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# Decreasing Respondent Heterogeneity by Likert Scales Adjustment via Multipoles

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**Abstract:** A description of Likert scales can be given using the multipoles technique known in quantum physics and applied to behavioral sciences data. This paper considers decomposition of Likert scales by the multipoles for the application of decreasing the respondents' heterogeneity. Due to cultural and language differences, different respondents habitually use the lower end, the mid-scale, or the upper end of the Likert scales which can lead to distortion and inconsistency in data across respondents. A big impact of different kinds of respondent is well known, for instance, in international studies, and it is called the problem of high and low raters. Application of a multipoles technique to the row-data smoothing via prediction of individual rates by the histogram of the Likert scale tiers produces better results than standard row-centering in data. A numerical example by marketing research data shows that the results are encouraging: while a standard row-centering produces a poor outcome, the dipole-adjustment noticeably improves the obtained segmentation results.

**Keywords:** Likert scale; high and low raters; respondents heterogeneity; quantum multipoles; marketing research

## 1. Introduction

A Likert scale is an ordinal limited rating scale of 5, 7, 10, or another finite number of levels. For example, a 7-point Likert scale in customer satisfaction studies can have the layers from the worst 1 to the best 7 values corresponding to: completely dissatisfied, very dissatisfied, somewhat dissatisfied, neither dissatisfied nor satisfied, somewhat satisfied, very satisfied, completely satisfied. This scale had been originated in works [1,2], and widely studied and used for statistical evaluations in applied psychological and sociological measurements, political and marketing research, and other fields [3–9].

Using Likert scales, researchers can encounter a problem of so-called high and low raters, when some respondents mostly use the upper end, and the others use the lower end of the scale for all questions. Sometimes responses even hit the endpoints of the scale, and a bunch of responses can be located at the middle point for all the questions. Such patterns of responses occur due to differences in cultures and languages, survey methodology, type of market, and specific scales formulation. Scale heterogeneity is an important issue, for example, in international studies which require consideration of differences between countries by mean value and top boxes frequency, variance and higher moments, with possible needed adjustments. Data integration and data fusion techniques suggest various possible solutions for adjusting different sources to a combined dataset [10–17]. The easiest way to reduce scale heterogeneity is row-centering each respondent by subtracting a mean value for all the questions.

The current paper is inspired by the recent works of Camparo [18] and Camparo and Camparo [19] who considered the Likert scale in a quantum-paradigm approach to obtain the so-called multipoles presentation of the scales with their specific features useful for the analysis of problems in behavioral sciences. Applications of quantum techniques in social, psychological, political, economics, marketing research, and other behavior studies could yield new valuable results [20–26]. A multipole approach [19] is employed for the description of different ethnic groups, and it can be also used for other applications in data mapping by dipole-quadrupole or other multipoles planes. For example, the respondents can be divided into four segments by the positive or negative dipole (direction of the trend across the scale boxes) and quadrupole (convex or concave curvature) parameters. These types of multipole values can be related to the respondents of various demographics, particularly, to the countries with different cultures and traditions revealing in evaluations by Likert scales.

Multipole presentation corresponds to a special kind of polynomial regression of the scale echelons by the observed frequencies of each box by multiple questions. This paper describes the multipoles approach and develops it for data smoothing via prediction of individual rates by the histogram of the Likert scale levels. This approach is applied for reducing the raters' heterogeneity due to scale usage in data segmentation. A numerical example from marketing research data is discussed.

## 2. Reducing Respondents' Heterogeneity

A multipole description for Likert scales is presented in the Appendix A. The state multipole parameters are convenient for the analysis of data by a Likert scale because the orthonormal functions in Appendix A Equation (A11) are free of the multicollinearity effects which can distort the individual parameters of a regression Equation (A10) if it is built using non-orthogonal basis functions. It is interesting to note that the multipole moments in Equation (A3) are the normalized versions of the known Gram polynomials which correspond to the orthogonal Chebyshev polynomials of the first type. They can be used both for the analysis and for prediction of the probability distribution by the scale boxes defined in the Equation (A10).

If the aim of the research consists not in the analysis of input of individual predictors but in the prediction of the dependent variable, then it is possible to use non-orthogonal functions because the predicted values and quality of prediction do not depend on the degree of multicollinearity and ill-condition of the covariance matrix between regressors [27]. This means that for prediction by Likert scales it is possible to apply regular non-orthogonal polynomials and to choose their degree by the needed level of the dependent variable fit by the regression model. This kind of modeling can be used for each respondent data by multiple Likert scales in order to reduce or relax a level of heterogeneity, for instance, in data for international studies. It can be used for column centering, and for double centering [28] as well.

Let us briefly describe the case-adjustment for a better homogeneity of the responses in a data. Suppose the respondents estimate various attributes by an n-point Likert scale. Row-centering corresponds to subtracting the mean value of this row from all the values in it, which already makes low and high raters less skewed to their poles. For the dipole adjustment in a row, we find how many times each box of the Likert scale is found in this row, build a pair regression of the box number by the box counts, and take the predicted value of each box as the adjustment made due to the distribution by the boxes, or by the shape of their histogram. Similarly, adjustment by the quadrupole consists in finding a quadratic model of the box number by each box counts and its squared value, and using the predicted value of each box as the adjustment made due to the distribution curvature.

For an explicit example, let us use a real segmentation study in a marketing research project with more than 27,000 respondents from about 30 countries all around the world (about 1000 respondents per country, familiar with the product) estimating consumer attitudes to a product by 57 attributes measured in 7-point Likert scales. The attributes can be named as, for instance: adventure, ambition, authenticity, … , wealth, working hard. The data includes respondents from countries known by various studies as high and low raters, respectively; for example, the mean values by all attributes for Brazil and Germany are 5.76 and 4.86, respectively. To diminish heterogeneity among the respondents, the row-centering was applied and clusters built. The six-cluster solution is used for the data segmentation, and prediction of assignment to each segment is performed in several approaches. Those include the linear discriminant analysis (LDA), multinomial-logit (MNL) regression, and also binary logistical (Logit) regressions of each segment versus all the others, adjusted by the sliding threshold for the optimum sensitivity and specificity in the receiver operating characteristic (ROC) curve [29]. These techniques are competitive, LDA uses linear and MNL non-linear estimation of the parameters of all segments together, while Logit with ROC sliding threshold considers each segment separately so it is easier for estimations, but they all produce similar results.

Table 1 in its left-hand half presents the results of prediction for six segments assignment obtained for the row-centered data. The original cluster and its prediction correspond to the rows and columns in each of the 6 × 6 cross-tables of the counts shown there. The upper cross-table is built in the LDA, the middle one—in the Logit-ROC approach, and the bottom one—in the MNL modeling. Below each of the tables, the hit-rate (HR, defined as the total in diagonal divided by the total number of observations) value is shown in percent. Ideally, the predicted assignments to clusters would coincide with the original ones, so the cross-tables will be given by diagonal matrices. We see that hit-rate for LDA is 81.3%, for Logit-ROC it is 77.9%, and for MNL it equals 76.4%. Judging by the off-diagonal counts, the worst prediction is made by any method for the last 6th segment in this row-centered data

**Table 1.** Segment predictions by linear discriminant analysis LDA, Logit-receiver operating characteristic (ROC), and multinomial-logit (MNL) models with the row-centered and dipole-adjusted data.

| **Segment Prediction with the Row-Centered Data** | | | | | | | **Segment Prediction with the Dipole-Adjusted Data** | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| LDA | 1 | 2 | 3 | 4 | 5 | 6 | LDA | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 5337 | 39 | 26 | 0 | 0 | 378 | 1 | 10121 | 2 | 11 | 0 | 41 | 0 |
| 2 | 117 | 4371 | 115 | 63 | 7 | 352 | 2 | 99 | 2906 | 216 | 11 | 167 | 17 |
| 3 | 107 | 199 | 4783 | 10 | 109 | 78 | 3 | 845 | 11 | 4335 | 2 | 67 | 0 |
| 4 | 0 | 452 | 78 | 3267 | 70 | 0 | 4 | 0 | 32 | 230 | 1823 | 18 | 37 |
| 5 | 0 | 105 | 140 | 91 | 3562 | 0 | 5 | 325 | 20 | 312 | 0 | 3082 | 7 |
| 6 | 1418 | 1004 | 101 | 2 | 0 | 683 | 6 | 3 | 51 | 160 | 26 | 150 | 1937 |
| Hit-rate, % | | | | | | 81.3 | Hit-rate, % | | | | | | 89.4 |
| Logit-ROC | 1 | 2 | 3 | 4 | 5 | 6 | Logit-ROC | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 5056 | 134 | 238 | 0 | 0 | 352 | 1 | 9812 | 76 | 126 | 0 | 160 | 1 |
| 2 | 96 | 3487 | 104 | 586 | 70 | 682 | 2 | 4 | 3240 | 62 | 16 | 65 | 29 |
| 3 | 45 | 184 | 4748 | 4 | 230 | 75 | 3 | 232 | 178 | 4262 | 252 | 278 | 58 |
| 4 | 0 | 288 | 79 | 3366 | 100 | 34 | 4 | 0 | 64 | 132 | 1895 | 15 | 34 |
| 5 | 0 | 73 | 178 | 82 | 3565 | 0 | 5 | 99 | 102 | 133 | 7 | 3140 | 265 |
| 6 | 1177 | 854 | 281 | 42 | 5 | 849 | 6 | 0 | 66 | 57 | 29 | 181 | 1994 |
| Hit-rate, % | | | | | | 77.9 | Hit-rate, % | | | | | | 89.9 |
| MNL | 1 | 2 | 3 | 4 | 5 | 6 | MNL | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 4814 | 143 | 169 | 1 | 0 | 653 | 1 | 10124 | 2 | 21 | 0 | 28 | 0 |
| 2 | 162 | 3738 | 164 | 477 | 90 | 394 | 2 | 14 | 3203 | 82 | 35 | 54 | 28 |
| 3 | 112 | 131 | 4716 | 61 | 239 | 27 | 3 | 158 | 33 | 4888 | 61 | 103 | 17 |
| 4 | 0 | 561 | 107 | 3063 | 132 | 4 | 4 | 0 | 40 | 52 | 2009 | 1 | 38 |
| 5 | 0 | 53 | 211 | 81 | 3553 | 0 | 5 | 50 | 38 | 85 | 12 | 3442 | 119 |
| 6 | 1173 | 963 | 242 | 38 | 8 | 784 | 6 | 0 | 33 | 50 | 55 | 43 | 2146 |
| Hit-rate, % | | | | | | 76.4 | Hit-rate, % | | | | | | 95.4 |

Table 1 in its right-hand's half is constructed similarly and presents the results of prediction obtained for the dipole-adjusted data. We see a noticeable improvement of the predictions by any method. The hit-rates grow to 89.4%, 89.9%, and 95.4% for LDA, Logit-ROC, and MNL, respectively, with the off-diagonal counts diminished sometimes even to zero. Thus, using much more homogeneous responses across various countries achieved in dipole adjustments in contrast to simple row-centering improves the results of clustering and predictions to belong to the correct segments. Quadrupole-adjusted data have been tried as well but they do not improve the results attained with the dipole row-adjustment in this data, and we present the main and most interesting findings.

## 3. Summary

The paper shows that application of the quantum-paradigm methodology of the so-called multipoles to the description of Likert scales can be useful in applied behavioral sciences, particularly in marketing research. The multipoles approach is applied for data smoothing via prediction of individual rates by the histogram of the Likert scale boxes. This technique is applied for decreasing the raters' heterogeneity in data segmentation. A numerical example with marketing research data demonstates that the dipole row adjustment noticeably improves segmentation results in comparison with a standard row-centering. The new technique can be convenient for decreasing data heterogeneity and useful for managerial decisions in various marketing research projects. Future research can include multipole presentation of the Likert scales for various other problems in behavioral sciences where these scales are widely applied.

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## Appendix A. Likert Scale in Multipole Description

Let us briefly describe Likert scales from the point of view of the multipole states, due to the work [19]. A quantum angular momentum of a value $L$ can have a finite number of projections $M_L$ on an oriented axis, from $-L$ to $L$ with the step 1 of the possible levels:

$$M_L = -L,\ -L+1,\ \ldots,\ L-1,\ L. \tag{A1}$$

A Likert scale with n points corresponds to the momentum:

$$L = (n-1)/2, \tag{A2}$$

which can be an integer or a half-integer number like a spin. In other words, a Likert scale corresponding to the levels in Equation (A1) has $n = 2L + 1$ points. For example, a 4-point Likert scale, $n = 4$, has $L = 1.5$, and $M_L = -1.5, -0.5, 0.5, 1.5$; or a 5-point Likert scale, $n = 5$, is related to $L = 2$, with the levels $M_L = -2, -1, 0, 1, 2$.

In physics, there are characteristics of the so-called multipoles associated with objects' angular momentum L and electro-magnetic phenomena. In behavioral sciences, these characteristics, called monopole, dipole, quadrupole, and higher order multipoles, correspond to mean level, linear trend, quadratic, and more complicated components of distribution of the respondents' answers by boxes of the Likert scales. The multipoles moments are defined as the following set of orthonormal functions:

$$g_K(L, M_L) = (-1)^{L-M_L}\sqrt{2K+1}\begin{pmatrix} L & L & K \\ M_L & -M_L & 0 \end{pmatrix}, \tag{A3}$$

where parameter $K$ defines an order of multipole. The function in parentheses at the right-hand side of Equation (A3) is the so-called Wigner 3j-symbol used for adding angular momenta [30]. In explicit form, the 3j-symbol for monopole ($K = 0$) equals:

$$\begin{pmatrix} L & L & 0 \\ M_L & -M_L & 0 \end{pmatrix} = \frac{(-1)^{L-M_L}}{\sqrt{2L+1}}, \tag{A4}$$

for dipole ($K = 1$) the 3j-symbol equals:

$$\begin{pmatrix} L & L & 1 \\ M_L & -M_L & 0 \end{pmatrix} = \frac{(-1)^{L-M_L} M_L}{\sqrt{(2L+1)(L+1)L}}, \tag{A5}$$

and for quadrupole ($K$ = 2) the 3j-symbol equals:

$$\begin{pmatrix} L & L & 2 \\ M_L & -M_L & 0 \end{pmatrix} = \frac{(-1)^{L-M_L} 2[3M_L^2 - L(L+1)]}{\sqrt{(2L+3)(2L+2)(2L+1)(2L)(2L-1)}}. \quad (A6)$$

Substituting the 3j-functions of Equations (A4)–(A6) into Equation (A3) yields the explicit forms for the first multipole moments. For instance, the monopole moment ($K$ = 0) is:

$$g_0(L, M_L) = \frac{1}{\sqrt{2L+1}}, \quad (A7)$$

the dipole moment ($K$ = 1) is:

$$g_1(L, M_L) = \frac{\sqrt{3} M_L}{\sqrt{(2L+1)(L+1)L}}, \quad (A8)$$

and the quadrupole moment ($K$ = 2) is:

$$g_2(L, M_L) = \frac{2\sqrt{5}[3M_L^2 - L(L+1)]}{\sqrt{(2L+3)(2L+2)(2L+1)(2L)(2L-1)}}. \quad (A9)$$

These first three and other multipoles of higher order comprise a basis by which the theoretical probability of a Likert scale distribution by its boxes in Equation (A1) can be given as:

$$\hat{p}(M_L) = \sum_{K=0}^{2L} a_K \cdot g_K(L, M_L), \quad (A10)$$

and the total set of functions satisfies the condition of orthonormality:

$$\sum_{M_L=-L}^{L} g_K(L, M_L) \cdot g_J(L, M_L) = \delta_{KJ}, \quad (A11)$$

where it is the Kronecker delta which equals one for $K$ = $J$ and zero otherwise.

The state multipole parameters $a_K$ of the probability decomposition in Equation (A10) by orthonormal functions of multipole moments are defined as follows:

$$a_K = \sum_{M_L=-L}^{L} p(M_L) \cdot g_K(L, M_L), \quad (A12)$$

so they are the multipoles averaged with the weights $p(M_L)$ of observed frequency of the distribution of the responses by the boxes in Equation (A1) of a Likert scale, with a normalization.

$$\sum_{M_L=-L}^{L} p(M_L) = 1. \quad (A13)$$

For example, using Equation (A7) in Equation (A12) yields the state monopole parameter:

$$a_0 = \sum_{M_L=-L}^{L} p(M_L) \cdot g_0(L, M_L) = \frac{1}{\sqrt{2L+1}} \sum_{M_L=-L}^{L} p(M_L) = \frac{1}{\sqrt{2L+1}} \quad (A14)$$

Similarly, using Equation (A8) in Equation (A12) gives the state dipole parameter:

$$a_1 = \sum_{M_L=-L}^{L} p(M_L) \cdot g_1(L, M_L) = \frac{\sqrt{3}}{\sqrt{(2L+1)(L+1)L}} \sum_{M_L=-L}^{L} p(M_L) \cdot M_L \quad \text{(A15)}$$

Substituting Equation (A9) into Equation (A12) produces the state quadrupole parameter:

$$\begin{aligned} a_2 &= \sum_{M_L=-L}^{L} p(M_L) \cdot g_2(L, M_L) \\ &= \frac{2\sqrt{5}}{\sqrt{(2L+3)(2L+2)(2L+1)(2L)(2L-1)}} \left\{ 3 \sum_{M_L=-L}^{L} p(M_L) \cdot M_L^2 - L(L+1) \right\} \end{aligned} \quad \text{(A16)}$$

where the Equation (A13) is accounted.

Values in Equations (A14)–(A16) and possible multipoles of higher order can serve for characterization of each respondent by multiple variables measured by the same Likert scale.

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