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Phase Diagram for Social Impact Theory in Initially Fully Differentiated Society

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Abstract: The study of opinion formation and dynamics is one of the core topics in sociophysics. In this paper, the results of computer simulation of opinion dynamics based on social impact theory are presented. The simulations are based on Latané theory in its computerised version proposed by Nowak, Szamrej and Latané. The active parameters of the model describe the volatility of the actors (social temperature T) and the effective range of interaction (governed by an exponent α in a scaling function of distance between actors). Initially, every actor i has his/her own opinion. Our results indicate that ultimately at least 90% of the initial opinions available are removed from the society. For a low social temperature and a long range of interaction, only one opinion survives. Also, a rough sketch of the system phase diagram is presented. It indicates a set of (α, T) leading either to (1) the dominance of the unanimity of the opinions or (2) mixtures of unanimity and polarisation, or (3) taking random opinions by actors, or (4) a mixture of the final fates of the systems. The drastic reduction of finally observed opinions vs. their initial variety may be generic for many sociophysical models of opinions formation but masked by assuming an initially small pool of available opinions (in the worst case, in models with only binary opinions).

Keywords: sociophysics; social impact; opinion dynamics; social temperature; clustering and polarisation

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1. Introduction

The studies of opinion formation and dynamics are one of the core topics in sociophysics [1–6]. For example, Galam models of opinion dynamics [7–20] are based on the reaction–diffusion model: the dynamics operates via local update rules and reshuffling. In these models, three kinds of actors correspond to floaters, contrarians, and inflexibles. The models assume two or three opinions available in the society [21]. Among other discrete models of opinion formation, one should mention the majority rule [8,10,22,23], voter [24–27], and Sznajd [28–32] models. In these models, usually only binary opinions are considered, which naturally causes society polarisation. However, modifications that allow for multiple opinions were also studied [33–45].

The Nowak–Szamrej–Latané model [46] is based on the Latané social impact theory [47–49]. Latané himself defined his theory as a "bulb theory" of social impact. According to this physical analogy, every actor plays simultaneously the role of an isotropic single wavelength light emitter and a multi-wavelength light detector. We assume that every actor can emit and detect easily distinguished K various light colours. Every discrete time step t actor i switches the emitted wave length (colour) $\lambda_i(t)$ to that perceptible illuminance is detected in his/her position as the strongest. The decision of which colour $\lambda_i(t+1)$ will be emitted by actor i depends on (i) the number of each colour sources, (ii) the distance from this point to every other source of light, (iii) and intensities (illuminance flux, "bulb" power) of each light source.

The earlier attempts to employ this model for sociophysical studies included: observing the influence of the strong leader on opinion formation [50]; studying the noise-induced order/disorder phase transition [35,51]; searching for self-organised criticality in opinion systems [36]; observing the disappearance of some opinions [37] (see Ref. [52] for review).

In this paper, with a computer simulation based on the Nowak–Szamrej–Latané model [46], we check how the initial diversity of opinions influences the possibility of reaching a unanimity of opinions. Namely, we build a phase diagram in the social temperature and the effective range of the interaction space based on the number of surviving opinions; numbers of clusters of opinions; and the probability distribution of the size of the largest cluster. Unlike previous works—where the number of available opinions was usually small (two [50,51], up to three [36], or up to five [35,37])—we proposed as a starting point a situation in which each actor has their own opinion.

2. Model Formalisation

Every actor i at time t has an opinion $\lambda_i(t)$. The social impact $\mathcal{I}_{i,k}(t)$ exerted in time t on an actor i by all actors who share opinions Λ_k is calculated as

$$\mathcal{I}_{i,k}(t) = \sum_{j=1}^{L^2} \frac{4s_j}{g(d_{i,j})} \cdot \delta(\Lambda_k, \lambda_j(t)) \cdot \delta(\lambda_j(t), \lambda_i(t)) \tag{1}$$

or

$$\mathcal{I}_{i,k}(t) = \sum_{j=1}^{L^2} \frac{4p_j}{g(d_{i,j})} \cdot \delta(\Lambda_k, \lambda_j(t)) \cdot [1 - \delta(\lambda_j(t), \lambda_i(t))], \tag{2}$$

where s_j is j-th actor supportiveness, p_j is j-th actor persuasiveness, $d_{i,j}$ stands for Euclidean distance between actors i and j, $g(\cdot)$ is an arbitrary distance scaling function, and Kronecker delta $\delta(x,y) = 0$ when $x \neq y$ and $\delta(x,y) = 1$ when $x = y$. The sum in Equations (1) and (2) reflects the increase in impact $\mathcal{I}_{i,k}$ by increasing the number of "bulbs" (point (i) of the model description in Section 1), the decrease in impact with the distance between "bulbs" (fraction denominator, point (ii) of the model description in Section 1), and the fraction nominator corresponds to the intensities of "bulbs" (point (iii) of the model description in Section 1). The terms with Kronecker's delta are equal to either zero or to one:

- Equation (1) applies to the calculation of the impact of actors currently sharing the opinion of actor i;
- while Equation (2) allows the calculation of the impact of all other opinions.

The parameters supportiveness s_i and persuasiveness p_i describe i-th actor intensity of interaction with actors sharing their opinions or with believers in opposite opinions, respectively. We decided to use $\forall i : p_i = s_i = 1/2$ (as in Ref. [35]) because whether it is easier to stick to our opinion or change it depends on numerous factors, such as the social context, emotions, beliefs, authorities, and persuasion strategies. People's decisions on this matter can vary widely and depend on individual circumstances and preferences. On the one hand, there are theories that explain the tendency to change opinions, such as: social influence theory and conformity [53]; motivation and belief theory [54]; authority influence theory [55] or persuasion theory [56]. On the other hand, there are theories that point to an advantage in trying to keep our opinions, such as cognitive consistency theory [57] or cognitive dissonance theory [58]. Furthermore, keeping the supportiveness and persuasiveness equal for each actor makes the initial variety of opinions (next to social temperature and range of interactions) the dominant factor in the results of our studies. With such assumption, the social impact (1) and (2) may be reduced to

$$\mathcal{I}_{i,k}(t) = \sum_{j=1}^{L^2} \frac{2 \cdot \delta(\Lambda_k, \lambda_j(t))}{g(d_{i,j})}. \tag{3}$$

To ensure a lower impact on the opinions of actors from a more distant neighbour, the distance scaling function $g(\cdot)$ must be an increasing function of its argument. Here, we assume that

$$g(x) = 1 + x^{\alpha}, \tag{4}$$

where the exponent α is a model control parameter while the first addition component ensures finite self-supportiveness $\mathcal{I}_{i,i}$. The parameter α qualitatively describes the effective range of interaction between the actors. Its quantitative meaning was delivered recently in Ref. [37] where it was shown that for $\alpha = 2$, about 25% of the impact comes from only nine nearest neighbours. This ratio increases to approximately 59%, 80% and 96% for $\alpha = 3, 4$ and 6, respectively. Calculating the relative impact exerted by 25 nearest neighbours gives about 39%, 76%, 92%, and 99% of the total social impact for $\alpha = 2, 3, 4$, and 6, respectively (see Ref. [37], Figure 2, Table 1). In Ref. [37], it is concluded that "the parameter α says how influential the nearest neighbours are with respect to the entire population: the larger α, the more influential the nearest neighbours are".

The example of calculating the social impact of nine actors and three colours is available as supporting information in Ref. [36].

In the deterministic version of the algorithm [37], the actor's opinion $\lambda_i(t+1) = \Lambda_k$, when $\mathcal{I}_{i,k}(t)$ has the maximum value among all impacts $\mathcal{I}_{i,j}(t)$ for $j = 1, \cdots, K$ (3). Following our previous studies [35–37], we employ the actors with "free will" by allowing them to avoid taking the opinions that believers exert the greatest impact on them. This scenario is realised in a probabilistic way by introducing a parameter T often termed "information noise" or "social temperature". Quoting Ref. [59]: "Using the statistical mechanical foundation, [$\cdots$] the most probable collective behaviour depends on a group's social temperature, a measure of the group's decision-making volatility. The extreme of zero temperature leads to stable, unchanging collective behaviour with pockets of minority and majority opinions. As group temperatures increase, the model's collective behaviour tends toward a uniform decision without clustering of minority opinions. When the social temperature exceeds a certain limit, the group will have a well defined average opinion, but individuals are no longer stable and vacillate in a nearly random manner between different possible opinions". Quoting Ref. [60] by the same authors: "Individuals are influenced by the group's temperature. When a group's social temperature is high, very little provocation is necessary to induce an individual to change opinion. At low group temperatures, individuals appear more phlegmatic or stubborn, and much greater provocation is required to induce a change in opinion. High social temperatures amplify the slightest excuse for change, whereas low temperatures diminish the arguments for change. Note that each individual's opinion strength is unaltered, but as the group's temperature changes, so does an individual's decision making abilities."

In our recent studies [36,37], we observed the exact same effects on the system evolution as described above, when social temperature T is introduced as a parameter in probabilities in time t of taking in the next time step opinion Λ_k by actor i based on Boltzmann-like factors,

$$p_{i,k}(t) = \begin{cases} 0 & \iff \mathcal{I}_{i,k} = 0, \\ \exp\left(\frac{\mathcal{I}_{i,k}(t)}{T}\right) & \iff \mathcal{I}_{i,k} > 0, \end{cases} \tag{5}$$

which yet requires proper normalisation,

$$P_{i,k}(t) = \frac{p_{i,k}(t)}{\sum_{j=1}^{K} p_{i,j}(t)}. \tag{6}$$

In contrast to our earlier attempt, due to a huge number of available opinions in the system, we reset to zero probability $p_{i,k}(t)$ when the impact from opinion Λ_k holders is zero, i.e., when believers of this opinion vanished.

Initially, every actor i has his/her opinion $\lambda_i(t = 0) = \Lambda_i = i$ among the $K = L^2$ opinions available in the system (see the Listing A1 in Appendix B). The time evolution for $L^2 = 21^2$ actors who decorate the nodes of the square network takes $\tau = 10^5$ time steps. The single step is completed when all L^2 actors attempt to change their opinions. The results are averaged over $R = 10^2$ independent simulations. The averaging procedure is marked by a $\langle \cdots \rangle$. The open boundary conditions are assumed.

To learn more about the spatial distribution of opinions, we detect, count, and measure the sizes of clusters of agents sharing the same opinion. To this end, we apply the Hoshen–Kopelman algorithm [61] (see also [62] (pp. 59–60), [63,64]) allowing the labelling of every actor in such a way that actors who share the same opinions in various clusters are labelled with various labels and actors belonging to a given cluster are labelled with the same label. The number of clusters and the size of the largest cluster at time $t = \tau$ are indicated as n_c and $S_{\max}$, respectively.

3. Results

In this Section, we present the results of computer simulation—based on a computer programme written in Fortran—for $\tau = 10^5$ and $R = 100$. Typically, simulation for this set of parameters τ and R and a single pair of (α, T) takes around 3 days of Central Processing Unit (CPU) time on the Dell Precision Rack 7920 Workstation with a 3.20 GHz CPU clock.

In Figure 1, the results of simulations concerning the average final (at $t = \tau$): number of opinions, $\langle n_o \rangle$ (Figure 1a), number of clusters $\langle n_c \rangle$ (Figure 1b), and their largest size, $\langle S_{\max} \rangle$ (Figure 1c), are presented. The values of $\langle n_o \rangle$ and $\langle n_c \rangle$ are normalised to the number K of opinions available in the system while $\langle S_{\max} \rangle$ is normalised to the system size L^2. Thus, these numbers are presented as percentages.

In Figure 2, examples of probability distribution function, F, for the size of the largest clusters $\langle S_{\max} \rangle$ are presented. Figure 2a shows the initial distribution (i.e., at $t = 0$, when $n_o = K$) of the largest cluster size, while Figure 2b–j show examples of the typical distribution obtained at the end of simulations, i.e., at $t = \tau$. In Figure 2b,c,h,i, the probability distribution of the largest cluster size for the “corners” of the parameter system plane (α, T)—see Figure 1—are presented. Furthermore, in Figure 2j, we present the function F for the limiting case $T \to \infty$, $\alpha \to \infty$.

In the system of opinions studied, independently of the control parameters of the model, at least about 90% initially available opinions are removed from the system. For low social temperature (small T) and effectively long range of interactions (small α), only a single opinion (when consensus on common opinion in society occurs) or two opinions survive (when system polarisation takes place). For high social temperature (large T) and effectively short range of interactions (large α), no clustering of opinions is observed (with their number reduced as mentioned above). Unlike many binary models, these effects are not embedded in the model rules themselves. With computer simulations of the opinion dynamics model, we have shown that successive disappearing of opinions are naturally associated with social impact theory, and the initial diversity of opinion vanishes in several time steps of system evolution. As mentioned in Section 2, the disappearing of any opinion in a given simulation is irreversible—this is not different from the sociological equivalent of Muller’s ratchet [65] observed also in Eigen’s quasi-species [66].

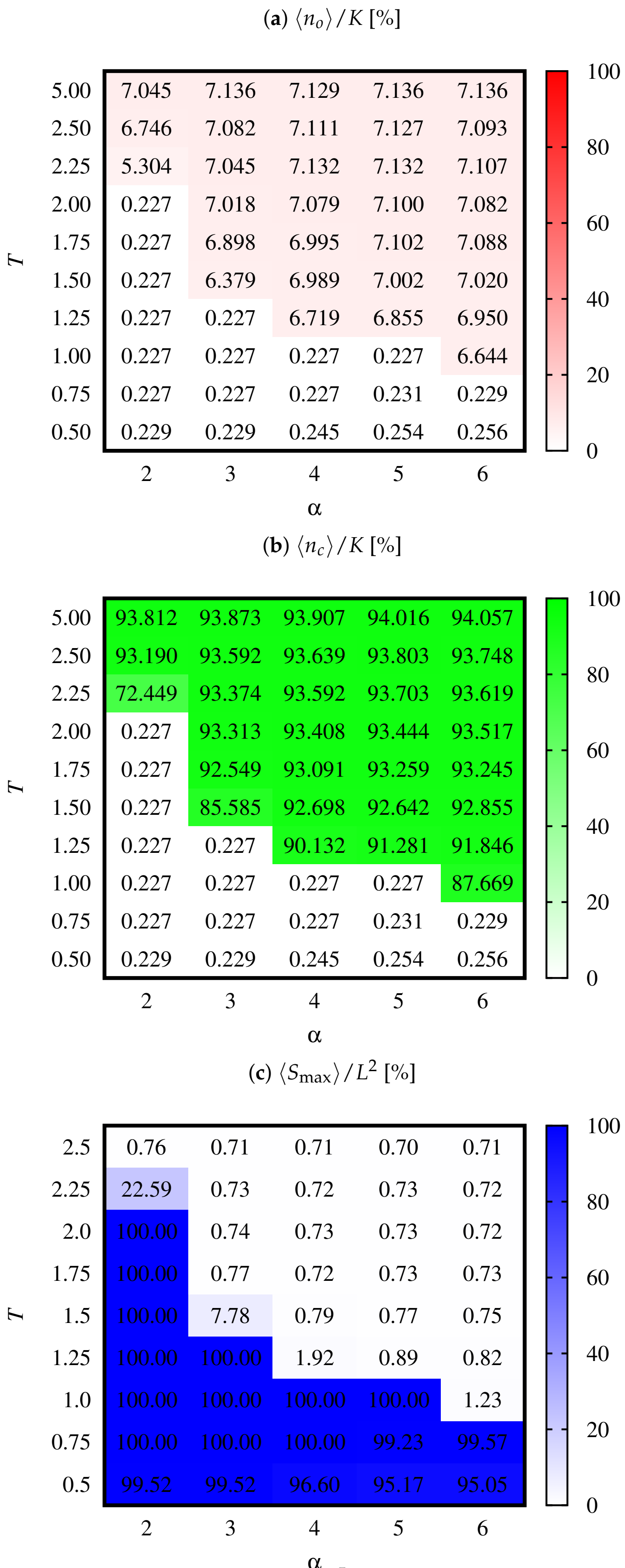

Figure 1. Ultimate (at time $t = 10^5$ steps) averaged (over $R = 10^2$ simulations) numbers (**a**) of observed opinions, $\langle n_o \rangle$, (**b**) clusters of opinions, $\langle n_c \rangle$, and (**c**) the largest cluster size, $\langle S_{\max} \rangle$. The data are normalised to the initial available number of opinions, K (**a**,**b**), and the system size, L^2 (**c**).

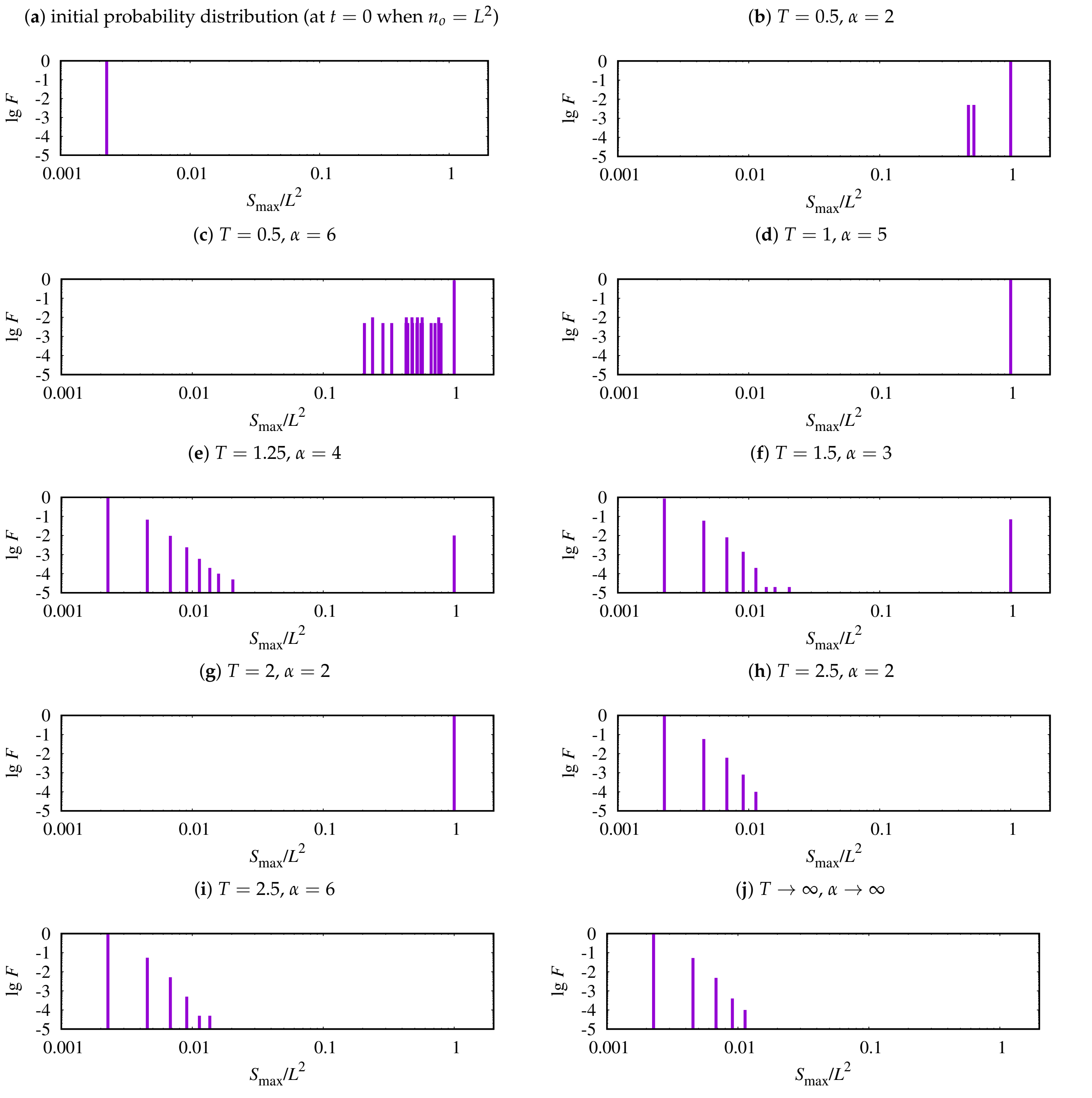

Figure 2. Examples of probability distribution function, F, of the size of the largest clusters $\langle S_{\max} \rangle$ for (**a**) initial configuration (at $t = 0$, $n_o = L^2$), (**b**) $T = 0.5$, $\alpha = 2$ (the lowest left corner of Figure 3), (**c**) $T = 0.5$, $\alpha = 6$ (the lowest right corner of Figure 3), (**d**) $T = 1$, $\alpha = 5$, (**e**) $T = 1.25$, $\alpha = 4$, (**f**) $T = 1.5$, $\alpha = 3$, (**g**) $T = 2$, $\alpha = 2$, (**h**) $T = 2.5$, $\alpha = 2$ (the highest left corner of Figure 3), (**i**) $T = 2.5$, $\alpha = 6$ (the highest right corner of Figure 3), (**j**) $T \to \infty$, $\alpha \to \infty$.

In Appendix A, we show six examples of the time evolution of the number n_o of opinions observed in the system. The presented results are for $T = 0.5$, $\alpha = 2$ (Figure A1a), $T = 2.5$, $\alpha = 6$ (Figure A1b), $T = 0.5$, $\alpha = 2$ (Figure A1c), and $T = 2.5$, $\alpha = 6$ (Figure A1d).

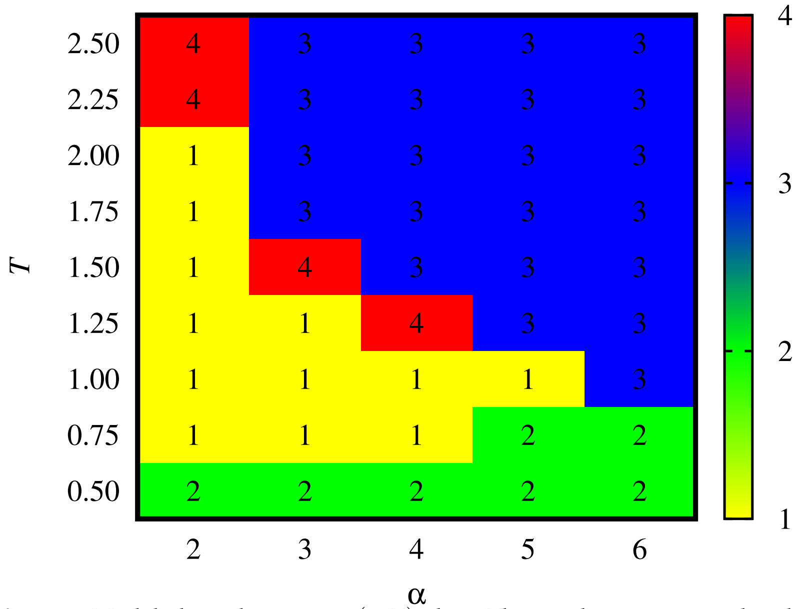

Figure 3. Model phase diagram on (α, T) plain. The numbers correspond to the list enumerator given in Section 4.

4. Discussion

In a single simulation, the number $n_o(t)$ is always a monotonically nonincreasing function of time (see Figure A1). The assumed simulation time $\tau = 10^5$ seems to be long enough to ensure reaching a plateau in the time evolution of n_o. Please note that vanishing during evolution any of the initially available opinion Λ_v, i.e., when at time t_v none of the actors share this opinion Λ_v, then for any $t \geq t_v$, this opinion Λ_v will not be restored. Here, we deal with the sociological equivalent of the famous Muller's ratchet [65] known in evolutionary genetics.

Changes in the number of observed opinions (surviving temporal evolution) $\langle n_o \rangle$ (Figure 1a) are accompanied by changes in the number of clusters $\langle n_c \rangle$ (Figure 1b) and their largest size $\langle S_{\max} \rangle$ (Figure 1c). These numbers bring complementary quantitative information on the system: for instance, for $\alpha = 2$ and $T = 0.5$, one simultaneously has $\langle n_o \rangle = 1$, $\langle n_c \rangle = 1$ and $\langle S_{\max} \rangle = L^2$—which are straightforward signatures of unanimity of opinion.

The analyses of the averages $\langle n_o \rangle$, $\langle n_c \rangle$, and $\langle S_{\max} \rangle$ presented in Figure 1 together with the analyses of the probability distribution function $P(S_{\max})$ shown in Figure 2 allow the identification of four possible phases observed in the system. These phases correspond to:

1. reaching unanimity of opinions ($\langle n_o \rangle = 1$, $\langle n_c \rangle = 1$, $\langle S_{\max} \rangle / L^2 = 1$, probability distribution function of the largest cluster size as in Figure 2d,g);
2. reaching unanimity of opinions or society polarisation (probability distribution function of the largest cluster size as in Figure 2b,c);
3. taking random opinions by actors (probability distribution function of the largest cluster size as in Figure 2h,i);
4. mixture of the phases mentioned above (probability distribution function of the largest cluster size as in Figure 2e,f).

These four scenarios, observed after system time evolution up to $\tau = 10^5$ time steps, can be mapped into (α, T) space to create the phase diagram of the computerised model of the Nowak–Szamrej–Latané social impact theory [46]. This diagram is presented in Figure 3 and the numbers there correspond to the list enumerator given above.

Looking for sociological theories that would explain the disappearance of some of the available opinions, one can refer to Nan Lin's hypothesis on the theory of social capital [67,68]. According to Lin's concept, opinions may be treated as a resource in a social network. The process by which resources in social networks become meaningful and valuable for members of these networks can be considered in relation to several principles ([67], pp. 30–33):

- The first principle has to do with consensus or influence developed or exercised within a group. Consent as to whether a resource is valuable or not can be achieved as a result of persuasion (communication and interaction without sanctions or penalties lead to the formation of an internal conviction in individuals as to the value of a given resource), request (appeals or lobbying result in recognising a given resource as valuable even when the individual does not understand its meaning but wants to remain a member of the group or identify with it), or coercion (an alternative to not recognising the value of a given resource is the threat of sanctions or penalties).
- The second mechanism that allows one to assign value to resources boils down to taking actions by all actors aimed at promoting their own interests by protecting or acquiring valuable resources. For example, it is in the community's well-understood interest to give a higher status to those who, in the opinion of its members (between whom consensus is reached), have valuable resources (knowledge, physical strength, knowledge of members of other communities, etc.). In this sense, the self-interest of individual members of the community becomes convergent with the collective interest (development, security, and cooperation). The devaluation of a given asset requires more than individual effort—it requires the consent of others who make similar demands.
- The third principle regarding valuable resources assumes that their maintenance and acquisition are the two basic motives of individuals' actions, although the former is more important than the latter. Only when the group's resources are secured can its members make an effort to acquire additional resources.

In the case of social resources, two types of mechanisms can be distinguished: network resources to which an individual has access by virtue of membership in that network and contact resources that an individual actually uses in the course of action. The first of them represents constantly available resources due to the durability of social relations in the network, the second represents resources that can be mobilised in order to achieve specific benefits. The nature of the resources contained in the social network to which an individual has access is determined by several factors. First, the range of resources in the network is important, that is, the "distance" between the most valuable and least valuable resource. Second, the most valuable resource available to an individual within the entire hierarchy of resources contained in the network is of importance. Third, the diversity or heterogeneity of resources in a social network plays an important role, and fourth, the composition of resources shaped by those of them that are average or the most typical composition is also significant [67] (p. 37), [68]. In the field of social sciences, Lin's theory is one of the most coherent and well-established theories of social capital. It deals with the exchange of resources in social networks. Like most sociological theories, it does not attempt to indicate how exchange occurs (in quantitative terms), but rather why it occurs. Therefore, it creates a context for understanding the complexity of interpersonal relationships in their social dimension.

The existence of these two mechanisms was confirmed by Luca Valori and coleeagues research [69], which used a large and detailed data set [69]. They have characterised the empirical properties of the large-scale distribution of individuals in multidimensional cultural space. By using simple models, they showed that ultrametricity has profound and nontrivial consequences on short- and long-term cultural dynamics. In the short term, they found the existence of a symmetry-breaking phase transition where collective behaviour arises out of purely local interactions. However, in the long term, the same ultrametric property suppresses cultural convergence by restricting it within disjoint domains, implying a strong sensitivity to the initial conditions. Thus, the apparent paradox of the coexistence of short-term collective social behaviour and long-term cultural diversity might have, as a simple and parsimonious explanation, the empirically observed hierarchical distribution of individuals in cultural space.

The character of actors' connections in social networks determines the availability of social resources and their size. If one treats information or opinion as a social network resource, then some members of the social network (opinion leaders) play a greater role in its propagation [70] than the sender of the message itself. In the first model (two-step flow), the key role is played by opinion leaders who mediate between the sender (mass media) and the rest of society. In this model, unlike the one-step or "hypodermic" model [71], individuals are not treated as atomised recipients of media influence.

Depending on whether the actors in social networks are similar or different from each other, the links between them can be bonding or bridging [72]. Ronald Burt [73,74] characterised two mechanisms of social contagion in the diffusion of innovation (opinions) in social networks depending on their structure: cohesion-induced contagion and equivalence-induced contagion. Cohesion-induced contagion occurs in cohesive networks between actors that maintain frequent and emphatic relationships. It is based on socialising communication. However, equivalence-induced contagion occurs in bridging networks as a result of competition between two actors who have similar relationships with other people. This applies to the competition of people who just use each other to evaluate their relative adequacy. Quoting Ref. [73] (p. 1291): "The more similar ego's and alter's relations with other persons are—that is, the more that alter could substitute for ego in ego's role relations, and so the more intense that ego's feelings of competition with alter are—the more likely it is that ego will quickly adopt any innovation perceived to make alter more attractive as the object or source of relations." Ultimately, a large number of bridges connecting diverse groups is essential for reducing opinion fractionalisation within societies [75]. A large number of bridges also has the effect of reducing distances between unconnected citizens [76].

5. Conclusions

In this paper, the Latané social impact theory is employed to build a model of opinion formation. With computer simulation, we investigate how the initial variety of opinions assigned to actors in such a way that initially every actor has his/her own opinion influences the final opinions number and their spatial distribution. The latter may be, to some extent, automatically checked (without direct analysis of snapshots from simulations) by means of techniques known from studies of site percolation phenomena.

As was pointed out in Refs. [36,37], a small noise dose (not too high a social temperature T) helps to reach consensus (not necessarily observed for the deterministic version of this model, i.e., for $T = 0$, cf. Figure 3 in Ref. [37] and Figure 7 in Ref. [36], where, however, the number of available opinions was restricted to several, namely $K = 3$ [36] and $K = 5$ [37]). This is well seen in Figure 1 and also in Figure 3 for $T = 1$ and $\alpha \leq 5$.

Independently of the model control parameters, at least 90% of the initially available opinions are removed from the system. In some cases, only two opinions (when society polarisation occurs) or even a single opinion (when consensuses on a common opinion takes place) survive. As explained by Lin [68], there are various mechanisms that connect the individual to the group around shared resources [67,68]. The group provides the individual with a more effective way of pursuing their interests than if the individual were to act individually. In order to remain a member of the group, one must agree to a consensus on the value of the resources held by the group. This consensus also applies to opinions. In Burt's theory [73,74], opinion reduction is caused by the action of opinion leaders. Opinion leaders are the people whose conversations trigger contagion across the social boundaries between status groups. As a consequence of such actions, groups can become more similar in terms of opinions.

Finally, it would be interesting to investigate if the observed vanishing of opinions is generic, i.e., if it may also be observed in other discrete models of opinion formation. Further studies of the model may also include investigating the influence of the network topology on obtained results: the studies may either deal with regular lattices—triangular (six neighbours) or honeycomb (three neighbours)—or complex networks (including small world networks). The latter requires, however, a redefinition of the distance, $d_{i,j}$, from its Euclidean definition to the shortest paths between actors.

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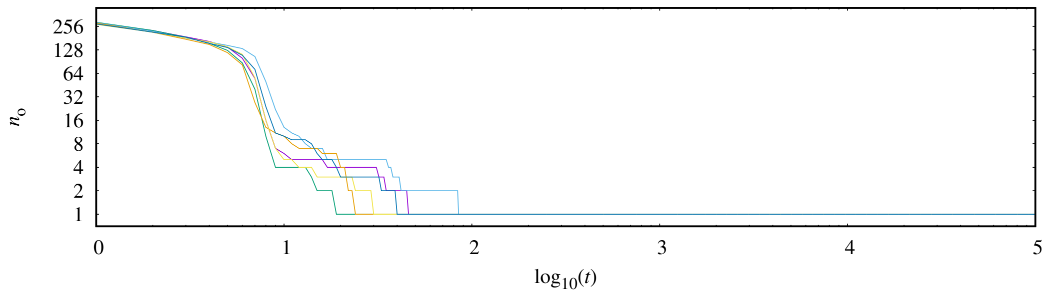
Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. Examples of Time Evolution of the Observed Number of Opinions

In Figure A1 the time evolution of the observed number of opinions $n_o(t)$ for $T = 0.5$, $\alpha = 2$ (Figure A1a), $T = 0.5$, $\alpha = 6$ (Figure A1b), $T = 2.5$, $\alpha = 2$ (Figure A1c) and $T = 2.5$, $\alpha = 6$ (Figure A1d).

(**a**) $T = 0.5$, $\alpha = 2$



(**b**) $T = 0.5$, $\alpha = 6$

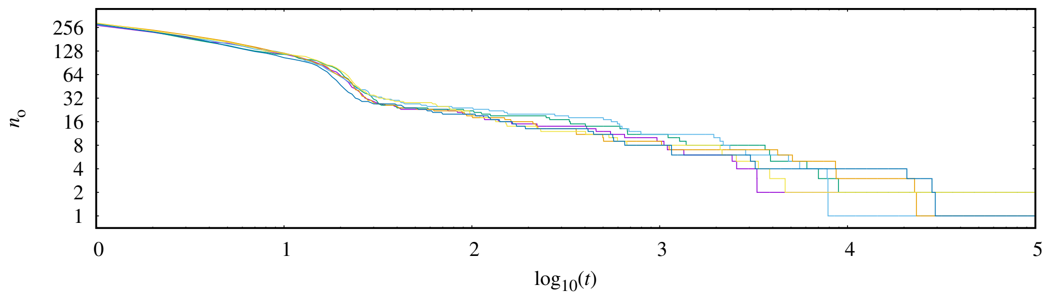
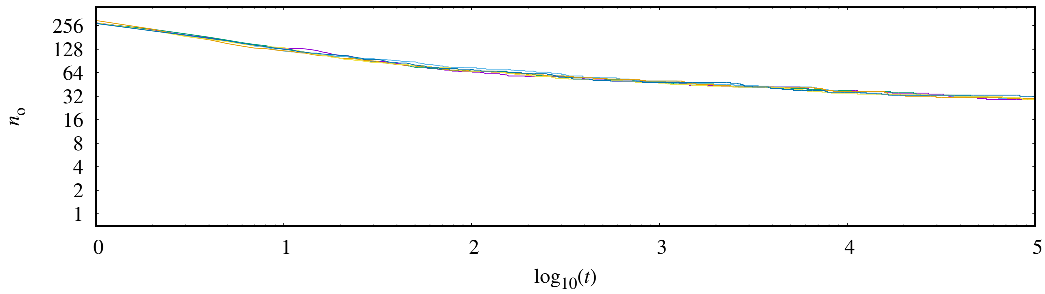


Figure A1. *Cont.*

(**c**) $T = 2.5$, $\alpha = 2$



(**d**) $T = 2.5$, $\alpha = 6$

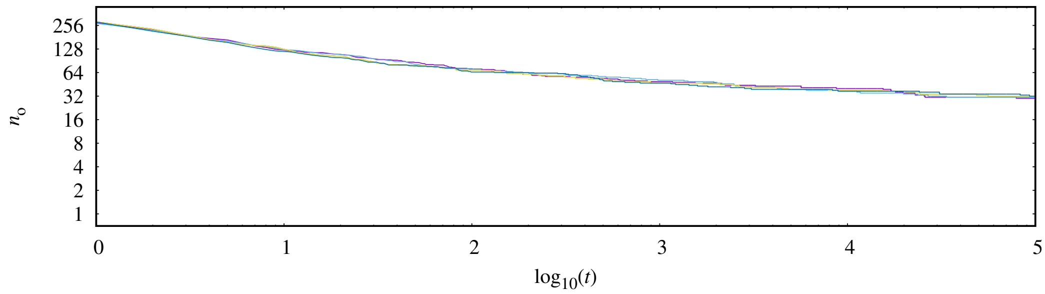


Figure A1. Time evolution of the observed number of opinions $n_o(t)$ for (**a**) $T = 0.5$, $\alpha = 2$, (**b**) $T = 0.5$, $\alpha = 6$, (**c**) $T = 2.5$, $\alpha = 2$ and (**d**) $T = 2.5$, $\alpha = 6$.

Appendix B. Examples of Spatial Opinion Distribution

An initial state of the system for $L = 21$ and $K = L^2$ is presented in Listing A1. In Listings A2–A7, examples of the final state of the system evolution for $L = 21$ after $\tau = 10^5$ time steps are presented. The numbers represent opinions. The examples are associated with four phases identified and presented in Figure 3. In Listing A2, the case of unanimity of opinions is presented. In Listings A3–A5, three variants of society polarization (with $n_o = 2$ and $n_c = 2$) are presented. In Listings A6 (with $n_o = 30$, $n_c = 399$) and A7 (with $n_o = 32$, $n_c = 415$), snapshots of the (still dynamical) state of the system are presented.

Listing A1. An initial state of the system for $L = 21$ and $K = L^2$. The numbers represent opinions. Every agent starts with his/her own opinion, which is different from the opinions of any other actor.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42
43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84
85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105
106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126
127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147
148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168
169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189
190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210
211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231
232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252
253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273
274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294
295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315
316	317	318	319	320	321	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336
337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357
358	359	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378
379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399
400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420
421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441

Listing A2. $\alpha = 2$, $T = 0.5$, $n_o = 1$, $n_c = 1$, $S_{\max}/L^2 = 100\%$.

```
# irun=          10
# t=      100001, lambda:
284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284
284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284
284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284
284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284
284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284
284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284
284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284
284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284
284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284
284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284
284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284
284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284
284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284
284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284
284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284
284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284
284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284
284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284
284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284
284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284
284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284  284
# histogram of cluster sizes:
#         441           1
### Smax=          441
###   nc=            1
###   no=            1
```

Listing A3. $\alpha = 6$, $T = 0.5$, $n_o = 2$, $n_c = 2$, $S_{\max}/L^2 \approx 52\%$.

```
# irun=           2
# t=      100001, lambda:
 50   50   50   50   50   50   50   50   50   50   50  387  387  387  387  387  387  387  387  387  387
 50   50   50   50   50   50   50   50   50   50   50  387  387  387  387  387  387  387  387  387  387
 50   50   50   50   50   50   50   50   50   50  387  387  387  387  387  387  387  387  387  387  387
 50   50   50   50   50   50   50   50   50   50  387  387  387  387  387  387  387  387  387  387  387
 50   50   50   50   50   50   50   50   50   50  387  387  387  387  387  387  387  387  387  387  387
 50   50   50   50   50   50   50   50   50   50  387  387  387  387  387  387  387  387  387  387  387
 50   50   50   50   50   50   50   50   50   50  387  387  387  387  387  387  387  387  387  387  387
 50   50   50   50   50   50   50   50   50   50  387  387  387  387  387  387  387  387  387  387  387
 50   50   50   50   50   50   50   50   50   50  387  387  387  387  387  387  387  387  387  387  387
 50   50   50   50   50   50   50   50   50   50  387  387  387  387  387  387  387  387  387  387  387
 50   50   50   50   50   50   50   50   50   50  387  387  387  387  387  387  387  387  387  387  387
 50   50   50   50   50   50   50   50   50   50  387  387  387  387  387  387  387  387  387  387  387
 50   50   50   50   50   50   50   50   50   50  387  387  387  387  387  387  387  387  387  387  387
 50   50   50   50   50   50   50   50   50   50  387  387  387  387  387  387  387  387  387  387  387
 50   50   50   50   50   50   50   50   50   50  387  387  387  387  387  387  387  387  387  387  387
 50   50   50   50   50   50   50   50   50   50  387  387  387  387  387  387  387  387  387  387  387
 50   50   50   50   50   50   50   50   50   50  387  387  387  387  387  387  387  387  387  387  387
 50   50   50   50   50   50   50   50   50   50  387  387  387  387  387  387  387  387  387  387  387
 50   50   50   50   50   50   50   50   50   50  387  387  387  387  387  387  387  387  387  387  387
 50   50   50   50   50   50   50   50   50   50  387  387  387  387  387  387  387  387  387  387  387
 50   50   50   50   50   50   50   50   50   50  387  387  387  387  387  387  387  387  387  387  387
# histogram of cluster sizes:
#         212           1
#         229           1
### Smax=          229
###   nc=            2
###   no=            2
```

Listing A4. $\alpha = 6$, $T = 0.5$, $n_o = 2$, $n_c = 2$, $S_{\max}/L^2 \approx 52\%$.

```
# irun=          23
# t=      100001, lambda:
 40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40
 40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40
 40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40
 40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40
 40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40
 40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40
 40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40
 40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40
 40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40
 40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40
 40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40   40
120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120
120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120
120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120
120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120
120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120
120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120
120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120
120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120
120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120
120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120  120
# histogram of cluster sizes:
#         210           1
#         231           1
### Smax=          231
###   nc=            2
###   no=            2
```

Listing A5. $\alpha = 4$, $T = 0.5$, $n_o = 2$, $n_c = 2$, $S_{\max}/L^2 \approx 67\%$.

```
# irun=          92
# it=      100001, lambda:
 96   96   96   96   96   96   96   96   96   96   96   96   96   96    2    2    2    2    2    2    2
 96   96   96   96   96   96   96   96   96   96   96   96   96   96    2    2    2    2    2    2    2
 96   96   96   96   96   96   96   96   96   96   96   96   96   96    2    2    2    2    2    2    2
 96   96   96   96   96   96   96   96   96   96   96   96   96   96    2    2    2    2    2    2    2
 96   96   96   96   96   96   96   96   96   96   96   96   96   96    2    2    2    2    2    2    2
 96   96   96   96   96   96   96   96   96   96   96   96   96   96    2    2    2    2    2    2    2
 96   96   96   96   96   96   96   96   96   96   96   96   96   96    2    2    2    2    2    2    2
 96   96   96   96   96   96   96   96   96   96   96   96   96   96    2    2    2    2    2    2    2
 96   96   96   96   96   96   96   96   96   96   96   96   96   96    2    2    2    2    2    2    2
 96   96   96   96   96   96   96   96   96   96   96   96   96   96    2    2    2    2    2    2    2
 96   96   96   96   96   96   96   96   96   96   96   96   96   96    2    2    2    2    2    2    2
 96   96   96   96   96   96   96   96   96   96   96   96   96   96    2    2    2    2    2    2    2
 96   96   96   96   96   96   96   96   96   96   96   96   96   96    2    2    2    2    2    2    2
 96   96   96   96   96   96   96   96   96   96   96   96   96   96    2    2    2    2    2    2    2
 96   96   96   96   96   96   96   96   96   96   96   96   96   96    2    2    2    2    2    2    2
 96   96   96   96   96   96   96   96   96   96   96   96   96   96    2    2    2    2    2    2    2
 96   96   96   96   96   96   96   96   96   96   96   96   96   96    2    2    2    2    2    2    2
 96   96   96   96   96   96   96   96   96   96   96   96   96   96    2    2    2    2    2    2    2
 96   96   96   96   96   96   96   96   96   96   96   96   96   96    2    2    2    2    2    2    2
 96   96   96   96   96   96   96   96   96   96   96   96   96   96    2    2    2    2    2    2    2
 96   96   96   96   96   96   96   96   96   96   96   96   96   96    2    2    2    2    2    2    2
# histogram of cluster sizes:
#         147           1
#         294           1
### Smax=          294
###   nc=            2
###   no=            2
```

Listing A6. $\alpha = 3$, $T = 1.5$, $n_o = 30$, $n_c = 399$, $S_{\max} = 4$.

```
# irun=          23
# t=      100001, lambda:
245   40  126  419   51  440   90  245  142   22  274  103  441  245  419  274   22  187  406  147   91
245  194  187  441   40  142  280   22  419  280  406  142  316   40  441  126  194  187  194   51  334
441  316   22   90   97  147   40  280  169  103   43  245   91  280  194  194  194   43   90  316  187
147   90   90   43  126  226  142  194   40  419  440   91  173  185  185  419  419  406  211  327   43
 40   97   90  419  419   51  274  334   97  245   91  147  147   43  274  173  185  194  334  126  334
316   22  173  327  316  419  440  194   51  187  419  327   43  419  406  245   90   40  280  280  194
245   22  187  226  194  327  187  441   90   97  280  440  334  334  173   40  327  226  185  316   90
173  173  316  173  235  406  316  406  185   22  142  147   97  406  327   22  327   22  334  245   22
 43  327  327   43  126   43  173  147   91  406  274   40  441  103   22  126  245  316   43  419  440
280  441   40  173  440   90  440  316  327  406   97  406  185  294  406  294   90  316  173  406  280
406  441  280  440   40   91   40  187  294  235  316   51   22   51   22  142  419   22   22  142  103
 51   22  294   51  126  294  187  440  226  187  169  280  406  441  327  316  185   22  406   40   43
211  294  334   97   22  294  173   40   91   51  235   51  441  316  142  245  274  211  147  235   43
 40   97   40  226  327  327  294  274  226  334   43  294  327  235   40   40   22  142  194   51  169
440   22   22  334  211  440   40  334  245  126  147  316  187   91  280  280  316  441  211  245  441
173  142  294  142  103   91  126  245  173   51  280  211  187  173  441   40  274   97  440   51  103
226  406  194  185   51   51  274  173  147  419   40  185  103  211  194  406  334  211  274  274  441
 43   40  294  185  327   90  327  334  419  316  245  103  419  211  185   40  226  280  235  280  103
142  294  194  280  142  142  274  334   43   51  187  185  334  327  406  126  103  226  142  419  441
187  294  334  441  441  211   97  211  226  334  294  173  147  235  406   43  103  406  142   51  103
194   51  169  327  327  126  245   90   97   22  142  169  441  185  185  142  440  185  245  142  194
# histogram of cluster sizes:
#           1          361
#           2           35
#           3            2
#           4            1
### Smax=          4
###   nc=        399
###   no=         30
```

Listing A7. $\alpha = 6$, $T = 2.5$, $n_o = 32$, $n_c = 415$, $S_{\max} = 3$.

```
# irun=          10
# t=      100001, lambda:
121  191   91  372   52  120  421  191  424  238  421  361  327  128  330  330  198  294  238  193  286
343  421   52  424  233  193  424  273  330  233  198  421  330  327  286  304  416  286  360  294  416
361  134  276  204  204  330  284  361  198  198  343  327  330  258  273  330  273  120  343  191    9
128  204  204  233  286  360  154  238  193  317  327  121  120  424  284  197  361  120  421  121  284
330  343  294  330  191  121  372    9  361  284  421  198  233  360  191  330  330  204  121  154  317
317  304  258  258  134  273  421  233  424  361  286  286   52  273  238  304  360  424  193  317  238
317  233  360  121   91  204   91  284  284  121  284  276  154  154  273    9  327  128  120  360  421
120  416  286  233  330  198  372  284  273  121   52  294  258  372  193  191  372  360  258  330  238
360  294  121   91  233  286  361  276   52  121  317  154  286  286  134    9  121  193  361  198  273
198  360  198  121  421  276  120  233  424  416  193  424  330  330  343  424  198  372  284  304  198
191  286  416  330  361  258   52  128  121  421  424  121  134  233  284  421   91   52  304  372  304
258    9  193  360  258  327  276  204  286  154  273  198  197  204  154  134  317  424  193  284  128
317  421  154  372  330  193   52  238  304  294  191  361  198  360  204  273  198  424  191  276  191
372  258    9  424  154  204   91  317   52  128  372  284  361  343  154  343  327  286  360  424  197
191   91  197  276  121  197  294  193    9  154  304    9  304  330  317  233  191  204  154  327  233
284  421  330  154  317  416  294  330  361  343    9  372  204  421  421  360  424  360  421  416  343
198  134  128  372  317  304    9  343  154  154   52  128  421  343  421  233  286  424  121  327  258
421   52  121  191   52  360  284  421  204  360  191  360  154  258  360  276  294  421  134  360  238
120  198  294  304  294  294  128  286  317  361  294  421  361  286   52  424  330  317  294  424  286
304  134  204  191  360  276  128  154  121  372  416  204  330  317  198  154  233  121  284  120  134
128  191   91  361   52    9  120  193  276  286  197  360  421  286  198  120  421  134  416  361  286
# histogram of cluster sizes:
#           1          392
#           2           20
#           3            3
### Smax=          3
###   nc=        415
###   no=         32
```

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