \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \delta \mathbf{E}}{\partial t^2} = \mu_0 e \rho_p \frac{\partial \mathbf{v}_p}{\partial t} + \frac{e}{\epsilon_0} \nabla \delta \rho_p \tag{56}$$

Next, we assume the longitudinal/transverse decomposition for the solution of the above equation:

$$\delta \mathbf{E}(\mathbf{r}) = e^{-i\Omega t} \sum_{n=-\infty}^{+\infty} \left(\mathbf{E}_n^L(\mathbf{r}) + \mathbf{E}_n^T(\mathbf{r}) \right) e^{-i\omega t} \tag{57}$$

$$\mathbf{E}_n^L(\mathbf{r}) = C_n^L \mathbf{L}_n, \quad \mathbf{E}_n^T(\mathbf{r}) = C_n^T \mathbf{N}_n \tag{58}$$

Due to (56), the amplitudes of the induced electric field can be expressed in terms of the amplitudes of the proton density and velocity fluctuations:

$$C_n^L = \left(k_n^2 - \frac{(\Omega + n\omega)^2}{c^2}\right)^{-1} \left[e\mu_0 \frac{(\Omega + n\omega)^2}{k_n} - k_n \frac{e}{\epsilon_0} \right] A_n \tag{59}$$

$$C_n^T = \left(k_n^2 - \frac{(\Omega + n\omega)^2}{c^2}\right)^{-1} e\mu_0 \rho_p \frac{(\Omega + n\omega)^2}{k_n} B_n^T \tag{60}$$

If we restrict ourselves to the case of no vorticity,

$$\nabla \times v_p = 0, \tag{61}$$

all the fluctuations of the velocity and electromagnetic fields are restricted to longitudinal components ($\sim L_n$), and by imposing the boundary condition (42), we obtain for each channel n the same equation that provides the overtones of the dipole mode:

$$j_1'(k_n R) = 0. \tag{62}$$

In Table 1, we present for a selection of four Skyrme parametrizations the characteristics related to the fundamental mode of the GDR for damping $\hbar\Gamma = 2.5$ MeV. In the last column, we list the experimental value of the corresponding first overtone. Whereas the older parametrization SIII predicts this value poorly, the SkX types provides a much better choice.

Table 1. Giant dipole fundamental mode, plasma frequency, and energy of the GDR fundamental overtone for various Skyrme parametrizations.

Skyrme Interaction	$\hbar\omega_p$ (MeV)	$\hbar\zeta_n$ (MeV)	$\hbar\Omega$ (MeV)	$\hbar\Omega \times A^{1/3}$ (MeV)	$\hbar\Omega_{\text{exp}} \times A^{1/3}$ (MeV)
SIII	8.4	10.9	10.8	63.9	
SkM	8.8	14.0	13.9	82.5	
SkA	8.7	14.2	14.2	84.0	80
SkP	8.9	14.7	14.6	86.7	

From the same boundary condition, we are left with non-vanishing components only in the $n = \pm 1$ channels of the density fluctuation (48),

$$A_{\pm 1} = \frac{1}{2} \sqrt{\frac{4\pi}{3}} \frac{e\rho_p R E_0}{m(c_{pp}^2 - c_{pn}^2)} [j_1(k_{\pm 1} R) - k_{\pm 1} R j_1'(k_{\pm 1} R)]^{-1}. \tag{63}$$

The electric field component of the laser pulse, oriented along the z-axis, produces a dynamic polarization of the $p - n$ “plasma”, and therefore induces a non-vanishing dynamic dipole moment only along this direction

$$D_z = \int d\mathbf{r} z \delta\rho_p(\mathbf{r}) = 2\sqrt{\frac{\pi}{3}} eR^4 \sum_{n=\pm 1} A_n \frac{j_2(k_n R)}{k_n R} e^{-i(\Omega+n\omega)t}. \tag{64}$$

Next, to calculate the classical absorption cross-section, we need the time-derivative of (64). Then, we take the average energy absorbed per unit time,

$$\sigma_{\text{abs}} \sim \frac{1}{T} \text{Re} \left\{ \int_0^T dt \dot{D}_z(t) E_0(t) \right\} \tag{65}$$

The derivation of the final form of σ_{abs} amounts to generalizing the derivation in the field-free case, as detailed in Ref. [17],

$$\sigma_{\text{abs}} = \frac{Ze^2}{m\epsilon_0} \left(1 + \frac{c_{np}^2 - c_{nn}^2}{c_{pp}^2 - c_{pn}^2} \right) \sum_{n=\pm 1} \frac{\Gamma/c}{\zeta_n^2 - 2} \left\{ \left[\Omega + n\omega + \frac{\omega_p^2 - \zeta_n^2}{\Omega + n\omega} \right]^2 + \Gamma^2 \right\}^{-1}. \quad (66)$$

and results in a two-peaked Lorentz function when $\omega \neq 0$. The action of a high-frequency laser field will split the one-peaked distribution in a manner analogous to the dipole strength in deformed nuclei [17]. It can be remarked from Figure 1 that the splitting of the GDR centroid of $\hbar\Omega_0 = 13.9$ MeV for the spherical heavy nucleus ^{208}Pb is visible for frequencies from the γ -ray domain. Within a quantum treatment of the problem, we expect the coming into play of higher terms ($|n| \geq 2$) in the sum over the number of exchanged photons, $\sigma_{\text{abs}} = \sum_n \sigma_n$. Depending on the frequency and the strength of the laser field, the broad resonance centered on the GDR fundamental mode Ω_0 will display a series of maxima corresponding to the satellites $\Omega_0 \pm \omega, \Omega_0 \pm 2\omega, \dots$. More precisely, the effect of a laser pulse in a continuous wave form produces a multiresonance shape of the absorption cross-section.

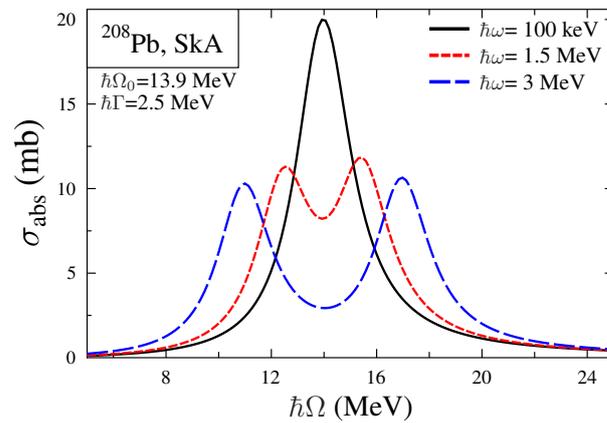


Figure 1. Photon absorption cross-section of ^{208}Pb in a laser field for three different frequencies: $\hbar\omega = 100$ keV (full line), 1.5 MeV (short dashes) and 3 MeV (long dashes). Calculations are made for the SkM parametrization of \mathcal{H}_{Sky} and the spreading width $\Gamma = 2.5$ MeV.

3. Conclusions and Outlook

Using the framework of nuclear hydrodynamics for two fluids interacting via nuclear Skyrme and Coulomb forces, the collective dipole response of a spherical nucleus to a strong incoming laser pulse was calculated. State-of-the-art laser pulses are still low in frequency in order to access directly collective states in atomic nuclei. A possible way to overcome the discrepancy between the photon energy of the available high-intensity lasers (<1 keV) and the energy scale of nuclear processes such as nuclear reactions, single-particle or collective excitations (~ 100 keV–20 MeV) were proposed by Mocken and Keitel [44]. They advanced the proposal to accelerate heavy ions at ultra-relativistic energies in the presence of a super-intense counter-propagating laser pulse. Later on, this proposal was discussed in [45] in the context of nuclear dipole mode excitations.

For example, the Ti:sapphire laser pulse with photon energy $\hbar\omega = 1.5$ eV propagating against a ^{208}Pb beam of ultra-relativistic energy $E_{\text{acc}} = 10$ TeV, is Doppler-shifted with an energy quanta $\hbar\omega' \approx 12.54$ keV and experiences a large amplification of the intensity in the ion-fixed frame: $I' \approx 7 \times 10^7 I$. A better choice would be a beam of hard X-rays with photon energy $\hbar\omega = 9.9$ keV and intensity $I = 10^{20}$ W/cm² colliding with the same ion beam, this time of energy $E_{\text{acc}} = 1$ TeV. In this case, we end-up with $\hbar\omega' \approx 8.3$ MeV and $I' \approx 7 \times 10^{25}$ W/cm². While the zero-photon absorption cross section displays a maximum at $\hbar\Omega = 13.5$ MeV, the energy centroid of the GDR, the one-photon σ_1 , displays two maxima

at 5.6 MeV and 22.3 MeV. By using intense laser pulses with photon frequencies shifted in the γ -ray range, nuclear collective modes ranging from low-energy dipole modes (pygmy resonance [46]) to binary fission can be made feasible.

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