

Article

# Vadasz Number Effects on Convection in a Vertical Rotating Porous Layer, Placed Far from Axis of Rotation, and Subjected to Internal Heat Generation and Centrifugal Jitter

Saneshan Govender <sup>1,2</sup>

<sup>1</sup> Eskom Holdings SOC, Generation, Asset Management, Johannesburg 2000, South Africa; govends@eskom.co.za; Tel.: +27-11-800-2467

<sup>2</sup> Department of Engineering, School of Mechanical Engineering, Howard College Campus, University of Kwa Zulu, Durban 4001, South Africa

**Abstract:** The flow and heat transfer in a rotating vertical porous layer, placed far from the axis of rotation, and subjected to internal heat generation and centrifugal jitter, is considered. The linear stability theory is used to determine the convection threshold, in terms of the critical Rayleigh number. Typical liquids used in engineering applications and heavy liquid metals are used to demonstrate conditions at which the Vadasz number is sufficiently small to warrant the retention of the time derivative in the momentum equation. When considering low amplitude and high frequency approximation, the results show that vibration has a stabilizing effect on the onset of convection. The impact of increasing the Vadasz number is to stabilize the convection, in addition to reducing the transition point from synchronous to subharmonic solutions. In summary, when the Vadasz number is large, centrifugal jitter has no impact on the convection stability criteria. In contrast, when the Vadasz number is small, centrifugal jitter impacts the convection stability criteria.

**Keywords:** centrifugal jitter; rotation; Mathieu functions; internal heat generation; Vadasz number



**Citation:** Govender, S. Vadasz Number Effects on Convection in a Vertical Rotating Porous Layer, Placed Far from Axis of Rotation, and Subjected to Internal Heat Generation and Centrifugal Jitter. *Physics* **2021**, *3*, 728–738. <https://doi.org/10.3390/physics3030044>

Received: 30 June 2021

Accepted: 23 August 2021

Published: 1 September 2021

**Publisher's Note:** MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



**Copyright:** © 2021 by the author. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

## 1. Introduction

The classical problem of gravity driven free convection in fluid saturated porous media has been widely researched for various configurations. The engineering applications include, inter alia, nuclear reactor technology, food and chemical processing industry, thermal storage systems, etc. Pioneering past studies for a constant basic temperature gradient have been investigated by past researchers. Previous pioneering study [1–4] for non-rotating porous media set the foundation for researchers in heat and mass transfer in porous media. Later, rotation was analyzed in detail, and these studies are presented in [5–11]. The Vadasz number was first proposed in [12], specifically for the body of study presented in [9]. In almost all of the papers dealing with flow and heat transfer in porous media, the Vadasz number is neglected on the basis that it is understood to be large for the typical fluids used in engineering. The analysis in [9] considers scenarios where the Vadasz number could assume small values, thereby warranting its retention together with the time derivative in the momentum equation. The results presented in [9] discover that only when the Vadasz number is small to moderate, is the oscillatory mode of convection possible. Later study [13–16] introduced the effects of g-jitter on the stability of convection in porous media. The study in [13,14], the shows that the transition from synchronous to subharmonic solutions is characterized by a spike in the curve of the synchronous mode, just prior to the transition. Furthermore [16] shows that (gravity) g-jitter effects only affect the convection stability criteria for small to moderate Vadasz numbers. For large Vadasz numbers, the g-jitter has no impact on the convection stability criteria, in the absence of internal heat generation.

The focus of the current study is to consider the effects of centrifugal jitter for a vertical porous layer with internal heat generation and placed far from the axis of rotation. The

readers are referred to pioneering studies for flow and heat transfer with internal heat generation with constant gravity [17,18]. Early study on g-jitter on porous media with internal heat generation are presented in [19]. Research involving specific configurations and internal heat generation in porous media are presented in [20–22]. The readers are also referred to an important book in porous media and applications [23] for further review/reading, together with an excellent reference list for further reading.

In typical engineering applications, one needs to cater for the impact of centrifugal jitter in a design or perhaps design the system to use the effects of centrifugal jitter to control the heat transfer. In the scenario, where the influence of centrifugal jitter on the stability of convection needs to be dialed out, one may opt to choose a fluid that allows for large Vadasz numbers. For large Vadasz numbers [16], vibration (g-jitter) was shown to have no influence on the stability of convection. The current study investigates this and determine if the same result is applicable for centrifugal jitter in the presence of internal heat generation. There could be scenarios where the stabilizing effect of vibration (g-jitter or centrifugal jitter) may be required in an engineering application. The fluid properties, and perhaps even the medium, may need to be designed in such a way that the resulting Vadasz number is small enough, to allow the effects of vibration (g-jitter or centrifugal jitter) to influence the stability criteria for convection.

The present research recovers the basic conduction flow and temperature profiles and then use the linear stability theory to solve for the characteristic Rayleigh number, for a rotating vertical porous layer, placed far from the axis of rotation, and subjected to internal heat generation. The equation for the centrifugal-jitter amplitude is cast into the canonical form of the Mathieu equation to ensure that the critical Rayleigh number (in terms of the Vadasz number) may be determined (for the low amplitude, high frequency vibration case). The results for the rotating vertical porous layer, placed far from the axis of rotation and subjected to centrifugal jitter is then be compared to the case of a horizontal porous layer subjected to g-jitter, with internal heat generation [24]. In principle, the current study extends the study of [24] to a vertical rotating porous layer that is placed far away from the axis of rotation, thereby allowing for a constant external centrifugal body force (that may be likened to gravitational body force in [24]). The study developed in [11] discovered that gravity plays a passive role in the stability of convection for the configuration studied in the current study. Therefore, although the gravity force is shown in the formulation, its role is passive. The stability criteria developed in the current study is then compared to the previous study [24] and a few important are drawn regarding the impact of the Vadasz number on the stability criteria. Based on the review of literature performed, the author is unable to locate study similar to that presented in this paper.

## 2. Problem Formulation

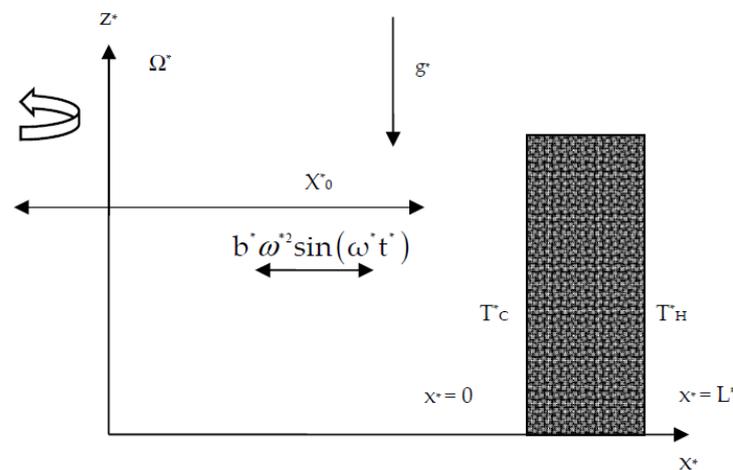
In the current study, the porous layer is placed far from the axis of rotation, in order to recover the case when the centrifugal effect is constant due to the offset distance  $X_0^*$ . When  $X_0^* \gg L^*$ , where  $L^*$  is the porous medium length, the variation of the centrifugal force within the porous layer is small compared to the centrifugal force due to  $X_0^*$ . From hereon, variables with the superscript "\*" refer to dimensional quantities, whilst those without the "\*" refer to dimensionless variables. For the special case of the porous layer placed far away from the axis of rotation, the constant centrifugal force due to offset distance  $X_0^*$  is analogous to the case of convection in a differentially heated porous layer subjected to g-jitter. In the following, a comparison is made between the current case and the case involving gravity only to illustrate this point. Figure 1 shows a vertical fluid saturated porous layer, placed far away from the axis of rotation, and subjected to vibration and internal heat generation. In addition to internal heat generation, the porous layer is also differentially heated, with a precise definition of the boundary conditions later in this section. Internal heat generation refers to heat generation (as a source) from within the porous layer per unit volume. A typical example of internal heat generation in an engineering application could be a nuclear reactor. The porous layer is sandwiched between two impermeable vertical boundaries (a

distance  $L^*$  apart), subjected to centrifugal effects  $\Omega^{*2}\chi_0^*$ , and vibration  $b^*\omega^{*2}\sin(\omega^*t^*)$ . At  $x^* = 0, T^* = T_C^*$ , and at  $x^* = L^*, T^* = T_H^*$ , and the Boussinesq approximation is applied to model the effects of the density variations. Here,  $T_H^*$  and  $T_C^*$  are the hot wall and cold wall temperatures,  $\Omega^*$  is the angular velocity,  $\omega^*$  represents the vibration frequency, and  $t^*$  is the time variable. This results in the following system of dimensional equations for continuity, momentum, and energy:

$$\nabla^* \cdot \mathbf{V}^* = 0, \tag{1}$$

$$\left(\frac{\rho_0^*}{\phi^*} \frac{\partial}{\partial t^*} + 1\right) \mathbf{V}^* = \frac{\chi^*}{\mu^*} \left\{ -\nabla^* p^* + \rho^* \left[ \Omega^{*2}\chi_0^* + b^*\omega^{*2}\sin(\omega^*t^*) \right] \hat{\mathbf{e}}_x - \rho^* \mathbf{g}^* \hat{\mathbf{e}}_z \right\}, \tag{2}$$

$$\frac{\partial T^*}{\partial t^*} + \mathbf{V}^* \cdot \nabla^* T^* = \kappa^* \left( \nabla^{*2} T^* + \frac{q^*}{k_m^*} \right). \tag{3}$$



**Figure 1.** Vertical rotating porous layer, placed far from axis of rotation, and subjected to centrifugal jitter and internal heat generation.  $T_H^*$  and  $T_C^*$  are the dimensional hot wall temperature and cold wall temperatures,  $t^*$  is the dimensional time variable, and  $\Omega^*$  and  $\omega^*$  are the rotational speed and vibration frequency. The length the porous layer is  $L^*$  and the offset distance is  $X_0^*$  whilst the gravity is defined as  $g^*$ .

In the system (1)–(3),  $\mathbf{V}^*$  is the fluid velocity,  $p^*$  is the pressure, the density is denoted by  $\rho^*$ , the temperature is denoted by  $T^*$ , the permeability of the porous medium is denoted by  $\chi^*$ ,  $\mu^*$  is the dynamic viscosity,  $k_m^*$  is the fluid thermal conductivity, and  $\hat{\mathbf{e}}_x$  and  $\hat{\mathbf{e}}_z$  are the unit vectors in the  $x$ - and  $z$ - directions.

The governing equations may be non-dimensionalized using the scaling variables  $\kappa^*/L^*$ ,  $\mu^*\kappa^*/k_0^*$ , and  $\Delta T^* = T_H^* - T_C^*$  for the filtration velocity components ( $u^*, v^*, w^*$ ), reduced pressure ( $p^*$ ), and the temperature variations  $T^* - T_C^*$ . For this scaling,  $\kappa^*$  is the thermal diffusivity, and  $k_0^*$  is a characteristic permeability associated with the porous medium. The height of the porous medium  $L^*$  is used to scale all the spatial lengths according to the relations  $x = x^*/L^*$ ,  $y = y^*/H^*$ , and  $z = z^*/L^*$ . Applying the scaling factors to Equations (1)–(3) yields the following governing equations for the specific case of constant permeability (i.e.,  $\chi = 1$ ):

$$\nabla \cdot \mathbf{V} = 0, \tag{4}$$

$$\left(\frac{1}{\text{Va}} \frac{\partial}{\partial t} + 1\right) \mathbf{V} = \{-\nabla p - \text{Ra}_{\omega 0} [1 + \delta \sin(\omega t)]\} T \hat{\mathbf{e}}_x + \text{Ra}_g T \hat{\mathbf{e}}_z, \tag{5}$$

$$\frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T = \nabla^2 T + 1. \tag{6}$$

The key non-dimensional parameters that emanate from the rescaling of the Equations (1)–(3) are the gravitational Rayleigh number,  $Ra_g = [\rho_0^* \beta^* g^* H^* k_0^* / (\mu^* \kappa^*)] (q^* H^{*2} / k_m^*)$ ; the centrifugal Rayleigh number,  $Ra_{\omega 0} = [\rho_0^* \beta^* \Omega^{*2} \chi_0^* H^* k_0^* / (\mu^* \kappa^*)] (q^* H^{*2} / k_m^*)$ ; and the vibration amplitude  $\delta = \eta Fr \omega^2$ , where  $\eta = b^* / H^*$  and the Froude number  $Fr = \kappa^{*2} / (\Omega^{*2} \chi_0^* H^{*3})$ . Here,  $H^*$  is the height of the horizontal porous layer. In Equation (5),  $Va$  is the Vadasz number, defined as  $Va = \phi^* Pr / Da$  (where  $Pr$  is the Prandtl number and  $Da$  is the Darcy number, defined as  $Da = k_0^* / L^{*2}$ ); and the symbols  $\mathbf{V}$ ,  $T$ , and  $p_r$  represent the dimensionless filtration velocity vector, temperature, and reduced pressure, respectively. In a previous study [9], the author has put forward a motivation for specific cases when the Vadasz number is small and is retained. In the current study, typical fluids used in engineering applications and selected heavy liquid metals were considered to determine the Vadasz number,  $Va$ , as a function of the porosity, as shown in Figures 2 and 3. Here, we use the Kozeny equation of the form  $k^* = L^{*2} \phi^{*3} / [172.8(1 - \phi^*)^2]$  to estimate the permeability when calculating the Darcy number, and the Vadasz number.

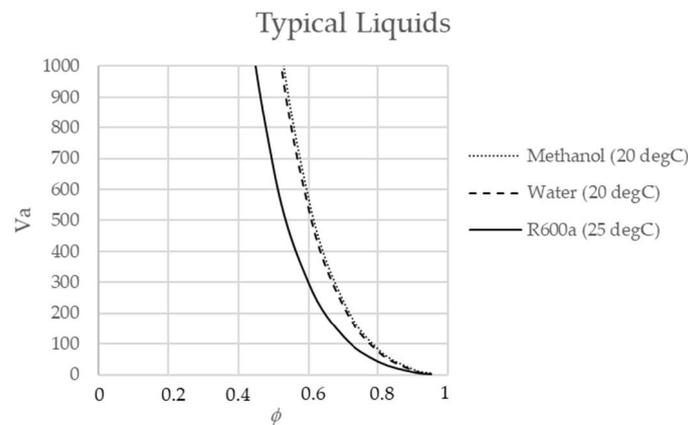


Figure 2. Vadasz number illustrated for typical liquids, used in engineering applications. Here,  $\phi$  is the porosity.

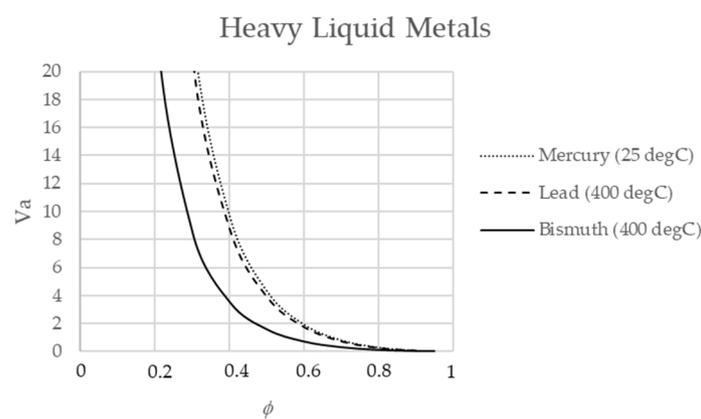


Figure 3. Vadasz number, illustrated for heavy liquid metals. Here,  $\phi$  is the porosity.

Figure 2 shows that when the porosity is low, the Vadasz number is very large, and this leads to the neglect of the time derivative in Equation (5). In the instance when the Vadasz number is very large, the presence of centrifugal jitter has no effect on the stability of convection. Figure 2 also shows that when the porosity is large, the Vadasz number could attain values that allow for the retention of the time derivative in Equation (5). In the case of high porosity media, the Brinkman model may need to be adopted in the formulation of the momentum equation. The introduction of the Brinkman term in Equation (2) is

the subject of current study, and the study presented in [25] for a rotating vertical porous layer, excluding gravity and Coriolis effects (for both the Darcy and Brinkman models), is consulted in that analysis.

In contrast, for heavy liquid metals, it is clearly demonstrated in Figure 3 that that Vadasz number is orders of magnitude smaller across the entire range of porosity values. Figure 3 shows that across the range of porosity values indicated, the Vadasz number is of an order of magnitude that allows for the retention of the time derivative in Equation (5).

Therefore, for systems using typical liquids with high porosity media, liquid metals allow for the retention of the time derivative in Equation (5), which, in turn, allows the centrifugal jitter to impact the convection stability criteria. The solutions for the basic temperature and flow field are given as  $T_B = 1/2(3x - x^2)$  and  $w_B = -1/2Ra_g(x^2 - 3x + 7/6)$ . In providing a solution to the system of equations, it is convenient to apply the curl operator ( $\nabla \times$ ) twice on Equation (5) and, using Equation (4), consider only the horizontal x-component of the result to yield,

$$\left(\frac{1}{Va} \frac{\partial}{\partial t} + 1\right) \nabla^2 u - Ra_{\omega 0} [1 + \delta \sin(\omega t)] \nabla_V^2 T + Ra_g \frac{\partial^2 T}{\partial x \partial z} = 0, \tag{7}$$

where  $\nabla_V^2 = \partial^2 / \partial y^2 + \partial^2 / \partial z^2$ .

### 3. Linear Stability Analysis

Assuming small perturbations around the basic solution of the form  $T = T_B + T'$  and  $w = w_B + w'$ , the linearizing Equations (4)–(7) yield the following equations:

$$\left(\frac{1}{Va} \frac{\partial}{\partial t} + 1\right) \nabla^2 u' - Ra_{\omega 0} [1 + \delta \sin(\omega t)] \nabla_V^2 T' + Ra_g \frac{\partial^2 T'}{\partial x \partial z} = 0, \tag{8}$$

$$\frac{dT_B}{dz} u' = \left(\nabla^2 T' - \frac{\partial T'}{\partial t} - w_B \frac{\partial T'}{\partial z}\right). \tag{9}$$

In Equations (8) and (9), the primes denote the perturbed quantities. Here, in any event, the cases of large and small to moderate Vadasz numbers are considered separately to demonstrate the effect of the Vadasz number on the stability of convection.

#### 3.1. Large Vadasz Numbers

When using typical engineering working fluids in systems with low porosity, large Vadasz numbers result. For large Vadasz numbers, Equations (8) and (9) may be written as follows:

$$\nabla^2 u' - Ra_{\omega 0} [1 + \delta \sin(\omega t)] \nabla_V^2 T' + Ra_g \frac{\partial^2 T'}{\partial x \partial z} = 0, \tag{10}$$

$$\frac{dT_B}{dz} u' = \left(\nabla^2 T' - \frac{\partial T'}{\partial t} - w_B \frac{\partial T'}{\partial z}\right). \tag{11}$$

Substituting Equation (11) in Equation (10) and simplifying yields the following equation for  $T'$ :

$$\nabla^2 \left(\frac{\nabla^2 T' - \frac{\partial T'}{\partial t} - w_B \frac{\partial T'}{\partial z}}{\frac{dT_B}{dz}}\right) - Ra_{\omega 0} [1 + \delta \sin(\omega t)] \nabla_V^2 T' + Ra_g \frac{\partial^2 T'}{\partial x \partial z} = 0. \tag{12}$$

Assuming an expansion into normal modes in the y- and z-directions of the form yields the following:

$$T' = \exp(is_y y + is_z z) \sum_{k=1}^N a_k(t) \sin(k\pi x). \tag{13}$$

which satisfies the boundary conditions  $T' = 0$  and  $w' = 0$  at  $x = 0$  and  $x = 1$ . In Equation (10).  $s^2 = s_y^2 + s_z^2$ , and  $k = 1, 2, 3, \dots N$ . It was shown in [11] that Equation (12) only yields real solutions when  $s_z = 0$ , thus indicating that gravity plays a passive role in the convection threshold criteria. Taking note of the findings in [11], substituting Equation (13) into Equation (12), and then multiplying the result by orthogonal functions and integrating over  $x \in [0, 1]$  yields the following system of equations:

$$\sum_{k=1}^N \left\{ \left( k^2 \pi^2 + s^2 + \frac{\partial}{\partial t} \right) \left( -\frac{1}{2} + \left( k^2 \pi^2 + s^2 \right) \left( \frac{D}{2} (D-1) + \frac{1}{6} - \frac{1}{\pi^2 (k+1)^2} \right) \right) - \text{Ra}_{\omega 0} s^2 (1 + \delta \sin(\omega t)) \left( \frac{1}{2} (D-1) \left( D^2 - \frac{D}{2} + \frac{1}{2} \right) + \frac{1}{8} + \frac{3}{2} \frac{(1-2D)}{\pi^2 (1+k)^2} \right) \right\} \xi_{lk} a_k + \text{Ra}_{\omega 0} s^2 (1 + \delta \sin(\omega t)) \left( \frac{6kl(D-1)^2}{\pi^2} - \frac{3kl(1-2D)}{\pi^2} - \frac{6}{\pi^4} \left( \frac{1}{(1-k)^4} - \frac{1}{(1+k)^4} \right) \right) \xi_{l+k, 2p-1} a_k = 0. \tag{14}$$

where  $a$  is the vibration amplitude,  $D = 3/2$ ,  $\xi$  is the Kronecker delta function defined as  $\xi_{lk} = 1$  when  $l = k$ , and  $\xi_{l+k, 2p-1}$  is 1 when  $(l+k)$  is odd, and zero otherwise. As can be observed, the system shown in Equation (14) becomes rather complicated when solved to higher ranks of  $N$ . A numerical solver may be used to solve Equation (14) to higher ranks, but that is outside the scope of the research. Useful information may be drawn from rank  $N = 1$ . For the current study, we consider rank  $N = 1$ , and Equation (14) may be presented as follows:

$$\frac{da_1}{dt} + F_0 s^2 [-(\text{Ra}_{\omega 0} - R_0) - \text{Ra}_{\omega 0} \delta \sin(\omega t)] a_1 = 0, \tag{15}$$

where

$$F_0 = \frac{\frac{5}{8} - \frac{3}{4\pi^2}}{\left[ (s^2 + \pi^2) \left( \frac{13}{24} - \frac{1}{4\pi^2} \right) - 1 \right]}, \tag{16}$$

and

$$R_0 = \frac{(s^2 + \pi^2) \left[ (s^2 + \pi^2) \left( \frac{13}{24} - \frac{1}{4\pi^2} \right) - 1 \right]}{s^2 \left( \frac{5}{8} - \frac{3}{4\pi^2} \right)}, \tag{17}$$

where Equation (17), incidentally, represents the Rayleigh number corresponding to internal heat generation with no vibration. Solving the first order differential equation in Equation (15) yields the following:

$$a_1 = b_0 \exp \left\{ F_0 s^2 \left[ \sigma t - \frac{\text{Ra}_{\omega 0} \delta}{\omega} \cos(\omega t) \right] \right\}, \tag{18}$$

where  $b_0$  is an integration constant and  $\sigma = \text{Ra}_{\omega 0} - R_0$ . For stationary convection,  $\sigma = 0$ , therefore,

$$\text{Ra}_{\omega 0} = R_0, \tag{19}$$

and

$$a_1 = b_0 \exp \left[ -F_0 s^2 \text{Ra}_0 \kappa \text{Fr} \omega \cos(\omega t) \right]. \tag{20}$$

In Equation (19),  $R_0$  represents the Rayleigh number when the vibration frequency is zero. From Equation (19) it is clear that for large Vadasz numbers, the stability criteria for convection are the same as that for the case of no vibration. This simply means that vibration has no effect on the stability criteria for a porous layer with internal heat generation when the Vadasz number, is large. In engineering practice, if the centrifugal jitter effect is required to stabilize the convection, then for a given fluid (as per Figure 2), one would need to increase the porosity of the matrix to reduce the Vadasz number. Conversely, if the effects of centrifugal jitter are present and one does not want it to affect the convection threshold, then one may choose a working fluid and porosity combination to ensure that a

large Vadasz number, results to ensure that the time derivative is neglected, thus resulting in the criteria presented by Equation (19).

### 3.2. Small to Moderate Vadasz Numbers

When using non-traditional working fluids, such as liquid metals (Figure 3), small to moderate Vadasz numbers result across the range of porosity values. For small to moderate Vadasz numbers, substituting Equation (9) in Equation (8) and simplifying yields the following equation for  $T'$ :

$$\left(\frac{1}{Va} \frac{\partial}{\partial t} + 1\right) \nabla^2 \left( \frac{\nabla^2 T' - \frac{\partial T'}{\partial t} - w_B \frac{\partial T'}{\partial z}}{\frac{dT_B}{dz}} \right) - Ra_{\omega 0} [1 + \delta \sin(\omega t)] \nabla_V^2 T' + Ra_g \frac{\partial^2 T'}{\partial x \partial z} = 0. \tag{21}$$

We assume an expansion into normal modes in the y- and z- directions as per Equation (13); substituting in Equation (21) and then multiplying the result by orthogonal functions and integrating over  $x \in [0, 1]$  yields the following system of equations:

$$\begin{aligned} \sum_{k=1}^N \left\{ \left( \frac{1}{Va} \frac{\partial}{\partial t} + 1 \right) \left( k^2 \pi^2 + s^2 + \frac{\partial}{\partial t} \right) \left( -\frac{1}{2} + \left( k^2 \pi^2 + s^2 \right) \left( \frac{D}{2} (D-1) + \frac{1}{6} - \frac{1}{\pi^2 (k+1)^2} \right) \right) - \right. \\ \left. Ra_{\omega 0} s^2 (1 + \delta \sin(\omega t)) \left( \frac{1}{2} (D-1) \left( D^2 - \frac{D}{2} + \frac{1}{2} \right) + \frac{1}{8} + \frac{3}{2} \frac{(1-2D)}{\pi^2 (1+k)^2} \right) \right\} \xi_{lk} a_k + \\ \sum_{k=1}^N \left\{ \left( k^2 \pi^2 + s^2 \right) \left( \frac{1}{Va} \frac{\partial}{\partial t} + 1 \right) \left( k^2 \pi^2 + s^2 + \frac{\partial}{\partial t} \right) \frac{2kl(2D-1)}{\pi^2 (k^2-l^2)^2} - \right. \\ \left. Ra_{\omega 0} s^2 (1 + \delta \sin(\omega t)) \left( \frac{6kl(D-1)^2}{\pi^2} - \frac{3kl(1-2D)}{\pi^2} - \frac{6}{\pi^4} \left( \frac{1}{(1-k)^4} - \frac{1}{(1+k)^4} \right) \right) \right\} \xi_{l+k, 2p-1} a_k = 0. \end{aligned} \tag{22}$$

As before, useful information may be drawn from rank  $N = 1$ , which may be presented as follows:

$$\frac{d^2 a_1}{dt^2} + (Va + s^2 + \pi^2) \frac{da_1}{dt} + Va F_0 s^2 [-(Ra_{\omega 0} - R_0) - Ra_{\omega 0} \delta \sin(\omega t)] a_1 = 0, \tag{23}$$

where  $F_0$  and  $Ra_0$  are as defined before. Equation (23) may be transformed into the Mathieu equation by taking  $a_1 = e^{-\sigma \tau} X_1(\tau)$  to ensure that the resulting equation may be presented as follows:

$$X''_1 + [A + 2Q \cos(2\tau)] X_1 = 0. \tag{24}$$

In Equation (24), the coefficients  $A$  and  $2Q$  are defined for stationary convection as follows:

$$A = - \frac{4Vas^2 \left( \frac{5}{8} - \frac{3}{4\pi^2} \right)}{\omega^2 \left[ (s^2 + \pi^2) \left( \frac{13}{24} - \frac{1}{4\pi^2} \right) - 1 \right]} (Ra_{\omega 0} - R_0), \tag{25}$$

$$2Q = \frac{4Vas^2 \left( \frac{5}{8} - \frac{3}{4\pi^2} \right) Ra_{\omega 0} \delta}{\omega^2 \left[ (s^2 + \pi^2) \left( \frac{13}{24} - \frac{1}{4\pi^2} \right) - 1 \right]}. \tag{26}$$

In previous studies [14,20], authors have related  $A$  and  $Q$  in Equations (25) and (26) via an indirect numerical method. Whilst that method recovers both the synchronous and subharmonic modes, it is quite cumbersome. In this paper, we propose the following asymptotic expansions for low amplitude and high frequency vibrations in an attempt to link  $A$  and  $Q$ :

$$X_1 = X_0 + QX_1 + Q^2 X_2 + \dots, \tag{27}$$

$$A = A_0 + QA_1 + Q^2 A_2 + \dots, \tag{28}$$

where  $Q$ , as defined in Equation (26), assumes small values for high vibration frequencies. The methodology used in the derivation of Equations (29) and (30) involves the use of simple asymptotic expansions, i.e., Equations (27) and (28), that are then applied on the Mathieu equation, Equation (24). The resulting equations are solved to the various orders

in  $Q$ , noting that one needs to apply the solvability condition to the resulting differential equations. The result shown in Equation (30), therefore, is the solvability condition. The relation between  $A$  and  $Q$  are then used in conjunction with their definitions in Equations (25) and (26) to yield the characteristic equation for the Rayleigh number. In the derivation of Equations (29) and (30), the coefficients of the exponential growth terms are set to zero and the constant,  $c_0 > 0$ , is an integration constant (and a real number). The resulting solutions to rank  $N = 1$  and the solvability condition may be represented as follows:

$$a_1 = X_1 = c_0 \left[ 1 - \frac{Q}{2} \cos(2\tau) \right], \tag{29}$$

$$A = -\frac{1}{2}Q^2. \tag{30}$$

Using Equations (25) and (26) and substituting in Equation (30) yields the following characteristic equation for the Rayleigh number:

$$\frac{Vas^2 \left( \frac{5}{16} - \frac{3}{8\pi^2} \right)}{\omega^2 \left[ (s^2 + \pi^2) \left( \frac{13}{24} - \frac{1}{4\pi^2} \right) - 1 \right]} (\kappa Fr \omega)^2 Ra_{\omega 0}^2 - Ra_{\omega 0} + Ra_0 = 0. \tag{31}$$

From Equation (31), one may observe that for the case when the vibration frequency approaches zero, i.e.,  $\omega \rightarrow 0$ , the Rayleigh number approaches the definition,  $Ra_{\omega 0} \rightarrow Ra_0$ . Equation (31) is solved for both zero and non-zero vibration frequencies. The Vadasz number and the Rayleigh numbers are henceforth scaled as  $\gamma = Va/\pi^2$ ,  $R = Ra_{\omega 0}/\pi^2$ , and  $R_0 = Ra_0/\pi^2$ .

#### 4. Results and Discussion

When the vibration is zero, the critical Rayleigh number for internal heat generation to rank  $N = 1$  for the horizontal porous layer subjected to gravity (and no rotation) is  $R_{0,cr.} = 3.819\pi^2$ , whilst the critical wavenumber is  $s_{cr.} = 0.49\pi$ . In contrast, for the vertical rotating porous layer, placed far from the axis of rotation and subjected to internal heat generation, the critical Rayleigh number is  $R_{0,cr.} = 3.391\pi^2$  and the critical wavenumber is  $s_{cr.} = 0.81\pi$ . These critical Rayleigh and wavenumbers are then compared to the critical Rayleigh number and wavenumber for the classical Benard convection stated as  $R_{0,cr.} = 4\pi^2$  and  $s_{cr.} = \pi$ . Figure 4 shows the three mentioned cases for comparison purposes, and it is clear from the results that the critical Rayleigh number for the rotating porous layer presents the most unstable case in terms of the convection threshold.

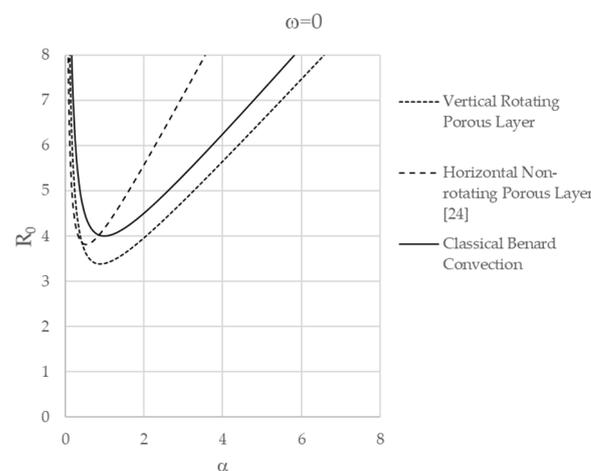
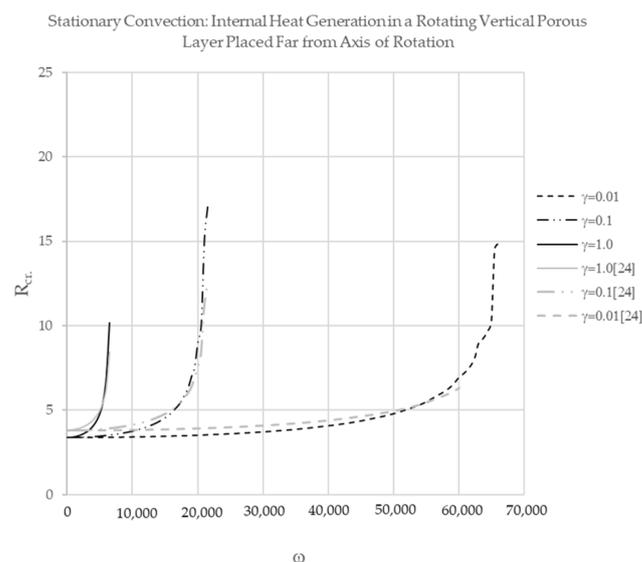


Figure 4. Comparison of characteristic Rayleigh number curves for the horizontal non-rotating porous layer [24], and vertical rotating porous layer, with internal heat generation, to the classical Benard convection characteristic curves.  $\alpha$  denotes the wavenumber.

Previously, the authors [24] showed that for the horizontal porous layer, subjected to internal heat generation and g-jitter, the critical Rayleigh number is significantly lower than the modified internal Rayleigh number of 470 ( $47.621\pi^2$ ) predicted in [17]. When comparing the critical Rayleigh number for the rotating porous layer, placed far from the axis of rotation and subjected to internal heat generation, one observes that although comparable to [24], the magnitude is still significantly smaller than [17].

Figure 5 shows the critical Rayleigh number versus the vibration frequency for selected values of the scaled Vadasz number, for the vertical porous layer placed far away from the axis of rotation and subjected to internal heat generation (black lines). Superimposed on Figure 5 are the data generated in [24] for a horizontal porous layer subjected to gravity (no rotation) and internal heat generation (gray lines). It is observed in Figure 5 that each of the curves end abruptly at specific frequency asymptotes, beyond which there are no more real solutions. These points also denote the end of the synchronous frequencies and the start of the subharmonic frequencies. As stated initially, the current solution approach does not recover solutions corresponding to subharmonic frequencies. The solution approach and methodology presented in [13,14] recovers the transition point from synchronous to subharmonic solutions. The impact of increasing the Vadasz number causes the onset of the subharmonic solutions at lower frequency values, as indicated in Figure 5. At very low-scaled Vadasz numbers, circa  $\gamma = 0.01$ , the onset of the subharmonic frequencies occurs at very large vibration frequencies.



**Figure 5.** Critical Rayleigh number versus vibration frequency for selected values of Vadasz number for a vertical porous layer placed far away from the axis of rotation (black lines). The results for the non-rotating horizontal porous layer with internal heat generation [24] (gray lines) superimposed.

The results show that the onset of convection for the vertical rotating porous layer placed far from the axis of rotation and subjected to centrifugal jitter occurs sooner across the bandwidth of vibration frequencies for a particular Vadasz number. As observed in Figure 5, increasing the Vadasz number stabilizes the convection.

## 5. Conclusions

The results for internal heat generation within a rotating porous layer placed far from the axis of rotation and subjected to centrifugal jitter is presented. Both the typical fluids used on engineering applications and heavy liquid metals were analyzed in the current study. The results of this analysis showed that for the typical fluids used in engineering applications, when the porosity is medium to low, the Vadasz number is large, thus neglecting the effects of the time derivative in the momentum equation. In principle, for large Vadasz numbers, the presence of centrifugal jitter has no impact on the stability

criteria for convection. However, even with typical engineering working fluids, it was established that a small Vadasz number could result when the porosity is high. In the instance of high porosity media, the Brinkman model may need to be considered when determining the stability criteria. For the instance when heavy liquid metals are considered, it was established that the Vadasz number is many orders of magnitude smaller across the entire range of porosity values when compared to the case of typical engineering working fluids. For large Vadasz numbers, it was shown that the convection stability criteria are independent of the centrifugal jitter.

For small to moderate Vadasz numbers, the results show that in comparison to the case of the horizontal (non-rotating porous layer) subjected to internal heat generation [24], the convection occurs sooner (i.e., at a lower Rayleigh number). However, although these two cases are comparable, the magnitude of the Rayleigh numbers are still significantly smaller than those predicted in [17].

**Funding:** This research received no external funding.

**Data Availability Statement:** Data sharing not applicable.

**Acknowledgments:** The author dedicates this work to his second daughter Vinayia-Chrisinithi Govender, for her interest and questions on the topics that I research.

**Conflicts of Interest:** The author declares no conflict of interest.

## References

1. Horton, C.W.; Rogers, F.T. Convection currents in a porous medium. *J. Appl. Phys.* **1945**, *16*, 367–370. [[CrossRef](#)]
2. Lapwood, E.R. Convection of a fluid in a porous medium. *Math. Proc. Camb. Philos. Soc.* **1948**, *44*, 508–521. [[CrossRef](#)]
3. Wooding, R.A. Rayleigh instability of a thermal boundary layer in a flow through a porous medium. *J. Fluid Mech.* **1960**, *9*, 183–192. [[CrossRef](#)]
4. Palm, E.; Weber, J.E.; Kvernfold, O. On steady convection in a porous medium. *J. Fluid Mech.* **1972**, *54*, 153–161. [[CrossRef](#)]
5. Vadasz, P. Natural convection in porous media induced by the centrifugal body force—The solution for small aspect ratio. *J. Energy Resour. Technol.* **1992**, *114*, 250–254. [[CrossRef](#)]
6. Vadasz, P. Centrifugally generated free convection in a rotating porous box. *Int. J. Heat Mass Transf.* **1994**, *37*, 2399–2404. [[CrossRef](#)]
7. Vadasz, P. Stability of free convection in a narrow porous layer subject to rotation. *Int. Commun. Heat Mass Transf.* **1994**, *21*, 881–890. [[CrossRef](#)]
8. Vadasz, P. Stability of free convection in a rotating porous layer distant from the axis of rotation. *Transp. Porous Media* **1996**, *23*, 153–173. [[CrossRef](#)]
9. Vadasz, P. Coriolis effect on gravity driven convection on a rotating porous layer heated from below. *J. Fluid Mech.* **1998**, *376*, 351–375. [[CrossRef](#)]
10. Govender, S. Oscillatory convection induced by gravity and centrifugal forces in a rotating porous layer distant from the axis of rotation. *Int. J. Eng. Sci.* **2003**, *41*, 539–545. [[CrossRef](#)]
11. Govender, S. Coriolis effect on the linear stability of convection in a porous layer placed far away from the axis of rotation. *Transp. Porous Media* **2003**, *51*, 315–326. [[CrossRef](#)]
12. Straughan, B. A sharp nonlinear stability threshold in rotating porous convection. *Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci.* **2001**, *457*, 87–93. [[CrossRef](#)]
13. Govender, S. Stability of convection in a gravity modulated porous layer heated from below. *Transp. Porous Media* **2004**, *57*, 103–112. [[CrossRef](#)]
14. Govender, S. Linear stability of gravity modulated convection in a porous layer heated from below: Transition from synchronous to subharmonic solutions. *Transp. Porous Media* **2005**, *59*, 215–225. [[CrossRef](#)]
15. Govender, S. Stability of gravity driven convection in a cylindrical porous layer heated from below. *Transp. Porous Media* **2006**, *63*, 489–502. [[CrossRef](#)]
16. Govender, S. Vibration effects on convection in a rotating fluid saturated porous layer distant from the axis of rotation. *Int. J. Heat Mass Transf.* **2019**, *141*, 112–115. [[CrossRef](#)]
17. Gasser, R.D.; Kazimi, M.S. Onset of convection in a porous medium with internal heat generation. *J. Heat Transf.* **1976**, *20*, 49–54. [[CrossRef](#)]
18. Tveitereid, M. Thermal convection in a horizontal porous layer with internal heat sources. *Int. J. Heat Mass Transf.* **1977**, *20*, 1045–1050. [[CrossRef](#)]
19. Bhadauria, B.S.; Hashim, I.; Siddheshwar, P.G. Study of heat transport in a porous medium under g-jitter and internal heating effects. *Transp. Porous Media* **2013**, *96*, 21–37. [[CrossRef](#)]

20. Yadav, D.; Wang, J.; Lee, J. Onset of Darcy-Brinkman convection in a rotating porous layer induced by purely internal heating. *J. Porous Media* **2017**, *20*, 691–706. [[CrossRef](#)]
21. Mahajan, A.; Nandal, R. Stability of an anisotropic porous layer with internal heat source and Brinkman effects. *Spec. Top. Rev. Porous Media* **2019**, *10*, 65–87. [[CrossRef](#)]
22. Storesletten, L.; Rees, D.A.S. Onset of convection in an inclined anisotropic porous layer with internal heat generation. *Fluids* **2019**, *4*, 75. [[CrossRef](#)]
23. Nield, D.; Bejan, A. *Convection in Porous Media*, 4th ed.; Springer: New York, NY, USA, 2017; pp. 1–983.
24. Govender, S. Vadasz number effects on convection in a horizontal porous layer subjected to internal heat generation and g-jitter. *Fluids* **2020**, *5*, 124. [[CrossRef](#)]
25. Saravanan, S.; Brindha, D. Global stability of centrifugal filtration convection. *J. Math. Anal. Appl.* **2010**, *367*, 116–128. [[CrossRef](#)]