

Appendices

The distribution of REPTree in the form of tree manner taking two different parameters are shown in Figure S1 for approach-2. Likewise, the Random tree and M5P distribution of approach-2 is shown in Figure S2 and S3. It shows the basic principle of tree distribution system to capture the variations and predict most accurate results.

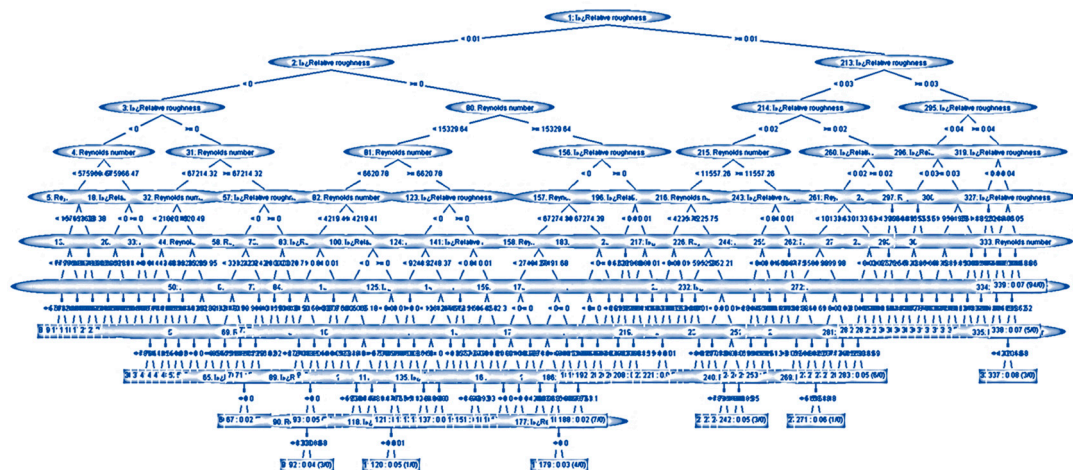
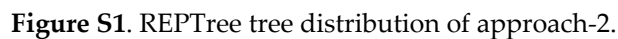


Figure S2. Random Tree distribution of approach-2.

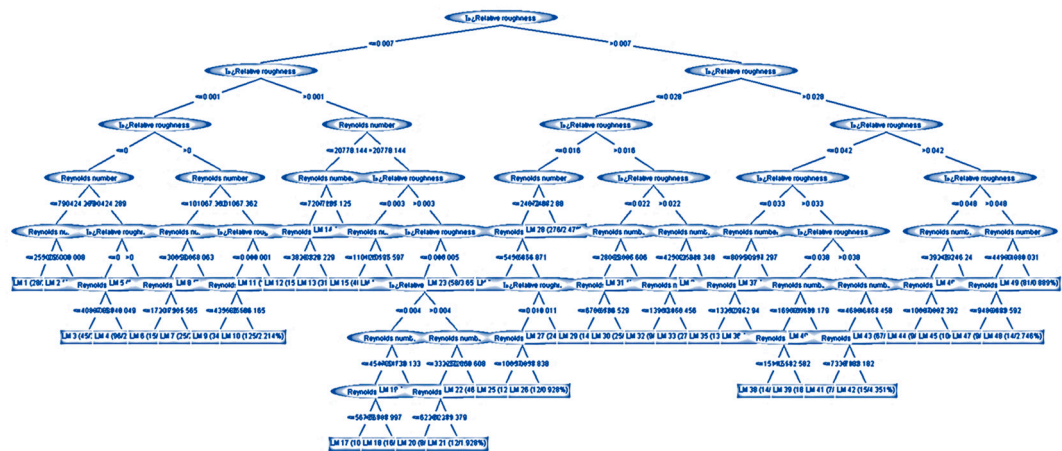


Figure. S3. M5P distribution of approach-2.

B) Comparison of Approach-1 Results

The training and testing datasets of approach-1 are compared with the expression given by [8] for turbulent region. It shows most accurate results with a relative error of $\pm 1\%$ in Figure S4. Taylor diagram represents correlation coefficients (CC) along with standard deviation of all the datasets. From Taylor diagram also similar kind of results are obtained. Simulated results from expression given by [8] predicted more accurate results as compared to all AI based models in Figure S5.

From this figure it can be concluded that the expression given by [8] predicts more accurate results as compared to AI tools. Another aspect comparison is relative error of observed and predicted datasets. Thus, relative error of all the predicted datasets from different AI models as well as analytical approaches are shown in Figure S6. Figure S6(a) displays that RF, RT, REPTree gives a relative error of $\pm 10\%$ whereas expression given by [8] shows a relative error within $\pm 1\%$ during training. Likewise, Figure S6(b) displays relative error of testing datasets of all models, where RF model shows relative error of $\pm 20\%$. But expression given by [8] shows a consistent result of relative error within $\pm 1\%$ during training as well as testing.

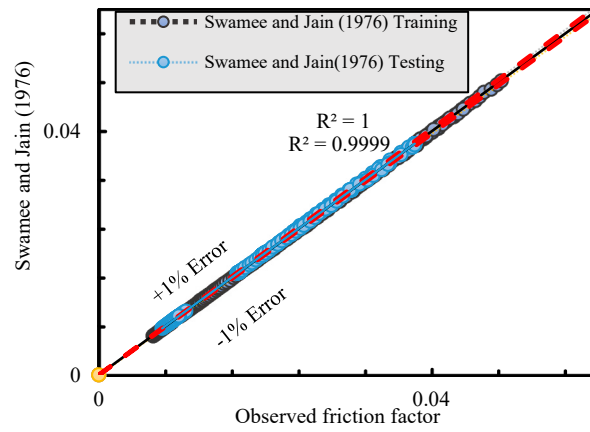


Figure. S4 Agreement diagram of observed and predicted friction factor of training and testing dataset of approach-1 by using expression of [8].

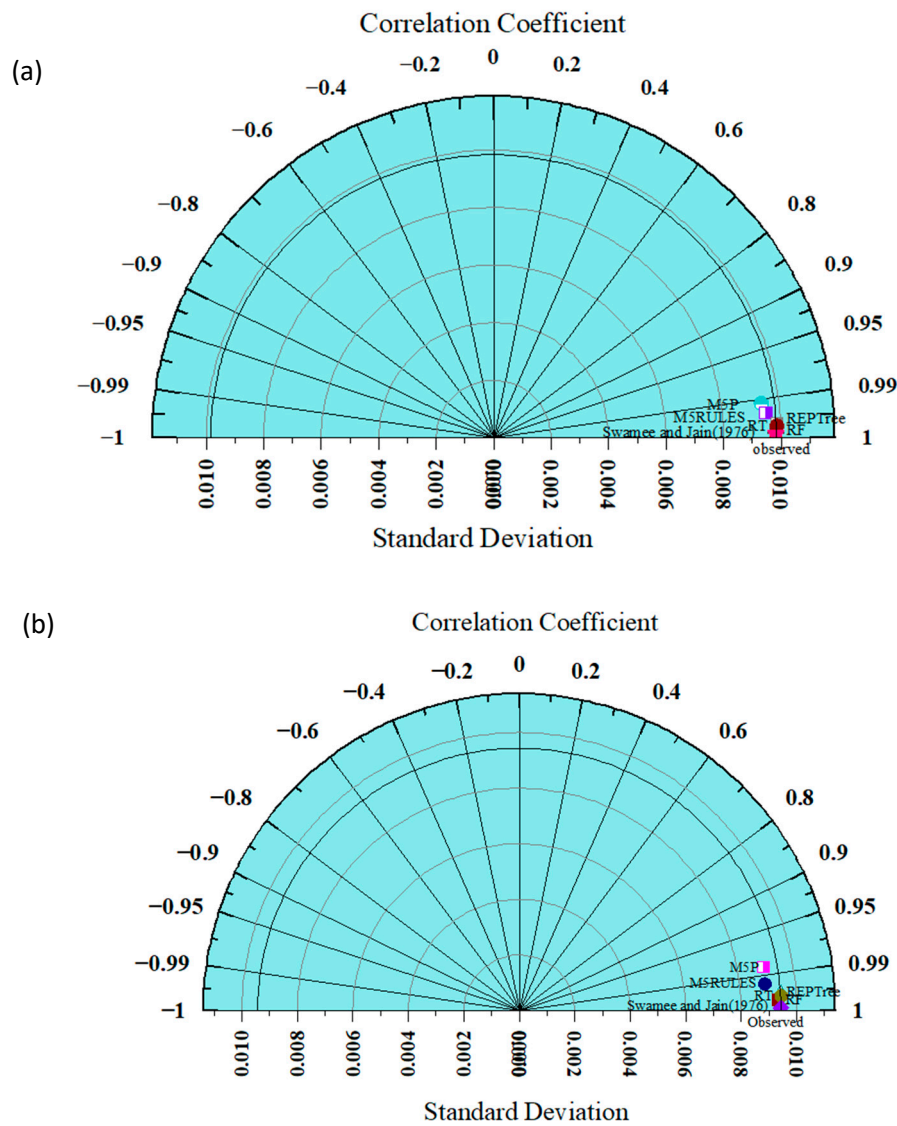
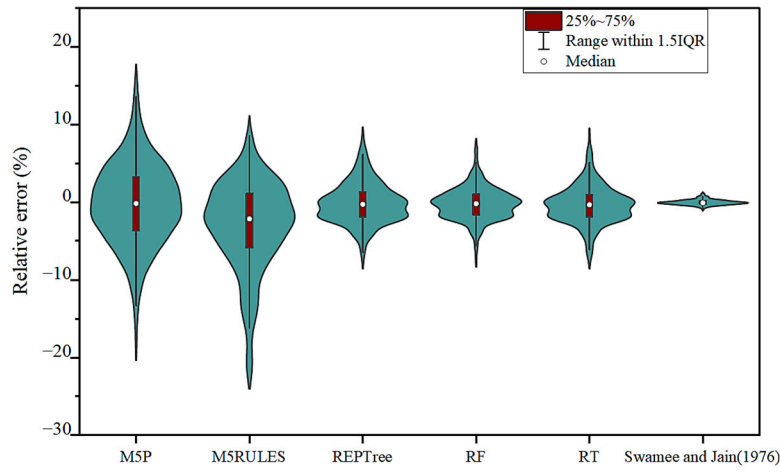
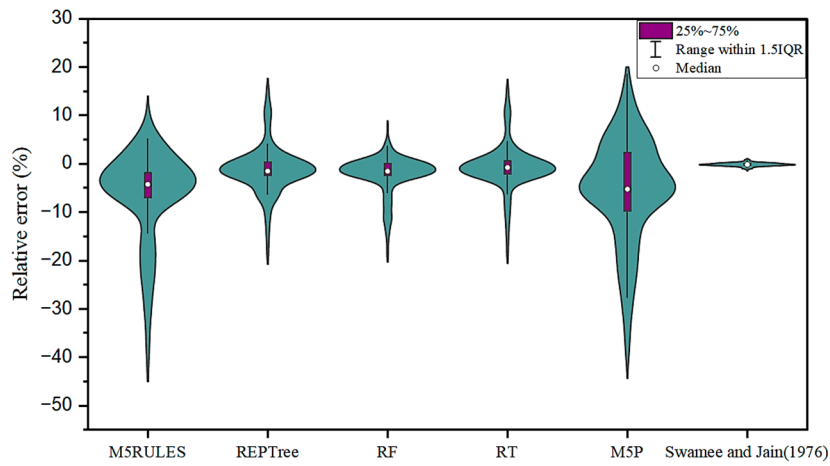


Figure. S5 Taylor diagram of approach-1 (a) Training; (b) Testing.



(a)



(b)

Figure. S6 Violin error box diagram of approach-1 (a) Training; (b) Testing.

C) Tree Distribution and Expressions

The distribution of REPTree and M5P distribution in the form of tree manner taking two different parameters are shown in Table S1 for approach-2. Likewise, the expressions to predict friction factor by using M5p and M5Rules of approach-2 is shown in Table S3 and S4. A number of linear expressions are given by M5p and M5Rules to predict friction factor.

Table S1. REPTree distribution system of approach-2.

Distribution of REPTree	
Relative roughness < 0.01	
	Relative roughness < 0
	Relative roughness < 0
	Reynolds number < 725848.75
	Reynolds number < 162744.16
	Reynolds number < 73437.92 : 0.02 (4/0) [2/0]

					Reynolds number >= 73437.92
					Reynolds number < 106294.26 : 0.02 (2/0) [2/0]
					Reynolds number >= 106294.26 : 0.02 (6/0) [2/0]
					Reynolds number >= 162744.16
					Reynolds number < 352410.66 : 0.02 (9/0) [10/0]
					Reynolds number >= 352410.66 : 0.01 (17/0) [11/0]
					Reynolds number >= 725848.75
					Relative roughness < 0
					Reynolds number < 3052891.44
					Reynolds number < 1305447.66 : 0.01 (8/0) [5/0]
					Reynolds number >= 1305447.66 : 0.01 (15/0) [10/0]
					Reynolds number >= 3052891.44 : 0.01 (75/0) [30/0]
					Relative roughness >= 0
					Relative roughness < 0
					Reynolds number < 4248853.13 : 0.01 (12/0) [4/0]
					Reynolds number >= 4248853.13 : 0.01 (25/0) [11/0]
					Relative roughness >= 0 : 0.01 (44/0) [19/0]
					Relative roughness >= 0
					Reynolds number < 66028.3
					Reynolds number < 20559.74
					Reynolds number < 11878.86
					Reynolds number < 8915.7 : 0.03 (3/0) [0/0]
					Reynolds number >= 8915.7 : 0.03 (2/0) [2/0]
					Reynolds number >= 11878.86
					Reynolds number < 13822.72 : 0.03 (2/0) [0/0]
					Reynolds number >= 13822.72 : 0.03 (5/0) [7/0]
					Reynolds number >= 20559.74
					Reynolds number < 35269.95
					Reynolds number < 29041.19
					Relative roughness < 0 : 0.03 (3/0) [2/0]
					Relative roughness >= 0 : 0.03 (9/0) [3/0]
					Reynolds number >= 29041.19 : 0.02 (6/0) [3/0]
					Reynolds number >= 35269.95
					Relative roughness < 0
					Reynolds number < 49004.56 : 0.02 (5/0) [4/0]
					Reynolds number >= 49004.56 : 0.02 (6/0) [2/0]
					Relative roughness >= 0
					Reynolds number < 49600 : 0.02 (6/0) [2/0]
					Reynolds number >= 49600 : 0.02 (4/0) [2/0]
					Reynolds number >= 66028.3
					Relative roughness < 0
					Reynolds number < 374102.5
					Reynolds number < 119170.99 : 0.02 (14/0) [0/0]
					Reynolds number >= 119170.99
					Reynolds number < 172624.1 : 0.02 (5/0) [4/0]
					Reynolds number >= 172624.1 : 0.02 (11/0) [6/0]
					Reynolds number >= 374102.5
					Relative roughness < 0 : 0.01 (43/0) [21/0]
					Relative roughness >= 0 : 0.02 (38/0) [26/0]
					Relative roughness >= 0
					Reynolds number < 249284.61

						Reynolds number < 123657.3 : 0.02 (11/0) [2/0]
						Reynolds number >= 123657.3 : 0.02 (8/0) [5/0]
						Reynolds number >= 249284.61 : 0.02 (73/0) [50/0]
						Relative roughness >= 0
						Reynolds number < 14974.27
						Reynolds number < 6620.78
						Reynolds number < 4219.41
						Relative roughness < 0.01
						Reynolds number < 3150.68 : 0.05 (3/0) [2/0]
						Reynolds number >= 3150.68
						Reynolds number < 3805.11 : 0.04 (5/0) [2/0]
						Reynolds number >= 3805.11 : 0.04 (2/0) [0/0]
						Relative roughness >= 0.01 : 0.05 (7/0) [0/0]
						Reynolds number >= 4219.41
						Relative roughness < 0 : 0.04 (12/0) [3/0]
						Relative roughness >= 0 : 0.04 (13/0) [0/0]
						Reynolds number >= 6620.78
						Relative roughness < 0
						Reynolds number < 9248.37
						Relative roughness < 0 : 0.03 (5/0) [0/0]
						Relative roughness >= 0 : 0.04 (3/0) [4/0]
						Reynolds number >= 9248.37
						Reynolds number < 12612.74
						Relative roughness < 0 : 0.03 (2/0) [2/0]
						Relative roughness >= 0
						Relative roughness < 0 : 0.03 (6/0) [1/0]
						Relative roughness >= 0 : 0.03 (3/0) [0/0]
						Reynolds number >= 12612.74 : 0.03 (3/0) [4/0]
						Relative roughness >= 0
						Reynolds number < 9542.3
						Relative roughness < 0.01 : 0.04 (2/0) [3/0]
						Relative roughness >= 0.01 : 0.04 (6/0) [0/0]
						Reynolds number >= 9542.3
						Relative roughness < 0.01 : 0.04 (3/0) [0/0]
						Relative roughness >= 0.01
						Relative roughness < 0.01 : 0.04 (4/0) [0/0]
						Relative roughness >= 0.01 : 0.04 (2/0) [1/0]
						Reynolds number >= 14974.27
						Relative roughness < 0
						Reynolds number < 39861.81
						Reynolds number < 23623.94
						Relative roughness < 0 : 0.03 (6/0) [4/0]
						Relative roughness >= 0
						Relative roughness < 0 : 0.03 (3/0) [1/0]
						Relative roughness >= 0 : 0.03 (2/0) [2/0]
						Reynolds number >= 23623.94
						Relative roughness < 0
						Relative roughness < 0 : 0.03 (8/0) [2/0]
						Relative roughness >= 0 : 0.03 (3/0) [2/0]
						Relative roughness >= 0 : 0.03 (3/0) [3/0]
						Reynolds number >= 39861.81

					Relative roughness < 0
					Reynolds number < 101748.5
					Reynolds number < 47909.23 : 0.03 (3/0) [0/0]
					Reynolds number >= 47909.23 : 0.02 (9/0) [5/0]
					Reynolds number >= 101748.5
					Relative roughness < 0
					Reynolds number < 326868.47 : 0.02 (9/0) [2/0]
					Reynolds number >= 326868.47 : 0.02 (37/0) [15/0]
					Relative roughness >= 0 : 0.02 (34/0) [25/0]
					Relative roughness >= 0
					Relative roughness < 0
					Reynolds number < 187974.55 : 0.03 (8/0) [4/0]
					Reynolds number >= 187974.55 : 0.02 (41/0) [14/0]
					Relative roughness >= 0 : 0.03 (44/0) [17/0]
					Relative roughness >= 0
					Relative roughness < 0.01
					Reynolds number < 65848.6
					Reynolds number < 27583.27 : 0.03 (3/0) [3/0]
					Reynolds number >= 27583.27 : 0.03 (3/0) [3/0]
					Reynolds number >= 65848.6 : 0.03 (34/0) [24/0]
					Relative roughness >= 0.01
					Relative roughness < 0.01
					Reynolds number < 36700.24 : 0.04 (4/0) [2/0]
					Reynolds number >= 36700.24 : 0.03 (38/0) [17/0]
					Relative roughness >= 0.01
					Reynolds number < 37163.68 : 0.04 (6/0) [0/0]
					Reynolds number >= 37163.68 : 0.04 (39/0) [18/0]
					Relative roughness >= 0.01
					Relative roughness < 0.03
					Relative roughness < 0.02
					Reynolds number < 7992.64
					Reynolds number < 5203.86
					Reynolds number < 3332.97 : 0.05 (2/0) [1/0]
					Reynolds number >= 3332.97 : 0.05 (5/0) [7/0]
					Reynolds number >= 5203.86
					Relative roughness < 0.01
					Reynolds number < 6318.47 : 0.05 (3/0) [0/0]
					Reynolds number >= 6318.47 : 0.05 (2/0) [2/0]
					Relative roughness >= 0.01 : 0.05 (2/0) [3/0]
					Reynolds number >= 7992.64
					Relative roughness < 0.01
					Relative roughness < 0.01
					Reynolds number < 18342.68
					Reynolds number < 11153.65 : 0.04 (2/0) [1/0]
					Reynolds number >= 11153.65 : 0.04 (3/0) [0/0]
					Reynolds number >= 18342.68 : 0.04 (36/0) [28/0]
					Relative roughness >= 0.01
					Reynolds number < 33373.13
					Reynolds number < 16251.13 : 0.05 (3/0) [3/0]
					Reynolds number >= 16251.13 : 0.04 (2/0) [3/0]
					Reynolds number >= 33373.13 : 0.04 (42/0) [35/0]

				Relative roughness >= 0.01
				Reynolds number < 71961.12
				Reynolds number < 21950.8 : 0.05 (3/0) [4/0]
				Reynolds number >= 21950.8 : 0.04 (8/0) [2/0]
				Reynolds number >= 71961.12 : 0.04 (44/0) [23/0]
				Relative roughness >= 0.02
				Relative roughness < 0.02
				Reynolds number < 10133.63
				Reynolds number < 5599.98
				Reynolds number < 3814.69 : 0.06 (3/0) [1/0]
				Reynolds number >= 3814.69 : 0.06 (4/0) [2/0]
				Reynolds number >= 5599.98
				Relative roughness < 0.02 : 0.05 (3/0) [3/0]
				Relative roughness >= 0.02
				Reynolds number < 7617.7 : 0.05 (2/0) [0/0]
				Reynolds number >= 7617.7 : 0.05 (2/0) [1/0]
				Reynolds number >= 10133.63
				Relative roughness < 0.02
				Reynolds number < 27337.03
				Reynolds number < 14569.34 : 0.05 (3/0) [1/0]
				Reynolds number >= 14569.34 : 0.05 (3/0) [2/0]
				Reynolds number >= 27337.03 : 0.05 (54/0) [18/0]
				Relative roughness >= 0.02
				Reynolds number < 28094.96 : 0.05 (5/0) [4/0]
				Reynolds number >= 28094.96 : 0.05 (52/0) [22/0]
				Relative roughness >= 0.02
				Reynolds number < 13960.46
				Reynolds number < 7071.75 : 0.06 (3/0) [1/0]
				Reynolds number >= 7071.75 : 0.06 (4/0) [1/0]
				Reynolds number >= 13960.46 : 0.05 (48/0) [29/0]
				Relative roughness >= 0.03
				Relative roughness < 0.04
				Relative roughness < 0.03
				Reynolds number < 12199.29
				Reynolds number < 4820.65 : 0.06 (3/0) [2/0]
				Reynolds number >= 4820.65 : 0.06 (7/0) [0/0]
				Reynolds number >= 12199.29 : 0.06 (60/0) [24/0]
				Relative roughness >= 0.03
				Reynolds number < 13172.75
				Reynolds number < 5291.54 : 0.07 (3/0) [2/0]
				Reynolds number >= 5291.54
				Reynolds number < 7592.67 : 0.07 (2/0) [1/0]
				Reynolds number >= 7592.67 : 0.06 (4/0) [1/0]
				Reynolds number >= 13172.75 : 0.06 (53/0) [32/0]
				Relative roughness >= 0.04
				Relative roughness < 0.04
				Reynolds number < 12520.47
				Reynolds number < 5064.06 : 0.07 (3/0) [1/0]
				Reynolds number >= 5064.06 : 0.07 (8/0) [0/0]
				Reynolds number >= 12520.47 : 0.07 (54/0) [23/0]
				Relative roughness >= 0.04

				Relative roughness < 0.05
				Reynolds number < 12038.11
				Reynolds number < 7065.54 : 0.07 (5/0) [1/0]
				Reynolds number >= 7065.54 : 0.07 (3/0) [1/0]
				Reynolds number >= 12038.11 : 0.07 (46/0) [28/0]
				Relative roughness >= 0.05
				Reynolds number < 11356.95
				Reynolds number < 4819.34 : 0.08 (3/0) [0/0]
				Reynolds number >= 4819.34
				Reynolds number < 6277.54 : 0.08 (2/0) [0/0]
				Reynolds number >= 6277.54 : 0.07 (3/0) [3/0]
				Reynolds number >= 11356.95 : 0.07 (63/0) [30/0]

Table S2. M5 pruned model tree of approach-2

Distribution of M5 pruned model tree
M5 pruned model tree: (using smoothed linear models)
Relative roughness <= 0.007 :
Relative roughness <= 0.001 :
Relative roughness <= 0 :
Reynolds number <= 790424.289 :
Reynolds number <= 255009.008 : LM1 (28/2.995%)
Reynolds number > 255009.008 : LM2 (41/4.368%)
Reynolds number > 790424.289 :
Relative roughness <= 0 :
Reynolds number <= 4088746.049 : LM3 (45/3.876%)
Reynolds number > 4088746.049 : LM4 (96/2.815%)
Relative roughness > 0 : LM5 (113/4.193%)
Relative roughness > 0 :
Reynolds number <= 101067.362 :
Reynolds number <= 30658.063 :
Reynolds number <= 17201.565 : LM6 (15/2.34%)
Reynolds number > 17201.565 : LM7 (25/2.873%)
Reynolds number > 30658.063 : LM8 (57/1.714%)
Reynolds number > 101067.362 :
Relative roughness <= 0.001 :
Reynolds number <= 435586.165 : LM9 (34/3.607%)
Reynolds number > 435586.165 : LM10 (125/2.214%)
Relative roughness > 0.001 : LM11 (139/4.751%)
Relative roughness > 0.001 :
Reynolds number <= 20778.144 :
Reynolds number <= 7207.125 :
Reynolds number <= 3828.229 : LM12 (15/1.913%)
Reynolds number > 3828.229 : LM13 (31/2.513%)
Reynolds number > 7207.125 : LM14 (62/2.569%)
Reynolds number > 20778.144 :
Relative roughness <= 0.003 :

- | | | | Reynolds number ≤ 110125.597 : LM15 (46/2.884%)
- | | | | Reynolds number > 110125.597 : LM16 (179/2.352%)
- | | | Relative roughness > 0.003 :
- | | | | Relative roughness ≤ 0.005 :
- | | | | | Relative roughness ≤ 0.004 :
- | | | | | | Reynolds number ≤ 454730.133 :
- | | | | | | | Reynolds number ≤ 56788.997 : LM17 (10/4.434%)
- | | | | | | | Reynolds number > 56788.997 : LM18 (16/2.685%)
- | | | | | | | Reynolds number > 454730.133 : LM19 (43/0.199%)
- | | | | | Relative roughness > 0.004 :
- | | | | | | Reynolds number ≤ 332250.608 :
- | | | | | | | Reynolds number ≤ 62281.379 : LM20 (8/3.988%)
- | | | | | | | Reynolds number > 62281.379 : LM21 (12/1.928%)
- | | | | | | | Reynolds number > 332250.608 : LM22 (46/0.481%)
- | | | | Relative roughness > 0.005 : LM23 (58/3.651%)
- Relative roughness > 0.007 :
- | Relative roughness ≤ 0.028 :
- | | Relative roughness ≤ 0.016 :
- | | | Reynolds number ≤ 24672.88 :
- | | | | Reynolds number ≤ 5456.871 : LM24 (22/1.509%)
- | | | | Reynolds number > 5456.871 :
- | | | | | Relative roughness ≤ 0.011 :
- | | | | | | Reynolds number ≤ 10657.838 : LM25 (12/1.765%)
- | | | | | | Reynolds number > 10657.838 : LM26 (12/0.928%)
- | | | | | Relative roughness > 0.011 : LM27 (24/2.597%)
- | | | | Reynolds number > 24672.88 : LM28 (276/2.47%)
- | | Relative roughness > 0.016 :
- | | | Relative roughness ≤ 0.022 :
- | | | | Reynolds number ≤ 28085.606 :
- | | | | | Reynolds number ≤ 6766.529 : LM29 (14/5.638%)
- | | | | | Reynolds number > 6766.529 : LM30 (25/1.842%)
- | | | | | Reynolds number > 28085.606 : LM31 (146/1.448%)
- | | | | Relative roughness > 0.022 :
- | | | | | Reynolds number ≤ 425021.348 :
- | | | | | | Reynolds number ≤ 13960.456 : LM32 (9/2.554%)
- | | | | | | Reynolds number > 13960.456 : LM33 (27/2.859%)
- | | | | | Reynolds number > 425021.348 : LM34 (50/0%)
- | | Relative roughness > 0.028 :
- | | | Relative roughness ≤ 0.042 :
- | | | | Relative roughness ≤ 0.033 :
- | | | | | Reynolds number ≤ 80954.297 :
- | | | | | | Reynolds number ≤ 13252.94 : LM35 (13/3.543%)
- | | | | | | Reynolds number > 13252.94 : LM36 (14/2.555%)
- | | | | | | Reynolds number > 80954.297 : LM37 (69/0.632%)
- | | | | | Relative roughness > 0.033 :
- | | | | | | Relative roughness ≤ 0.038 :
- | | | | | | | Reynolds number ≤ 169691.179 :
- | | | | | | | Reynolds number ≤ 15112.582 : LM38 (14/4.104%)
- | | | | | | | Reynolds number > 15112.582 : LM39 (18/1.96%)
- | | | | | | | Reynolds number > 169691.179 : LM40 (66/0.41%)
- | | | | | Relative roughness > 0.038 :

| | | | | Reynolds number ≤ 46864.458 :
 | | | | | Reynolds number ≤ 7338.182 : LM41 (7/1.623%)
 | | | | | Reynolds number > 7338.182 : LM42 (15/4.351%)
 | | | | | Reynolds number > 46864.458 : LM43 (67/0.674%)
 | | Relative roughness > 0.042 :
 | | | Relative roughness ≤ 0.048 :
 | | | | Reynolds number ≤ 39246.24 :
 | | | | Reynolds number ≤ 10687.392 : LM44 (9/1.129%)
 | | | | Reynolds number > 10687.392 : LM45 (10/1.379%)
 | | | | Reynolds number > 39246.24 : LM46 (65/1.07%)
 | | | Relative roughness > 0.048 :
 | | | | Reynolds number ≤ 44980.031 :
 | | | | Reynolds number ≤ 9489.592 : LM47 (9/1.978%)
 | | | | Reynolds number > 9489.592 : LM48 (14/2.746%)
 | | | | Reynolds number > 44980.031 : LM49 (81/0.889%)

Table S3. M5 pruned model tree equations of approach-2.

Expression for approach-2		
Rules Expression	M num: 1 friction factor = $8.3034 * \text{Relative roughness} - 0 * \text{Reynolds number} + 0.0186$	LM num: 11 friction factor = $1.464 * \text{Relative roughness} - 0 * \text{Reynolds number} + 0.0179$
Rules Expression	LM num: 2 friction factor = $8.3034 * \text{Relative roughness} - 0 * \text{Reynolds number} + 0.0145$	LM num: 12 friction factor = $1.1322 * \text{Relative roughness} - 0 * \text{Reynolds number} + 0.0466$
Rules Expression	LM num: 3 friction factor = $5.9166 * \text{Relative roughness} - 0 * \text{Reynolds number} + 0.0109$	LM num: 13 friction factor = $1.2658 * \text{Relative roughness} - 0 * \text{Reynolds number} + 0.0424$
Rules Expression	LM num: 4 friction factor = $5.9166 * \text{Relative roughness} - 0 * \text{Reynolds number} + 0.0097$	LM num: 14 friction factor = $1.5708 * \text{Relative roughness} - 0 * \text{Reynolds number} + 0.0348$

Rules Expression	LM num: 5 friction factor = 6.5466 * Relative roughness - 0 * Reynolds number + 0.0118	LM num: 15 friction factor = 2.6046 * Relative roughness - 0 * Reynolds number + 0.023
Rules Expression	LM num: 6 friction factor = 2.0093 * Relative roughness - 0 * Reynolds number + 0.0307	LM num: 16 friction factor = 3.4905 * Relative roughness - 0 * Reynolds number + 0.0171
Rules Expression	LM num: 7 friction factor = 2.0093 * Relative roughness - 0 * Reynolds number + 0.0288	LM num: 17 friction factor = 0.7937 * Relative roughness - 0 * Reynolds number + 0.026
Rules Expression	LM num: 8 friction factor = 4.0169 * Relative roughness - 0 * Reynolds number + 0.0234	LM num: 18 friction factor = 0.7937 * Relative roughness - 0 * Reynolds number + 0.0254
Rules Expression	LM num: 9 friction factor = 3.6508 * Relative roughness - 0 * Reynolds number + 0.0173	LM num: 19 friction factor = 0.7937 * Relative roughness - 0 * Reynolds number + 0.0245
Rules Expression	LM num: 10 friction factor = 9.4025 * Relative roughness - 0 * Reynolds number + 0.0129	LM num: 20 friction factor = 0.8044 * Relative roughness - 0 * Reynolds number + 0.027
Rules Expression	LM num: 21 friction factor = 0.8044 * Relative roughness - 0 * Reynolds number + 0.0267	LM num: 31 friction factor = 0.9727 * Relative roughness - 0 * Reynolds number + 0.0297

Rules Expression	LM num: 22 friction factor = 0.8044 * Relative roughness - 0 * Reynolds number + 0.0259	LM num: 32 friction factor = 0.2 * Relative roughness - 0 * Reynolds number + 0.0515
Rules Expression	LM num: 23 friction factor = 0.671 * Relative roughness - 0 * Reynolds number + 0.029	LM num: 33 friction factor = 0.2 * Relative roughness - 0 * Reynolds number + 0.0494
Rules Expression	LM num: 24 friction factor = 0.8967 * Relative roughness - 0 * Reynolds number + 0.0422	LM num: 34 friction factor = 0.2 * Relative roughness - 0 * Reynolds number + 0.0486
Rules Expression	LM num: 25 friction factor = 1.0911 * Relative roughness - 0 * Reynolds number + 0.0359	LM num: 35 friction factor = 0.1651 * Relative roughness - 0 * Reynolds number + 0.0564
Rules Expression	LM num: 26 friction factor = 1.0998 * Relative roughness - 0 * Reynolds number + 0.035	LM num: 36 friction factor = 0.1651 * Relative roughness - 0 * Reynolds number + 0.0544
Rules Expression	LM num: 27 friction factor = 0.969 * Relative roughness - 0 * Reynolds number + 0.0361	LM num: 37 friction factor = 0.1651 * Relative roughness - 0 * Reynolds number + 0.0527
Rules Expression	LM num: 28 friction factor = 1.1573 * Relative roughness - 0 * Reynolds number + 0.0268	LM num: 38 friction factor = 0.2048 * Relative roughness - 0 * Reynolds number + 0.0575

Rules Expression	LM num: 29 friction factor = 0.6688 * Relative roughness - 0 * Reynolds number + 0.0426	LM num: 39 friction factor = 0.2048 * Relative roughness - 0 * Reynolds number + 0.0556
Rules Expression	LM num: 30 friction factor = 0.9175 * Relative roughness - 0 * Reynolds number + 0.035	LM num: 40 friction factor = 0.2048 * Relative roughness - 0 * Reynolds number + 0.0544
Rules Expression	LM num: 46 friction factor = 0.1766 * Relative roughness - 0 * Reynolds number + 0.0609	LM num: 41 friction factor = 0.2118 * Relative roughness - 0 * Reynolds number + 0.0598
Rules Expression	LM num: 47 friction factor = 0.1621 * Relative roughness - 0 * Reynolds number + 0.0668	LM num: 42 friction factor = 0.2118 * Relative roughness - 0 * Reynolds number + 0.0583
Rules Expression	LM num: 48 friction factor = 0.1621 * Relative roughness - 0 * Reynolds number + 0.0654	LM num: 43 friction factor = 0.2118 * Relative roughness - 0 * Reynolds number + 0.0568
Rules Expression	LM num: 49 friction factor = 0.1621 * Relative roughness - 0 * Reynolds number + 0.064	LM num: 44 friction factor = 0.1766 * Relative roughness - 0 * Reynolds number + 0.0634
Rules Expression		LM num: 45 friction factor = 0.1766 * Relative roughness - 0 * Reynolds number + 0.0624

Table S4. M5Rules model rules of approach-2.

Expression for approach-2			
Rules	Expression	Rules	Expression
Rule: 1		Rule: 6	
IF		IF	
	Relative roughness > 0.007		Relative roughness <= 0.009
	Relative roughness <= 0.028		Relative roughness <= 0.001
	Relative roughness <= 0.016		Reynolds number > 145612.536
	Reynolds number > 24672.88		Relative roughness <= 0
THEN			Reynolds number > 360716.151
		THEN	
friction factor =		friction factor =	
	1.1573 * Relative roughness		1.7865 * Relative roughness
	- 0 * Reynolds number		- 0 * Reynolds number
	+ 0.0268 [276/2.47%]		+ 0.0143 [119/4.565%]
Rule: 2		Rule: 7	
IF		IF	
	Relative roughness <= 0.007		Relative roughness > 0.009
	Relative roughness > 0.001		Relative roughness > 0.038
	Reynolds number > 20778.144		Relative roughness > 0.048
	Relative roughness <= 0.003		Reynolds number > 44980.031
	Reynolds number > 110125.597		
THEN		THEN	
friction factor =		friction factor =	
	3.4905 * Relative roughness		0.1347 * Relative roughness
	- 0 * Reynolds number		- 0 * Reynolds number
	+ 0.0171 [179/2.217%]		+ 0.0653 [81/0.909%]
Rule: 3		Rule: 8	
IF		IF	
	Relative roughness > 0.007		Relative roughness > 0.009
	Relative roughness <= 0.033		Relative roughness <= 0.033
	Relative roughness <= 0.022		Relative roughness > 0.028
	Reynolds number > 24578.467		Reynolds number > 80954.297
THEN		THEN	
friction factor =		friction factor =	
	0.922 * Relative roughness		0.1373 * Relative roughness
	- 0 * Reynolds number		- 0 * Reynolds number
	+ 0.0307 [148/1.539%]		+ 0.0535 [69/0.673%]
Rule: 4		Rule: 9	
IF		IF	
	Relative roughness <= 0.007		Relative roughness > 0.009
	Relative roughness <= 0.001		Relative roughness > 0.028
	Relative roughness > 0		Relative roughness <= 0.042
	Reynolds number > 101067.362		Relative roughness > 0.038
	Relative roughness > 0.001		Reynolds number > 46864.458

<p>THEN</p> <p>friction factor = $1.4629 * \text{Relative roughness}$ $- 0 * \text{Reynolds number}$ $+ 0.0179 [139/4.251\%]$</p> <p>Rule: 5</p> <p>IF</p> <p>Relative roughness ≤ 0.007 Relative roughness ≤ 0.001 Reynolds number > 277567.489 Relative roughness ≤ 0 Reynolds number > 1453170.59</p> <p>THEN</p> <p>friction factor = $31.4699 * \text{Relative roughness}$ $- 0 * \text{Reynolds number}$ $+ 0.0096 [231/3.068\%]$</p> <p>Rule: 11</p> <p>IF</p> <p>Relative roughness ≤ 0.007 Relative roughness ≤ 0.001 Reynolds number ≤ 110049.882 Reynolds number > 35269.947</p> <p>THEN</p> <p>friction factor = $4.1374 * \text{Relative roughness}$ $- 0 * \text{Reynolds number}$ $+ 0.0235 [63/1.428\%]$</p> <p>Rule: 12</p> <p>IF</p> <p>Relative roughness ≤ 0.007 Relative roughness > 0.001 Reynolds number > 12612.735 Relative roughness ≤ 0.005 Reynolds number > 25249.58 Relative roughness > 0.004</p> <p>THEN</p> <p>friction factor = $0.5236 * \text{Relative roughness}$ $- 0 * \text{Reynolds number}$ $+ 0.0274 [64/4.491\%]$</p> <p>Rule: 13</p> <p>IF</p>	<p>THEN</p> <p>friction factor = $0.1774 * \text{Relative roughness}$ $- 0 * \text{Reynolds number}$ $+ 0.0582 [67/0.709\%]$</p> <p>Rule: 10</p> <p>IF</p> <p>Relative roughness > 0.007 Relative roughness > 0.028 Relative roughness ≤ 0.038 Reynolds number > 169691.179</p> <p>THEN</p> <p>friction factor = $0.1966 * \text{Relative roughness}$ $- 0 * \text{Reynolds number}$ $+ 0.0547 [66/0.44\%]$</p> <p>Rule: 16</p> <p>IF</p> <p>Relative roughness ≤ 0.007 Reynolds number ≤ 13244.206 Reynolds number > 7207.125</p> <p>THEN</p> <p>friction factor = $1.4437 * \text{Relative roughness}$ $- 0 * \text{Reynolds number}$ $+ 0.0377 [45/1.617\%]$</p> <p>Rule: 17</p> <p>IF</p> <p>Relative roughness ≤ 0.007 Relative roughness ≤ 0.001 Reynolds number > 72609.962 Reynolds number > 1604846.936</p> <p>THEN</p> <p>friction factor = $0.7192 * \text{Relative roughness}$ $- 0 * \text{Reynolds number}$ $+ 0.0181 [43/0.285\%]$</p> <p>Rule: 18</p> <p>IF</p> <p>Relative roughness ≤ 0.007 Reynolds number > 16584.458 Relative roughness > 0.004 Reynolds number > 228578.181</p>
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<p>Relative roughness > 0.007 Relative roughness > 0.028 Relative roughness > 0.038 Reynolds number > 48106.519</p> <p>THEN</p> <p>friction factor = 0.2467 * Relative roughness - 0 * Reynolds number + 0.058 [63/1.035%]</p> <p>Rule: 14</p> <p>IF</p> <p>Relative roughness > 0.007 Relative roughness <= 0.028 Reynolds number > 425021.348</p> <p>THEN</p> <p>friction factor = 0.2197 * Relative roughness - 0 * Reynolds number + 0.0483 [50/0%]</p> <p>Rule: 15</p> <p>IF</p> <p>Relative roughness <= 0.007 Relative roughness <= 0.001 Reynolds number > 118410.343 Reynolds number <= 2722775.132 Reynolds number > 209787.328</p> <p>THEN</p> <p>friction factor = 1.2377 * Relative roughness - 0 * Reynolds number + 0.0174 [47/4.849%]</p> <p>Rule: 21</p> <p>IF</p> <p>Relative roughness <= 0.014 Reynolds number <= 13268.003 Reynolds number > 3875.168 Relative roughness > 0.005</p> <p>THEN</p> <p>friction factor = 1.0049 * Relative roughness - 0 * Reynolds number + 0.0402 [37/3.73%]</p> <p>Rule: 22</p> <p>IF</p>	<p>THEN</p> <p>friction factor = 0.4669 * Relative roughness - 0 * Reynolds number + 0.0302 [43/0.493%]</p> <p>Rule: 19</p> <p>IF</p> <p>Relative roughness <= 0.007 Reynolds number > 15232.855 Reynolds number <= 394112.24 Relative roughness > 0.001 Relative roughness <= 0.003 Reynolds number > 26988.916</p> <p>THEN</p> <p>friction factor = 2.1073 * Relative roughness - 0 * Reynolds number + 0.0252 [44/2.504%]</p> <p>Rule: 20</p> <p>IF</p> <p>Relative roughness <= 0.009 Reynolds number > 13010.198 Reynolds number > 454730.133</p> <p>THEN</p> <p>friction factor = 0.6883 * Relative roughness - 0 * Reynolds number + 0.0256 [43/0.232%]</p> <p>Rule: 26</p> <p>IF</p> <p>Relative roughness > 0.011 Relative roughness <= 0.038 Relative roughness <= 0.022 Reynolds number > 6641.383</p> <p>THEN</p> <p>friction factor = 0.8441 * Relative roughness - 0 * Reynolds number + 0.0383 [38/2.494%]</p> <p>Rule: 27</p> <p>IF</p> <p>Relative roughness <= 0.011 Reynolds number <= 10241.628 Reynolds number <= 3875.168</p>
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<p>Relative roughness > 0.011 Relative roughness > 0.028 Relative roughness > 0.038 Relative roughness <= 0.048 Reynolds number > 6778.211</p> <p>THEN</p> <p>friction factor = 0.6954 * Relative roughness - 0 * Reynolds number + 0.0408 [31/2.409%]</p> <p>Rule: 23 IF</p> <p>Relative roughness <= 0.009 Reynolds number > 10240.691 Relative roughness <= 0.001 Reynolds number > 72609.962</p> <p>THEN</p> <p>friction factor = 4.6265 * Relative roughness - 0 * Reynolds number + 0.0225 [29/3.271%]</p> <p>Rule: 24 IF</p> <p>Relative roughness <= 0.011 Reynolds number > 10240.691 Relative roughness <= 0.004 Relative roughness <= 0.001</p> <p>THEN</p> <p>friction factor = 0.8148 * Relative roughness - 0 * Reynolds number + 0.0302 [24/2.77%]</p> <p>Rule: 25 IF</p> <p>Relative roughness > 0.011 Relative roughness <= 0.028 Reynolds number > 23325.425</p> <p>THEN</p> <p>friction factor = 0.3576 * Relative roughness - 0 * Reynolds number + 0.0463 [24/2.852%]</p> <p>Rule: 31 IF</p>	<p>THEN</p> <p>friction factor = 0.9123 * Relative roughness - 0 * Reynolds number + 0.0479 [21/2.671%]</p> <p>Rule: 28 IF</p> <p>Relative roughness <= 0.011 Reynolds number > 10241.628 Relative roughness <= 0.004 Reynolds number <= 73638.781 Relative roughness > 0.001 Reynolds number > 18283.072</p> <p>THEN</p> <p>friction factor = 1.3146 * Relative roughness - 0 * Reynolds number + 0.0291 [20/1.935%]</p> <p>Rule: 29 IF</p> <p>Relative roughness <= 0.011 Reynolds number > 10241.628 Relative roughness <= 0.004 Reynolds number > 18844.945</p> <p>THEN</p> <p>friction factor = 0.7375 * Relative roughness - 0 * Reynolds number + 0.0291 [22/2.997%]</p> <p>Rule: 30 IF</p> <p>Relative roughness <= 0.011 Reynolds number > 10241.628</p> <p>THEN</p> <p>friction factor = 1.1254 * Relative roughness - 0 * Reynolds number + 0.0309 [51/5.242%]</p> <p>Rule: 36 IF</p> <p>Relative roughness > 0.038 Relative roughness > 0.045</p> <p>THEN</p>
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<p>Relative roughness ≤ 0.016</p> <p>THEN</p> <p>friction factor =</p> $0.8455 * \text{Relative roughness} - 0 * \text{Reynolds number} + 0.0452 [35/3.841\%]$ <p>Rule: 32</p> <p>IF</p> <p>Relative roughness ≤ 0.038</p> <p>Relative roughness ≤ 0.028</p> <p>Reynolds number ≤ 8902.62</p> <p>THEN</p> <p>friction factor =</p> $0.6984 * \text{Relative roughness} - 0 * \text{Reynolds number} + 0.0444 [20/6.928\%]$ <p>Rule: 33</p> <p>IF</p> <p>Relative roughness ≤ 0.038</p> <p>Reynolds number ≤ 6895.428</p> <p>THEN</p> <p>friction factor =</p> $0.7873 * \text{Relative roughness} - 0 * \text{Reynolds number} + 0.0408 [15/5.043\%]$ <p>Rule: 34</p> <p>IF</p> <p>Relative roughness ≤ 0.038</p> <p>Relative roughness > 0.033</p> <p>Reynolds number > 24926.002</p> <p>THEN</p> <p>friction factor =</p> $0.4 * \text{Relative roughness} - 0 * \text{Reynolds number} + 0.0492 [15/3.513\%]$ <p>Rule: 35</p> <p>IF</p> <p>Relative roughness > 0.038</p> <p>Reynolds number ≤ 18280.96</p> <p>Relative roughness > 0.042</p> <p>Reynolds number ≤ 8311.997</p> <p>THEN</p>	<p>friction factor =</p> $0.4002 * \text{Relative roughness} - 0 * \text{Reynolds number} + 0.0547 [15/3.06\%]$ <p>Rule: 37</p> <p>IF</p> <p>Relative roughness ≤ 0.033</p> <p>Relative roughness > 0.028</p> <p>Reynolds number > 21200.117</p> <p>THEN</p> <p>friction factor =</p> $0.5595 * \text{Relative roughness} - 0 * \text{Reynolds number} + 0.0435 [11/2.732\%]$ <p>Rule: 38</p> <p>IF</p> <p>Relative roughness ≤ 0.033</p> <p>THEN</p> <p>friction factor =</p> $0.8152 * \text{Relative roughness} - 0 * \text{Reynolds number} + 0.0382 [15/4.379\%]$ <p>Rule: 39</p> <p>IF</p> <p>Relative roughness ≤ 0.038</p> <p>THEN</p> <p>friction factor =</p> $0.6457 * \text{Relative roughness} - 0 * \text{Reynolds number} + 0.0434 [9/6.067\%]$ <p>Rule: 40</p> <p>friction factor =</p> $-0 * \text{Reynolds number} + 0.0759 [6/19.756\%]$
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friction factor =	
0.6173 * Relative roughness	
- 0 * Reynolds number	
+ 0.0463 [14/3.683%]	