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A Revisit to Sunk Cost Fallacy for Two-Stage Stochastic Binary Decision Making

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Abstract: This paper undertakes a revisit of the sunk cost fallacy, which refers to the tendency of people to persist investing resources into something, even if it is destined to have no good outcome. We emphasize that the utilities associated with different alternatives are not static for decision makers, which is exactly opposite to the traditional perspective. This paper argues that the utility of an option may change due to the choice of another option, suggesting that decisions considered irrational by the traditional analytical method, i.e., sunk cost fallacy, may be rational. We propose a novel analytical method for decision making with sunk cost when considering the utility change and validate the effectiveness of this method through mathematical modeling and computational experiments. This paper mathematically describes such decision-making problems, analyzing the impact of changes in the utilities across different alternatives on decision making with a real-world example. Furthermore, we develop a two-stage stochastic optimization model for such decision-making problems and employ the sample average approximation (SAA) method to solve them. The results from computational experiments indicate that some decisions traditionally considered irrational are, in fact, rational when the utility of an option changes as a result of choosing another option. This paper, therefore, highlights the significance of incorporating utility changes into the decision-making process and stands as a valuable addition to the literature, offering a refreshed and effective decision-making method for improved decision making.

Keywords: sunk cost fallacy; decision analysis; two-stage stochastic optimization; sample average approximation

MSC: 90-10

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1. Introduction

Sunk costs refer to costs that have already been incurred and cannot be recovered, including time, money, resources, etc., invested in a project or decision option. These costs should not influence current decision making because they are irretrievable [1]. Notably, sunk cost is different from opportunity cost, which refers to the loss of potential gain from other alternatives when one alternative is chosen. Previous research believes that rational decisions should focus on future potential costs and benefits, rather than past expenditures [2,3]. There are numerous examples of sunk costs in everyday life. For instance, if someone has purchased a movie ticket and the ticket is non-refundable, the money spent on the ticket becomes a sunk cost, as it cannot be recovered regardless of whether the movie is attended. In the financial domain, sunk costs refer to the funds that have been invested and cannot be recovered, such as the costs that cannot be recouped through selling stocks or bonds. In corporate management, they cover resources that have been invested by the company, such as machinery and equipment purchases, which cannot be recovered by ceasing production or shutting down factories.

1.1. Literature Review

Research on sunk costs and related biases spans various domains, shedding light on decision-making processes. Several studies confirm the presence of the sunk cost effect, highlighting its intricate relationship with cognitive processes. Ronayne et al. [4] underscore the roles of cognitive reflection and crystallized intelligence in decision making influenced by sunk costs. However, Ott et al. [5] present alternative explanations, challenging the traditional understanding of this effect. Moreover, individual differences in decision-making strategies are explored by Yan and Otto [6], while Guan et al. [7] link impulsive behavior to anticipated regret and after-sale risk, revealing nuanced factors contributing to the sunk cost phenomenon. Leal et al. [8] emphasize its impact on business decisions, while Duin et al. [9] highlight the importance of certainty and uncertainty in shaping decision-making outcomes.

Studies also delve into diverse contexts, enriching our understanding of sunk costs' influence. Sehl et al. [10] contrast children's and adults' anticipation of sunk costs, indicating developmental differences in decision making. Jhang et al. [11] connect childhood environments to the sunk-cost fallacy, while Perignat and Fleming [12] explore its manifestation in research project termination, elucidating its implications in various settings. Beyond these, research extends to education, sports, and industry concentration, offering a comprehensive view of sunk costs' effects. Bhayani [13] de-bates the impact of payment methods on higher education, while Farah and Baker [14] analyze its implications in talent development within the NHL draft. Lieberman [15] examines its effects on industry dynamics, and Deslatte and Stokan [16] investigate its role in local government sustainability efforts. Schmitzer-Torbert [17] examines mind-fulness's influence, while Devoto and DeFulio [18] compare behavior-based and scenario-based procedures. Jain and Chen [19] investigate its implications in pricing decisions, and Ma [20] studies its effect on car insurance policyholders' behavior. Moreover, age-related differences in sunk cost decision making are highlighted by Huai et al. [21], while leadership skills in mitigating the sunk cost fallacy are emphasized by Smith [22]. Additionally, neural mechanisms underlying sunk cost effects are revealed by Wang et al. [23], and consumer behavior tendencies in life insurance pricing are uncovered by Carson et al. [24]. Finally, Negrini et al. [25] contributes to understanding risk and loss aversion dynamics through the discovery of a reverse sunk cost effect in investment decisions.

Economists and psychologists have long been interested in how sunk costs influence decision makers' behavior [26]. Many individuals have an aversion to losses and worry about wasting resources, as illustrated by the scenario where people may compel themselves to watch a movie that they have no desire to see, simply because they fear to waste the money spent on tickets. Sunk costs often sway people's decisions, leading them to exhibit a tendency to persist investing resources such as money, effort, or time, a phenomenon known as the sunk cost fallacy [27,28]. However, rational decision making should be based on whether one desires to continue watching the movie, rather than the amount paid for the tickets. The decision at hand should not take into account the act of purchasing tickets but rather be made with the mindset of watching a free movie. If the utility of the movie is lower than expected, rational individuals should exit and engage in more meaningful activities, considering that they would only incur a minor loss, whereas continuing to watch the movie would prolong the sense of loss. Sunk cost fallacy, also referred to as the Concorde effect, highlights the situation in the 1960s when the British and French governments continued to fund the development of the Concorde aircraft despite its apparent lack of economic viability, and ultimately leading to a commercial disaster [29]. Therefore, excessive focus on sunk costs when we make decisions, whether by businesses or individuals, may result in erroneous choices [30], such as persisting with investments in a failing project solely due to prior substantial investments of capital and time. Therefore, rational decision makers should eliminate the influence of sunk costs when making investment decisions. However, classical economic theories consider the sunk cost fallacy under the premise that the utilities of different alternatives are static and invariant for decision makers [31]; that is, if one is chosen, the utilities of the other alternatives remain unchanged.

1.2. Contributions and Organization

Interestingly, we notice that the utilities of different alternatives may not be static for decision makers, i.e., the utility of an option may change due to the choice of another option. For example, a rational decision maker has two alternatives: going to cinema for a movie or staying at home. Choosing to watch an unsatisfactory movie instead of going back home may not be driven solely by the fear of wasting the ticket money. Rather, due to the need to commute back home from the cinema, the utility of staying at home decreases and is actually lower than that of continuing to watch the movie. Thus, seemingly irrational decisions indicated by the sunk cost fallacy may actually be rational in certain circumstances.

Traditionally, in decision-making problems related to the choice of different alternatives, the utilities of alternatives are treated as deterministic and static for decision makers. However, this paper proposes that the utilities are actually dynamic; that is, the utilities of different alternatives may change during the decision-making process. This research, thus, revisits sunk cost fallacy and contributes a novel perspective to the existing literature, considering the impact of utility changes on decision-making problems with sunk cost.

Specifically, this paper analyzes the changes in the utilities of two alternatives during the proceeding of decision making and their impacts on the decision making. By contrasting with the traditional analytical method, this paper argues that if the utilities of different alternatives are altered by decision making, then decisions perceived as irrational indicated by the sunk cost fallacy may, in fact, be rational decisions. For a quantitative study of this framework, a two-stage stochastic optimization model is developed, and the paper analyzes how optimal decisions should be made in situations where utilities fluctuate [32,33]. The results of computational experiments validate our proposed model and solution method. Specifically, in some scenarios, our proposed method leads to the rational and optimal decision, while the traditional analytical method results in a loss of utility.

Therefore, our proposed method distinguishes itself from the existing literature in the following three aspects:

(i) We consider the utility changes of different alternatives during the decision-making process for decision makers.
(ii) We propose a novel mathematical model for solving our studied decision-making problem, which is a two-stage stochastic binary optimization model.
(iii) We verify the effectiveness of our proposed method through comprehensive computational experiments.

The remainder of this paper is organized as follows. Section 2 states the decision-making problem with sunk cost and illustrates it through a real-world example. Section 3 establishes a two-stage stochastic optimization model for this decision-making problem. Section 4 designs four sets of simulation experiments, analyzes the experimental results, and draws insights. Section 5 concludes this paper.

2. Problem Statement

We consider a decision-making problem with sunk cost as follows [6]. For a rational decision maker with the objective of maximizing the utility, there are two options A and B at time point one with the utilities $\widetilde{a}$ and b, respectively, where $\widetilde{a}$ represents a random utility for option A, and b is a deterministic utility for option B. Choosing option A requires a deterministic cost, denoted as c. Once option A is invested, it cannot be refunded. Before choosing option A, the decision maker has an estimated utility value of $\widetilde{a}$, denoted by $\hat{a}$. Furthermore, the realized value of $\widetilde{a}$, denoted by a, can only be observed after choosing option A. The decision maker faces the decision of choosing between options A and B at time point one. If option A is chosen and the realized value a is observed, there remains an opportunity to switch to option B; that is, the decision maker can choose to continue with option A or switch to option B at time point two. However, if option B is initially chosen, the opportunity to switch to option A is lost, which means that the decision maker obtains a utility b.

Given the above introduction, when a decision maker is confronted with two options at time point one, it is difficult to assign specific numerical values to $\hat{a}$ and b; instead, they often analyze the difference between them. Standing at time point one, it is straightforward to see that as long as $\hat{a} - b > c$, that is, the utility increment generated by option A exceeds the utility of option B, a rational decision maker should choose option A.

If choosing option A at time point one and observing the realized value a, the decision maker needs to make the further decision between continuing with option A or switching to option B at time point two. According to the classical economic theory, if $a \geq b$, a rational decision maker should continue with option A; otherwise, the decision maker should switch to option B. Conversely, if the decision maker realizes that $a < b$ but feels compelled to continue with option A because the cost has already been invested and cannot be recovered, the decision is deemed irrational, falling into the sunk cost fallacy.

However, the traditional sunk cost fallacy mentioned above overlooks some factors. For the decision maker, choosing option A may, under some circumstances, result in the utility of option B changing from b to b', and in general $b' < b$. In other words, the utilities of options are sometimes not static, and the utility of option B may change due to the choice of option A. Considering the change of utility from b to b' for option B, the rational decision should be: if $a \geq b'$, the decision maker should continue with option A; if $a < b'$, the decision maker should switch to option B. Therefore, if $a < b$ but $a \geq b'$, the seemingly irrational decision of the decision maker under the traditional analytical method is actually rational. For better illustration, the flowchart of the decision-making process for the problem mentioned above considering the utility change is illustrated in Figure 1.

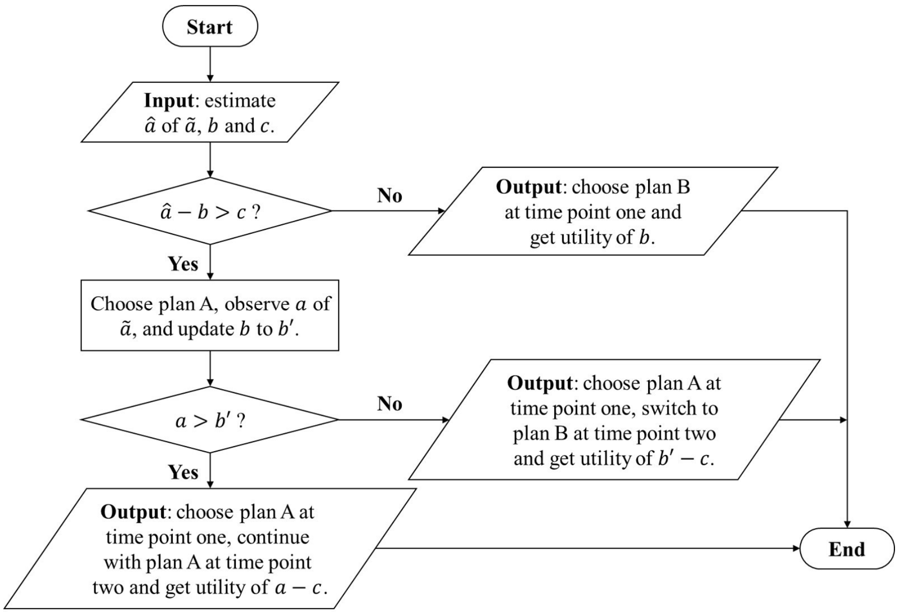

Figure 1. Flowchart of decision-making process for the problem considering the utility change.

2.1. An Illustrative Example of a Manufacturing Enterprise

We consider a real-world example of a manufacturing enterprise, who is currently facing a decision between two options: plan A producing on the new production line, while plan B continuing production on the current old production line. $\widetilde{a}$ denotes the profit obtained per unit time when producing on the new production line, and $\widetilde{a}$ is considered a random variable. Before choosing plan A, the enterprise has an estimated value $\hat{a}$ of $\widetilde{a}$, and the realized value a of $\widetilde{a}$ can only be observed after choosing plan A. c denotes the investment cost per unit time for the new production line, and b denotes the profit obtained per unit time when producing on the current old production line, and b is considered a deterministic value. It is assumed that for this enterprise, if plan A is chosen and the benefit of the new production line is not as expected, there is still an opportunity to switch back to the original old production line; however, if plan B is chosen initially, the opportunity to update the production line will be missed.

It is straightforward to see that as long as $\hat{a} - b > c$, that is, the expected increase in profit brought by the new production line exceeds the input cost, a rational enterprise should choose plan A to update the production line. Suppose the enterprise decides to choose plan A and, after a period of production, observes the realized profit value a is lower than $\hat{a}$. Subsequently, according to the classical economic theory, if $a \geq b$, the enterprise should continue producing on the new production line; if $a < b$, the enterprise should switch back to the original old production line. If the enterprise realizes that $a < b$ but feels compelled to continue with plan A because the cost of updating the old production line has already been invested and cannot be recovered, the decision is irrational, falling into the sunk cost fallacy.

However, some practical factors are ignored for the enterprise in the traditional sunk cost fallacy mentioned above. For instance, if the enterprise chooses plan A to update the production line, it is necessary to recruit new technical personnel, spend time and energy training workers, and require some time to adapt to the new mode of production. Consequently, if the enterprise finds that the profit after updating the production line is lower than before, and decides to switch back to the old production line, the enterprise needs to lay off the new technical personnel, spend time and energy converting the new production line back to the old one, and after a series of disturbances, the productivity of workers may decrease. Therefore, the profit of plan B would change from b to b', and $b' < b$. In this case, the rational decision should be: if $a \geq b'$, the enterprise should continue producing on the new production line; if $a < b'$, the enterprise should opt for plan B, i.e., switch back to the original old production line. Therefore, if $a < b$ but $a \geq b'$, the seemingly irrational decision of the enterprise under the traditional analytical method is actually rational.

2.2. Other Real-World Examples

There are various decision-making problems with sunk cost in daily life, such as Examples 1 and 2 shown below.

Example 1. *For a rational decision maker, consider that there are two alternatives for spending Friday evening: plan A is to go to the cinema to watch a movie; and plan B is to stay at home and watch TV. We assume that, for the decision maker, if plan A is chosen and the movie turns out to be unsatisfactory, there is still an opportunity to switch to plan B and watch TV at home. However, if plan B is chosen initially, the opportunity to switch to plan A is lost due to missing the movie. If the decision maker chooses plan A to go to the cinema, he may need to spend time changing clothes and commuting from home to the cinema. Consequently, when the decision maker chooses plan A and intends to return home to watch TV after finding the movie unsatisfactory, he also needs to spend time commuting back home, etc., resulting a decline in the utility of plan B. In other words, the utilities of both plans are not static, and the utility of plan B may change due to the choice of plan A. Therefore, the seemingly irrational decision of the decision maker under the traditional analytical method is actually rational in some situations.*

Example 2. *Upon graduation, a university student may receive multiple job offers but can only choose one. If the chosen job turns out to be unsatisfactory upon starting, the student may not necessarily seek out other job opportunities. The reason is that the student may not necessarily be influenced by sunk cost fallacy but rather by the fact that other job opportunities have been filled, or* *he would not be able to get the high salary as before if switching to another job; that is, the utilities of different alternatives have changed*.

In summary, through abstracting and mathematically describing real-world problems, we demonstrate that the utilities of different alternatives in decision making are not static and are subject to change. That is, the utility of one alternative may change with the implementation of another alternative. Therefore, what may appear as an irrational decision (i.e., the sunk cost fallacy) under traditional views may actually be rational in certain circumstances. We also present a decision-making flowchart considering utility changes in this section, and Section 3 establishes a mathematical programming model for a detailed analysis of this issue.

3. Mathematical Models and Solution Approach

This section develops a two-stage stochastic optimization model for the research problem in Section 2 and employs the SAA method to solve the model.

3.1. Two-Stage Stochastic Optimization Model

The studied decision-making process unfolds in two stages: the first stage is to make decisions prior to the occurrence of uncertainty, while the second stage involves decision making after the uncertainty is realized while considering the utility change. Therefore, a two-stage stochastic optimization method is employed to model the above problem. The assumptions of the optimization model are as follows:

(i) The model considers that there are two options A and B for the decision maker.
(ii) The utility of option A is a random parameter, while the utility of option B is deterministic.
(iii) Choosing option A requires a deterministic cost, which cannot be refunded once option A is chosen.
(IV) If option A is chosen, there remains an opportunity to switch to option B. However, if option B is initially chosen, the opportunity to switch to option A is lost.
(V) After choosing option A, if the decision maker wants to switch to option B, the utility of option B has changed relative to its original utility.

In the first stage, the decision maker is tasked with choosing between options A and B, with the objective of maximizing the overall expected utility of both stages. The parameters and variables for the first stage decision problem are defined in Table 1. If option A is chosen in the first stage, and the decision maker observes the realized value a of the random parameter $\widetilde{a}$, the traditional analytical method assumes that the utility of option B remains unchanged at b. However, the method proposed in this paper suggests that, due to the choice of option A, the utility of option B undergoes a decrease, updated to b' and $b' < b$. Subsequently, in the second stage, the decision maker is required to choose between continuing with option A and switching to option B to maximize the current total utility if the decision maker chooses option A at the first stage. The parameters and variables for the second-stage decision-making problem are defined in Table 2.

We further denote by $u(\widetilde{a})$ the overall utility obtained by the decision maker when choosing option A at the first stage. Since the decision in the first stage must be made before the uncertainty realizes, $u(\widetilde{a})$ is an expression containing random parameter $\widetilde{a}$. After the random parameter is realized, denoted by a, we represent the overall realized utility by $u(a)$ if the decision maker chooses option A at the first stage.

Table 1. Definition of parameters and variables for the first-stage decision-making problem.

Parameters	
$\widetilde{a}$	Utility of option A, treated as a random parameter.
b	Utility of option B, treated as deterministic.
Decision Variables	
x^1	Decision under the traditional analytical method: $x^1 \in \{0,1\}$; $x^1 = 1$ indicates choosing option A; $x^1 = 0$ indicates choosing option B.
x^2	Decision under the method proposed in this paper: $x^2 \in \{0,1\}$; $x^2 = 1$ indicates choosing option A; $x^2 = 0$ indicates choosing option B.

Table 2. Definition of parameters and variables for the second-stage decision-making problem.

Parameters	
a	Utility observed by the decision maker after choosing option A in the first stage.
b'	Utility of option B updated due to the choice of option A.
c	Cost invested for option A, which cannot be recovered.
Decision Variables	
$y^1(a)$	Decision under the traditional analytical method: $y^1(a) \in \{0,1\}$; $y^1(a) = 1$ indicates continuing with option A when the observed utility of option A is a; $y^1(a) = 0$ indicates switching to option B when the observed utility of option A is a.
$y^2(a)$	Decision under the method proposed in this paper: $y^2(a) \in \{0,1\}$; $y^2(a) = 1$ indicates continuing with option A when the observed utility of option A is a; $y^2(a) = 0$ indicates switching to option B when the observed utility of option A is a.

The two-stage stochastic optimization model under the traditional analytical method is defined as follows:

Model I-1:

$$\max \left(1 - x^1\right) b + x^1 \mathbb{E}[u(\widetilde{a})] \tag{1}$$

subject to

$$x^1 \in \{0,1\}.$$

Model I-2:

$$u(a) = \max y^1(a)a + \left[1 - y^1(a)\right] b - c \tag{2}$$

subject to

$$y^1(a) \in \{0,1\}.$$

The two-stage stochastic optimization model under the method proposed in this paper is defined as follows:

Model II-1:

$$\max \left(1 - x^2\right) b + x^2 \mathbb{E}[u(\widetilde{a})] \tag{3}$$

subject to

$$x^2 \in \{0,1\}.$$

Model II-2:

$$u(a) = \max y^2(a)a + \left[1 - y^2(a)\right] b' - c \tag{4}$$

subject to

$$y^2(a) \in \{0,1\}.$$

The objective functions (1)–(4) maximize the total expected utility in respective stages under different analytical methods.

To evaluate the decision effectiveness of the traditional analytical method and the method proposed in this paper, we define the decision utility loss. When $x^1 = x^2 = 0$, the decision maker chooses option B in the first stage; therefore, there is no sunk cost. When $x^1 = x^2 = 1$, the decision maker chooses option A in the first stage. For the observed value a of the random parameter $\widetilde{a}$ in the second stage, we define $u^1(a)$ and $u^2(a)$ as the realized utilities for the decisions under the traditional analytical method and the method proposed in this paper, respectively, where

$$u^1(a) = \max y^1(a)a + \left[1 - y^1(a)\right]b' - c,$$

$$u^2(a) = \max y^2(a)a + \left[1 - y^2(a)\right]b' - c.$$

Then the decision utility loss of the traditional analytical method compared to the method proposed in this paper is defined as

$$L = u^2(a) - u^1(a). \tag{5}$$

3.2. Solution Method: SAA

Since $u(a)$ is nonlinear of a in the first stage decision-making problem, using $u[\mathbb{E}(\widetilde{a})]$ instead of $\mathbb{E}[u(\widetilde{a})]$ may lead to incorrect decisions [34]. In this case, we should consider the distribution of the random parameter $\widetilde{a}$ and employ methods such as SAA for solution [35]. The main idea of the sample average approximation (SAA) is to replace $\mathbb{E}[u(\widetilde{a})]$ with the empirical average utility of finite samples. Specifically, we draw a certain number of samples from the probability distribution of the uncertain parameter $\widetilde{a}$ for approximating $\mathbb{E}[u(\widetilde{a})]$. To this end, we generate N observations $a_1, \ldots, a_N$ from the distribution of the random parameter $\widetilde{a}$ and obtain the estimated $\hat{\mathbb{E}}[u(\widetilde{a})] = N^{-1}\sum_{i=1}^{N} u(a_i)$. Based on this estimator, decisions x^1 and x^2, i.e., choosing option A or B, can be made for the first stage. If option A is chosen in the first stage, the decision maker then makes the second-stage decisions $y^1(a_i)$ and $y^2(a_i)$, i.e., continuing with option A or switching to option B, based on the observed values a_i $(i \in \{1, \ldots, N\})$ of the random parameter $\widetilde{a}$.

Therefore, the approximate models for Model I and II are denoted as Models I′ and II′, respectively, shown as follows:

Model I′-1:

$$\max \left(1 - x^1\right)b + x^1 \frac{\sum_{i=1}^{N} u(a_i)}{N} \tag{6}$$

subject to

$$x^1 \in \{0,1\}.$$

Model I′-2 ($i \in \{1, 2, \ldots, N\}$):

$$u(a_i) = \max y^1(a_i)a_i + \left[1 - y^1(a_i)\right]b - c \tag{7}$$

subject to

$$y^1(a_i) \in \{0,1\}.$$

Model II′-1:

$$\max \left(1 - x^2\right)b + x^2 \frac{\sum_{i=1}^{N} u(a_i)}{N} \tag{8}$$

subject to

$$x^2 \in \{0,1\}.$$

Model II′-2 ($i \in \{1, 2, \ldots, N\}$):

$$u(a_i) = \max y^2(a_i)a_i + \left[1 - y^2(a_i)\right]b' - c \tag{9}$$

subject to

$$y^2(a_i) \in \{0, 1\}.$$

For the observed value a_i ($i \in \{1, 2, \ldots, N\}$) of the random parameter $\widetilde{a}$ in the second stage, we define $u^1(a_i)$ and $u^2(a_i)$ as the realized utility for the decisions under the traditional analytical method and the method proposed in this paper respectively, where

$$u^1(a_i) = \max y^1(a_i)a_i + \left[1 - y^1(a_i)\right]b' - c,$$

$$u^2(a_i) = \max y^2(a_i)a_i + \left[1 - y^2(a_i)\right]b' - c.$$

Subsequently, the estimated decision utility loss of the traditional analytical method relative to the method proposed in this paper is

$$\hat{L} = L_N = \frac{1}{N}\sum_{i=1}^{N}\left[u^2(a_i) - u^1(a_i)\right]. \tag{10}$$

4. Computational Experiments

In this section, we conduct computational experiments on the example of the manufacturing enterprise provided in Section 2.1 and solve the two-stage stochastic optimization model using the SAA method. By comparing with the traditional analytical method, we verify the effectiveness of the proposed method in this paper.

For the manufacturing enterprise mentioned in Section 2.1, we assume that the random parameter $\widetilde{a}$ in plan A follows a uniform distribution, i.e., $\widetilde{a} \sim U(20, 100)$. The profit of plan B at time point one is set to $b = 50$. If plan A is chosen at time point one, the profit of plan B changes to $b' = 40$ actually. Four sets of simulation experiments are conducted with $N = 1000$ in each experiment. Experiments 1 and 3 simulate the traditional analytical method without considering the parameter b', while experiments 2 and 4 simulate our proposed method in this paper with the parameter $b' = 40$. In experiments 1 and 2, we set the plan A's investment cost to be $c = 20$ that mimics the high-cost scenario, while $c = 10$ in experiment 3 and 4 mimics the low-cost scenario. The setup of experimental data in this study is designed based on the real situation of a manufacturing company, which reflects the essence of a class of two-stage decision-making problems that enterprises face. The experiment results are presented in Table 3.

Table 3. Results of the four sets of simulation experiment.

Experiment No.	Model	$\widetilde{a}$	b	b'	c	x	$\sum_{i=1}^{N} y(a_i)$
1	Model I′	$U(20, 100)$	50	/	20	0	/
2	Model II′	$U(20, 100)$	50	40	20	0	/
3	Model I′	$U(20, 100)$	50	/	10	1	610
4	Model II′	$U(20, 100)$	50	40	10	1	746

In experiments 1 and 2, due to the relatively high cost of updating the old production line in plan A, the optimal decision for the enterprise at time point one is to choose plan B, with profit $b = 50$. Experiment 3 simulates the traditional analytical method, which considers that the profit of plan B remains unchanged at b. Therefore, the optimal decision at time point one is to choose plan A; among the 1000 scenarios at time point two, the enterprise continues producing on the new production line under 610 scenarios. Experiment 4 simulates the method proposed in this paper, with $b' = 40 < b$, indicating a decrease in the profit of plan B after choosing plan A. In this case, the optimal decision at time point one is still to choose plan A; among the 1000 scenarios at time point two, the enterprise continues producing on the new production line under 746 scenarios. The estimated decision utility loss of the traditional analytical method compared to the proposed method in this paper is

$$L_N = \frac{1}{1000}\sum_{i=1}^{1000}\left[u^2(a_i) - u^1(a_i)\right] = 0.711.$$

Comparing experiments 3 and 4, there are 136 scenarios at time point two where the decisions differ, after the enterprise chooses plan A and finds that the profit of the new mode of production is lower than expected. The traditional analytical method suggests that the enterprise should switch to plan B, while the method proposed in this paper suggests that the enterprise should continue with plan A. Among these 136 scenarios, the decision to continue producing on the new production line, considering the change in the profit of plan B due to the choice of plan A, is actually rational according to the method proposed in this paper. However, the traditional analytical method considers the decision of continuing producing on the new production line as irrational, implying a sunk-cost fallacy, thus resulting in a loss of profit.

Therefore, when faced such a decision-making problem, the enterprise should take into account the profit change of plan B to maximize the expected profit. In order to demonstrate the impact of the profit change of plan B on decision making, we plot the variation diagram of the optimal decision at time point one for the enterprise using our proposed method, i.e., considering the change of profit of plan B. Under the settings of $\widetilde{a} \sim U(20, 100)$ and $c = 10$, Figure 2 displays the decision that the enterprise should make between plan A and plan B at time point one under different parameter combinations of b and b'. For parameter combinations within the region shaded with red diagonal lines, such as $b = 45$ and $b' = 40$, the enterprise should choose plan A at time point one, and then make the optimal decision at time point two based on the realized profit values a, b' and c using Model II′-2 and obtain the corresponding profit of $\max\{a - c,\ b' - c\}$. For those combinations within the region filled with blue cross lines, such as $b = 55$ and b' = 30, the enterprise should choose plan B at time point one and obtain profit of b.

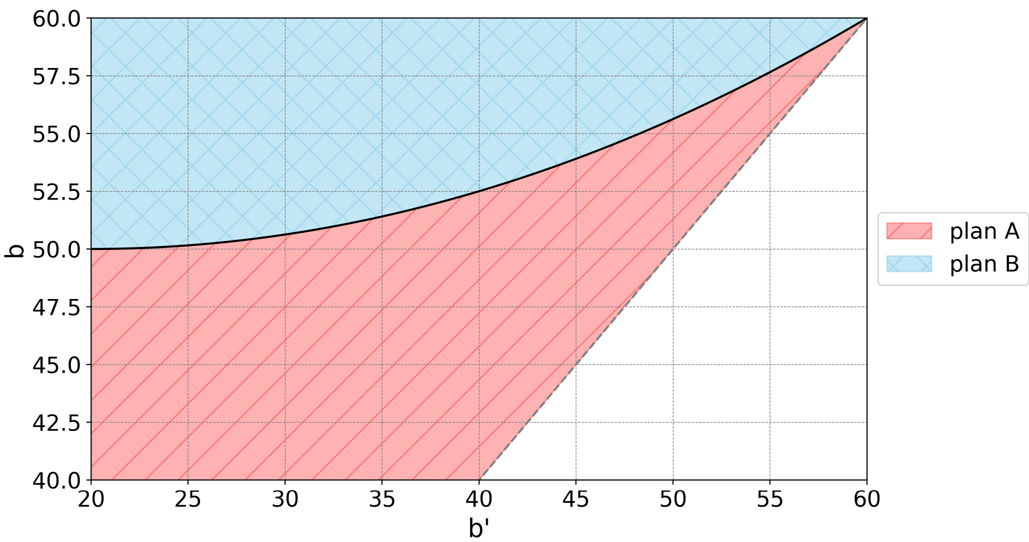

Figure 2. Decision at time point one under different parameter combinations of b and b' considering the change of profit.

In summary, we conduct computational experiments with data simulated from the real situation of a manufacturing enterprise. The enterprise faces decision-making problems related to choosing between plan A and plan B, with the goal of maximizing the profit.

The simulation experiments compare the effectiveness of our proposed method with the traditional analytical method. Results show that the proposed method outperforms the traditional method in decision making in some circumstances. The experiments demonstrate the importance of considering the variations of profit, especially in scenarios where the sunk cost fallacy may lead to a loss of profit. The decision area map at time point one further illustrates the impact of profit variations on the optimal decision-making strategies. Overall, the experiments highlight the significance of incorporating profit variations into the decision-making process for improved outcomes in manufacturing enterprises.

5. Conclusions

This paper revisits the sunk cost fallacy and proposes that the utilities of different alternatives are not static for decision makers; that is, the utility of an option may change due to the choice of another option. Therefore, what appears to be an irrational decision under the traditional analytical method is actually rational. This paper, therefore, stands as a valuable addition to the existing literature, offering a refreshed and effective decision-making method with sunk cost considering the utility changes.

We provide a mathematical description for such decision-making problems, analyze the impact of changes in the utility of different alternatives on decision making, and illustrate the proposal with a real-world example of the manufacturing enterprise. Furthermore, this paper establishes a two-stage stochastic optimization model for such decision-making problems, defines decision utility loss, an indicator to evaluate the effectiveness of different decisions, and solves the model using the SAA method. The results of computational experiments indicate that considering the change in the utility of an option due to the choice of another option, decisions considered irrational by traditional analytical methods are, in fact, rational in some scenarios, and the decisions obtained from the traditional analytical method may actually lead to a loss of utility.

This paper considers two options in the discussion of the decision-making problem with sunk cost, and the decision-making process in our study is divided into two stages. There are generally three directions for future research. Firstly, consider other types of decision-making scenarios, such as multiple options and multi-stage decision making, and researchers can make comprehensive comparative analysis with other decision-making models based on the real-world data. Secondly, when the utility variations of multiple alternatives become more complicated, the modeling and solving algorithms of decision problems considering sunk costs become more challenging, which are directions worth investigating in the future. Thirdly, future research can explore the psychological aspects of decision making influenced by sunk cost fallacy in fields like cognition, organizational behavior, and team management.

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