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Stability Analysis and Prediction of Traffic Flow of Trucks at Road Intersections Based on Heterogenous Optimal Velocity and Artificial Neural Network Model

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Abstract: The evolution of traffic-related accidents caused by long, short, and medium trucks at signalized road intersections have been underemphasized in the last few years. Far, little attention has been paid to the modelling of trucks traffic flow using an artificial neural network model and evaluating the stability analysis of trucks depending on the heterogenous optimal velocity. This research evaluates the effect of trucks on some specific traffic flow features. Over the years, it has been deduced that trucks, irrespective of their sizes, significantly impact their surrounding traffic flow due to their body sizes and operational features. In this study, we focused on modelling the traffic flow of trucks at signalized road intersections using traffic flow variables such as speed, traffic volume, traffic density, and time as our inputs and outputs. The truck traffic data was collected using up-to-date equipment such as video cameras and inductive loop detectors from the South Africa transportation network. During the ANN modelling of the truck traffic flow, we used 956 traffic datasets divided into 70% for training and 15% each for testing and validation. The ANN model results show testing regression values of R^2 (0.99901). This shows that the inputs and output are well correlated and the ANN model's superiority in predicting truck traffic flow at signalized road intersections. Based on the HEOV model results, the result of the research indicates that in the mixed traffic flow of trucks in real-life scenarios, the proportion of different trucks on the signalized road intersections rather than the proportions of types of trucks can be used in the determination of traffic flow stability of each truck. This research extends our knowledge of truck traffic flow modelling and provides a blueprint for examining the stability analysis of long, short, and medium trucks in their immediate driving environment.

Keywords: traffic flow; signalized road intersection; artificial neural network; traffic volume; heterogenous optimal velocity

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1. Introduction

Traffic flow is a problematic occurrence comprising many movable vehicles, and there is a complex relationship among these vehicles. The research on traffic flow can assist individuals in understanding the traffic flow features of pedestrians and drivers and can also offer clarity in other areas of human life [1,2]. Transportation researchers from both the side of the aisle in physics and transportation engineering have been creating models to tackle the problem of traffic congestion caused by recurrent and non-recurrent actions of pedestrians and commuters using road transportation networks. According to researchers such as [3–6], which have done various studies on traffic flow models, pre-existing models comprise the hydrodynamic, gas kinetic, cellular automata (CA), car-following, and coupled-map lattice models. In present studies, the heterogenous occurrences of the traffic flow of vehicles have attracted considerable attention among transportation researchers [3].

Existing research has primarily aimed at three distinct types of heterogeneous traffic flow in the last few decades: driver, vehicle, and traffic environment heterogeneity. Taylor et al. [3] stated in their research that driver heterogeneity occurs within a distance by applying a dynamic time warping technique. Ossen and Hoogendoorn [4], in their research, explored the effect of different styles of a car-following model on the traffic flow of vehicles. Davis [5], in his study, focused on the impact of adaptive cruise control percentages on vehicles' traffic flow when on-ramps on freeways, and Islam et al. [6] evaluated traffic flow features characterized by motorized and non-motorized vehicles. The effectiveness and efficiency of prediction of traffic flow of vehicles is a significant study in intelligent transportation systems, which not only assist urban planners and transportation engineers in understanding the theoretical framework behind traffic flow on freeways and road intersections but also assist pedestrians and drivers in traffic flow information which in turn assist commuters in planning their travel route in advance [7–10]. Therefore, an efficient traffic flow predictive model would be appropriate for traffic management in developed and developing countries.

Presently, different types of methods are used to predict traffic flow [11–13]. These methods can be categorized into two; they are statistical learning methods (ARIMA) [14], Kalman filter [15], and local linear regression [16]. These statistical learning methods are also known as the conventional methods of predicting traffic flow. They are used for the construction of implementable predictive models to evaluate the performances of these models. These conventional methods depend on numerous assumptions, such as stationary traffic flow variables, which can be non-dependent or distributed evenly as a normal distribution, causing different types of limitations based on their applications. The second category is called machine learning methods; they comprise of support vector machine (SVM) [11], *K*-nearest neighbour [17], and artificial neural network model [18].

In comparison with statistical learning methods, machine learning models have been widely acknowledged to be effective based on their efficient results when it comes to modelling and because of their effectiveness in achieving the randomness of conditions of traffic flow of vehicles; however, their efficacy is significantly based on the traffic data obtained. However, it is important to note that the conventional ANN model is constructed like a "black box," which is efficient in the prediction of nonlinear systems. Still, it primarily focuses on the learning error, which is commonly caused due to overfitting. Furthermore, depending on the reduction of risks, the ANN model does not possess the required theoretical framework for the construction of a neural network structure. They are easily prone to local extreme points.

Therefore, it is difficult to evaluate the neural network's structure and determine the black box's operations [19]. To evaluate the inherent defects in the ANN model, using the theoretical framework of the combination of dimension and structural risk minimization theory, SVM would occur; this type of model possesses efficient predictive performances, and it is also effective when compared to ANN models that are using small data for the training of the ANN model. When compared to the ANN model, SVM has numerous merits when it comes to modelling traffic flow, merits such as (1) SVM is known as convex quadratic programming, and they obtain their optimal global solution; (2) SVM is used for severe nonlinear issues via kernel method; and (3) SVM can be used to exchange the fitness of the data and complexity of the model by using structural risk minimization to reduce the problem of overfitting.

In the last few years, transportation researchers have researched the different types of conventional models to enhance traffic flow prediction [10,20]. However, most of this research can be divided into two significant groups; they are parametric and non-parametric models. Parametric models comprise of time series and Kalman filtering models [21]. The ARIMA conventional model, also known as the autoregressive integrated moving average, represents a time series model. It was first discovered by transportation researchers, such as [22], for the prediction of traffic flow of vehicles. Researchers such as [23] used Kalman filtering for forecasting traffic flow. However, due to the stochastic features of the traffic flow system of vehicles, the estimation of parameters in ARIMA and Kalman filtering are too complex for traffic flow prediction. The comparison of the artificial neural network model with conventional models was conducted by [2], in which they concluded that the artificial neural network model is capable of traffic flow prediction efficiently and effectively when compared to the Markov chain model. In comparison with other research on traffic flow prediction in the literature, machine learning-dependent models are flexible and intelligent, which means that these models can comprehensively explain structural insights into the obtained traffic data. Because SVM is a typical example of a conventional machine learning technique, it is dependent on the theory of statistics and possesses exceptional recognition performance. Support vector regression (SVR) [24] applies SVM to find a solution to the problem of regression estimation. One of SVR's objectives is to look for a regression functionality consisting of maximum deviation derived from the actual data from the training data obtained. The primary problem of SVR is the elevated computational problems. This has caused limitations to its ability to analyse extensive traffic data. To solve this problem, least-squares SVR (LSSVR) is recommended by applying the least squares method [25]. In the application of LSSVR, inequality constraints are converted to equality constraints in SVR. This method enhances the learning speed; however, the robustness of LSSVR is not exceptional compared to SVR. Peng [26] recommended a new non-parallel plane regression model by replicating SVM [23] called TSVR, which encourages the reduction of the computational cost of the SVR.

The objective of the TSVR is to create a pair of functions in which one of the pair of functions is a crucial determinant for the ε-insensitive down and up bounds of an unknown regressor. For example, the twin SVM, also known as TSVR, can be used to solve two smaller problems that are quadratic in nature compared to a single large one in SVR. This method makes TSVR approximation four times quicker than SVR based on the theoretical framework. However, the demerits of TSVR should not be neglected, including demerits such as: (1) TSVR solutions are impacted by the constraints of memory, especially when evaluating enormous data [27]. (2) There is no difference in the penalties given to TSVR data points. It is important to note that data points discovered at distinct positions significantly impact the bound functions. To reduce the TSVR cost, Huang et al. [27] suggested that the least squares twin SVR techniques should be applied (LSTSVR) using the least-squares sense technique. According to the LSTSVR method, constraints of inequality of TSVR are converted into equality constraints. Additionally, LSTSVR can be used to directly find a solution to a linear equation system using the primal space compared to quadratic programming issues in dissimilar space, which can be used to evaluate enormous data by not using external optimization. Summarily, this research signifies that the SVR techniques can be applied for the prediction of traffic flow.

The evolution of technological innovation in transportation engineering has made traffic flow data obtained from freeways and road intersections easier for research on traffic flow prediction. In short-term traffic flow prediction based on historical data, traffic flow is driven by non-replicable patterns of traffic flow that change regularly. Therefore, traffic data is very significant in conducting investigative analysis on the traffic flow conditions of vehicles on freeways and the usage of intelligent transportation systems in urban traffic management [28,29]. Like many traffic flow prediction models, SVR can be used to determine the regression values of traffic data without factoring in external interference such as bad weather or road accidents. The present study on traffic flow prediction is riddled with numerous practical issues, such as the traffic data of vehicles being impacted by weather and accidents, which will lead to missing traffic data and errors during the collection of the traffic data. These abnormality points are also known as outliers. These are different from other points in the traffic data. Some conventional models are used for the modelling of incomplete traffic data, especially with predictive models [30]. However, it is important to know that there is no basis that the traffic data missing will be replaced. However, these outliers may be significant characteristics in the prediction of the traffic flow of vehicles.

Some researchers have stated that measurements of inefficient traffic flow variables can be corrected depending on conventional or machine learning theories [31]. It is important to note that machine learning models applied criteria such as the Euclidean distance metric to suggest that every characteristic of the data inputs is significant and not dependent on other variables. In summary, it means that conventional machine learning models are not useful to traffic data that comprises outliers based on enormous square errors, mainly found in the overall error. However, many related studies have tried to enhance the performance of TSVR and LSTSVR [27], ε-TSVR [32], and modified TSVR [33]. However, there are still existing research gaps in dealing with outliers used for robust regression values. Presently, to the best of our knowledge, no research has applied artificial neural network model for predicting the traffic flow of long, short, and medium trucks. From previous studies, the artificial neural network model has been identified as an effective and efficient machine learning model that can be used for the prediction of traffic flow on freeways. In this research, we attempt to enhance previous studies by taking into consideration the long, short, and medium trucks at a signalized road intersections using heterogenous optimal velocity due to its ability to evaluate the microscopic driving features of trucks based on the work done by [34]. Moreover, due to a strong historical background and behaviour, we proposed Bando's optimal velocity (OV) car following model [34], which is used to develop a heterogenous model to form a long-short-medium truck heterogenous traffic flow at signalized road intersection. Furthermore, to the best of our knowledge, there has been no research that focuses on the development of an heterogenous model for the traffic flow of long, short, and medium trucks. The research contributes the following to the field of road transportation, especially road intersections and modelling traffic flow of trucks:

- This research proposed a heterogenous optimal velocity model to systematically evaluate the long-short-medium truck traffic flow at signalized road intersections.
- This research proposed a Levenberg–Marquardt artificial neural network to model the traffic flow of long-short-medium trucks at signalized road intersections using traffic flow parameters such as speed, time, traffic density, and traffic volume.
- This research developed an ANN model for the stability analysis of long-short-medium trucks at signalized road intersections.

The main paper has been divided into four parts. The first part begins by laying out this study's theoretical, fundamental dimensions and objectives. The second part is concerned with the methodologies used for this research, such as the location of the study, data collection, and ANN model development. It also presents the findings of the stability analysis of the trucks using heterogenous optimal velocity (HEOV). The third part focuses on the prediction of the traffic flow of the trucks using an artificial neural network model. Finally, the fourth part draws upon the entire research, tying up the various theoretical methodologies and results to give a summary and future recommendations.

2. Materials and Methods

2.1. Data Collection and Study Location

The traffic data used in this research was made available by the South Africa Ministry of Transportation; this is part of their collaboration. The data collection equipment used for the collection is the video cameras and inductive loop detectors that capture video images from parts of the signalized road intersections between the N1 route between Johannesburg and Pretoria. Moreover, well-detailed traffic datasets were created via video image processing. The image of the study location is shown in Figure 1. The section of these road intersections is more than 500 m long and consists of five significant lanes with less than two auxiliary lanes. These road intersections comprise a section used for on-ramp (when joining the freeway), and another used for off-ramp (when exiting the freeway). It is important to note that heavy trucks have no restrictions (long, short, or medium trucks), and the grade level is closer to zero. The traffic data was obtained during a 24-h cycle, and the video cameras were applied to capture the number of trucks at these road intersections at 15 frames per 60 s. The traffic data was obtained under certain conditions such as visible weather, good driving visibility, and non-wet pavement conditions. In the datasets, we have classified them as long, short, and medium trucks (based on the traffic data collected on the South African road transportation networks). Table 1 illustrates the traffic flow parameters obtained from the road intersections. The datasets provide traffic flow information of the trucks and the surrounding road intersection features (speed limit, pavement type, and directions). This dataset provides significant information that can be used to evaluate the physical features (length and width of the trucks), traffic density, speeds, and acceleration of individual trucks and their surroundings—not excluding features such as space gaps between these trucks and the speed between each truck at a certain time of the day.

Figure 1. An overview of the N1 route in South Africa (The N1 route is circled in green).

Figure 1 shows the location of the site where the traffic data was collected. This data was collected during the on-peak and off-peak periods depending on the traffic volume of the long, short, and medium trucks. Moreover, environmental factors such as severe rainfall and heatwave were considered.

Table 1. Features and Schedule of Collection of traffic data.

Signalized Road Intersections	Video Camera	Position of the Camera	Time and Date	Total Duration	Speed Limit (km/h)	Road Type	Number of Lanes
Road intersection 1	1	First View Second View	22/07/2019 (Monday: 12:00 am to 12 pm) 23/07/2019 (Tuesday: 12:00 am to 12:00 pm)	24 h	120	Pavement	4
Road Intersection 2	2	First View Second View	22/07/2019 (Monday: 12:00 am to 12 pm) 23/07/2019 (Tuesday: 12:00 am to 12:00 pm)	24 h	120	Pavement	5
Road Intersection 3	3	First View Second View	24/07/2019 (Wednesday: 12:00 am to 12 pm) 25/07/2019 (Thursday: 12:00 am to 12:00 pm)	24 h	120	Pavement	4
Road Intersection 4	4	First View Second View	25/07/2019 (Thursday: 12:00 am to 12 pm) 26/07/2019 (Friday: 12:00 am to 12:00 pm)	24 h	120	Pavement	5

However, it is important to note that apart from video cameras that were used in the collection of the traffic data, we also used the secondary means of traffic data collection, which involves intermittent interviews with the transportation engineers in the South Africa Ministry of transportation for information about the traffic networks in the South Africa transportation systems and government regulations when it comes to road networks. The division of the traffic flow variable into inputs and output was based on the research done by [8,18] and based on the significance of knowing the traffic volume of a particular intersection in determining the level of traffic congestion that usually occurs during the on-peak period of the day. The definition of the traffic flow variables used in this research is explained below. Figure 2a–c illustrates different types of heavy trucks you can find at road intersections and freeways. Additionally, Figure 3 shows the key traffic flow variables used for the artificial neural network modelling.

- **Traffic density:** This is the number of vehicles per unit length. It is calculated as:

$$Traffic\ density = \frac{Number\ of\ vehicles}{length}$$

- **Traffic volume:** This is the number of vehicles depending on a specific period.

$$Traffic\ volume = \frac{Number\ of\ vehicles}{time}$$

- **The number of short/medium/long trucks:** This is the total number of different types of trucks on a specific road depending on the time of the day and traffic volume.
- **Time of day of the short/medium/long trucks:** This parameter depends on the speed of the vehicles or trucks and the distance of the specific road site. For example, the road sites used as a case study in this research study have their own distance. Its mathematical expression is:

$$speed = \frac{distance}{time}\ therefore,\ time = \frac{distance}{speed}$$

- **The average speed of the short/medium/long trucks:** This is the speed of the vehicles on the road at a specific period. Each road has its speed limit. The road sites used for this study all have a speed limit of 120 km/h.

  In this research, we focused on three types of trucks:

- **Long trucks:** These types of trucks are also called heavy trucks. They usually weigh between 26,000 lb to 32,000 lb. Examples of long trucks are log carrier trucks, refrigeration trucks, and environmental refuse trucks.
- **Short Trucks:** They are also known as mini trucks, bigger but smaller when compared to medium trucks. They are usually minivans, SUVs, and tow trucks.
- **Medium Trucks:** They are also known as large trucks but smaller when compared to heavy trucks. They weigh between 14,000 and 26,000. They are usually fire service trucks and box trucks.

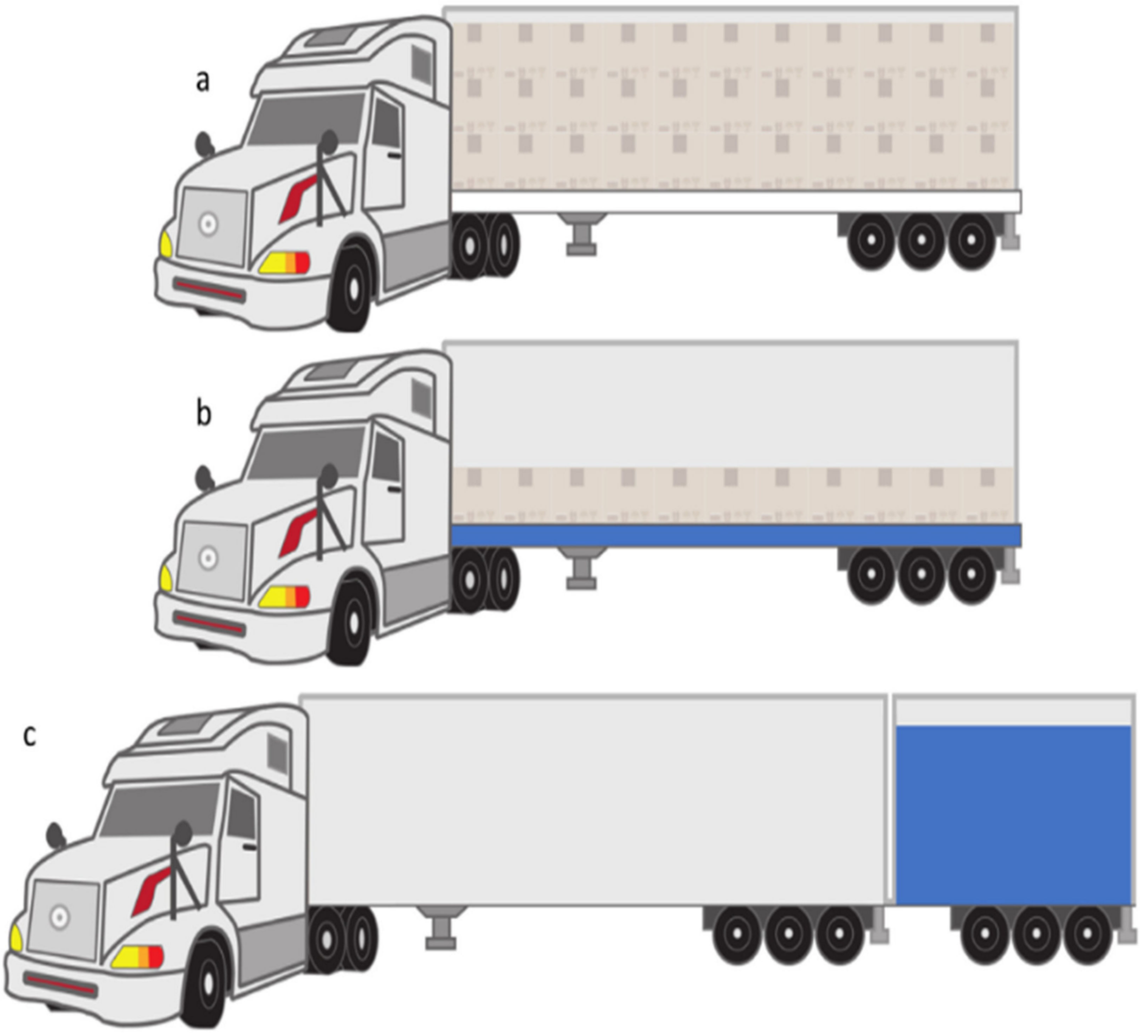

Figure 2. Different types of trucks. Reprinted with permission from ref. [35]. Published by Elsevier B.V.

2.2. Method of Data Collection

The method adopted for the collection of data includes the primary and secondary methods. This research's primary method involves collecting traffic data from the South Africa N1 road intersections through inductive loop detectors, video cameras, and road-wide stationed GPS-controlled equipment. The secondary data was obtained by interviewing the South Africa Ministry of Transportation traffic engineers.

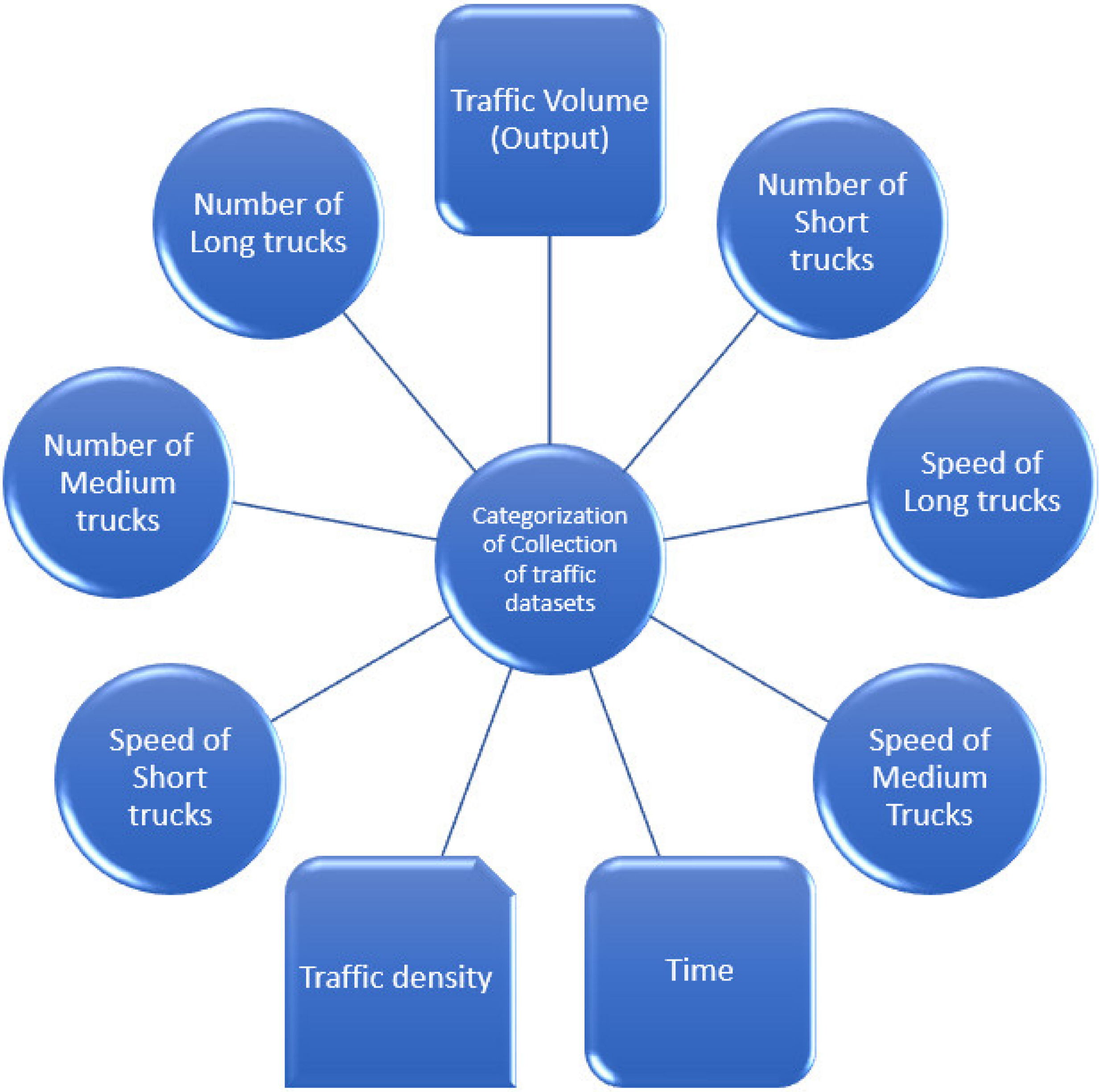

Figure 3. Division of the traffic flow variables used for the ANN modelling.

- **Data loggers**: These loggers are operated electronically and usually installed near a traffic light at signalized road intersections to obtain information from vehicles on a 24-h cycle.
- **Loop detectors:** These detectors, also called the inductive loop detectors, are usually installed inside road bumps, zebra crossings, and road intersections. They are used to know the number of vehicles and the distance travelled by each vehicle on the road.
- **Video Cameras:** These cameras have a wide range of views and are usually installed on traffic light poles in underground tunnels, freeways, and on rare occasions at roundabouts. Sometimes, governments use them to monitor traffic offenders and for security purposes. However, they are majorly installed to monitor the traffic situations at congested road intersections.

2.3. Development of the Artificial Neural Network Model

An artificial neural network model is defined as a model comprising mathematical and computational features and is motivated by the human brain, which has a technique based on pattern recognition and machine learning methods. A neural network consists of processing parameters, input and output layers, weighting features, activation functionalities, and learning functions. Neural network models are regarded as a complete system of neurons with special relationships and relationships between inputs, outputs, and hidden neurons. Unfiltered information, such as raw data, usually passes through the neurons in the inputs to make connections between the hidden and input neurons by using weights and biases. The primary function of the output neurons is to provide information by applying the relationships between the hidden and output neurons (Figure 4). In the past decades, neural networks have been known for their wide applications in engineering fields. This is due to their powerful features such as recognition of patterns, innovative adaptive learning, and real-life scenarios. However, when compared to other machine learning methods, artificial neural networks usually provide training patterns and create a relationship between the input and output datasets [36]. Additionally, the ANN model will not compulsorily put a restrictions embargo on the distribution of the input datasets without depending on the existing relationships.

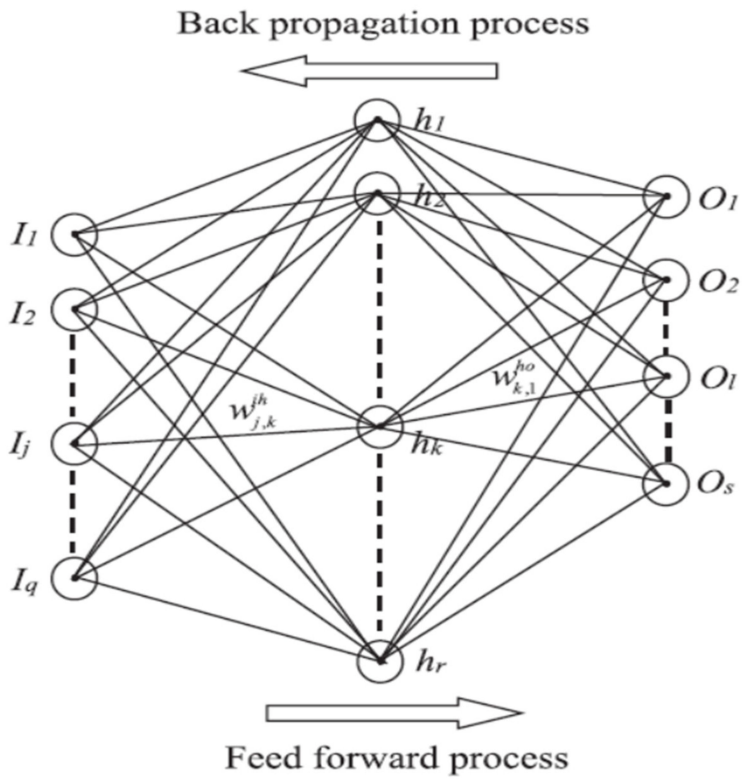

Figure 4. Example of a feed-forward back propagation neural network. Reprinted with permission from ref. [37]. Published by Elsevier B.V. Copyright© 2019 Elsevier Ltd. All rights reserved.

2.4. Input and Output Traffic Flow Parameters of the Trucks

This research aims to develop an ANN model for the prediction of traffic flow of long, short, and medium trucks by using the traffic flow variables in Figure 4. It is important to understand that the parameters of the inputs have enormous effects on the parameter output (traffic volume) for the ANN development. In this current research, the traffic volume, density, number, and speed of long, short, and medium trucks are significant input and output parameters. Based on the previous ANN-related research carried out by [2,11], we noticed that most of the input data variables used for the ANN model training and testing significantly impact the output data variables. The datasets used in this research comprise both the input and output variables. However, the ANN training is imperative for the effectiveness of the performance of the ANN model, and there is a need for an enormous amount of traffic data to be used for the ANN training, testing, and validation. This is to ensure the efficiency of the ANN model performance. The ANN model's performance depends on the generalization and effectiveness of the model prediction, which is dependent tremendously on the training, validation, and testing of the traffic data sets and the origin of the traffic data.

2.5. Performance, Training, and Architecture of the Artificial Neural Network Model

To achieve an optimal performance of the ANN model, it is appropriate to reduce the input and output variables within a suitable range that is appropriate for the desired activation function. Normalized data sets might improve the model's learning speed and enhance the ANN model performance, effectiveness, and stability during the ANN training of the datasets. The linear activation function is applied between two hidden and output layers. However, because the log-sigmoid activation function is applied between the input and hidden layers, the traffic data must undergo normalization to ensure that the parameters are within 0.0–1.0, based on the following expression in Equation (1):

$$Y^{norm} = \frac{\left(Y^{act} - Y^{min}\right)}{\left(Y^{max} - Y^{min}\right)} \tag{1}$$

Note:
Y^{act} *represents the actual values of the traffic volume*
Y^{min} *represents the minimum values of the traffic volume*
Y^{max} *signifies the maximum values of the traffic volume*
Y^{norm} *represents the normalization values of the input and output traffic flow data*

The ANN model applied in this research uses the supervised backpropagation feed forwarded multi-layered network, which comprises nodes with layers that are all connected. This is shown in Figure 4. The ANN model training and testing are conducted by applying the neural network toolbox, which can be found in the MATLAB environment. The logistic sigmoid functions, also known as logsig, are applied as a form of activation function for the hidden neurons compared to the linear transfer, also known as the purelin, which is used as an activation function for the neurons in the output. We used the Levenberg–Marquardt learning algorithm based on the research done by [13]. They explained the advantages of using this type of algorithm due to its quick and stable convergence and not excluding its high effectiveness. The ANN model's rate of learning depends on the traffic flow variables because it prevents error during the model training, and it usually occurs between 0 and 1. High-performance model results lead to different oscillations compared to inefficient results, which causes slow training of the model. Figure 5 shows the artificial neural network was developed from the traffic data collected to the performance metrics of the ANN model.

The input layer in the ANN model comprises several nodes: the speed, time, and the number of long, short, and medium trucks, which are needed to predict the traffic flow of long, short, and medium trucks. The nodes or neurons in a targeted layer are obtained with all the nodes of the subsequent layers, also known as the hidden layer. The signal found in every node in the hidden layer is then created. The function results from the linear relationship between the incoming inputs. Such as the output parameter of y, which is obtained by the addition of the combination of individual weighted variables (xw) and bias (b), these two have a significant effect on the activation function f which is shown in Equation (2):

$$x_1w_1 + x_2w_2 + \ldots\ldots\ldots\ldots x_nw_n + b = y = f = \sum_{i=1}^{n}\left(x_iw_{ij} + b_j\right) \tag{2}$$

where

f represents the activation function that allows the passage of input signals ($x_1, x_2 \ldots\ldots\ldots x_n$) and weighted parameters ($w_1, w_2 \ldots\ldots\ldots w_n$). The mathematical formulation of the ANN model sigmoid is shown in Equation (3):

The mathematical formulation of Figure 6 is shown in Equation (3):

$$y_j = f\left(\sum_{i=1}^{n} w_{ij} x_i + b_j\right) \tag{3}$$

x_i and y_j are known as the nodal values in previous layer i and the current layer j.
n is the overall number of the nodal values from the previous layer.
w_{ij} and b_j are known as the weights and biases of the network.

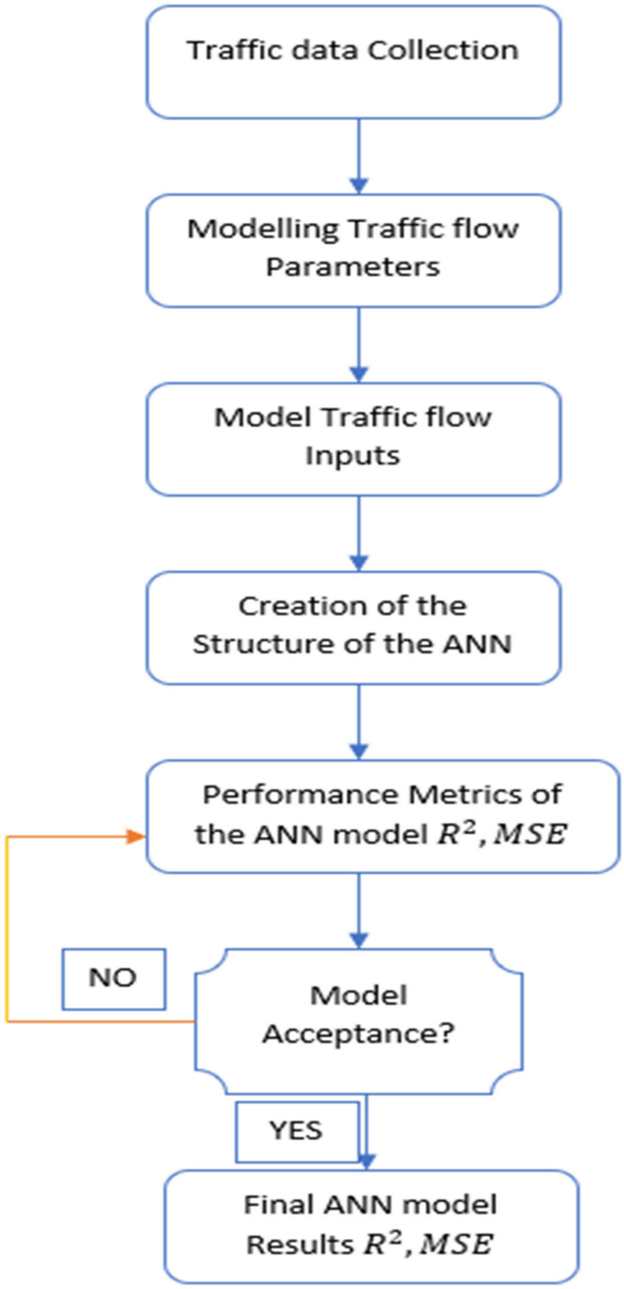

Figure 5. Schematic diagram of the development of the ANN model.

The value of these inputs combined with a value of bias is transformed by an activation function, as explained figuratively in Figure 6. Finally, the output signal is transferred to the neurons in the next layer. The fundamental conceptual framework is improving the efficiency in the prediction of the traffic flow of long, short, and medium trucks by achieving a function that reduces the error that can be found between the input (actual) and output (predicted) variables.

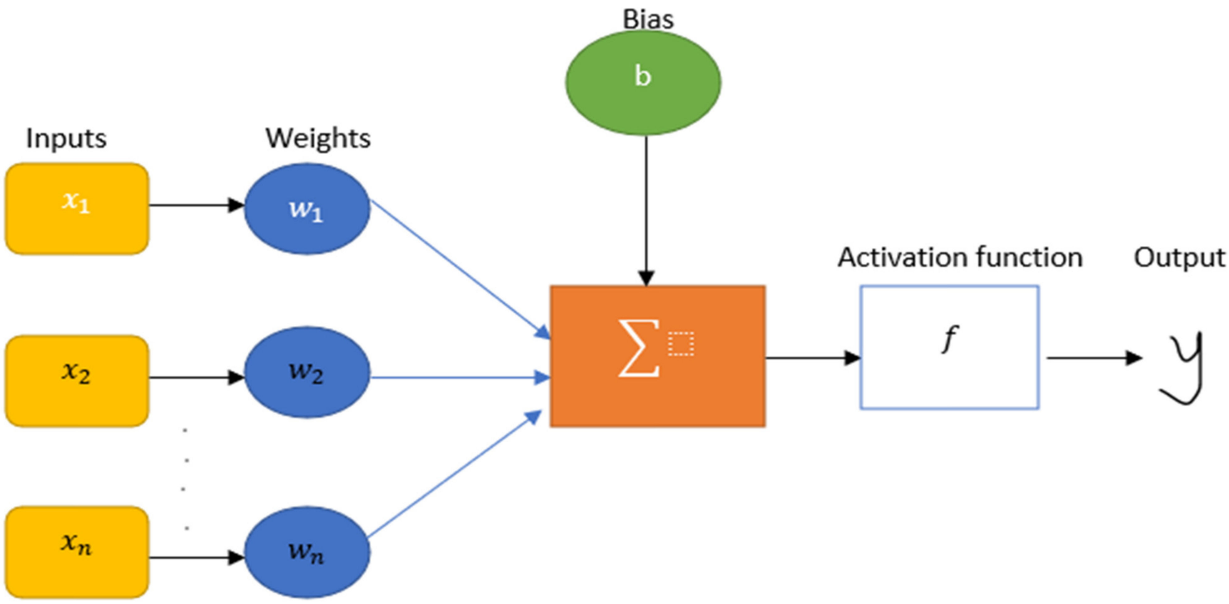

Figure 6. A neuron in an artificial neural network.

During each training of the ANN model, the neural network uses numbers of the weights and biases that are random and not fixed and analyses iteratively. The most significant problem in the ANN model is evaluating the hidden layers of the neural network model. It is important to analyse the adequate number of neurons to prevent the problem of over or underfitting, which is common in an artificial neural network model. The overall number of the traffic flow of long, short, and medium trucks traffic flow variables are not random numbers of neurons that can be found in the input and output layers. Trial and error techniques are applied to determine the overall number of neurons in the neural network model. In the case of historical input and output data, the significant dependent factors in the case of optimal network topology are the overall number of models undergoing training, testing, and validation data percentages and the quantity of noise in the truck traffic flow data and functional complexities. The neural fitting in the MATLAB 2020a was used in this research. This fitting consists of a three-layer feed-forward neural network consisting of hidden sigmoid neurons and output neurons that are linear, which are applied in analysing the problem of fitting in the input and output. The ANN model in this research was trained using the Levenberg–Marquardt training algorithm [13] due to its quick connection among numeric inputs and targets. The neural fitting which can be found in the MATLAB 2020a environment chooses traffic flow data, network creation, and train these neural networks and not excluding the validation of the ANN model performance, which is used to determine the mean square error (MSE) and the evaluation of what we called the goodness of fit.

2.6. Evaluation of the ANN Model

The R^2 is evaluated using the following equations:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (Y_{0,i} - Y_{m,i})^2}{\sum_{i=1}^{n} \left(\overline{Y}_0 - Y_{m,i}\right)^2} \tag{4}$$

$\sum_{i=1}^{n} (Y_{0,i} - Y_{m,i})^2$ represents the residual sum of the traffic flow of long, short, and medium trucks.

$\sum_{i=1}^{n} \left(\overline{Y}_0 - Y_{m,i}\right)^2$ represents the overall sum of traffic flow of long, short, and medium trucks.

$\overline{Y}=\frac{1}{n}\sum\limits_{i=1}^{n} Y_0, i$ represents the mean of observed values

Y_0 represents the simulated traffic flow values
Y_m indicates the ANN model values.
n represents the number of traffic flow data
Using the equation above, the MSE is evaluated as:

$$MSE = \frac{\sum_{i=1}^{n} (Y_{0,i} - Y_{m,i})^2}{n} \tag{5}$$

Note: All the equations above are validated on Windows 10 operating systems, and the 2020a of MATLAB version was used.

2.7. Model Description of the Heterogenous Optimal Velocity of Long, Short, and Medium Trucks at the Signalized Road Intersection

In a homogenous traffic flow of vehicles at a signalized road intersection, both the drivers and trucks on these road intersections have similar features. However, commonly, the trucks have dissimilarities, and the drivers of these trucks will have different driving behaviour. Equation (6) illustrates an example of a heterogeneous traffic flow comprising long-short-medium trucks. All three types of trucks consist of these combinations: LT-ST-MT. Bando's optimal velocity model assumes the following:

The truck driver is the only person who can decide the truck acceleration, which will ensure that the truck's velocity is directly proportional to the optimal velocity of these trucks, depending on the spacing in-between these trucks.

Bando's OV model is shown in Equation (6).

$$\begin{cases} d^2 Y_n(t) = t\left\{ f(\Delta Y_n(t)) - \frac{dY_n(t)}{dt} \right. \\ f(\Delta Y_n(t)) = \left(\frac{V^{max}}{2} \right) [tanh(\Delta Y_n(t) - d_s) + tanh(d_s)] \end{cases} \tag{6}$$

Note
$Y_n(t)$ *represents the position of the long-short-medium trucks n at a certain time t*
t is the coefficient of sensitivity
$\Delta Y_n(t)$ *signifies the headway of long-short-medium trucks "n"*
$\Delta Y_n(t) = Y_{n-1}(t) - Y_n(t), f(\Delta Y_n(t))$ *represents the optimal velocity of the long-short-medium trucks*
V^{max} *represents the maximum velocity of the long-short-medium trucks*
d_s *indicates the safe distance to drive between the long-short-medium trucks.*
This research considers two significant features in a long-short-medium truck traffic flow at a signalized road intersection. The features are:

- The difference in reaction time between the trucks is based on the research done by [38].
- The difference in the required velocity of the long-short-medium trucks.

According to the bando optimal velocity model, the sensitivity coefficient 't' increment depends on the reaction time decrement. In summary, the truck driver with a 't' parameter. The relationship between parameters of Bando's optimal velocity and the long-short-medium truck following real-life occurrences is listed in Table 2.

Table 2. The associated relationships between the optimal velocities of long, short, and medium trucks.

	LT	ST	MT
Responsive Sensitivity	MH	ML	L
Maximum Velocity	H	L	L

MH = Medium high; ML = Medium low, H = High, L = Low.

It is important to note that the difference between the maximum velocity between medium, long, and short trucks can be determined by V^{max}. The table illustrates the various levels of a combination of maximum velocity and sensitivity of the response of each truck. For the modelling of long-short-medium trucks traffic flow, the homogenous bando's optimal velocity car-following model is re-arranged into a heterogenous Bando's optimal velocity, also known as HEOV, by representing the subscripts as 't' and 'V^{max}' in Equation (7).

$$\begin{cases} d^2Y_n(t) = t_n\left\{f_n(\Delta Y_n(t)) - \frac{dY_n(t)}{dt}\right. \\ f_n(\Delta Y_n(t)) = \left(\frac{V_n^{max}}{2}\right)[tanh(\Delta Y_n(t) - d_s) + tanh(d_s)] \end{cases} \quad (7)$$

Note:

n represents the '$\boldsymbol{n}$th' truck, which can be long, short, or medium.

t_n and V_n^{max} have similar representation in the homogenous model, but their parameters are based on the features of the different long, short, and medium truck combinations.

t_n comprises of four options, they are t_{LT}, $t_{ST,}$ and t_{MT}

V_n^{max} only possesses three options, they are V_{LT}^{max}, V_{MT}^{max}, and V_{ST}^{max}

Evaluation of the Linear Stability of the Heterogenous Bando's Optimal Velocity Model

- *Uniform flow in the long, short, and medium truck traffic flow*

It is significant to evaluate the uniform traffic flow of long, short, and medium trucks at signalized road intersections to investigate the stability analysis of the heterogeneous model; based on the state of homogenous equilibrium, both the long, short, and medium trucks have no acceleration, the velocities are equal to each other, and there is appropriate spacing in-between them. However, in the case of heterogeneous traffic flow at signalized road intersections, they achieve zero acceleration. It is important that all trucks have a headway distance that is not constant among the long, short, and medium trucks. In addition, at the state of equilibrium of the heterogeneous flow of the long, short, and medium trucks, all trucks possess zero acceleration but similar velocity. These trucks also have dissimilar distance headways. The HEOV state of equilibrium is known as:

$$t_n \begin{cases} t_n\left\{f_n(\Delta Y_n(t)) - \frac{dY_n(t)}{dt}\right\} = 0 \\ f_n(\Delta Y_n(t)) = \left(\frac{V_n^{max}}{2}\right)[tanh(\Delta Y_n(t) - d_s) + tanh(d_s)] \end{cases} \quad (8)$$

Equation (8) represents the relationship between uniform velocity and headway of the long, short, and medium trucks in the HEOV model (which is the optimal velocity function). In summary, three (3) uniform traffic flow functions only occur for the combination of the long, short, and medium truck, and this is shown in Equations (9)–(11).

$$h_{LT}^{*} = d_s - \sqrt[log]{\frac{V_{LT}^{max} + V_{LT}^{max}e^{2ds}}{V^* + V_{LT}^{max} + V^*e^{2ds}} - 1} \quad (9)$$

$$h_{ST}^{*} = d_s - \sqrt[log]{\frac{V_{ST}^{max} + V_{ST}^{max}e^{2ds}}{V^* + V_{ST}^{max} + V^*e^{2ds}} - 1} \quad (10)$$

$$h_{MT}^{*} = d_s - \sqrt[log]{\frac{V_{MT}^{max} + V_{MT}^{max}e^{2ds}}{V^* + V_{MT}^{max} + V^*e^{2ds}} - 1} \quad (11)$$

h_{MT}^{*} represents the uniform headway of medium trucks corresponding to the uniform velocity of medium trucks at a signalized road intersection V^*

h_{LT}^{*} represents the uniform headway of long trucks corresponding to the uniform velocity of long trucks at a signalized road intersection V^*

h_{ST}^{*} represents the uniform headway of short trucks corresponding to the uniform velocity of short trucks at a signalized road intersection V^*

Conclusively, the uniform traffic flow of the HEOV model corresponds to:

$$V_n = V^* \tag{12}$$

$$t_n = 0 \tag{13}$$

The headway of the long, short, and medium trucks (n) depends on the type of trucks.

- *The stability evaluation of the HEOV model*

According to bando's optimal velocity model, the stability of the homogenous model is:

$$f < \frac{t}{2} \tag{14}$$

f indicate the optimal velocity derivatives function at the uniform headway distance of long, short, and medium trucks.

$$f = V^1(h^*) \tag{15}$$

Conditions

- The long, short, and medium trucks possess different features.
- Satisfying Equation (15) using the long, short, and medium trucks traffic flow can ensure traffic flow stability at signalized road intersections. However, it is too rigid to ensure the everlasting stability of these trucks at road intersections.
- All the truck-following pairs (LS, ST, and MT) do not need to be stable to ensure stability. Their stability depends on the suppression of the effects of unstable pairs by the stable pairs.
- It is important to create less strict stability criteria when compared to Equation (15).

In this research, we applied ward's technique [32] to formulate the HEOV model's stability conditions. Based on the following assumptions:

- N trucks are travelling along a signalized road intersection.
- The N can be small or in a more significant number

The two types of perturbation that were used for the heterogenous Bando's optimal velocity model in this research are:

- Headway Perturbation
- Velocity Perturbation

$$h_x = h_x^* + \widetilde{h_x} \tag{16}$$

$$V_x = V_x^* + \widetilde{V_x} \tag{17}$$

$\widetilde{h_x}$ represents the small headway perturbation of the long, short, and medium trucks at the signalized road intersection

$\widetilde{V_x}$ represents the velocity perturbation of the long, short, and medium trucks at the signalized road intersection.

The relationship between the location and the headway perturbation is stated as follows:

$$\widetilde{h}_x = \widetilde{X}_{x-1} - \widetilde{X}_x \tag{18}$$

Using first and second-order derivatives on Equation (18) based on the combination of velocity and headway perturbations.

$$\dot{\widetilde{h}}_x = \widetilde{V}_{x-1} - \widetilde{V}_x \tag{19}$$

$$\ddot{\widetilde{h}}_x = \dot{\widetilde{V}}_{x-1} - \widetilde{V}_x \tag{20}$$

Linearization of the heterogenous bando's optimal velocity:

$$\dot{\widetilde{h}}_x = t_x f_x \widetilde{h}_x - t_x \widetilde{V}_x \tag{21}$$

f_x represents the function of the optimal velocity of the long, short, and medium trucks 'x' at uniform headway.

$$f_x = V_n^1(h_n^*) \tag{22}$$

Differentiating Equation (21) of the trucks and substituting into Equations (19) and (20), we have:

$$\ddot{\widetilde{h}}_x + t_x \dot{\widetilde{h}}_x + t_x f_x \widetilde{h}_x = t_x f_x \widetilde{h}_{x-1} \tag{23}$$

Considering the ansatz for the evaluation of the development of the spatiotemporal perturbation at a signalized road intersection.

$$h_x = R_e(A_x exp(i\theta x + \lambda t)) \tag{24}$$

h_x represents the Fourier modes
R_e signifies the actual part of the signalized road intersection
A_x is the constant (and not dependent on 't')
θ can be formulated as $\theta = \frac{2\pi k}{N}$ (which is known as the discrete wave mathematical number)
Considering $k = 1, 2, 3, 4 \ldots\ldots\ldots N/2$
Substitute Equation (24) into Equation (23).

$$\lambda^2 A_x + t_x \lambda A_x + t_x f_x A_x = t_x f_x A_{x-1} e^{-i\theta} \tag{25}$$

Rewritten as

$$\lambda^2 \begin{bmatrix} A_1 \\ . \\ A_N \end{bmatrix} = M \begin{bmatrix} A_1 \\ . \\ A_N \end{bmatrix} \tag{26}$$

Then,

$$M = \begin{bmatrix} -t_1\lambda - t_1 f_1 & 0 & t_1 f_1 e^{-i\theta} \\ t_2 f_2 e^{-i\theta} & -t_2\lambda - t_2 f_2 & 0 \\ 0 & t_3 f_3 e^{-i\theta} & 0 \\ & -t_N\lambda - t_N f_N & \end{bmatrix} \tag{27}$$

Therefore Equation (26) must fulfil Equation (27).

$$\begin{bmatrix} \lambda^2 t_1 \lambda + t_1 f_1 & 0 & -t_1 f_1 e^{-i\theta} \\ -t_2 f_2 e^{-i\theta} & \lambda^2 + t_2\lambda + t_2 f_2 & 0 \\ 0 & 0 & 0 \\ 0 & & \lambda^2 + t_N\lambda - t_N f_N \end{bmatrix} = 0 \tag{28}$$

It represents the determinant. Equation (28) can be rewritten as:

$$\prod_{x=1}^{X} \left(\lambda^2 + t_x\lambda + t_x f_x\right) - e^{iX\theta} \prod_{x=1}^{X} t_x f_x = 0 \tag{29}$$

However, Equation (29) can be re-formulated as:

$$\lambda(\theta) = \lambda_R(\theta) + i\lambda_I(\theta) \tag{30}$$

This research focuses on larger X parameters; we consider the range which is continuous $0 < \theta < \pi$. A small, non-negative value of θ is directly proportional to long-wavelength fluctuations. θ equals to 0, which leads to the longest wavelength using discrete settings [26,27,29,30]. The research carried out by Wilson's research [27] stated that the occurrence of instability is at a long wavelength (λ = 0, θ = 0) which is used to solve Equation (21). The system can become unstable if the rate of growth ($\gamma_R\theta$) bends upwards

$(\theta = 0)$ [27]. Using the substitution method Equation (30) into Equation (29) illustrates the symmetry.

$$\lambda(-\theta) = \overline{\lambda(\theta)} \tag{31}$$

$\lambda_R(\theta)$ represents an even functionality
$\lambda_I(\theta)$ represents the odd function
To establish this occurrence, we applied the perturbation expansion (λ)

$$\lambda = i\lambda_1\theta + \lambda_2\theta^2 + 0\left(\theta^2\right) \tag{32}$$

To evaluate the wavelength using marginal stability, substitute Equation (32) into Equation (29). It is reformulated as:

$$\prod_{x=1}^{X}\left([i\lambda_1\theta + \ldots]^2 + t_x\left[i\lambda_1\theta + \lambda_2\theta^2 \ldots\right] + t_x f_x\right) - \left(1 + iX\theta - \frac{X^2}{2}\theta^2 + \ldots\right)\prod_{x=1}^{X}(t_x f_x) = 0 \tag{33}$$

Removing all θ, the imaginary part equation is:

$$i\lambda_1\sum_x\left(t_x\prod_{m\neq x}(t_m f_m)\right) - iN\prod_x(t_x f_x) = 0 \tag{34}$$

Evaluating Equation (34), we have:

$$\lambda_1 = X\prod_x t_x f_x \Big/ \sum_x\left(t_x\prod_{m\neq x}(t_m f_m)\right) \tag{35}$$

Moreover, the real part can be written as follows:

$$\sum_x\left(\left(-\lambda_1^2 + t_x\lambda_2\right)\prod_{m\neq x} t_m f_m\right) - \lambda_1^2\sum_{i\neq j, j>1}\left((t_i \times t_j)\prod_{m\neq i,j} t_m f_m\right) + \frac{X^2}{2}\prod_x(t_x f_x) = 0 \tag{36}$$

In summary, Equation (36) can be reformulated as:

$$\lambda_2\sum_x\left(t_x\prod_{m\neq x} t_m f_m\right) = \lambda_1^2\sum_x\left(\prod_{m\neq x} t_m f_m\right) + \lambda_1^2\sum_{i\neq j, j\neq i}\left((t_i \times t_j)\prod_{m\neq i,j} t_m f_m\right) - \frac{N^2}{2}\prod_x(t_x f_x) \tag{37}$$

Further deviation of Equation (37):

$$\sum_x\left(\prod_{m\neq x} t_m f_m\right) = \frac{\sum_x\left(t_x f_x\left(\prod_{m\neq x} t_m f_m\right)^2\right)}{\prod_x(t_x f_x)} \tag{38}$$

Substitute Equation (38) into Equation (37).

$$\begin{aligned}\lambda_2\sum_x&\left(t_x f_x\prod_{m\neq x} t_m f_m\right)\\ &= \frac{\lambda_1^2}{\prod_x(t_x f_x)}\left\{\sum_x\left(t_x f_x\left(\prod_{m\neq x} t_m f_m\right)^2\right)\right.\\ &\left.+\left(\prod_x(t_x f_x)\right)\sum_{i\neq j, j\neq i}\left((t_i \times t_j)\prod_{m\neq i,j} t_m f_m\right) - \frac{1}{2}\left(\sum_x\left(t_x\prod_{m\neq x} t_m f_m\right)\right)^2\right\}\end{aligned} \tag{39}$$

Another identity can be applied for the simplification of Equation (39).

$$\sum_{i\neq j,j\neq i}\left((t_i\times t_j)\prod_{m\neq i,j}t_mf_m\right)=\frac{1}{2\prod_x t_xf_x}\left\{\left[\sum_x\left(t_x\prod_{m\neq x}t_mf_m\right)\right]^2-\left(\sum_x\left(t_x^2\prod_{m\neq x}t_mf_m\right)^2\right)\right\} \tag{40}$$

Mathematical expression of λ_2 can be rewritten as:

$$\lambda_2=\frac{X^2\prod_x t_xf_x}{\left[\sum_x\left(t_x^2\prod_{m\neq x}t_mf_m\right)\right]^2}\left\{\sum_n\left(t_xf_x-\frac{1}{2}t_x^2\right)\left(\prod_{m\neq x}t_mf_m\right)^2\right\} \tag{41}$$

In accordance with the linear stability theoretical framework, when $\lambda_2<0$, this means that the system is stable; therefore, the conditions of stability of these trucks can be written as follows:

$$\sum_x\left(t_xf_x-\frac{1}{2}t_x^2\right)\left(\prod_{m\neq x}t_mf_m\right)^2<0 \tag{42}$$

All the definitions of the notations used in the formulation of the equations can be found in Table A1 in the Appendix A.

In this research, Bando's optimal velocity car-following model applied in this research encompasses the heterogenous optimal velocity, which is also known as the HEOV. This was used in describing the heterogeneous traffic flow occurrences of trucks on signalized road intersections with the three different types of trucks used in this research: the (long truck-short truck-medium truck) LT-ST-MT. The linear stability technique was applied for deriving the Bando's optimal velocity model's stability criteria, followed by the model's verification using numerical simulations. An analysis of the stability contributions of the long, short, and medium trucks are conducted. During the development of the HEOV model for the different types of trucks, different scenarios occurred based on three different types of combinations. Using different truck traffic flow scenarios as an example, the combination of stable and unstable impacts on the traffic flow of long, short, and medium trucks. Additionally, the proportion of the combinations of both the long, short, and medium trucks are evaluated using analytical examinations and simulation techniques.

3. Results and Discussions

Artificial Neural Network Model

The Artificial neural network model training, testing, and validation results are shown in Figure 7. From the figure, we can deduce that the best ANN model is selected depending on how low the MSE values are and how close are the regression values to 1 for the testing, training, and validation of the model. Taking into consideration that the ANN architecture of 13-5-1-1 gives the best optimal results of the ANN model, the overall best structure of the model was able to achieve the learning capability of an overall regression value of 0.9993 (Figure 7). It is important to note that after choosing the appropriate thirteen input truck traffic flow variables, various types of neurons undergo testing to evaluate the overall optimal performance of the ANN model. The input layer comprises of the number, time, and speed of both long, short, and medium trucks, not excluding the traffic density, and the output layer contains the traffic volume of the long, short, and medium trucks. The overall number of neurons in the ANN model results is constant at five. The training algorithm used was the Levenberg–Marquardt algorithm due to its faster convergence to 1 and accuracy. The ANN model's best performing structure was applied to obtain an accurate evaluation of the neural network model. The traffic datasets obtained from both the long, short, and medium truck traffic flow were divided into 70% training, 15% testing, and the remaining 15% for validation of the model. The overall best validation performance of the ANN model on the traffic datasets during training and testing was obtained at epoch 18, shown in Figure 8.

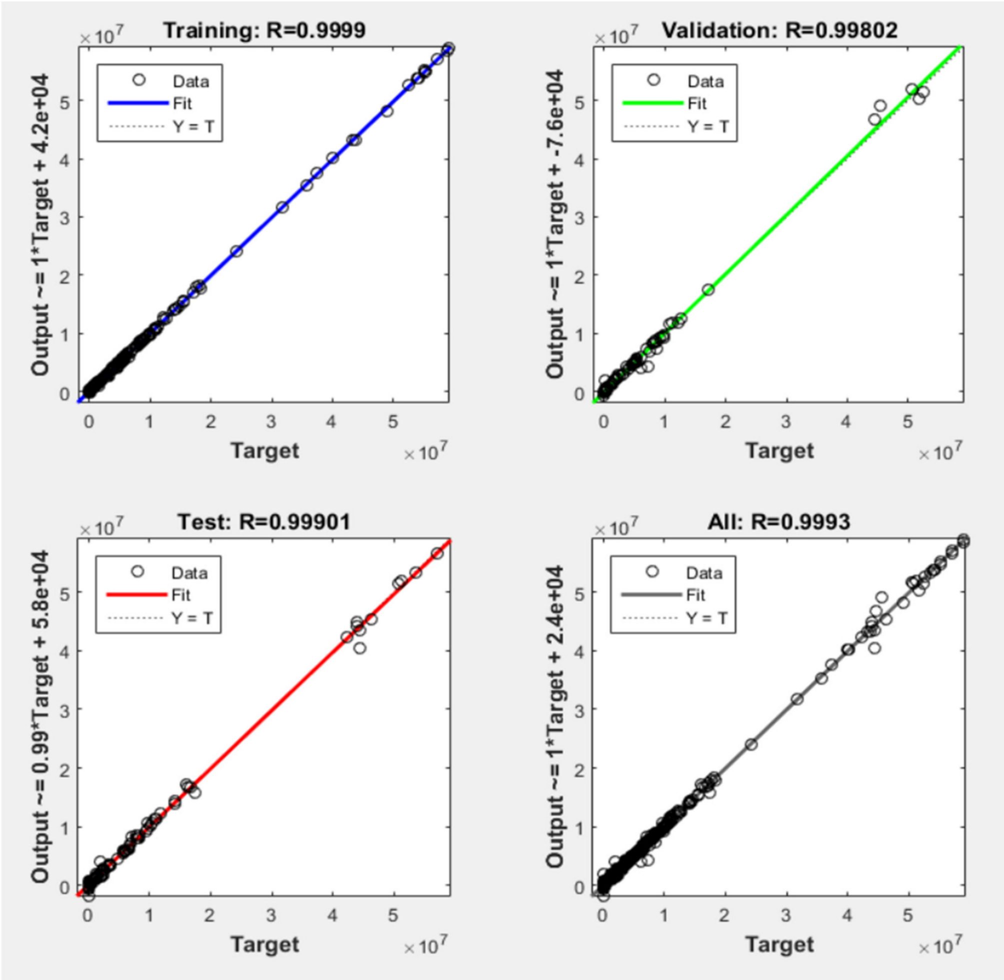

Figure 7. The regression values for the training, testing, validation, and overall.

From Figure 3, only 14 truck traffic flow parameters were used, they are obtained from the road intersections and are used as the inputs and output of the ANN model. Between these traffic flow variables, it can be deduced that the dependent variables, traffic volume, are well correlated with the other inputs, representing the number, speed, time, and traffic density of long, short, and medium trucks. The bias and weights in the MATLAB environment were created randomly using the neural network tool in the MATLAB 2020a environment. The model's accuracy is significantly dependent on how lower the MSE and the closeness of the regression values are to 1. This can all be achieved during the training of the ANN model. Figure 8 illustratively shows the architecture of the ANN model applied in the prediction of the traffic flow of trucks. Evidently, from Figure 9, most of the training, testing, and validation data points are on the linear line, which shows an optimal performance for the developed ANN model. Based on the ANN results, the evaluation of the coefficients for both training and testing of the truck's traffic flow datasets can be compared to the increment in the number of neurons.

Figure 7 shows a regression value for training, testing, validation, and overall performance of 0.9999, 0.99901, 0.99802, and 0.9993; it can be deduced that the input and output are well correlated. Figure 9 shows that the validation of the ANN model is optimal at 6 and an epoch at 24. This is because, during the training of the traffic datasets, the neural network changes the weight parameters of the inputs and output to ensure the best fitness for the model; however, during the testing of the model, the output indicates the performance of the model that undergo training without changing the weights during the creation of the ANN architecture.

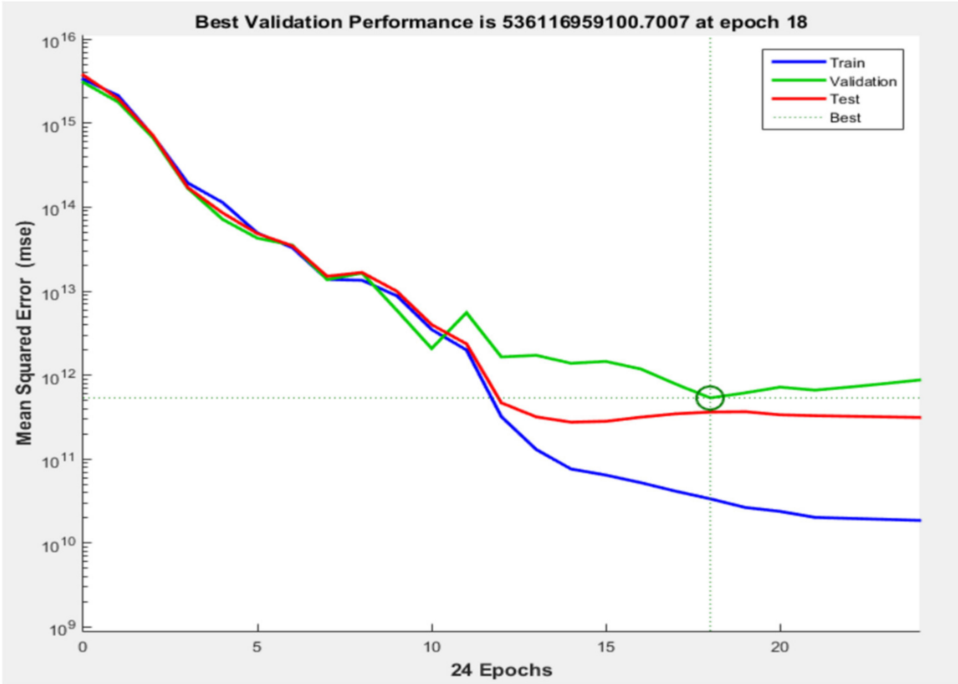

Figure 8. The ANN model validation performance.

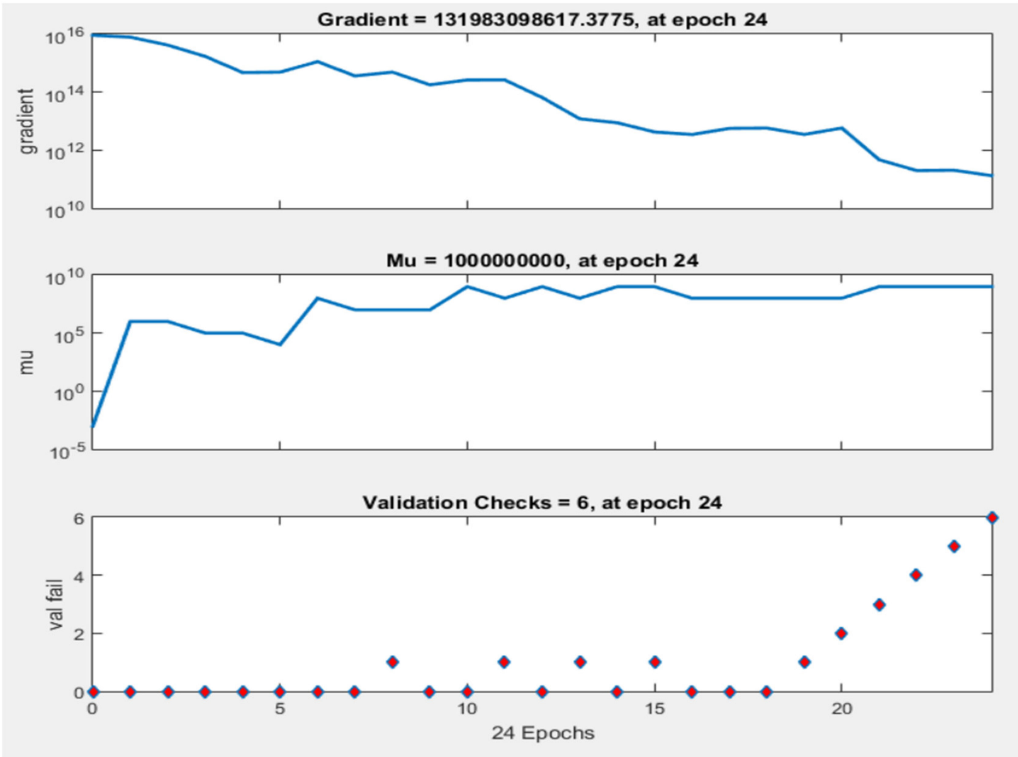

Figure 9. The validation checks of the ANN model.

4. Conclusions and Recommendations

4.1. Conclusions

This research aims to propose a heterogenous optimal velocity model to explain the long-short-medium truck traffic flow at signalized road intersections and develop a Levenberg–Marquardt artificial neural network model capable of predicting the traffic flow of long, short, and medium trucks at signalized road intersections. The following conclusions can be drawn from the present research.

- One of the most significant findings to emerge from this research is that the stability of the long-short-medium trucks linearly is evaluated by the different proportions of the LT-ST-MT traffic flow occurrences in comparison to using the overall number of trucks on the road.
- The second significant finding was that the stabilization and destabilization of the long, short, and medium trucks at the signalized road intersections depend on the trucks' density at the road intersections and on the traffic flow parameters of the HEOV model.
- The relevance of the Levenberg–Marquardt training algorithm in the artificial neural network model is clearly supported by the current findings of the $R^2 = 0.9993$ (which is closer to 1).
- This study has shown that the traffic density, speed, and time are significant in modelling trucks' traffic flow at signalized road intersections.
- The most apparent findings to emerge from this research are that traffic density and speed of long, short, and medium trucks significantly impact the traffic flow of other vehicles at the road intersections.
- This research extends our knowledge of traffic flow modelling of long, short, and medium trucks at signalized road intersections using machine learning (artificial neural network model).
- The current findings of this study also add to the growing body of knowledge regarding the application of a heterogenous optimal velocity model for traffic flow of long, short, and medium trucks.
- Finally, the results of this study have shown that the stabilization and destabilization of truck traffic flow at road intersections depend on the traffic density of both the long, short, and medium trucks.

4.2. Recommendations

- It would be interesting to assess the impacts of long, short, and medium trucks on the prevention analysis of accidents on freeways.
- Further research might explore the varying speed and traffic density of long and short trucks and their effects on driver behavior and pedestrians.
- Future research investigating the usage of particle swarm optimizations and genetic algorithms to predict the traffic flow of trucks on freeways would be interesting.
- Transportation researchers should also focus on comparing artificial neural network models with other emerging deep neural network models.
- Future researchers can explore other ways of validating heuristics models by using the 10-fold cross-validation.

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Data Availability Statement: Due to the sensitivity of the traffic data. The traffic data is available upon reasonable requests from the corresponding author.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Table A1. List of Notations for the heterogenous optimal velocity of long, short, and medium trucks.

Notations	Explanation
$Y_n(t)$	The position of the long-short-medium trucks n at a certain time t.
t	Coefficient of sensitivity
$\Delta Y_n(t)$	The headway of long-short-medium trucks "n"
$\Delta Y_n(t) = Y_{n-1}(t) - Y_n(t), f(\Delta Y_n(t))$	The optimal velocity of the long-short-medium trucks
V^{max}	The maximum velocity of the long-short-medium trucks
d_s	The safe distance to drive between the long-short-medium trucks
N	The 'nth' truck, which can be long, short, or medium
h^*_{MT}	The uniform headway of medium trucks corresponding to the uniform velocity of medium trucks at a signalized road intersection V^*
h^*_{LT}	The uniform headway of long trucks corresponding to the uniform velocity of long trucks at a signalized road intersection V^*
h^*_{ST}	The uniform headway of short trucks corresponding to the uniform velocity of short trucks at a signalized road intersection V^*
f	The optimal velocity derivatives function at the uniform headway distance of long, short, and medium trucks
$\widetilde{h_x}$	The small headway perturbation of the long, short, and medium trucks at the signalized road intersection
$\widetilde{V_x}$	The velocity perturbation of the long, short, and medium trucks at the signalized road intersection.
f_x	The function of the optimal velocity of the long, short, and medium trucks 'x' at uniform headway
h_x	The Fourier modes
R_e	The actual part of the signalized road intersection
A_x	The constant (and not dependent on 't')
$\lambda_R(\theta)$	Even functionality
$\lambda_I(\theta)$	Odd function

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