Supplemental Materials

Minho Lee, Yon Soo Suh, and Minjeong Jeon

S1 Equivalence Between Model Parameterizations in `mirt` and `Mplus`

For Rasch StrMixIRMs, three outputs are compared: a) `mirt` with the new parameterization (Equation (5)), b) `Mplus` with the new parameterization, and c) `Mplus` with the original parameterization (Equation (3)). For Rasch with $s\tau_{hk}$ and 2PL StrMixIRMs, refer to `GitHub` repository (https://github.com/ysuh09/StrMixIRM).

    `Mplus` 8.10 was used for ECPE and verbal aggression data analyses, and detailed data analysis strategies were as follows. First, we used the EM algorithm with 49 quadrature points in lieu of the default setting of 15 points to increase precision. This was set by `INTEGRATION = GAUSS (49);` in `Mplus` input. The number of quadrature points is set smaller in `Mplus` because `Mplus` adapts the class-specific quadrature points to the provisional estimates of mean and variance parameters during estimation. Second, we started with 50 starting values and 10 final optimizations to promote convergence, which was `STARTS = 50 10;` in the input. If convergence was not achieved, we increased them and refit the data to get the converged result. Third, to match the labels between the two programs, the direction of some parameters was constrained using `MODEL CONSTRAINTS` of `Mplus`. For example, for the verbal aggression data analysis (see Table 8), τ_{22} was positive, so the parameter was constrained to be positive (i.e., `t22>0;`) as well. We followed `Mplus`'s default estsimation settings with convergence criterion of the change in log-likelihood being less than 10^{-3}, the change in relative log-likelihood being less than 10^{-6}, the minimum derivative being smaller than 10^{-3}, and the M-step convergence criterion being less than 10^{-3}.

    As a StrMixIRM in `Mplus` with the original parameterization was presented in [16], here we only present the Rasch StrMixIRM syntax for the analysis of the verbal aggression data with the new parameterization. Interested readers can refer to our `GitHub` repository for the input files of all models. Listing S1 shows the `Mplus` input file (i.e., '.inp' file).

Listing S1: `Mplus` input syntax for the Rasch StrMixIRM

```
1  TITLE: Verbal_poly_uni_rasch_grm; ! Title of the analysis
2  DATA: FILE =Verbal_poly.dat; ! Data file
3
4  !! Define variables in dataset
5  VARIABLE:
6  NAMES= y1-y24; ! name of items
7  CATEGORICAL= y1-y24; ! polytomous responses (0, 1, 2)
8  USEVAR= y1-y24; ! what will be used in the analysis
9  MISSING = all(999); ! missing data coding
10 CLASSES = c(2); ! name of the latent class(the number of latent classes);
11
```

```
12 !! Estimation settings
13 ANALYSIS:
14 TYPE = mixture; ! mixture models
15 ALGORITHM = integration; ! numerical integration with quadrature
16 INTEGRATION = GAUSS (49); ! 49 Gaussian quadrature (default is 15)
17 STARTS = 50 10 ; ! 50 random starting / 10 final optimizations among 50 startings
18 PROCESSOR = 4; ! the number of processors
19
20 !! Model specification
21 MODEL:
22 %OVERALL% ! overall model
23 ! Latent trait
24 f by y1-y24@1; ! fix loadings to 1
25 f; ! freely estimate factor variances
26
27 ! D parameters (for y13 to y24)
28 ig2 by y13*; ! free loading of ig2
29 ...
30 ig2 by y24*; ! free loading of ig2
31
32 ! For discrete latent variable
33 ig2@0; ! variance = 0
34 f with ig2@0; ! covariance with f = 0
35
36 ! Equal Item difficulty
37 [y1$1-y24$1] (d1_1-d1_24); ! Idiffs for cat 1
38 [y1$2-y24$2] (d2_1-d2_24); ! Idiffs for cat 2
39
40 %c#1% ! Reference Class model
41 ! Latent Trait distribution
42 f (c1v); ! estimate factor variance
43 [f@0]; ! factor mean at 0 -- model identification
44
45 ! D parameters (for y13 to y24)
46 ig2 by y13@0; ! fix the loading to 0
47 ...
48 ig2 by y24@0; ! fix the loading to 0
49 [ig2@0]; ! value = 0 (discrete latent variable)
50
51 %c#2% ! Focal Class 1 model
52 ! Latent Trait distribution
53 f (c2v); ! estimate factor variance
54 [f] (c2m); ! estimate factor mean
55
56 ! D parameters (for y13 to y24)
57 ig2 by y13* (t22); ! free loading of ig2
58 ...
59 ig2 by y24* (t22); ! free loading of ig2
60 [ig2@1]; ! value = 1 (discrete latent variable)
61
62 !! For model constraints
63 MODEL CONSTRAINTS:
64 t22 >0;
```

- Line 2 is the data file.

- Lines 5 to 10 define the variable types and latent classes.

- Lines 13 to 18 define the estimation settings. The number of quadrature points is 49 (Line 16), and the numbers of starting values and optimizations are 50 and 10 (Line 17).

- Lines 21 to 60 define the overall and class-specific models.

- Lines 22 to 38 define the overall model (`%overall%`).

- Lines 24 and 25 define θ_{ih}, named `f` with fixed slopes of 1 and freely estimated variances.

- Lines 27 to 30 define the latent variable a_{ih2} on items 13 to 24 named `ig2` .

- Lines 33 and 34 define a_{ih2} to be the discrete latent variable by setting the variance to 0 and covariance with `f` to 0.

- Lines 37 and 38 define the item difficulties for all items in each category. Naming the difficulties in parentheses (e.g., `d1_1`) results in equal item difficulties across latent classes.

- Lines 40 to 49 define the Class 1 model (`%c#1%`).

- Lines 42 and 43 define the mean and variance of θ_{i1}. The mean is fixed at 0 (`[f@0]`) for model identification.

- Lines 46 to 48 fix the slopes at 0 for the focal item group (items 13 to 24) on the discrete latent variable a_{i12} for model identification.

- Lines 49 fixes the value of a_{i12} at 0.

- Lines 51 to 60 define the Class 2 model (`%c#2%`).

- Lines 53 and 54 define the mean and variance of θ_{i2}. Both are freely estimated.

- Lines 57 to 59 define the differentiation parameter (τ_{22}) by giving the slopes that same name (`(t22)`).

- Line 60 fixes the value of a_{i22} at 1.

- Lines 63 and 64 impose a constraint on τ_{22} to prevent potential label switching so that the results match that of the `mirt` package.

The results from three analyses in `mirt` and `Mplus` on the Rasch StrMixIRM are presented in Tables S1 and S2 for the ECPE dataset and Tables S3 and S4 for the verbal aggression dataset. From the tables, we first observe that the two parameterizations are empirically equivalent. The model fit indices and parameter estimates are almost equal across the three analyses. This is consistent with the theoretical results in Section 2.2. Second, `mirt` results in smaller standard errors for all parameters than `Mplus`. This may be because each program utilizes different methods to calculate standard errors. `mirt` uses Oakes' method [43,44], while `Mplus` uses a robust method with a sandwich estimator as the default setting [24]. A future research topic would be to investigate the source of differences in standard errors by method and/or program.

Table S1: ECPE: Model fit and structural parameter comparison using `mirt` and `Mplus`

		mirt		Mplus1		Mplus2	
1. Model fit		Value		Value		Value	
N.par		39		43		66	
Log−lik		−42635.12		−42635.12		−42635.12	
AIC		85348.25		85348.24		85348.24	
BIC		85581.47		85581.46		85581.461	
SBIC		85457.55		85457.54		85457.544	
2. Model parameters		EST	SE	EST	SE	EST	SE
Class probabilities	π_1	0.56	-	0.57	-	0.56	-
	π_2	0.32	-	0.32	-	0.32	-
	π_3	0.12	-	0.12	-	0.12	-
Latent trait distributions	μ_1	0.00	-	0.00	-	0.00	-
	σ_1^2	0.34	0.01	0.34	0.14	0.34	0.14
	μ_2	0.43	0.04	0.43	0.29	0.43	0.29
	σ_2^2	1.09	0.05	1.09	0.19	1.09	0.19
	μ_3	−1.60	0.02	−1.61	0.14	−1.61	0.15
	σ_3^2	0.19	0.01	0.19	0.07	0.19	0.07
Differentiation parameters	τ_{22}	0.03	0.15	0.03	0.25	0.03	0.25
	τ_{23}	0.88	0.08	0.88	0.22	0.88	0.22
	τ_{32}	0.99	0.13	1.00	0.24	1.00	0.24
	τ_{33}	0.89	0.08	0.89	0.15	0.89	0.15

Note. `mirt`, `Mplus1`, and `Mplus2` each represent a) `mirt` with the new parameterization (Equation 5), b) `Mplus` with the new parameterization, and c) `Mplus` with the original parameterization (Equation 3). Comparing results of `mirt`, Classes 2 and 3 were switched in the `Mplus` original output. We rearranged the class numbers to match the values in this table because, in the StrMixIRMs, the permutation of labels between focal latent classes does not change the parameter values when the reference latent class is fixed.

Table S2: ECPE: Item intercept comparison using `mirt` and `Mplus`

Item	mirt EST	mirt SE	Mplus1 EST	Mplus1 SE	Mplus2 EST	Mplus2 SE
1	1.34	0.06	1.35	0.13	1.34	0.13
2	1.71	0.07	1.71	0.09	1.71	0.09
3	0.10	0.05	0.10	0.13	0.10	0.13
4	1.09	0.05	1.09	0.09	1.09	0.09
5	2.45	0.07	2.44	0.10	2.44	0.10
6	2.11	0.06	2.11	0.10	2.11	0.10
7	0.84	0.05	0.84	0.13	0.84	0.13
8	2.34	0.08	2.33	0.10	2.33	0.10
9	1.07	0.05	1.07	0.09	1.07	0.09
10	0.50	0.05	0.50	0.13	0.50	0.13
11	0.83	0.05	0.83	0.13	0.83	0.13
12	−0.61	0.05	−0.61	0.13	−0.61	0.13
13	1.03	0.05	1.03	0.13	1.03	0.13
14	0.46	0.05	0.46	0.13	0.46	0.13
15	2.39	0.06	2.39	0.10	2.39	0.10
16	0.74	0.05	0.74	0.13	0.74	0.13
17	2.20	0.08	2.20	0.09	2.20	0.09
18	2.05	0.06	2.04	0.10	2.04	0.10
19	1.12	0.05	1.12	0.09	1.12	0.09
20	−0.47	0.05	−0.47	0.13	−0.47	0.13
21	1.04	0.05	1.04	0.13	1.04	0.13
22	0.69	0.05	0.69	0.09	0.69	0.09
23	1.57	0.07	1.57	0.09	1.57	0.09
24	0.10	0.06	0.10	0.08	0.10	0.08
25	0.30	0.05	0.30	0.13	0.30	0.13
26	1.07	0.05	1.07	0.09	1.07	0.09
27	−0.54	0.05	−0.54	0.13	−0.54	0.13
28	1.83	0.06	1.83	0.09	1.83	0.10

Note. `mirt`, `Mplus1`, and `Mplus2` each represent a) `mirt` with the new parameterization (Equation 5), b) `Mplus` with the new parameterization, and c) `Mplus` with the original parameterization (Equation 3). Item intercept is $-\beta_{jk}$.

Table S3: Verbal aggression: model fit and structural parameter comparison using `mirt` and `Mplus`

		`mirt`		`Mplus1`		`Mplus2`	
1. Model fit		Value		Value		Value	
N.par		53		53		53	
Log−lik		-6255.40		-6255.31		-6255.30	
AIC		12616.81		12616.62		12616.61	
BIC		12815.86		12815.67		12815.66	
SBIC		12647.76		12647.57		12647.56	
2. Model parameters		EST	SE	EST	SE	EST	SE
Class	π_1	0.48	-	0.47	-	0.47	-
probabilities	π_2	0.52	-	0.53	-	0.53	-
	μ_1	0.00	-	0.00	-	0.00	-
Latent trait	σ_1^2	3.02	0.42	3.06	0.75	3.07	0.73
distributions	μ_2	-0.11	0.07	-0.09	0.31	-0.11	0.30
	σ_2^2	0.84	0.06	0.85	0.21	0.85	0.20
Differentiation parameter	τ_{22}	1.74	0.13	1.74	0.15	1.74	0.15

Note. `mirt`, `Mplus1`, and `Mplus2` represent a) `mirt` with the new parameterization (Equation 5), b) `Mplus` with the new parameterization, and c) `Mplus` with the original parameterization (Equation 3), respectively.

Table S4: Verbal Aggression: Item Intercept Comparison in `mirt` and `Mplus`

| | Intercept 1 | | | | | | Intercept 2 | | | | | |
| | mirt | | Mplus1 | | Mplus2 | | mirt | | Mplus1 | | Mplus2 | |
Item	EST	SE	EST	SE	EST	SE	EST	SE	EST	SE	EST	SE
1	1.26	0.16	1.24	0.22	1.25	0.22	−0.39	0.16	−0.41	0.22	−0.40	0.21
2	0.65	0.15	0.63	0.22	0.64	0.22	−0.85	0.16	−0.87	0.22	−0.85	0.21
3	0.13	0.15	0.11	0.22	0.12	0.22	−1.75	0.18	−1.76	0.23	−1.75	0.23
4	1.76	0.17	1.74	0.24	1.76	0.23	−0.28	0.15	−0.30	0.22	−0.28	0.22
5	0.75	0.16	0.73	0.22	0.75	0.22	−0.86	0.16	−0.88	0.22	−0.86	0.22
6	0.09	0.15	0.08	0.22	0.09	0.22	−1.45	0.17	−1.47	0.22	−1.46	0.21
7	0.56	0.15	0.54	0.22	0.56	0.22	−1.60	0.18	−1.62	0.22	−1.60	0.22
8	−0.61	0.16	−0.63	0.22	−0.61	0.21	−2.92	0.24	−2.93	0.27	−2.92	0.27
9	−1.40	0.17	−1.42	0.23	−1.40	0.23	−3.85	0.33	−3.87	0.34	−3.85	0.34
10	1.10	0.16	1.08	0.22	1.10	0.22	−1.09	0.17	−1.11	0.23	−1.09	0.22
11	−0.27	0.16	−0.29	0.22	−0.27	0.21	−2.13	0.20	−2.14	0.23	−2.13	0.23
12	−0.95	0.16	−0.97	0.22	−0.95	0.22	−2.59	0.22	−2.61	0.26	−2.59	0.25
13	0.40	0.19	0.36	0.40	0.38	0.40	−1.67	0.19	−1.71	0.43	−1.69	0.42
14	−0.50	0.18	−0.53	0.41	−0.51	0.40	−2.36	0.20	−2.40	0.42	−2.38	0.42
15	−1.85	0.19	−1.89	0.43	−1.87	0.42	−3.48	0.23	−3.52	0.47	−3.50	0.46
16	0.00	0.19	−0.03	0.41	−0.01	0.40	−1.82	0.19	−1.85	0.43	−1.83	0.42
17	−0.98	0.19	−1.02	0.41	−1.00	0.40	−2.84	0.21	−2.88	0.43	−2.86	0.43
18	−2.44	0.20	−2.47	0.43	−2.45	0.42	−4.11	0.27	−4.15	0.48	−4.13	0.47
19	−1.17	0.19	−1.21	0.42	−1.19	0.41	−3.55	0.24	−3.59	0.45	−3.56	0.44
20	−2.44	0.20	−2.48	0.44	−2.46	0.43	−4.64	0.31	−4.68	0.50	−4.66	0.49
21	−3.87	0.25	−3.90	0.49	−3.88	0.48	−6.25	0.56	−6.30	0.68	−6.27	0.68
22	−0.22	0.19	−0.25	0.41	−0.23	0.40	−2.39	0.20	−2.42	0.43	−2.40	0.43
23	−1.31	0.19	−1.35	0.42	−1.33	0.41	−3.33	0.23	−3.37	0.45	−3.34	0.44
24	−2.93	0.21	−2.97	0.44	−2.94	0.44	−4.79	0.33	−4.82	0.50	−4.80	0.50

Note. `mirt`, `Mplus1`, and `Mplus2` represent a) `mirt` with the new parameterization (Equation 5), b) `Mplus` with the new parameterization, and c) `Mplus` with the original parameterization (Equation 3). Item intercepts are $-\beta_{jkm}$.

S2 Verbal Aggression Data: Multidimensonal Rasch StrMixIRM in `mirt`

In this analysis, we mimicked Jeon's [17] model specification for the verbal aggression data. Specifically, we followed Jeon [17] hypothesis (a) that respondents' verbal aggression differs based on the situational types of 'Self-to-blame' and 'Other-to-blame,' so that the two situational types each make up a separate latent trait. The ability of 'Do' items to differentiate respondents into latent classes (i.e., reference versus focal) would differ across the two dimensions. In the data, items 1-6 and 13-18 are related to the 'Self-to-blame' situation, while items 7-12 and 19-24 relate to the 'Other-to-blame' situation. Items 13-18 and 19-24 ('Do' items) are used to differentiate respondents on each of these two dimensions. The `mirt.model` syntax for this model is presented in Listing S2.

Listing S2: Verbal aggression: multidimensional Rasch StrMixIRM syntax

```
1  # mirt model syntax for Rasch multidimensional model
2  mod.syn <- mod.syn <- "F1 = 1-6, 13-18
3             F2 = 7-12, 19-24
4             COV = F1*F2
5             D1T2 = 13-18
6             D2T2 = 19-24
7
8             CONSTRAINB = (1-24, d1)
9             CONSTRAINB = (1-24, d2)
10
11            START[MIXTURE_1, MIXTURE_2] = (1-6, 13-18, a1, 1), (7-12, 19-24, a2,
    1)
12            FIXED[MIXTURE_1, MIXTURE_2] = (1-6, 13-18, a1), (7-12, 19-24, a2)
13
14            START[MIXTURE_1] = (13-18, a3, 0), (19-24, a4, 0)
15            FIXED[MIXTURE_1] = (13-18, a3), (19-24, a4)
16
17            CONSTRAIN[MIXTURE_2] = (13-18, a3), (19-24, a4)
18
19            FREE[MIXTURE_1] = (GROUP, COV_11), (GROUP, COV_12), (GROUP, COV_22)
20            FREE[MIXTURE_2] = (GROUP, MEAN_1), (GROUP, MEAN_2), (GROUP, COV_11), (
    GROUP, COV_12), (GROUP, COV_22)"
```

- Lines 2 and 3 define items loaded on each domain. `F1` and `F2` correspond to 'Self-to-blame' and 'Other-to-blame', respectively.

- Line 4 sets the covariance to be estimated.

- Lines 5 and 6 define items loading on discrete latent variables for the differentiation parameters on each dimension. `D1T2` and `D2T2` relate 'Do' items for 'Self-to-blame' and 'Other-to-blame', respectively.

- Lines 8 and 9 constrain intercepts to be equal across latent classes.

- Lines 11 and 12 fix the slope of `F1` and `F2` at 1.

- Lines 14 and 15 fix the differentiation parameters to 0 for the reference class.

- Lines 17 constrains the slopes of latent discrete variables to be equal in the focal class for the differentiation parameters.

- Lines 19 and 20 define the freely estimated parameters of the latent trait distributions. The covariance accounting for the multidimensionality is involved by `(GROUP, COV_12)`.

Model calibration code is presented in Listing S3.

Listing S3: Verbal aggression: muldimensional model calibration

```
1 # Quadrature points: 31s for each dimension [-6, 6] / 1 for two discrete latent
      variables
2 quad <- data.matrix(expand.grid(seq(-6, 6, length.out = 31), seq(-6, 6, length.out
      = 31))) # for multiple latent trait
3 quad <- cbind(quad, 1, 1) # for discrete latent variables
4
5 # run models with 20 starting points
6 multi.out <- vector(mode = "list", length = 20)
7 multi.ll <- NULL
8 for(i in 1:20){
9   multi.out[[i]] <-   mirt::multipleGroup(data = verb.wide, # response data object
10                                      model = mod.syn, # mirt model syntax
11                                      SE=T, # calculate standard errors
12                                      dentype = "mixture-2", # mixture IRT for 2
     classes
13                                      technical = list(customTheta = quad, #
     specified quadratures
14                                                    NCYCLES = 2000), # max number
      of iterations
15                                      GenRandomPars = T) # random starting values
16
17   multi.ll <- c(multi.ll, mirt::extract.mirt(multi.out[[i]], what="logLik"))
18 }
```

- Lines 2 and 3 define the quadrature points manually. Quadrature points for the two latent traits are created first (Line 2), followed by the addition of quadrature points for the two discrete latent variables (Line 3).

- The other lines are similar to those in the manuscript.

Listing S4 shows how to extract the outputs once model calibration is completed.

Listing S4: Verbal Aggression: Multidimensional Model Output

```
1 # result with the highest log-likelihood
2 highest <- which(multi.ll == max(multi.ll))
3
4 # select the best output
5 out <- multi.out[[highest]]
6
7 # model information
8 mirt::extract.mirt(out, what="nest") # the number of unique parameters
9 mirt::extract.mirt(out, what="logLik") # log-likelihood
10 mirt::extract.mirt(out, what="AIC") # AIC
```

```r
mirt::extract.mirt(out, what="BIC") # BIC
mirt::extract.mirt(out, what="SABIC") # SABIC

# Estimates and SE
out.coef <- coef(out, printSE=T)

# item intercept (a negative of item difficulty)
int1 <- NULL
int2 <- NULL
for(i in 1:24){
  int1 <- rbind(int1, out.coef[["MIXTURE_1"]][[i]][, "d1"])
  int2 <- rbind(int2, out.coef[["MIXTURE_1"]][[i]][, "d1"])
}

# Intercept Tau parameter
out.coef[["MIXTURE_2"]][[13]][, "a3"] # tau122
out.coef[["MIXTURE_2"]][[19]][, "a4"] # tau222

# Latent trait distribution
out.coef[["MIXTURE_1"]][[24+1]] # for the class 1 (reference)
out.coef[["MIXTURE_2"]][[24+1]] # for the class 2 (focal 1)

# class probability
exp(c(out.coef[["MIXTURE_1"]][[24+1]][1, "PI"],
      out.coef[["MIXTURE_2"]][[24+1]][1, "PI"]))/
  sum(exp(c(out.coef[["MIXTURE_1"]][[24+1]][1, "PI"],
            out.coef[["MIXTURE_2"]][[24+1]][1, "PI"])))
```

S3 Item Parameters in ECPE and Verbal Aggression Data Analyses

Since the item intercept parameters of the Rasch StrMixIRMs are presented in the tables above, here we present item parameters of the Rasch with $s\tau_{hk}$ and 2PL StrMixIRMs. Only results from the `mirt` package are included. For the results of `Mplus`, refer to the `GitHub` repository.

Table S5: ECPE: item parameters from `mirt`

| | **Rasch with** $s\tau_{hk}$ | | **2PL** | | | |
| | Intercept | | Slope | | Intercept | |
Item	EST	SE	EST	SE	EST	SE
1	1.71	0.07	0.47	0.04	1.31	0.07
2	1.60	0.08	0.50	0.04	1.79	0.09
3	0.47	0.06	0.44	0.04	0.09	0.05
4	1.19	0.05	0.45	0.03	1.16	0.06
5	2.54	0.07	0.51	0.04	2.61	0.10
6	2.21	0.06	0.44	0.03	2.17	0.08
7	1.20	0.07	0.72	0.04	1.07	0.07
8	2.24	0.08	0.59	0.05	2.53	0.11
9	1.17	0.05	0.34	0.03	1.05	0.05
10	0.86	0.06	0.53	0.04	0.53	0.06
11	1.20	0.07	0.64	0.04	0.98	0.07
12	−0.24	0.06	0.74	0.05	−0.52	0.06
13	1.40	0.07	0.52	0.04	1.06	0.06
14	0.82	0.06	0.43	0.04	0.42	0.05
15	2.48	0.06	0.56	0.04	2.64	0.10
16	1.11	0.06	0.63	0.04	0.87	0.06
17	2.10	0.08	0.60	0.05	2.40	0.10
18	2.14	0.06	0.38	0.03	2.02	0.07
19	1.21	0.05	0.49	0.03	1.22	0.06
20	−0.11	0.06	0.68	0.04	−0.38	0.06
21	1.40	0.07	0.65	0.04	1.20	0.07
22	0.78	0.05	0.60	0.04	0.85	0.06
23	1.47	0.08	0.66	0.05	1.83	0.09
24	−0.02	0.08	0.55	0.04	0.23	0.07
25	0.66	0.06	0.35	0.04	0.22	0.05
26	1.17	0.05	0.28	0.02	1.00	0.05
27	−0.18	0.06	0.50	0.04	−0.50	0.05
28	1.93	0.06	0.49	0.03	1.95	0.07

Table S6: Verbal aggression: item parameters from `mirt`

| | **Rasch with** $s\tau_{hk}$ | | | | **2PL** | | | | | |
| | Intercept 1 | | Intercept 2 | | Slope | | Intercept 1 | | Intercept 2 | |
Item	EST	SE	EST	SE	EST	SE	EST	SE	EST	SE
1	1.18	0.17	−0.50	0.16	1.75	0.25	1.63	0.21	−0.10	0.17
2	0.56	0.16	−0.96	0.16	2.16	0.29	1.16	0.21	−0.60	0.19
3	0.03	0.16	−1.86	0.18	1.31	0.19	0.34	0.15	−1.47	0.17
4	1.70	0.18	−0.38	0.16	1.58	0.23	2.05	0.21	−0.01	0.16
5	0.67	0.16	−0.97	0.16	1.90	0.25	1.16	0.19	−0.61	0.18
6	0.00	0.16	−1.56	0.18	1.36	0.21	0.32	0.15	−1.18	0.17
7	0.47	0.16	−1.71	0.18	1.23	0.18	0.73	0.15	−1.32	0.16
8	−0.71	0.16	−3.03	0.24	1.47	0.21	−0.35	0.15	−2.65	0.25
9	−1.51	0.18	−3.95	0.33	0.97	0.19	−1.08	0.15	−3.28	0.30
10	1.02	0.17	−1.20	0.17	1.07	0.17	1.16	0.16	−0.84	0.15
11	−0.37	0.16	−2.24	0.20	1.58	0.22	−0.01	0.16	−1.92	0.20
12	−1.06	0.17	−2.71	0.22	1.02	0.18	−0.69	0.14	−2.18	0.20
13	0.39	0.19	−1.68	0.19	1.94	0.23	0.90	0.20	−1.20	0.18
14	−0.51	0.18	−2.36	0.20	2.51	0.27	0.10	0.21	−1.95	0.21
15	−1.86	0.19	−3.46	0.23	2.05	0.24	−1.39	0.19	−3.06	0.24
16	−0.01	0.19	−1.83	0.19	1.92	0.22	0.49	0.19	−1.35	0.18
17	−1.00	0.19	−2.83	0.21	2.53	0.27	−0.42	0.20	−2.49	0.23
18	−2.43	0.20	−4.08	0.27	2.18	0.26	−2.01	0.20	−3.77	0.29
19	−1.18	0.19	−3.53	0.23	1.63	0.20	−0.73	0.17	−3.04	0.22
20	−2.44	0.20	−4.60	0.31	2.09	0.25	−2.00	0.20	−4.30	0.33
21	−3.83	0.25	−6.19	0.56	1.59	0.26	−3.32	0.25	−5.68	0.57
22	−0.23	0.19	−2.39	0.20	1.58	0.20	0.19	0.17	−1.91	0.18
23	−1.33	0.19	−3.31	0.23	1.99	0.23	−0.84	0.18	−2.89	0.22
24	−2.92	0.21	−4.75	0.32	1.76	0.24	−2.45	0.20	−4.30	0.33

S4 Simulation Study on Parameter Recovery

In addition to the comparison of the parameter estimates of ECPE and verbal aggression data, a small simulation study was conducted to verify the performance of the new parameterization in `mirt` and `Mplus`. For the sake of simplicity, a Rasch model with dichotomously scored items was used as the data generating model. True parameter values were borrowed from the results of the verbal aggression dataset. More specifically, the first intercept values were used as the item threshold values of the Rasch model for simulation. 30 response datasets were generated.

We tried to utilize the default estimation settings in the two programs as much as possible as we assumed practitioners will most likely utilize those settings. For the `mirt` package, the quadrature points of the primary latent trait ranged from -6 to 6 with 61 equidistant points, which is the default setting of `mirt` for unidimensional IRT models. The convergence criterion was the maximum absolute difference in parameter estimates between adjacent iterations being less than 10^{-4}. Each simulated dataset was run 15 times to ascertain global maxima, same as in the empirical data applications. Estimation settings of `Mplus` were equivalent to those described in Section S1.

Evaluation criteria consisted of Bias, standard error of estimates (SEE), and root mean square error (RMSE). Each criterion was defined as follows:

$$Bias_p = \frac{\sum_{r=1}^{30}(\hat{\gamma}_{pr} - \gamma_p)}{30}. \tag{1}$$

$$SEE_p = \sqrt{\frac{\sum_{r=1}^{30}(\hat{\gamma}_{pr} - \bar{\gamma}_p)^2}{30}}. \tag{2}$$

$$RMSE_p = \sqrt{\frac{\sum_{r=1}^{30}(\hat{\gamma}_{pr} - \gamma_p)^2}{30}}. \tag{3}$$

For the pth parameter ($p = 1, \cdots, 29$), $\hat{\gamma}_{pr}$ was the estimate at the rth replication ($r = 1, \cdots, 30$), γ_p is the true value, and $\bar{\gamma}_p$ is the average of the estimates over 30 replications.

Table S7 presents the recovery results of individual parameters for the `mirt` package and `Mplus`. The averages of running time over 30 replications are also included in the last row. As expected, parameter recovery in terms of Bias, SEE, and RMSE was satisfactory. Somewhat large Bias, SEE, and RMSE in the variance parameters might be attributable to the small sample size (316). It is well known that stable estimation of such random effect parameters usually requires more samples than fixed effect parameters. In both programs, the patterns and sizes of parameter recovery indices were almost equivalent, and thus their differences were negligible.

Table S7: Simulation: Parameter Recovery in Both Programs

Par	True	mirt			Mplus		
		Bias	SEE	RMSE	Bias	SEE	RMSE
π_1	0.48	-0.03	0.09	0.09	-0.03	0.09	0.09
σ_1^2	3.02	-0.21	0.57	0.60	-0.16	0.62	0.64
μ_2	-0.11	0.05	0.32	0.32	0.03	0.32	0.32
σ_2^2	0.84	0.08	0.33	0.34	0.07	0.32	0.33
τ_{22}	1.74	0.04	0.19	0.20	0.04	0.20	0.20
β_1	-1.26	-0.01	0.23	0.23	0.00	0.23	0.23
β_2	-0.65	0.00	0.26	0.26	0.01	0.26	0.26
β_3	-0.13	0.00	0.28	0.28	0.01	0.28	0.28
β_4	-1.76	0.02	0.29	0.29	0.03	0.29	0.29
β_5	-0.75	0.06	0.29	0.30	0.07	0.29	0.30
β_6	-0.09	-0.04	0.27	0.27	-0.04	0.26	0.27
β_7	-0.56	-0.04	0.25	0.25	-0.03	0.25	0.25
β_8	0.61	-0.02	0.29	0.29	-0.01	0.29	0.29
β_9	1.40	0.03	0.23	0.23	0.04	0.22	0.23
β_{10}	-1.10	-0.02	0.29	0.29	-0.01	0.29	0.29
β_{11}	0.27	-0.04	0.27	0.27	-0.03	0.27	0.27
β_{12}	0.95	0.01	0.26	0.26	0.02	0.25	0.26
β_{13}	-0.40	-0.03	0.33	0.33	-0.02	0.32	0.32
β_{14}	0.50	-0.04	0.35	0.36	-0.04	0.35	0.35
β_{15}	1.85	0.00	0.29	0.29	0.01	0.29	0.29
β_{16}	0.00	-0.10	0.38	0.39	-0.09	0.37	0.38
β_{17}	0.98	-0.04	0.34	0.34	-0.03	0.33	0.33
β_{18}	2.44	-0.01	0.43	0.43	0.00	0.43	0.43
β_{19}	1.17	-0.06	0.36	0.37	-0.06	0.36	0.36
β_{20}	2.44	-0.01	0.36	0.36	0.00	0.36	0.36
β_{21}	3.87	-0.11	0.60	0.61	-0.09	0.60	0.61
β_{22}	0.22	-0.04	0.34	0.34	-0.03	0.33	0.34
β_{23}	1.31	-0.02	0.39	0.39	-0.01	0.39	0.39
β_{24}	2.93	-0.04	0.38	0.39	-0.03	0.38	0.38
Running Time (min)			2.73			0.45	

Note. Running Time (min) in the last row indicates the average running time over 30 replications per program. In mirt, the computation time of standard errors is included in the running time. In Mplus, standard errors are always computed, so the running time includes the computation of standard errors.