

Article

The Influence of Lobbies: Analyzing Group Consensus from a Physics Approach

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Abstract: In this paper, we study the influence of a small group of agents (i.e., a lobby) that is trying to spread a rumor in a population by using the known model proposed by Serge Galam. In particular, lobbies are modeled as subgroups of individuals who strategically choose their seating in the social space in order to protect their opinions and influence others. We consider different social gatherings and simulate, using finite Markovian chains, opinion dynamics by comparing situations with a lobby to those without a lobby. Our results show how the lobby can influence opinion dynamics in terms of the prevailing opinion and the mean time to reach unanimity. The approach that we take overcomes some of the problems that behavioral economics and psychology have recently struggled with in terms of replicability. This approach is related to the methodological revolution that is slowly changing the dominant perspective in psychology.

Keywords: opinion dynamics; sociophysics; Markovian chain physics models; social influence

1. Introduction

Besides the known incident [1] and the more recent case [2] in behavioral economics, replicability has also been questioned in psychology since at least 2015 [3], and alternative approaches to analyzing the phenomena studied by these disciplines could help to shed light on human behavior in groups.

Social influence is a topic that has attracted great interest in both social psychology [4] and behavioral economics [5]. Social psychology in particular has provided some groundbreaking experiments in which the effect of groups in changing opinions [6] has been investigated. Of additional relevance is the research on the influence of minorities using a simulation approach on lattices [7] and on networks [8,9]; for a discussion of spatial interactions in agent-based models, see [10]. Another interesting example can be found in studies on the influence of small groups on majority consent [11]. These studies analyze the influence of groups at the microscopic level and provide some insights that can be applied in the analysis of political change [12]. Previous research has focused in particular on two different aspects: the influence of the majority (i.e., conformity, Ref. [6]) and the influence of the minority [13]. Studies on conformity show that the most common opinion in a group is able to exert an enormous influence on individuals and leads them to give inaccurate answers [6]. By contrast, studies on minority influence have focused on how people resist group pressure and how individuals are able to change the opinion upheld by the dominant position [14]. Minority studies have been applied to innovation and social change [15] and also to the study of influence on dominant positions [12]; for reviews on the influence of small groups, see [11,16,17]. Further experimental studies have been conducted in order to shed light on the psychological processes of minority influences [18] and the strategies of influencing dominant positions [19]. In the field of social epistemology, both mathematical and computational models of social influence are being developed to compare the ability to secure consensus in different types of social arrangements [20] and



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how agents may fail to converge to the truth [21]. Another important research branch is devoted to the topic of strategic influence. In particular (see, e.g., [22,23]), strategic influence is applied to voting schemes. In this paper, we consider lobbies as groups of individuals who strategically choose their seats in the social space to protect their opinions and influence other individuals. Finally, some contributions in the literature on group decision making and consensus building can be found in Refs. [24,25].

More recently, the spread of conspiracy theories [26] on various topics [27], but also on science [28,29] and the social processes underlying their spread [30], show how relevant these phenomena can be to the society. Given the problems of replicability mentioned above, analytical approaches or replicable simulation models can be highly useful. Therefore, we introduce minorities in the known model of Serge Galam [31], who analyzed the spread of hoaxes twenty years ago using sociophysics.

The paper is structured as follows. In Section 2, we discuss some possible approaches to modeling opinion spreading. In Section 3, we summarize Galam's model [31] and illustrate how we extend it by introducing lobbies. The results of our simulation are analyzed in Section 4. Finally, Section 5 gives the conclusions and suggests further research.

2. Analytical and Computational Approaches to Opinion Spreading

Although experimental studies provided by social psychology offer important contributions in understanding the proximate mechanisms that influence human behavior, this approach presents some intrinsic limits [13]. One of these limits is that experimental studies cannot investigate the temporal dimensions and the long-run effects of opinion dynamics on a population. Therefore, it is crucial to integrate the knowledge of the proximate processes derived from social psychology with methods that can focus on a more dynamic dimension. In other words, once it is known that a local majority opinion can influence individuals in small groups, how will this effect spread to the population? One attempt to fill this gap has been provided by sociophysics [32]. A sociophysics approach focuses on the dynamic dimension, i.e., the time needed for a minority to reach a consensus in a population. One of the first attempts to integrate social psychological theories with the methods of sociophysics was proposed in Ref. [33]. This study provides a different context to test theories developed in an experimental setting and allows for the validation of these concepts on a broader scale. In particular, this study focuses on the phenomenon of polarization in groups, i.e., the tendency of individuals to hold more radical opinions in the presence of others.

Other approaches that can be integrated with social psychology for the study of opinion dynamics are mathematical modeling [34–36] and agent-based simulations [37,38]. Recently, the integration of other approaches, such as agent-based modeling, has become more popular (see, e.g., [39]), because they make it possible to consider several phenomena in social psychology “as emergent results of dynamically interactive processes taking place in their contexts” [40]. This is part of the quiet methodological revolution referred to in [41], as statistics moves from the mechanical application of a series of procedures to the building and the evaluation of models.

Among other disciplines, sociophysics offers some important insights into the analysis of social influence on opinion dynamics and consensus [37,42,43]. One of the main interesting models is proposed in [31]. Galam's model focuses on the analysis of the propagation or diffusion of hoaxes in public debates. To be more specific, in Galam's model, individuals interact together on a specific topic in groups of different dimensions. The model shows that even if, at the beginning, an opinion represents the minority, after a certain amount of discussion, this opinion can be diffused to the population. The configuration of different tables of different sizes where people discuss is called a *social space* [31,44] for details). An example of interaction in Galam's model can be found in Figure 1. Further research has introduced more complexity in this model [45]. For example, the agents proposed in Ref. [45] hold always the same opinion and have the ability to influence all the agents sitting at the table. In this research, we introduce the presence of lobbies in Galam's model.

Lobbies are modeled as minorities that are able to influence other opinions with a specific strategy. As a matter of fact, members of the lobby have the ability to find and seat, when possible, at a table where they can be the majority and exert their influence more efficiently.

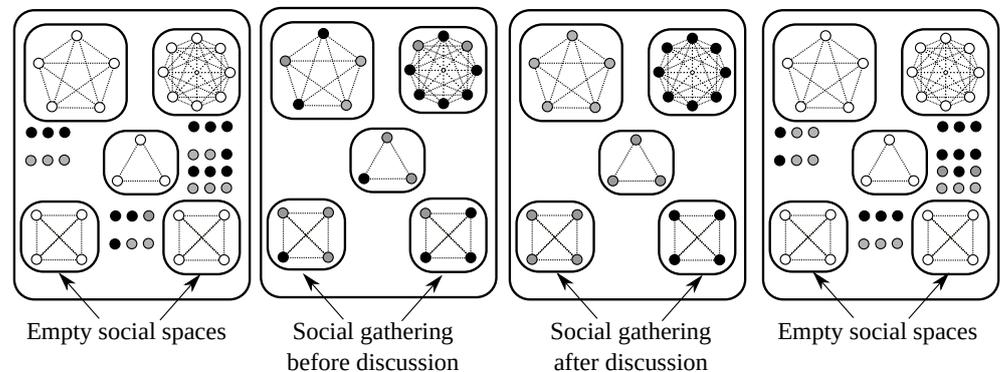


Figure 1. An exemplification of Galam's model: a one-step opinion dynamics. **Left to right:** First stage: individuals (black and gray circles) sharing the two opinions are moving around in order to sit at the available seats (white circles) of the tables. Gray circle individuals are "in favor of opinion \mathcal{X} ", while black circle individuals are "against \mathcal{X} ". Second stage: individuals are partitioned into groups of various sizes (tables). Before the discussion occurs, there are eleven individual in favor (gray circles) and thirteen against (black circles). Third stage: within each group, the discussion occurs and consensus has been reached. As a result, there are now twelve individuals in favor, and twelve individuals against. Last stage: individuals are again moving around with no discussion.

3. Rumors and Lobbies in Galam's Model

Galam's model can be considered a dynamic application of the conformity influence in different social gatherings. In Ref. [31], individuals have the same characteristics (i.e., they all meet randomly and change their opinion according to the majority) and a consensus can be reached when a specific minority threshold is reached. Therefore, Galam's model does not explain the dynamics in which small organized groups such as lobbies can implement some strategies to influence others' behavior. Lobbies can be modeled in different ways and can have a wide range of strategies, behaviors or interactions. In this paper, we present a model of a lobby capable of influencing the dynamics of opinions at strategic tables. More specifically, as in other studies examining seating positions at specific tables [46–48], lobbies are groups of individuals who always sit at tables where they can influence others. The influence rule is the same as in Galam's model [31] but, in the case considered, one has a small part of the population who strategically sit at the tables where they possibly represent the majority and then persuade others. To summarize, we consider a Galam-like model with a lobby, where the lobby is modeled as a subgroup of individuals who are "against opinion \mathcal{X} ", and coordinate their sitting at the tables of the social space with the aim of influencing as many agents as possible of the opposite opinion without being overridden. As a matter of fact, in the case here considered, the lobby members are subject to the table majority rule too; in other words, they are different from the opinion leaders considered in [45], who are always able to persuade all agents at their table. The strategic choice of the lobby depends on the social space configuration. As it is impossible to consider all the possible social space configurations for any number of individuals, we consider an example to study opinion dynamics.

In Galam's model [31], gatherings take place at different times, and each individual holds one of two possible opinions, e.g., in favor of \mathcal{X} , or against \mathcal{X} . When the individuals meet in a specific gathering configuration (a table with 3, 4, ... n individuals), they change their opinion according to the majority. Then, they will discuss again randomly in different social gatherings and change always their opinion according to this rule.

3.1. The Model without Lobby

We consider a population of N individuals who, at time t , have either opinion \mathcal{X} (in favor) or $\bar{\mathcal{X}}$ (against). At each time t , j denotes the number of individuals with the opinion \mathcal{X} and defines the state of the system; therefore, the set of the possible states of the system is $J = \{0, 1, \dots, N\}$, where $j = 0$ indicates a consensus on the opinion $\bar{\mathcal{X}}$ and $j = N$ on the opinion \mathcal{X} . In particular, as in Ref. [44], individuals interact in a social space consisting of a set of available tables $T_r \in \mathcal{N} = \{T_1, T_2, \dots, T_L\}$, whose size is denoted s_r . The sizes of the tables in \mathcal{N} are represented as a vector $\mathbf{s} = (s_1, s_2, \dots, s_L) \in \mathbb{R}^L$ (the case $L = N$, i.e., all tables of dimension 1, is trivial and therefore is not considered), where $L < N$ and $\sum_{T_r \in \mathcal{N}} s_r = N$. The number of individuals who hold the same opinion at a given table of size s is denoted by $n(\mathcal{X})$ and $n(\bar{\mathcal{X}})$, respectively, so that $n(\mathcal{X}) + n(\bar{\mathcal{X}}) = s$. The distribution of individuals in favor of opinion \mathcal{X} across the L tables is represented by the vector $\mathbf{n}(\mathcal{X}) = (n_1(\mathcal{X}), n_2(\mathcal{X}), \dots, n_L(\mathcal{X}))$ and $j = \sum_{r=1}^L n_r(\mathcal{X})$ is the state of the system.

At each time step t , the dynamics of each individual’s opinion is governed by the following set of rules:

- individuals are randomly distributed to the available seats;
- individuals change their opinion according to a local or table majority rule, i.e., at each table, if $n(\mathcal{X}) > n(\bar{\mathcal{X}})$, then all individuals sitting at the table will change to the opinion \mathcal{X} ; vice versa, if $n(\bar{\mathcal{X}}) \geq n(\mathcal{X})$, then all the individuals sitting at the table will change to opinion $\bar{\mathcal{X}}$.

This approach leads to a stochastic process where the individuals submit to the local majority with certainty; this is different, for instance, from Ref. [49], where individuals may also change opinion, with a certain probability that depends on a propensity parameter.

3.2. The Model with Lobby

We assume that an interest group, with the goal of influencing the population’s opinion, is able to strategically choose seats. Organized interests quite often target opponents and undecideds [50]. We therefore assume that, within the population of N individuals, there exists a subset of size $N^* \leq N$ —called a *lobby*—that strategically distributes itself across the available tables in order to maximize the number of new allies who share its opinion $\bar{\mathcal{X}}$. The number of lobbyists seated at a table T_r is denoted as n_r^* . The distribution of lobbyists across the L tables is represented by the vector $\mathbf{n}^* = (n_1^*, n_2^*, \dots, n_L^*)$. Taking into account the table majority rule defined above in Section 3.1, the dynamics rules are as follows:

- the lobbyists sit at the tables, possibly occupying half of the seats available at each table (if s is an even number), or $\lceil s/2 \rceil$ (if the seats are an odd number) (recall that the symbol $\lceil \cdot \rceil$ indicates the ceiling function);
- non-lobbyist individuals are randomly distributed across the remaining seats;
- the table majority rule is applied to both lobbyists and non-lobbyists and the opinions are updated.

3.3. Simulation Model

The opinion dynamics can be modeled as a finite Markov chain [44], i.e., a discrete-time process in which the future state depends only on the present state and not on the past states [51]. As a matter of fact, all possible opinion configurations can be considered as states and the probability of a transition between any couple of states can be computed as shown in Ref. [44]:

$$P_j(\mathbf{n}(\mathcal{X})) = \frac{\binom{j}{n_1(\mathcal{X}), n_2(\mathcal{X}), \dots, n_L(\mathcal{X})} \binom{N-j}{s_1 - n_1(\mathcal{X}), s_2 - n_2(\mathcal{X}), \dots, s_L - n_L(\mathcal{X})}}{\binom{N}{s_1, s_2, \dots, s_L}}, \tag{1}$$

where $\binom{b}{b_1, b_2, \dots, b_L}$ is the common notation for multinomial coefficients.

When considering lobbies, the previous formula can be adapted as follows, conditional on the lobbyists' position:

$$P_j(\mathbf{n}(\mathcal{X})|\mathbf{n}^*) = \frac{\binom{j}{n_1(\mathcal{X}), n_2(\mathcal{X}), \dots, n_L(\mathcal{X})} \binom{N-N^*-j}{s_1-n_1^*-n_1(\mathcal{X}), s_2-n_2^*-n_2(\mathcal{X}), \dots, s_L-n_L^*-n_L(\mathcal{X})}}{\binom{N-N^*}{s_1-n^*, s_2-n^*, \dots, s_L-n^*}}. \quad (2)$$

In our study, we consider a population with $N = 24$ individuals as it offers a large enough number of social spaces; even in this case, the partition number is too large for a systematic analysis of all the possible spaces since, for $N = 24$, the partition number is 1575 [52] (sequence A000041). Therefore, we consider the special case in which all the tables have the same size. In order to be able to assess the role of the lobby, we limit its size to a maximum of six individuals. The social spaces that we consider, together with the strategic seating choices of the lobby members, are shown in Figure 2. Similar to that in Galam's model, when a table has the same number of contrasting opinions, the bias is in favor of the individuals who are "against \mathcal{X} ".

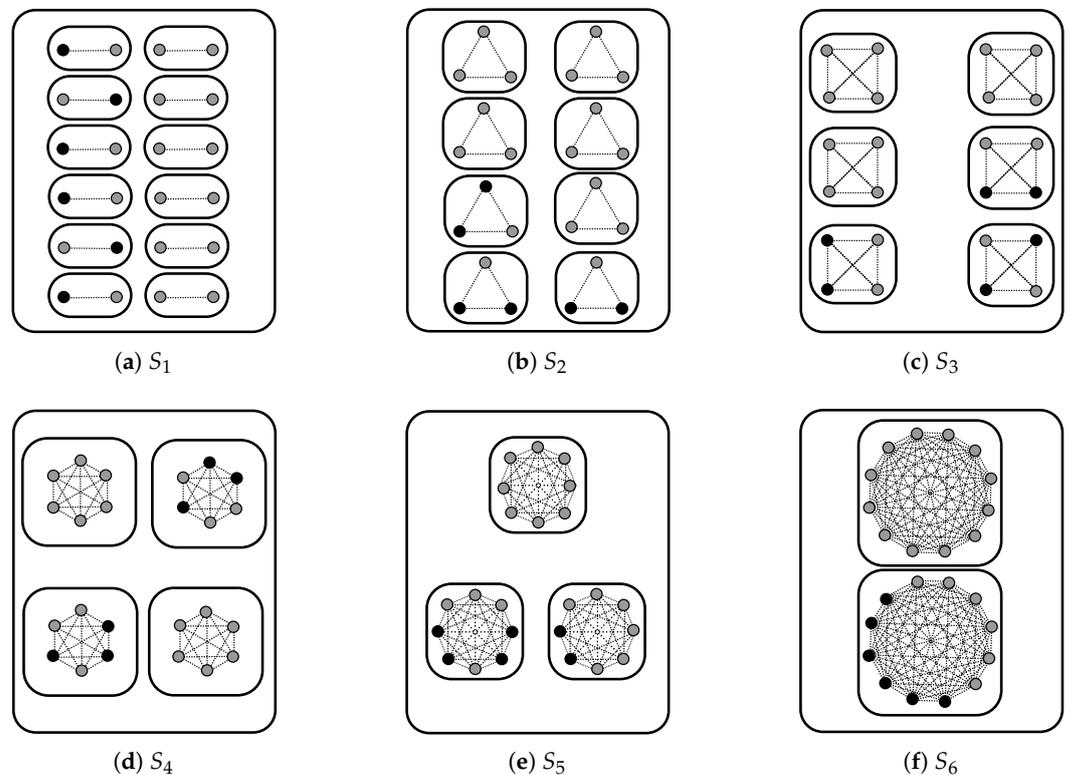


Figure 2. Population of twenty-four people seated in different social spaces. Lobby consists of at most six people (black dots) who are "against \mathcal{X} " seating at prefixed strategic tables for different social spaces S_h as follows. (a) S_1 : twelve tables with two seats each; lobby members sit individually at six tables. (b) S_2 : eight tables with three seats each; lobby members sit in couples at three tables. (c) S_3 : six tables with four seats each; here, too, lobby members sit in couples at three tables. (d) S_4 : four tables with six seats each; lobby members sit in groups of three at two tables. (e) S_5 : four tables with six seats each; four lobby members sit at one table and are always able to influence, while the two remaining sit at another table. (f) S_6 : two tables with twelve seats each; lobby members sit together at one table.

As is typical with finite Markov chains, the transition probabilities are arranged in a matrix [53] (p. 374). For each social space S_h (with $h = 1, \dots, 6$) of Figure 2, we compare the two scenarios, one with and the other without a lobby. The transition matrices can be computed using Formulas (1) and (2) as illustrated in the following example.

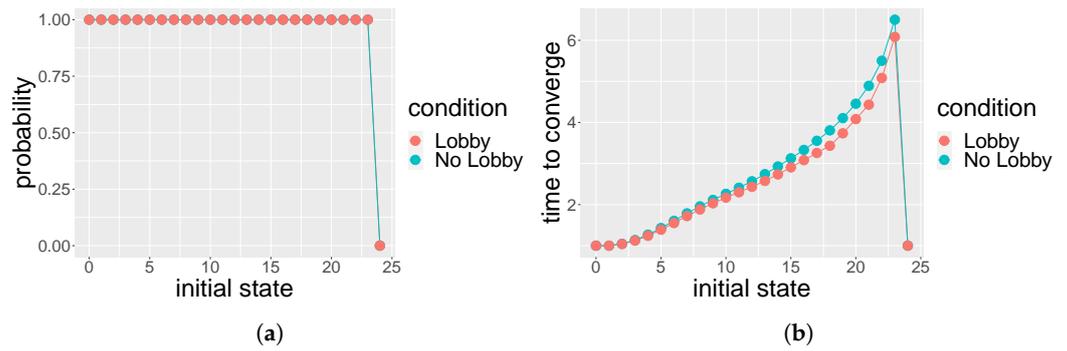


Figure 3. Social space S_1 : twelve tables with two seats each and the members of the lobby sitting strategically, as shown in Figure 2a. The probability of convergence (a) to the unanimous opinion $\bar{\mathcal{X}}$ and the absorption time (b) to any unanimous opinion as functions of the initial number of individuals with opinion \mathcal{X} for cases with (red) and without (blue) a lobby.

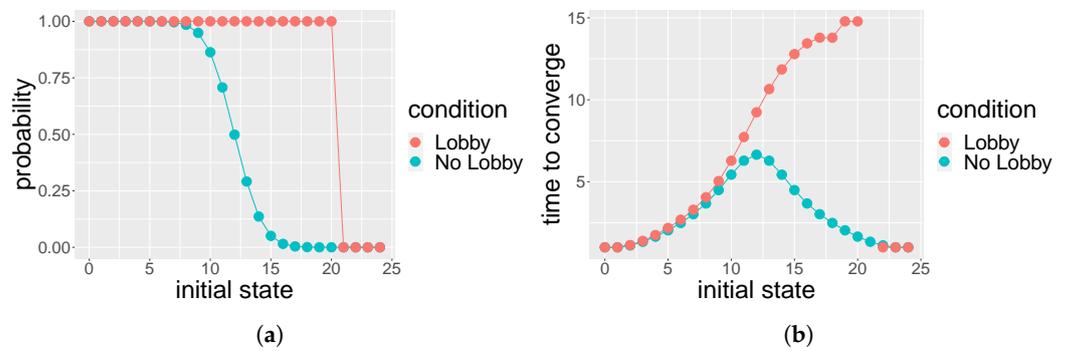


Figure 4. Social space S_2 : eight tables with three seats each and the members of the lobby sitting strategically, as shown in Figure 2b. The probability of convergence (a) to the unanimous opinion $\bar{\mathcal{X}}$ and the absorption time (b) to any unanimous opinion as functions of the initial number of individuals with opinion \mathcal{X} for cases with (red) and without (blue) a lobby.

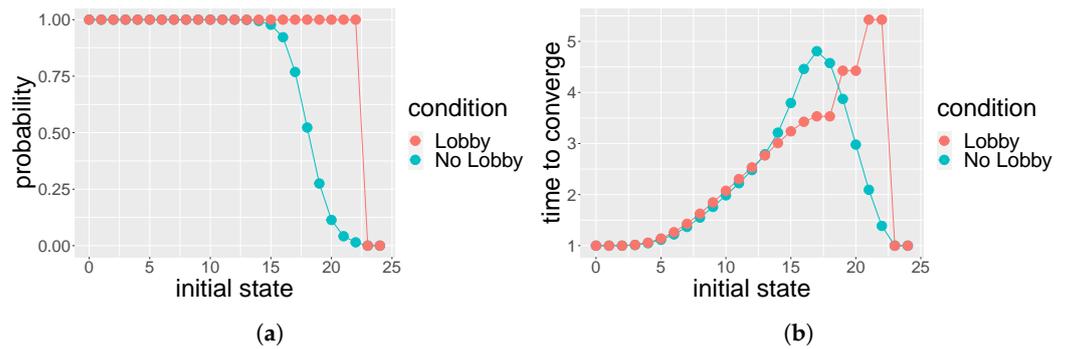


Figure 5. Social space S_3 : six tables with four seats each and the members of the lobby sitting strategically, as shown in Figure 2c. The probability of convergence (a) to the unanimous opinion $\bar{\mathcal{X}}$ and the absorption time (b) to any unanimous opinion as functions of the initial number of individuals with opinion \mathcal{X} for cases with (red) and without (blue) a lobby.

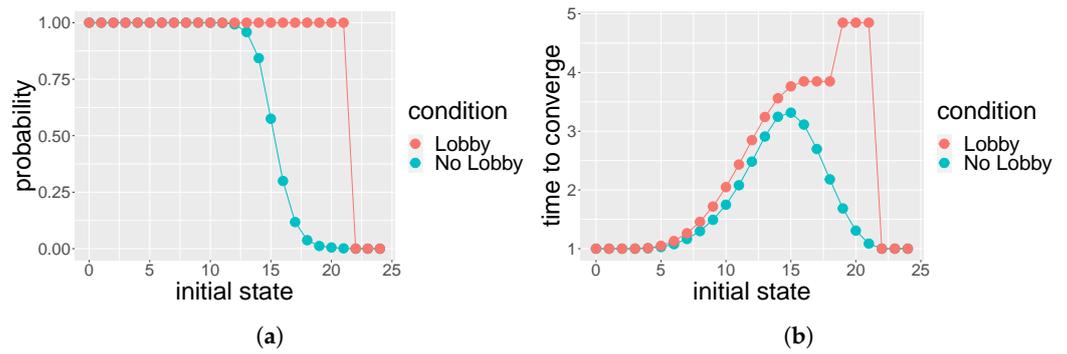


Figure 6. Social space S_4 : four tables with six seats each and the members of the lobby sitting strategically, as shown in Figure 2d. The probability of convergence (a) to the unanimous opinion $\bar{\mathcal{X}}$ and the absorption time (b) to any unanimous opinion as functions of the initial number of individuals with opinion \mathcal{X} for cases with (red) and without (blue) a lobby.

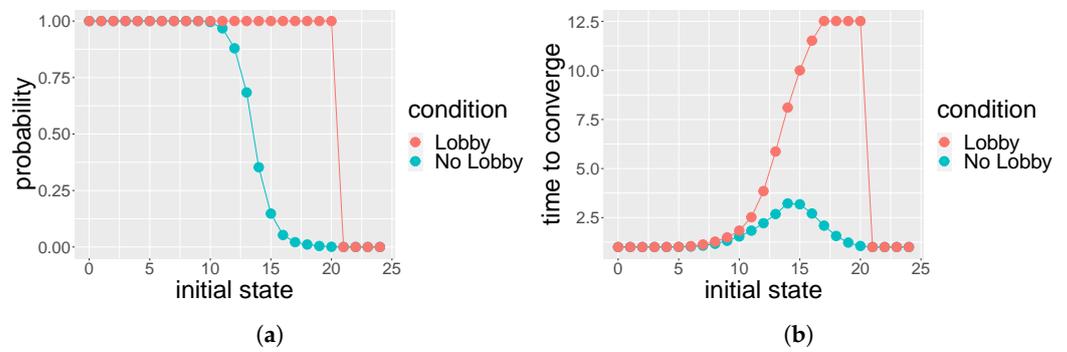


Figure 7. Social space S_5 : three tables with eight seats each and the members of the lobby sitting strategically, as shown in Figure 2e. The probability of convergence (a) to the unanimous opinion $\bar{\mathcal{X}}$ and the absorption time (b) to any unanimous opinion as functions of the initial number of individuals with opinion \mathcal{X} for cases with (red) and without (blue) a lobby.

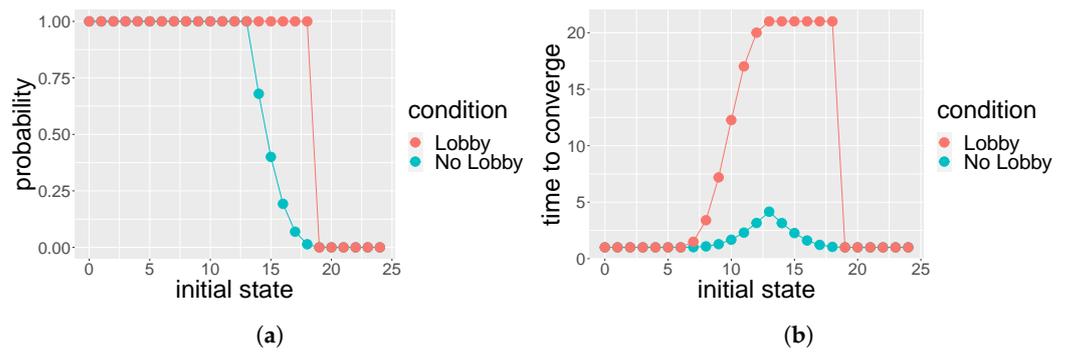


Figure 8. Social space S_6 : two tables with twelve seats each and the members of the lobby sitting strategically, as shown in Figure 2f. The probability of convergence (a) to the unanimous opinion $\bar{\mathcal{X}}$ and the absorption time (b) to any unanimous opinion as functions of the initial number of individuals with opinion \mathcal{X} for cases with (red) and without (blue) a lobby.

4. Results and Discussion

Formulas (1) and (2) are general and can be written for finite populations and any social space; moreover, the simulation approach—which is used to avoid the tedious calculation of all possible cases—can also be used in general.

The results show that, across all the social spaces considered, lobbies are able to influence populations with increased sample sizes. This is true for all social spaces except

for the simplest one, where there are no differences in dynamics between the situation with and that without a lobby. Moreover, if one considers the time of convergence in each of Figures 3–8, i.e., the time to converge to the opinion held by the lobby (“against opinion \mathcal{X} ”), the lobbies increase rather than significantly decrease the average number of steps to convergence to an absorbing state, i.e., to a state that cannot be left once entered [51] (p. 35). This means that even if strategic lobbies are able to influence a larger part of the population, their presence can shorten the time to change the public opinion on a given issue. From these two findings, one can conclude that the presence of lobbies can ensure influence on a larger scale, but at the cost of time. In Figures 3–8, we summarize the dynamics with and without lobbies for each of the social spaces considered.

In the social space S_1 , the results for convergence are identical for both models with and without lobbies; however, the average convergence time decreases when there are lobby members (Figure 3).

The social space S_2 exhibits a new absorbing state that can be deduced from Figure 2b when the initial state of the population has a lobby with three members. Two of these members strategically sit at the same table, leaving the other member alone at a different table. According to the table majority rule, the pair of lobbyists will persuade the third person at the table in favor of opinion $\bar{\mathcal{X}}$, while the lone lobby member will be persuaded in favor of opinion \mathcal{X} , thus maintaining the size of the lobby as originally three members; this is an impasse situation similar to that discussed in Ref. [44]. However, it must be noted that, in this social space, the impasse can only occur when the lobby is active. In Figure 4b, the mean time of convergence to this new absorbing state is not reported as it is not a unanimous state. According to Galam’s definition (see [31] (p. 577) for details), a killing point is the unstable fixed point, producing the flow and its direction, determined by the stochastic dynamical system describing the time evolution of the hoax support. In S_2 , one can see in Figure 4a that the state $j = 12$ is the killing point when there is no lobby; in contrast, in this social space, when the lobby is present, there is no killing point and this case suggests a bang-bang result [54]. The mean absorption time of unanimity is higher around the killing point; the process is even slower when generated by the lobby, as shown in Figure 4b.

The social space S_3 (Figure 5) has a killing point at state $j = 18$, i.e., at the initial state with eighteen individuals with opinion \mathcal{X} against a lobby of six members. As for space S_2 , the mean time of absorption to a unanimous opinion increases when the initial number of individuals in favor of \mathcal{X} is near the killing point. In contrast to space S_2 , however, the process triggered by lobbyists near the killing point is faster than in the case without a lobby. Only a further reduction in the size of the lobby leads to a slowdown in the process of convergence to unanimity. This is because the lobbyists’ strategy favors the preservation of their opinion over its diffusion in initial states that are smaller than the killing point. That is, one has a type of *echo chamber*, defined as “environments in which the opinion, political leaning, or belief of users about a topic gets reinforced due to repeated interactions with peers or sources having similar tendencies and attitudes” [55]. Expectedly, these cases of social influence generate opinion cascades [56] that reduce individual noise at the expense of group noise [57].

In the social spaces S_4 , S_5 , and S_6 , the average absorption times in the case with a lobby are uniformly higher than without; that is, the process of absorption is slower overall regardless of the killing points (see Figures 6–8). In other words, except for the particular case of the social space S_1 , the lobby generally makes a hoax $\bar{\mathcal{X}}$ prevalent even in the cases where it would be impossible; however, this process takes longer on average.

In all the social spaces that we analyzed, except S_1 , which is trivial, the killing point is lower in the case without a lobby than in the case with a lobby. This means that with the lobby, in order for the hoax $\bar{\mathcal{X}}$ not to spread, many more people are needed who do not believe it. This is not cost-less; we observe an increase in the mean time of absorption to a unanimous opinion in almost every case. Without lobbying, if the number of individuals in favor of the opinion \mathcal{X} is greater than the number at the killing point, the entire population

would also be in favor; with lobbying, however, the attractor changes and the population would end up believing the opinion $\bar{\mathcal{X}}$ but at a slower rate of convergence. This is always the case, except in the social space S_3 , as we have already discussed just above. To summarize, we have seen that the lobby increases the killing point and transforms the opinion dynamics into a form of *bang-bang* outcome. In general, however, the lobby increases the mean time to unanimity, as the strategic seating arrangement is primarily aimed at preserving the lobbyists' opinion rather than riskily spreading it. This becomes clearer when analyzing some particular examples. For example, let us consider the social space S_6 and compare the transition probabilities to the states $j = 0$, $j = 12$ and $j = 24$ with a lobby and without a lobby. One can see from Figure 9 that for $7 \leq j \leq 12$, even if the lobby is not present, a unanimous opinion in favor of the opinion \mathcal{X} cannot be reached and, at the same time, a unanimous opinion against \mathcal{X} is more likely without a lobby. In these cases, the lobby strategy is too conservative and slows down the achievement of its goal. If, on the other hand, $14 \leq j \leq 18$, the lobby is effective in avoiding reaching a unanimous opinion in favor of opinion \mathcal{X} .

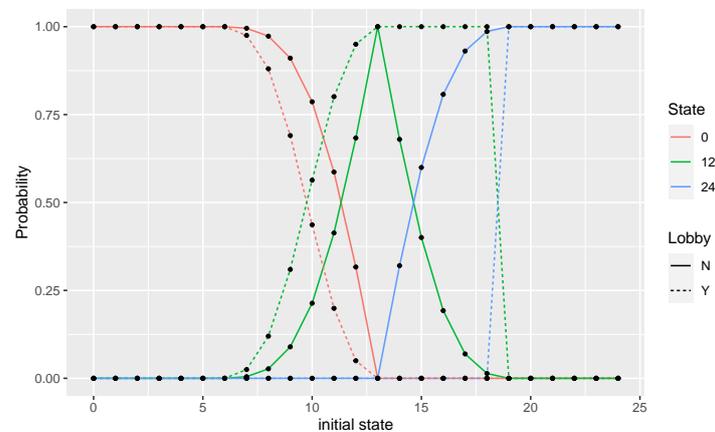


Figure 9. Transition probabilities to the states $j = 0$, $j = 12$ and $j = 24$ with (dashed lines) or without (solid lines) lobby in the social space S_6 versus the number of individuals who are in favor of opinion \mathcal{X} .

The same reasoning applies to the other social spaces, even if the number of states with positive probabilities makes the analysis less immediate; see, Figure 10 for the social space S_5 as an example.

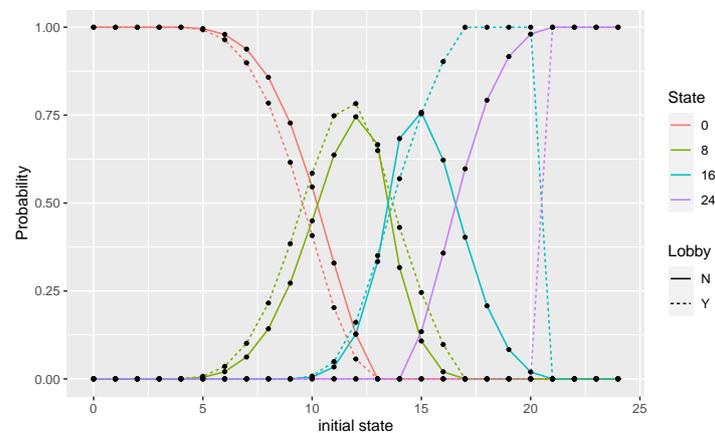


Figure 10. Transition probabilities to the states $j = 0$, $j = 12$ and $j = 24$ with (dashed lines) or without (solid lines) lobby in the social space S_5 versus the number of individuals who are in favor of opinion \mathcal{X} .

To avoid the slowing effect of the lobby that occurs in some social spaces, the strategic behavior of the lobby should add what [58] calls the *third dimension* in negotiation—that is, being able to determine the proper sequence of tables to be seated at, depending on the state; see also [59] for features that help groups in generating knowledge.

Finally, as we have seen in some special cases, such as social space S_2 , the lobby can generate some impasses that do not occur without lobbies.

5. Conclusions

In this paper, we addressed the influence of lobbies on opinion dynamics considering a population of individuals in different social gatherings. In particular, we considered a lobby that can interact in strategic positions. Extending a known sociophysics model of opinion spreading [31], we introduced lobbies to understand when and how they can be effective in different social spaces. In this case, opinion dynamics can also be analyzed as a Markov chain. Since the calculation of transition probabilities can become tedious, these were calculated using simulations. The results show that lobbies with the investigated strategy are able to overturn the majority opinion, even if this requires a certain amount of time to reach a consensus.

The limitations of the results that we obtained highlight the known impossibility of a theory of social behavior to be simultaneously general, accurate and simple [60]; in particular, in order to have a more accurate model of opinion diffusion in a group, agents should be modeled as heterogeneous entities rather than atoms [32] (p. 29). However, in view of the “quiet methodological revolution” discussed in Ref. [41], the sociophysics perspective [32] should be considered as another possible way to overcome the problem of replicability in psychology, as mentioned in the Introduction.

The model and the analysis of the influence of lobbies can be extended in several ways. First, we have looked at a particular strategy of a lobby and a particular decision rule (i.e., the local majority). Future research would need to examine alternative strategies and decision rules to better understand how opinions can spread in social spaces. Second, we considered different twenty-four -individual social spaces consisting of separate tables; these are special types of networks in which each component is a clique. In future research, we plan to consider more general social networks similar to what was studied in Ref. [61]. In particular, it is interesting to investigate whether the influence of lobbies remains the same and how the time of convergence to unanimity is affected by their presence.

Author Contributions: The authors contributed equally to this work: Conceptualization, U.M. and A.D.F.; methodology, U.M. and A.D.F.; software, U.M.; validation, U.M. and A.D.F.; formal analysis, A.D.F.; investigation, U.M. and A.D.F.; writing—original draft preparation, U.M. and A.D.F.; writing—review and editing, U.M. and A.D.F.; visualization, U.M. and A.D.F.; supervision, A.D.F. All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement: The simulated transition matrices \mathbf{M}_h , $\overline{\mathbf{M}}_h$, the R code and the libraries for the replication of our results are available from the authors upon request.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

The pseudocode to compute the transition matrices \mathbf{M}_h without a lobby is reported in Algorithm A1 and the pseudocode to compute the transition matrices $\overline{\mathbf{M}}_h$ of the population with the lobby is reported in Algorithm A2, for each social space S_h .

The parameters that we considered for the simulations in Section 3.3 and the results discussed in Section 4 are the following:

- NumAgents := $N = 24$;
- LobbySize := $N^* = 6$;
- NumberSimulations := 1,000,000.

Algorithm A1 Compute Transition Frequencies with No Lobby

```

1: procedure TRANSITION
2:   M ← 0
3:   for InitialState:=0 to NumAgents do
4:     repeat
5:       randomly choose InitialState agents
6:       set their opinion 'in favor of X'
7:       set remaining agents opinion to 'against X'
8:       agents take seats
9:       agents discuss according to the local majority rule
10:      compute new state
11:      update M
12:    until NumberSimulations

```

Algorithm A2 Compute Transition Frequencies with Lobby

```

1: procedure TRANSITIONLOBBY
2:   M ← 0
3:   for InitialState:=0 to NumAgents do
4:     repeat
5:       if InitialState < NumAgents – LobbySize then
6:         set lobby opinions to 'against X'
7:         randomly choose InitialState – LobbySize agents
8:         set their opinion 'in favor of X'
9:         set remaining agents opinion to 'against X'
10:      else
11:        set first NumAgents – InitialState lobby opinions to 'against X'
12:        set remaining lobby opinions to 'in favor of X'
13:        set remaining opinions to 'in favor of X'
14:      lobby agents take seats
15:      other agents take seats
16:      all agents discuss according to the local majority rule
17:      compute new state
18:      update M
19:    until NumberSimulations

```

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