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Validation of a Manual Methodology for Measuring Roundness and Cylindricity Tolerances

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Abstract: This paper describes a practice-based methodology applied to roundness and cylindricity concepts. Traditionally, technicians encounter difficulties in understanding this topic, especially when they analyze the tolerances involved due to the complexity of their graphic interpretation. Additionally, it is mandatory for industrial engineers to acquire competencies that allow them to validate form and make decisions in this field. With the goal of enhancing the handling of measurement techniques involved in this topic and analyzing the results obtained, a methodology has been designed to address the roundness and cylindricity tolerance evaluation from different perspectives. Firstly, based on a conceptual analysis, an analytical procedure is introduced. Secondly, an engaging manual practice is implemented by using simple measuring instruments that allow the user to be involved in the development of the procedure applied. The conditions that tried to optimize the manual procedure to get good results were analyzed. Moreover, the results obtained under this perspective allow us to ensure that better skills can be acquired regarding the typical method based on the coordinate measuring machines. This experience has been validated based on the practice with ongoing users under a training period.

Keywords: manual measuring techniques; roundness and cylindricity; manufacturing

Citation: García-Martínez, E.; García-González, N.; Manjabacas, M.C.; Miguel, V. Validation of a Manual Methodology for Measuring Roundness and Cylindricity Tolerances. *Appl. Sci.* **2023**, *13*, 9702. https://doi.org/10.3390/app13179702

Academic Editor: Paolo Renna

Received: 27 July 2023
Revised: 25 August 2023
Accepted: 25 August 2023
Published: 28 August 2023

1. Introduction

In the industrial field, the search for constant improvement of manufacturing processes and the quality of manufactured parts due to technological evolution leads to a demand for qualified professionals in the field of dimensional metrology, responsible for the control and verification of product shapes and tolerances. The industrial engineer emerges as a key figure in this type of task, as they have versatile training in terms of manufacturing, materials science, metrology, process control, and automation [1]. For these reasons, curricula and training methodologies in a high-level technical environment need to be updated, as well as methods sought to encourage conceptual understanding, which reinforces users' working skills and brings them closer to the conditions typically found in the industrial background [2,3]. It is therefore essential that the training activities carried out herein provide professionals with a comprehensive knowledge of different procedures to deal with design, production, and their relationship with the standard system of Geometrical Product Specifications (GPS) [4].

As future users in the field of dimensional metrology, some Mechanical Engineering students have been chosen to experiment with the efficiency of the technological proposal that is explored in this paper.

Dimensional tolerances are used in manufacturing to control the actual sizes, shapes, and geometrical characteristics of parts to meet their functional requirements and to ensure parts' interchangeability [5]. Therefore, tolerancing has evolved as an important activity that is related to design, manufacturing organization, and inspection, significantly affecting the economics of manufacturing processes, which has a significant effect on the development of precision products [6,7].

However, it has been detected that the understanding of some of these concepts, especially those related to roundness and cylindricity tolerances, is not easy for technicians, as it is not clear how to visualize them graphically, i.e., it is not easy to try to make them understand the theoretical and analytical procedures if there is no practical activity involved. In addition, there is no commercial software on this topic that offers a training approach and supports virtual learning. Thus, the calculation and testing of roundness and cylindricity tolerances pose a problem when they must be applied to the verification of solids of revolution.

Typical methods to measure roundness and cylindricity tolerances are those that use a table, and particularly, specific measuring machines are common in industrial environments or metrological verification laboratories [8]. In some cases, it is necessary to control workpieces either during the manufacturing process without being disassembled from the machine, or on working parts. For those cases, the V-block method [9–11], systems based on the laser [12], or inductive or capacity sensor systems [13], have been developed. The V-block method is an adaptation of the typical one based on V-blocks on a table, but some sensors and monitoring computers are necessary for this to be performed [10,11]. Laser triangulation is a typical non-contact method that has the advantage of high accuracy and efficiency. Nowadays, even some laser systems supported by digital twin technology are being examined [14]. Some recent research has been carried out to apply machine vision to determine deviations of rotational parts [15], but there is no standard equipment in the industry and more research is necessary in this context [16].

Specifically, automated cylindricity measuring equipment (CME) is designed for the evaluation of roundness and cylindricity tolerances. Basically, a stylus takes a lot of readings on the surface of the part with respect to a reference axis while it rotates. The measurement process is usually simple and even intuitive, but not the mathematical and conceptual one. The machines are shown as "black boxes" to the user, offering a numerical tolerance result, but without showing the procedure carried out for its acquisition, and many of the implications involved. For this reason, the simple use of these instruments lacks validity, as it does not provide the knowledge and reasoning bases necessary for understanding the theoretical concepts applied to other situations and even for having real control of the measuring process. Furthermore, current standards do not provide clear guidelines about working methodologies when using CME [17]. One question that may be asked is how roundness and cylindricity tolerances can be obtained, at least approximately, without using a specifically designed instrument. Therefore, it is highly recommended to establish training manual methods for obtaining these tolerances, which also allow the technicians to acquire the necessary metrological and conceptual fundamentals for the development of verification tasks in industrial environments [18].

Furthermore, the mathematical approach of cylindricity tolerance is not sufficiently described in the literature, neither in dimensional metrology nor in specific training resources [8]. The analytical calculation procedure needs to be described for the industrial community, especially those that are concerned with cylindricity, as it is more complex to be implemented analytically. For all these reasons, this paper aims to revisit the concepts and the analytical mathematical fundamentals necessary for the verification of roundness and cylindricity tolerances. Additionally, it proposes a practical experience of laboratory activity in order to establish a methodology that can be tested by future users based on three key points: the theoretical review of the relevant roundness and cylindricity principles and their mathematical definition, a laboratory approach that follows the established manual measurement procedure by using typical metrological instruments, and, finally, the verification of the obtained results by means of specifically designed instruments, i.e., CME. The main objective of this research is to improve technical skills in the field of revolution shape metrology and to demonstrate that MM permits us to obtain an acceptable accuracy if some conditions are defined.

2. Materials and Methods

2.1. Theoretical and Analytical Fundamentals and Development

2.1.1. Roundness Tolerance

ISO 1101 standard [19] defines the roundness tolerance zone of a considered plane section as the area bounded by two concentric circumferences that contain all the points of the profile, Figure 1a. As is well known, the distance between these circumferences is the roundness tolerance. The control of this tolerance is crucial for the manufacturing of rotational workpieces taking part in assemblies [20].

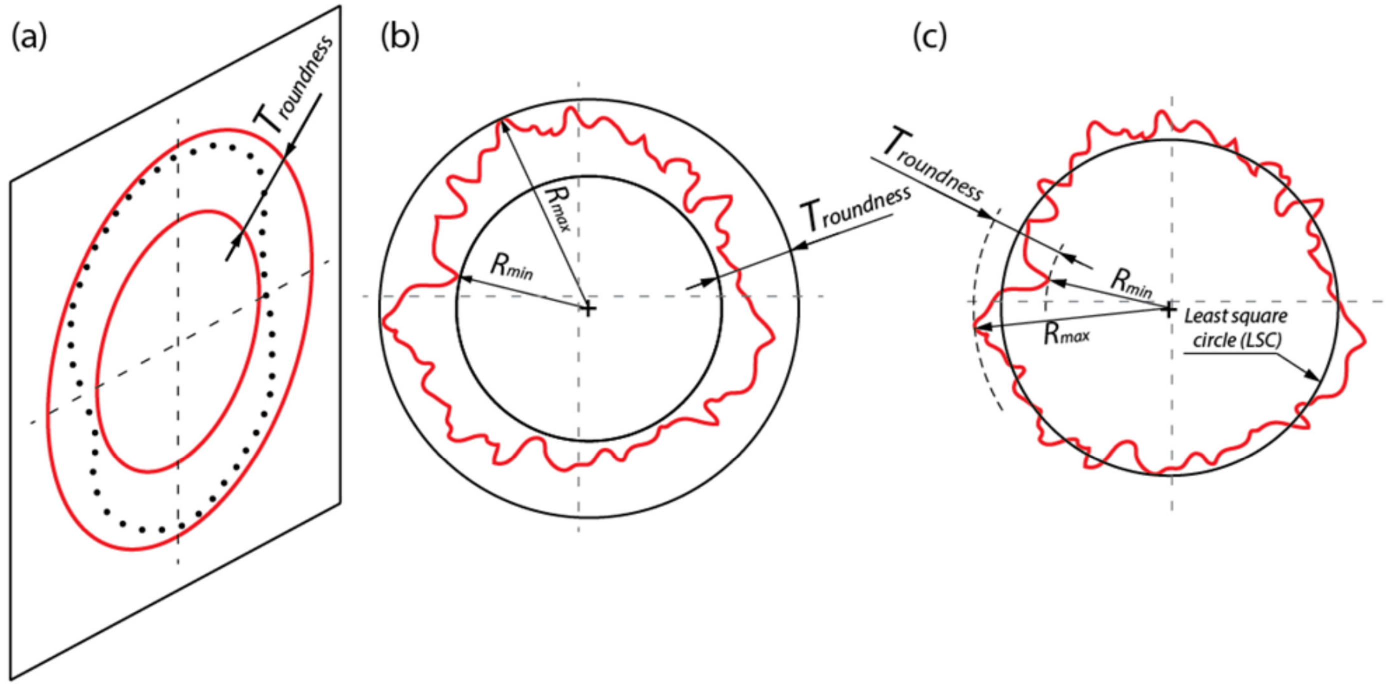


Figure 1. Roundness tolerance zone definition according to (**a**) UNE-EN ISO 1101 standard [19]; (**b**) MZC method; and (**c**) LSC method.

Several different methods have been suggested in the ISO 12181-1 standard [21] for the evaluation of roundness tolerance according to four different reference circles. These evaluation methods include: the minimum circumscribed circle (MCC), the maximum inscribed circle (MIC), the least squares circle (LSC), and the minimum zone circles (MZC). Various researchers have attempted to find analytical methods or algorithm-based ones for establishing reference features [8].

Among the cited methods, only MZC considers exactly the definition of roundness tolerance established by ANSI and ISO standards, since it involves the determination of two concentric circles by using all points of the profile, so that the radial distance between them is minimum [22], as shown in Figure 1b. However, this method requires the use of numerical procedures with calculus algorithms [8], which is not considered to be the most appropriate for educational environments.

On the other hand, the authors consider that LSC is the most convenient method for understanding the concepts of tolerance, as well, it is the most used in engineering design. It allows users to apply and reinforce methodologies of statistical analysis and regression, acquired previously, focused on a real meaningful objective. Furthermore, although LSC sometimes leads to higher tolerance values than MZC, which might cause one to discard of some valid parts, it is commonly used at the industrial level due to its ease and robustness [22].

In LSC, the least squares circle is fitted, and taking into consideration the measured part profile, the sum of the squares of radial distances between the circle and each point of the surface must be minimized. The solution is unique, so the center of the LSC circle can be used to define an inscribed and a circumscribed circle with respect to the profile. The radial distance between them is defined as the roundness tolerance, see Figure 1c.

The analytical determination of the roundness tolerance according to LSC consists of determining the position of the LSC center since it is the reference that allows the roundness tolerance to be calculated. The surface of the evaluated profile can be discretized into a succession of points, P_i, located from the origin of coordinates, O, by a distance r_i, forming an angle θ with respect to the horizontal axis, Figure 2. The cartesian coordinates of each point $P_i(x_i, y_i)$ can be obtained by Equation (1).

$$\left.\begin{array}{l} x_i = r_i \cos\theta_i \\ y_i = r_i \sin\theta_i \end{array}\right\} \quad (1)$$

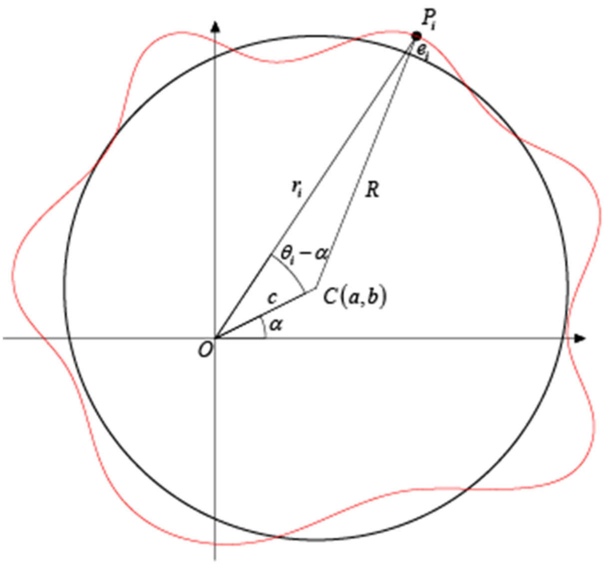


Figure 2. Geometric representation of a section (red line) and the least squares circle (in black line).

The position of the LSC center, $C(a, b)$, is defined by $a = c \cos\alpha$, $b = c \sin\alpha$, where c is the distance from the origin, O, and α is the angle with respect to the horizontal axis.

By means of trigonometry, it is possible to obtain a relationship between the distance from each point of the profile to the origin, r_i, the position of the center of LSC, and the radial error between the considered point and LSC, e_i, Equation (2).

$$r_i = \sqrt{(R + e_i)^2 - c^2 \sin^2(\theta_i - \alpha)} + c\cos(\theta_i - \alpha) \quad (2)$$

In addition, according to the industrial application of the roundness tolerance concept, the LSC center is very close to the origin, as the roundness tolerance is 2 or 3 orders of magnitude smaller than the dimension of the parts. For that reason, the distance c is very small and the term c^2 is negligible. Consequently, Equation (3) is obtained.

$$r_i = R + c\cos(\theta_i - \alpha) + e_i \quad (3)$$

Moreover, taking into consideration the definition of LSC circle, the sum of the squares of the radial errors between the circle and each point, S, Equation (4), must be minimized. The full derivation of the expression for S is shown in Appendix A.

$$S = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (R + a\cos\theta_i + b\sin\theta_i - r_i)^2 \tag{4}$$

For that purpose, partial derivatives with respect to the position of LSC center and the radius, i.e., with respect to a, b, and R, must be calculated. The detailed mathematical procedure is depicted in the Appendix A. Thus, considering that the measured points of the profile are equidistant, the solution is found using Equations (5) and (6).

$$\left.\begin{aligned} a &= \frac{2\sum_{i=1}^{n} r_i\cos\theta_i}{n} = 2\overline{x} \\ b &= \frac{2\sum_{i=1}^{n} r_i\sin\theta_i}{n} = 2\overline{y} \end{aligned}\right\} \tag{5}$$

Note that the coordinates of the center, a and b, are the double of the mean values of the x and y coordinates of the profile points. Once the center has been obtained, the radius of LSC is equal to the average of the distances between each point on the profile and LSC center, Equation (6).

$$R = \frac{\sum_{i=1}^{n} \sqrt{(x_i + a)^2 + (y_i + b)^2}}{n} = \frac{\sum_{i=1}^{n} R_i}{n} = \overline{R} \tag{6}$$

As observed, the coordinates of the LSC center and its radius are easily determined from the cartesian coordinates of the points of the profile. Finally, the roundness tolerance, T_{round}, is obtained, according to Equation (7), i.e., it is the difference between the maximum and minimum distances from LSC center and the measured points on the profile.

$$T_{roundness} = R_{\max} - R_{\min} \tag{7}$$

2.1.2. Cylindricity Tolerance

ISO 1101 standard [19] defines the cylindricity tolerance zone of a considered part as the volume bounded by two coaxial cylinders containing all points of the profile. The radial distance between these cylinders is the cylindricity tolerance, Figure 3. Cylindricity tolerance is closely related to the concepts of roundness tolerance and coaxiality. It goes without saying that the determination of the cylindricity tolerance involves the previous calculation of the roundness tolerance in various sections of the part. The more sections are evaluated, the more accurate this evaluation is, although it is much more time-consuming.

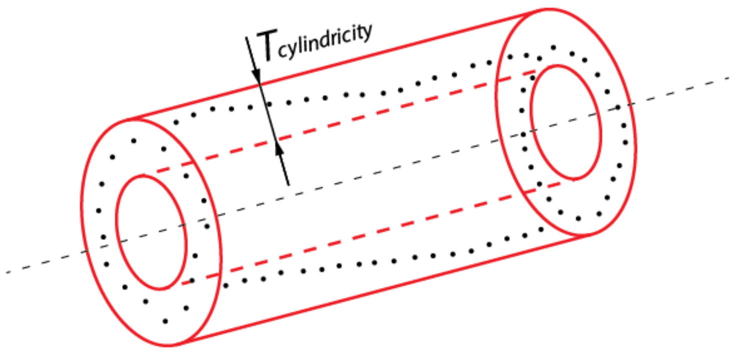


Figure 3. Cylindricity tolerance zone definition according to UNE-EN ISO 1101 standard [19].

As in the previous study of roundness, the cylindricity tolerance can be obtained by considering four reference cylinders, as well as cylindricity deviations. So, in a similar way to roundness, the ISO standard suggests four methods: the minimum zone cylinders (MZCy), the least squares cylinder (LSCy), the maximum inscribed cylinder (MICy), and the minimum circumscribed cylinder (MCCy). Again, according to the cylindricity tolerance definition, only the MZC method leads to the minimum value of tolerance. Nevertheless, this method implies complex algorithms based on "simplex approximations" or iterative calculus [23,24]. For that reason, it is more adequate for educational environments the use of LSCy, widely accepted in the industrial field, with tolerance values that are very close to those obtained by MZC [24].

LSCy fits the least squares cylinder to the measured profile of the whole part so that the sum of the squares of radial distances between the fitted cylinder and each point of the surface is minimized. It is well known that the difference between the largest and smallest distances from the points on the surface of the part to the axis of the least squares cylinder is the cylindricity tolerance.

Typically, the determination of the least squares cylinder that fits a set of points on the workpiece is equivalent to obtaining the equation of the axis that best fits the centers of the least squares circles of each considered section of the cylinder, see Figure 4. Therefore, to evaluate the cylindricity tolerance of a part, it is necessary to assess the roundness corresponding to different sections in a previous step. And it can be checked that the more planes are measured, the greater the accuracy is obtained.

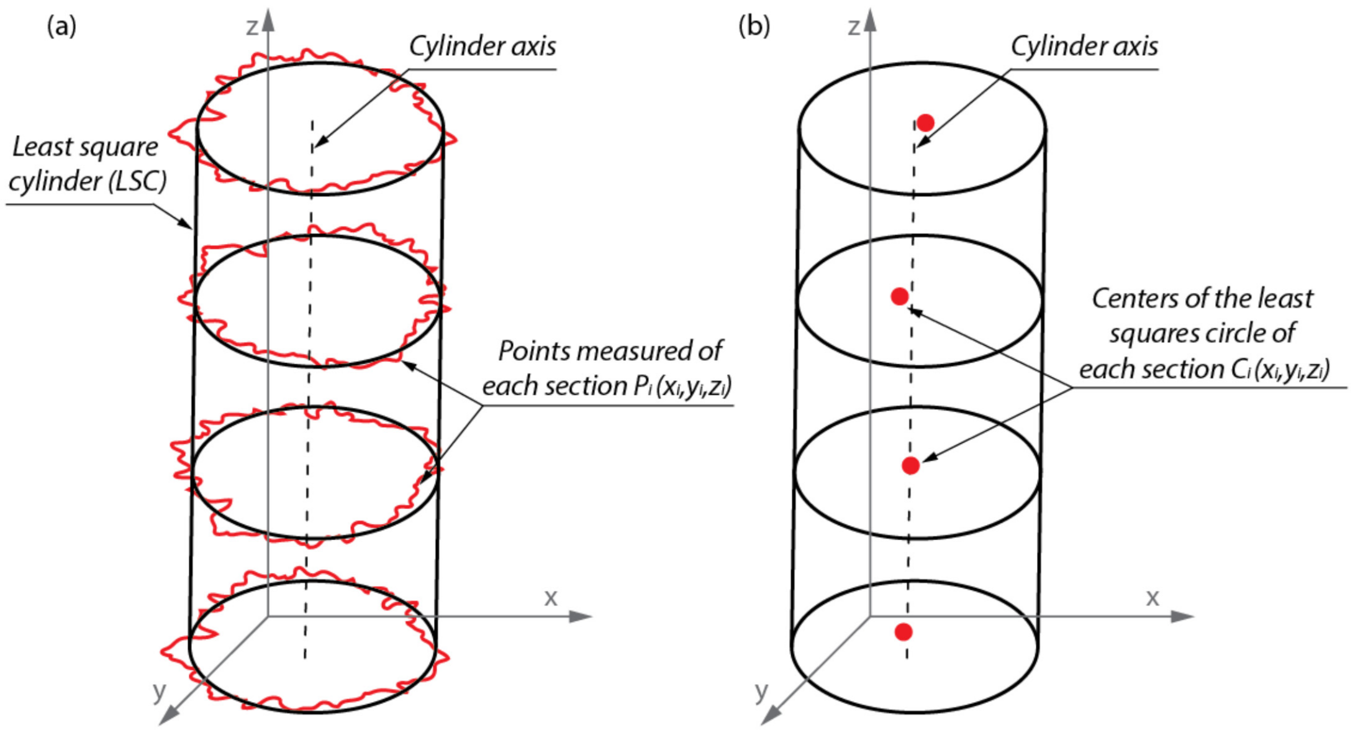


Figure 4. (**a**) Representation of the least squares cylinder fitting several sections of the same part; (**b**) axis of the least squares cylinder by adjusting the centers of the LSC of each section.

Then, it is assumed that the centers of the least squares circles of each section of the part have been calculated according to the method described above, resulting in a set of points, $C_i(x_i,y_i,z_i)$. The mathematical problem consists of finding the equation of the straight line, r, that fits this series of points. The cylinder axis follows the general Equation (8), where $P_0(x_0,y_0,z_0)$ and $\vec{V} = (u, v, w)$ are a point and a vector of the axis, respectively.

$$r: \ \frac{x - x_o}{u} = \frac{y - y_o}{v} = \frac{z - z_o}{w} \tag{8}$$

It can be mathematically demonstrated that the point $\overline{X}(\overline{x}, \overline{y}, \overline{z})$, whose coordinates are the average values of those corresponding to all points to be fitted, is a point on the cylinder axis, see Appendix B. Thus, a director vector of the axis can be written as $\vec{V} = (u, v, 1)$, obtaining the Equation (9).

$$r: \ \frac{x - \overline{x}}{u} = \frac{y - \overline{y}}{v} = \frac{z - \overline{z}}{1} \tag{9}$$

The straight line described by Equation (9) can be expressed as the intersection of two planes, Equation (10).

$$r: \ \begin{cases} x = \overline{x} + u(z - \overline{z}) \\ y = \overline{y} + v(z - \overline{z}) \end{cases} \tag{10}$$

The objective is to minimize the sum of squared distances, ε, between the straight line describing the cylinder axis and each of the centers, C_i, of each section, Equation (11).

$$\varepsilon = \sum_{i=1}^{n} d_i(C_i, r)^2 = \sum_{i=1}^{n} \left[(x_i - \overline{x} + u(z - \overline{z}))^2 + (y_i - \overline{y} + v(z - \overline{z}))^2 \right] \tag{11}$$

By setting the partial derivatives of ε with respect to the coordinates of the vector, u and v, equal to zero, Equations (12) and (13) are obtained. It can be appreciated that they include the values that make ε minimized.

$$u = \frac{\sum_{i=1}^{n} (x_i - \overline{x})(z_i - \overline{z})}{\sum_{i=1}^{n} (z_i - \overline{z})^2} \tag{12}$$

$$v = \frac{\sum_{i=1}^{n} (y_i - \overline{y})(z_i - \overline{z})}{\sum_{i=1}^{n} (z_i - \overline{z})^2} \tag{13}$$

Note that the covariance between x and z and between y and z, as well as the variance of z, are obtained from Equations (14)–(16).

$$\sigma_{xz} = \frac{\sum_{i=1}^{n} (x_i - \overline{x})(z_i - \overline{z})}{n} \tag{14}$$

$$\sigma_{yz} = \frac{\sum_{i=1}^{n} (y_i - \overline{y})(z_i - \overline{z})}{n} \tag{15}$$

$$\sigma_z^2 = \frac{\sum_{i=1}^{n} (z_i - \overline{z})^2}{n} \tag{16}$$

Taking into account Equations (14)–(16) and the solution obtained for the u and v components, the director vector of the axis of the least squares cylinder can be expressed according to Equation (17).

$$\vec{V} = \left(\frac{\sigma_{xz}}{\sigma_z^2}, \frac{\sigma_{yz}}{\sigma_z^2}, 1 \right) \tag{17}$$

Once the equation of the axis of the cylinder is known, the distance from each point on the surface of the measured part with respect to the axis is calculated according to Equation (18).

$$d_i(P_i, r) = \frac{\left|\overrightarrow{MP_i} \times \overrightarrow{V}\right|}{\left|\overrightarrow{V}\right|} = \frac{\sqrt{(x_i - \overline{x})^2 + (y_i - \overline{y})^2 + (z_i - \overline{z})^2 - [u(x_i - \overline{x}) + v(y_i - \overline{y}) + (z_i - \overline{z})]^2}}{\sqrt{u^2 + v^2 + 1}} \tag{18}$$

The cylindricity tolerance, $T_{cylindricity}$, is finally obtained as it has been mentioned before, i.e., as the difference between the largest and smallest distance between the points on the surface and the cylinder axis, Equation (19).

$$T_{cylindricity} = d_{\max} - d_{\min} \tag{19}$$

This analytical procedure for cylindricity tolerance evaluation can be easily implemented in an Excel spreadsheet.

2.2. Experimental Procedure

2.2.1. Manual Measurement Methodology in the Lab Environment (MM)

The application of manual measurement methods in the laboratory environment is essential to ensure that technicians understand the concepts of roundness tolerance and cylindricity according to the experience of the teaching staff. Practical activities, consisting of measuring with a standard roundness machine, did not allow the future technicians to acquire the metrology competencies completely, i.e., the fundamentals about the evaluation of roundness and cylindricity. After the experience, users focus properly on the management of the equipment software, selecting functions from the different menus to find solutions that are automatically given by the program. Thus, sometimes, the technical concepts are applied without a critical analysis, for example, it can be highlighted by the magnitude of the results obtained.

Therefore, with MM, users are forced to acquire skills regarding the manipulation of metrology instruments, in order to control the different sections to be measured and to obtain the coordinates of the part profile, later used for the analytical calculation. Furthermore, these methods have industrial applications, especially in job-shop layouts, where it is not possible to have costly metrology equipment that is high in quality [25,26].

The manual method for roundness and cylindricity tolerances evaluation established in this paper follows the recommendations of the ISO standards and can be easily carried out. The materials needed to take the measurement with its purpose are listed below:

- Two V-blocks to hold the part to be measured.
- A dial indicator to register the variations in the radius of the part at each measured point. The resolution used was 0.001 mm.
- A support for the dial indicator is also required. It is recommended to use guided support, so that the dial indicator can be moved parallel to the cylinder axis, reducing positional errors.
- A standard granite surface plate to locate the measurement assembly in order to control and reduce possible errors in measurement readings due to surface waviness. If this is not available, it is sufficient to use a smooth table, although measurement errors may be added.

The assembly for the measurement procedure is shown in Figure 5. Although any part can be used for the evaluation of roundness and cylindricity tolerances, from an instructive perspective, a cylinder previously machined and then subjected to intentional deformations to produce large roundness and cylindricity defects should be selected. For that purpose, it is recommended that the extremes of the cylinder should not be deformed to ensure correct positioning on the V-blocks.

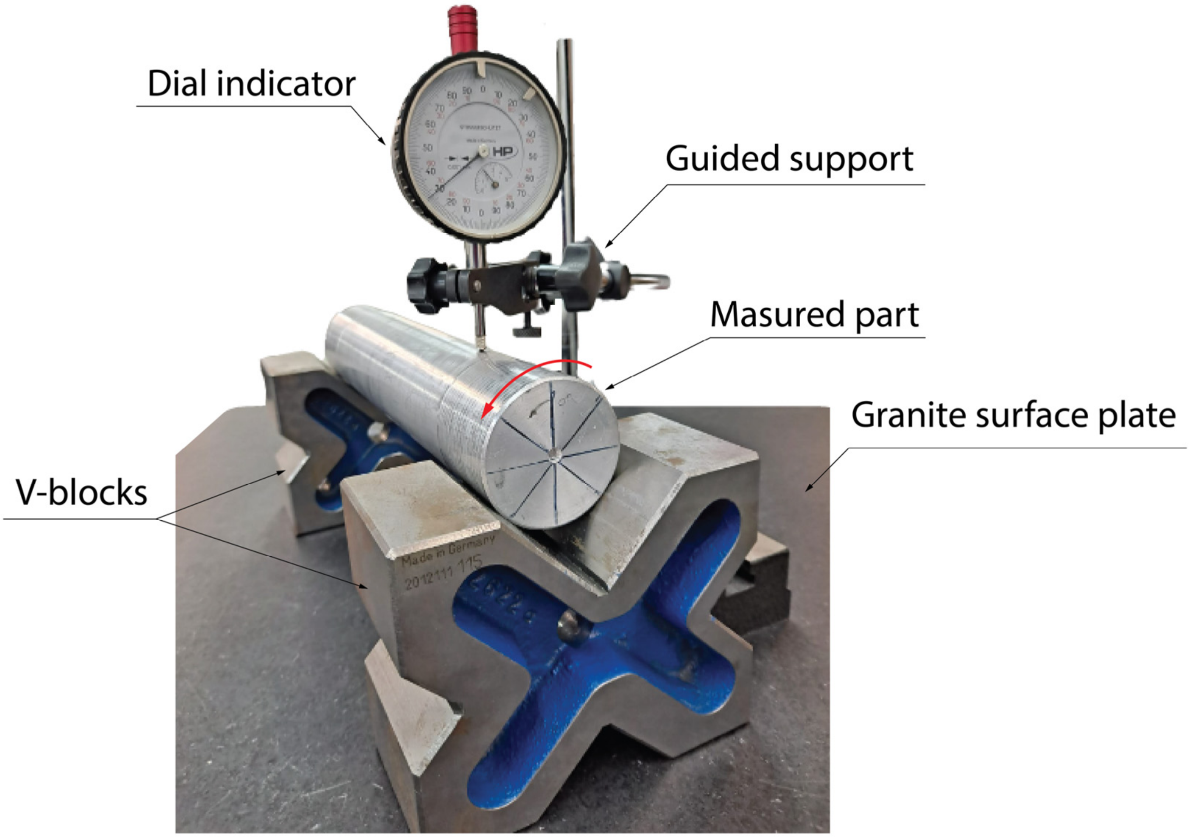


Figure 5. Assembly made to develop the manual measurement method (MM). Red arrow represents the rotation direction of the workpiece.

The measuring process for roundness and cylindricity evaluation consists of the following steps:

1. First, marks are made on the front face of the cylinder using an indelible pen. The marks must be located at a constant angular relationship, so that angle gauge blocks can be used as a support. Although it is not strictly necessary, their use introduces technicians to the handling of laboratory instruments. A minimum of 8 divisions are recommended.
2. Before assembly, a reference diameter value, D_{ref}, of the part must be considered on both endings (it could be an average), just in the supporting area on the V-block, as this area is assumed to have the best roundness quality and it is considered ideal as a reference for the optimum surface.
3. After placing the cylinder in the V-blocks, the dial indicator must be positioned vertically on the ideal reference section previously mentioned, recording the reference value in the dial indicator, dc_{ref}. In this way, the corresponding set-up operation has been performed.
4. Using the guided support, the dial indicator is moved to the first section to be evaluated. The reading of the dial indicator, c_i, must be recorded at 8 equally spaced points of the section by manual rotation of the cylinder (it is possible to increase the number of points if necessary).
5. The radius of each point in the assessed section is obtained from the reference values and the dial indicator measurement through Equation (20).

$$r_i = \frac{D_{ref}}{2} + \frac{c_i - c_{ref}}{1000} \tag{20}$$

6. From the radius obtained for each point, r_i, the cartesian coordinates are calculated, according to the position angle, by using Equation (1).
7. The dial indicator is then moved along the cylinder axis to each section to be analyzed, repeating the measuring procedure. It is recommended to evaluate a minimum of 5 sections to calculate the cylindricity tolerance.

2.2.2. Verification with the Cylindricity Measuring Equipment (CME Methodology)

In the work plan proposed in this paper, an automated roundness and cylindricity measuring equipment Talyrond 131C from Taylor Hobson is employed, see Figure 6. This equipment is a coordinate measuring machine that allows the roundness and cylindricity tolerances to be evaluated, according to the ISO 1101 standard [19], with a measurement range of 2 mm. It consists of a rotating table with a stylus that makes contact with the workpiece. A total of 3600 points are recorded in each section. The axial positioning of the stylus is defined as the number of sections to be evaluated and the distance between them.

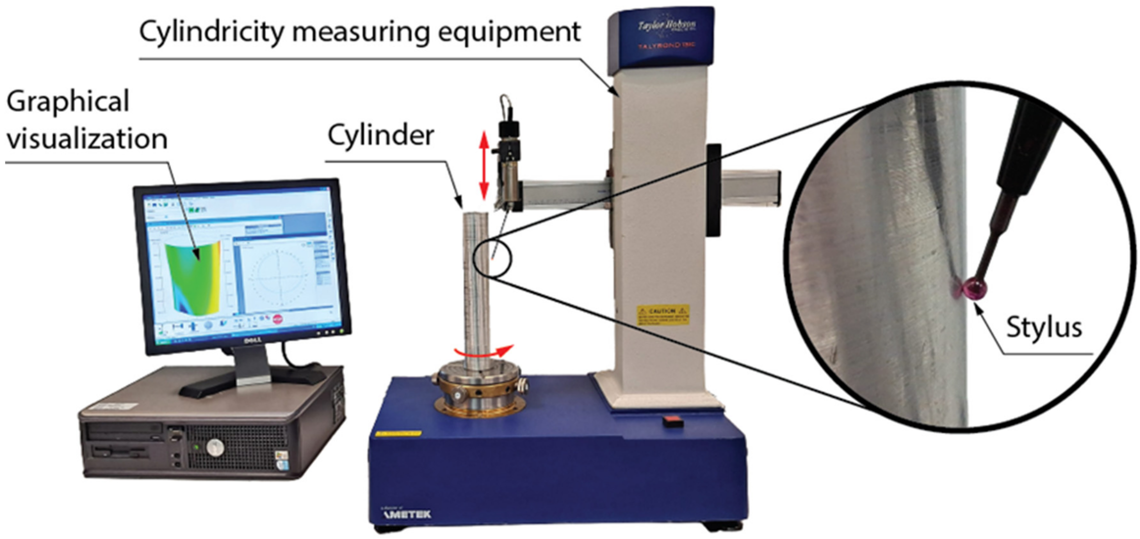


Figure 6. Talyrond 131C cylindricity measuring equipment by TaylorHobson. Red arrows indicate the possible movement directions for the location of the stylus and the rotation of the table.

The CME methodology requires application to some previous operations: to level the table, to calibrate the axis of the part with respect to the center of the table, to select the correct position of the stylus, and to choose the plane to be worked. The use of the software that controls the equipment is also an important feature to take into consideration, at least to select the main parameters from a quantitative viewpoint: exterior or inner roundness or cylindricity and the planning of the vertical stylus position, especially for cylindricity measuring. Nevertheless, the most important fact is the visualization of the results, as users can consider the four reference circles defined by the ISO 12181-1 standard [21]. The results menu offers the total description of the roundness tolerance of a selected cross section, as well as the cylindricity tolerance for the whole part. Moreover, one of the aspects that can be highlighted in this context is the possibility of evaluating the roundness and cylindricity in a qualitative way by observing the 3D sketch of the part, where the deviations of the shape are magnified and supported with colors and a bar scale.

Particularly, the CME methodology consists of testing the shape of the part according to the least squares circle and least squares cylinder criteria, for the same sections that have been measured by the manual method. This allows the measurements performed manually to be compared and contrasted with the results obtained using high-precision equipment.

3. Results

The equipment is expected to be able to perform a roundness analysis on at least five sections of every part, obtaining the roundness tolerance by programming the analytical method. The objective of the research is to draw one of the analyzed sections based on the eight registered points and compare it with the ideal circle obtained by least squares regression. In this way, it is expected to visualize which parts of the section exhibit the largest deviations from LSC, letting the tolerance zone be described.

It is essential to compare the results obtained by MM with those obtained by CME. Figure 7 shows the profile of a section of a part measured by both methods, MM and CME. As it can be observed, these methods throw a different roundness tolerance value due to the low number of profile points taken into consideration by MM (red points). The results obtained by MM are significantly lower than those recorded by the automated machine (blue profile). This causes variations in the tolerance results, although the general shape obtained by both methods is similar. The positions of the eight points selected for MM are placed out of line from the profile registered by the machine due to the reference value of the dial indicator. This causes the least squares circles (green for the manual method and black for the CME) to be offset by the same distance. However, this relative displacement does not change the tolerance results, as the calculation of the roundness tolerance refers to the relative distance between the center of the least squares circle and the points used to determine it, independently of their absolute location.

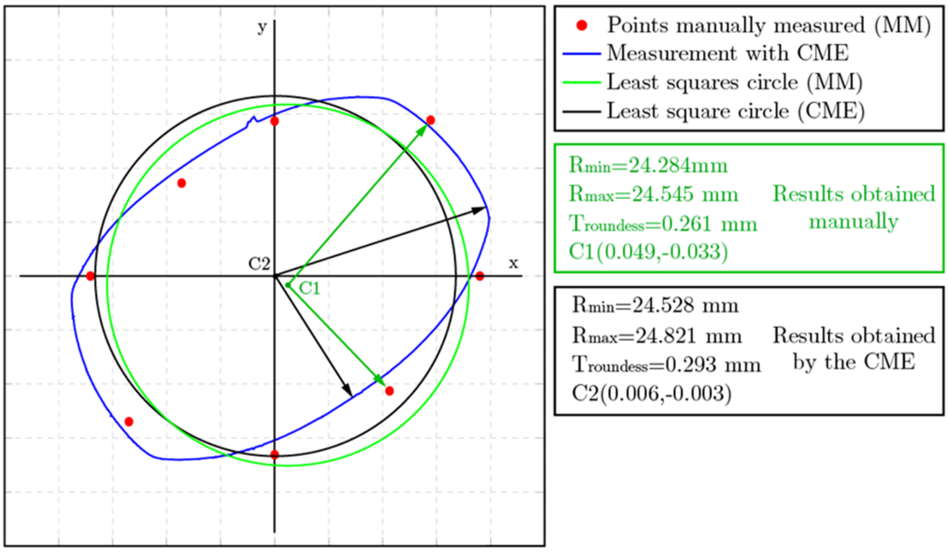


Figure 7. Expected representation for the roundness analysis of any cross section. Comparison between the two methods used.

Moreover, MM carried out on a granite surface plate leads to obtaining a tolerance error of less than 10% (30 µm for this example). Conclusions can be drawn from the resulting accuracy related to the suitability of the methodology, which depends on the quality requirements of the workpieces.

Similarly, the value of the cylindricity tolerance of every part from all the sections considered for roundness must be obtained. However, MM has limitations on this point, due to the difficulty of 3D graphing, regarding the plot of the part deviations with respect to the adjusted least squares cylinder. Nevertheless, this aspect will be supported by the 3D representation obtained by CME. For the example shown in Figure 8, the manual measurement procedure of seven cross sections of the cylinder yields a cylindricity tolerance, $T_{cylindricity}$, of 303.66 µm according to Equations (17)–(19). The same cross sections measured by CME lead to a cylindricity tolerance of 349.68 µm. Thus, the error found is 46 µm, which represents a relative value of approximately 13%. Again, this may be explained by the smaller number of points analyzed, which does not allow the real maximum and minimum points of the profile to be identified.

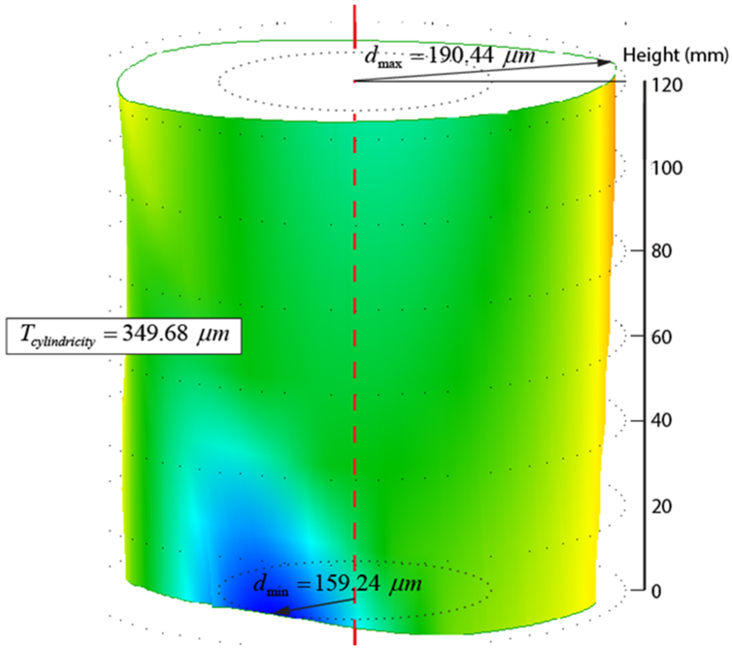


Figure 8. Result obtained for the cylindricity analysis of a whole part using the CME.

To verify the proposed analytical method, the coordinates of the points measured by CME were exported for the seven cross sections, carrying out the analytical calculation with the same 3600 points used with the machine for each section. A value of $Tc_{ylindricity}$ of 349.77 µm validates the analytical procedure. The small difference, less than 0.1 µm, is because the machine uses approximation algorithms to determine the axis of the least squares cylinder, rather than performing an exact analytical calculus.

4. Discussion

After having obtained the results using MM and CME, some technological considerations should be pointed out.

The first feature focuses on the measurement process fundamentals. The CME method is a one-point measurement system as the workpiece is supported vertically on one of its front faces, the stylus contacts at a point on the workpiece surface, and positive or negative variations with respect to an initial reference are registered. In other words, there is no influence of any point of the surface on the one that is being measured at the moment. Comparatively, the manual procedure consists of a three-point measurement, as the workpiece is supported on two points on the V-blocks. These contact points vary for each position during the rotation of the part influencing the signal of the dial indicator [27], see Figure 9. This becomes important when analyzing lobed forms as the actual measurement of the dial indicator can be slightly varied. This influence is really a measuring error source that might be more important for high-angled V-blocks. To compensate for this effect, correction factors based on the number of sides of the lobed shape are traditionally applied [28]. Fortunately, although measurement correction is necessary, the three-point measurement with V-blocks makes it possible to evaluate lobe forms, which would not be noticeable if a two-point measurement method, i.e., diameter measurements, was applied.

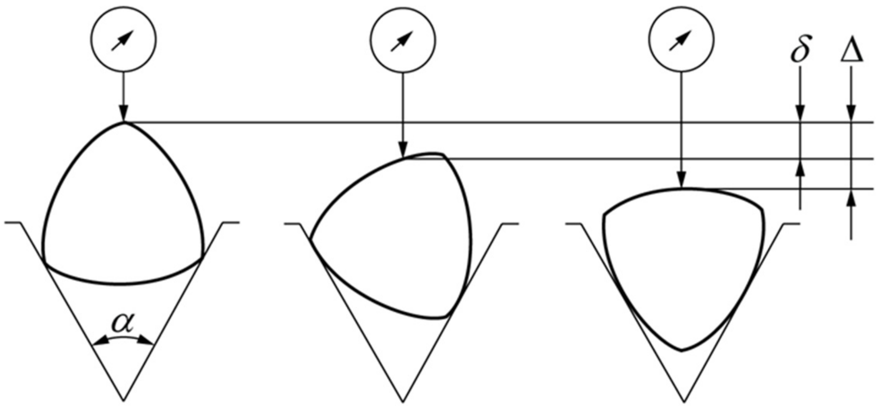


Figure 9. Contact point variation when assessing deviations of lobed forms in V-blocks (Adapted from [28]). Arrows represent the contact part-dial indicator.

The second aspect that can be established is related to the minimum number of points that are necessary to be considered in the manual procedure. Regarding this issue, it was decided to measure three cross sections of one of the parts using the CME, see Figure 10. For this purpose, several points used for the roundness analysis according to the analytical procedure established by Excel® have been filtered out. The evolution of the relative error with respect to the total measured number of points of the profile (i.e., 3600 points) is shown in Figure 11. No significant variations were found as the number of points decreased up to 1000. Thereafter, slight variations occur as the number of measured points decreases up to 40 points, as the maximum difference in the results is less than 1% in any case. To consider a lower number of points leads to an unstable behavior of the roundness tolerance value. Particularly, for only eight points, the differences found among readings were higher than 12%. Logically, using fewer points for the measurement leads to lower roundness tolerance values. This is due to the lower probability of detecting the position of the furthest and closest points of the profile from the center of the LSC, which are the ones that really define the tolerance value. It is concluded that it would be possible to obtain good measurement accuracy using the manual procedure by taking at least 40 equidistant points.

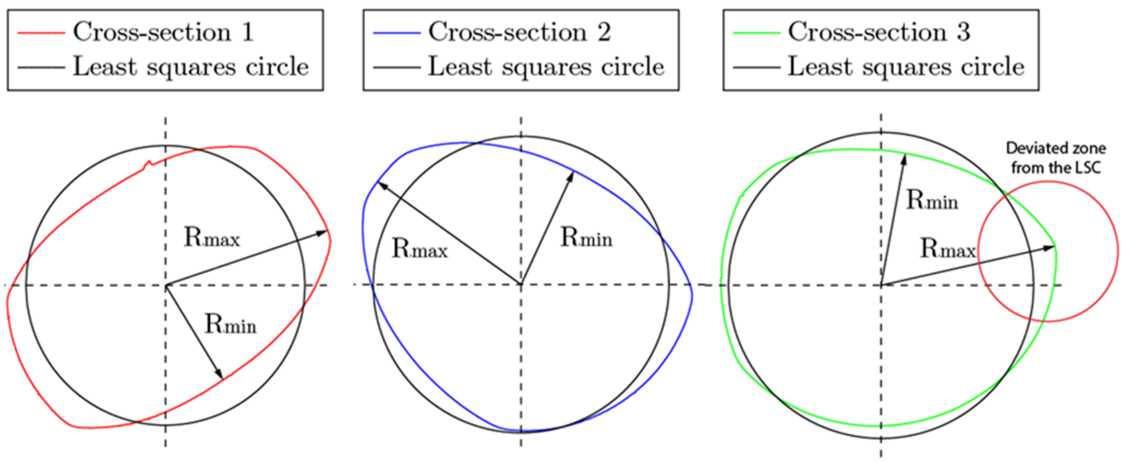


Figure 10. Cross sections selected for the analysis.

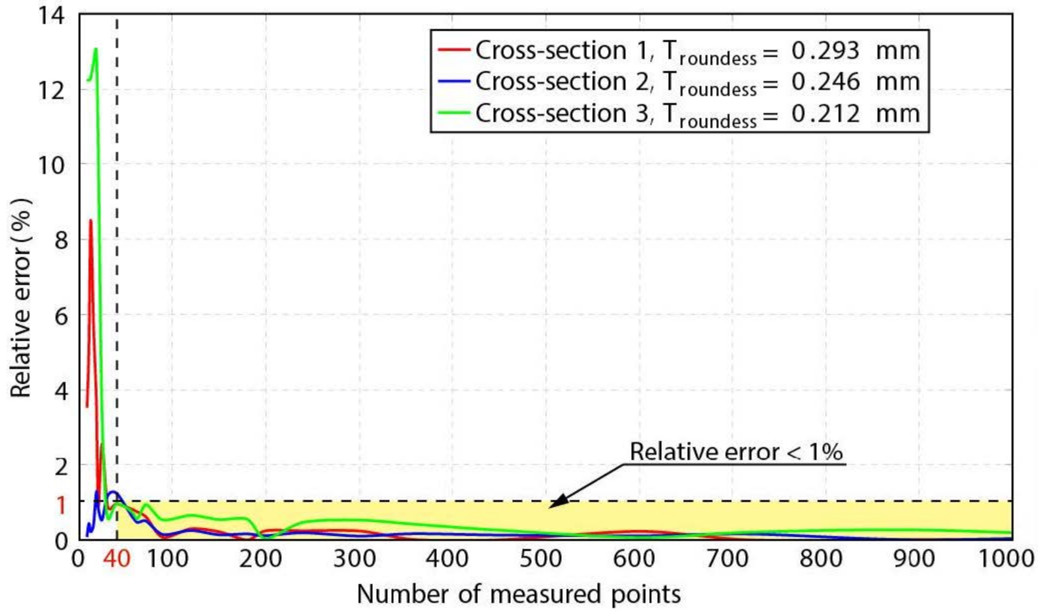
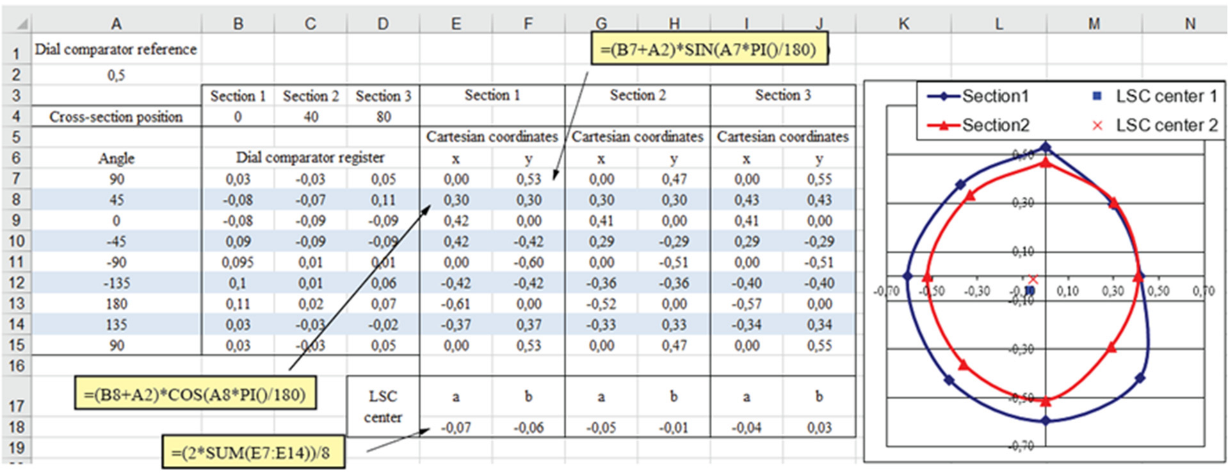


Figure 11. Relative error of the roundness tolerance according to the number of points considered in the analysis.

The shape of the profile also influences the error made as the number of points is reduced, which is established as the third conclusion. Cross section 3 in Figure 12 has the smallest roundness tolerance as the points of the profile are closer to LSC. However, the value of the tolerance is mostly defined by the deviated zone of LSC. Therefore, as the probability of registering this zone with a smaller number of points decreases, the error made in the measurement increases significantly. On the contrary, oval or lobed profiles, such as cross sections 1 and 2, respectively, that differ from LSC in several zones, are more likely to be correctly defined with a smaller number of points.



	A	B	C	D	E	F	G	H	I	J
1	Dial comparator reference									
2	0,5									
3		Section 1	Section 2	Section 3	Section 1		Section 2		Section 3	
4	Cross-section position	0	40	80						
5					Cartesian coordinates		Cartesian coordinates		Cartesian coordinates	
6	Angle	Dial comparator register			x	y	x	y	x	y
7	90	0,03	-0,03	0,05	0,00	0,53	0,00	0,47	0,00	0,55
8	45	-0,08	-0,07	0,11	0,30	0,30	0,30	0,30	0,43	0,43
9	0	-0,08	-0,09	-0,09	0,42	0,00	0,41	0,00	0,41	0,00
10	-45	0,09	-0,09	-0,09	0,42	-0,42	0,29	-0,29	0,29	-0,29
11	-90	0,095	0,01	0,01	0,00	-0,60	0,00	-0,51	0,00	-0,51
12	-135	0,1	0,01	0,06	-0,42	-0,42	-0,36	-0,36	-0,40	-0,40
13	180	0,11	0,02	0,07	-0,61	0,00	-0,52	0,00	-0,57	0,00
14	135	0,03	-0,03	-0,02	-0,37	0,37	-0,33	0,33	-0,34	0,34
15	90	0,03	-0,03	0,05	0,00	0,53	0,00	0,47	0,00	0,55
16										
17				LSC center	a	b	a	b	a	b
18					-0,07	-0,06	-0,05	-0,01	-0,04	0,03

Figure 12. Example of a screenshot of a team's excel spreadsheet. The formulae have been superimposed afterwards. In this example, "," indicates the decimal point ".".

MM methodology presents clear advantages to the understanding and clarifying of the concepts involved. Thus, the first aspect to consider is the handling of specific instrumentation that requires a valuable learning process for a technician. For example, when the users face a dial comparator, they usually think that this instrument is properly used for measuring length, and they do not guess the final goal inherent in the instrument. Some characteristics, such as sensitivity, resolution, range, and others, are perfectly understood considering the MM method. On the contrary, the only use of CME does not lead to acquiring these relevant concepts completely.

For the assembly to be carried out with V-blocks, it is also fundamental to take into consideration aspects as the geometrical vs. rotation axes, which are hardly visible with CME method. Conclusions are easily drawn about the influence of the V-Blocks on that feature and the way of improving the measuring process.

On the other hand, the MM method is essential to guarantee that data management is embedded. Thus, for instance, according to the experience carried out, one of the most highlighted aspects is that drawing the roundness tolerances of a workpiece is difficult to put into practice. Although CME gives directly the real geometric shape of the piece, it is recommended to be capable of building the shape of the workpieces from the numerical data, which must be trained in a specific way. Interpretation of the results is not the same as calculating them, which is something that is completed mechanically from a known formula. From this viewpoint, the MM approach is once again much more robust than the use of CME.

In addition, although it is true that cylindricity is difficult to figure out through MM, the different sections of the workpiece measured and interpreted during the practice allowed the gist of this topic to be understood. CME provides a more realistic result of cylindricity, but, again, some aspects can go unnoticed.

In Figure 12, an example of the results obtained by a team is shown. In that example, the formulae used are written in an explicit way and they are detailed in Table A1 of Appendix C.

Validation of the MM Proposed as a Useful Tool for Measuring Roundness and Cylindricity

The problems that the users involved in this research found during the activity, and the corresponding solutions applied, have been summarized in Table 1. Some aspects related to transversal competencies have also been included.

Table 1. Summary of the difficulties found in the experience and implemented solutions.

Difficulties Found	Implemented Solutions
Related to MM procedure	
Selection of necessary instruments and building the correct assembly	Reading lectures notes, guides, or a reference book. Supervisor support.
Experimental measuring procedure	Punctual support among the participants and/or the supervisor.
Elemental mathematical programming in MS Excel®	Supervisor support.
Related to MMC procedure	
To manage with the equipment	Continuous supervision and users' debates.
Interpretation of the results	
Related to transversal competencies	
Work in groups: disagreements	Negotiation among the participants.
Autonomous work	Debate of the results.

The methodology applied was evaluated by Mechanical Engineering students with the aim of checking the suitability and learning results from the user perspective. A satisfaction questionnaire whose questions can be collected into three conceptual dimensions was implemented: relative to the acquisition of the concepts, the suitability of the methodology applied, and the training skills. In Table 2, the questionnaire is shown, indicating after each question the educational dimensions involved, noted as C, M, and S, respectively. As can be observed, some questions take into account more than one dimension. Finally, question 10 is related to a global evaluation of the experience.

Table 2. Satisfaction survey about the experience (11 users). Educative dimensions involved: C, relative to the concepts acquisition; M, relative to the suitability of the methodology applied; S, relative to the skills training.

	Question	Average	Standard Deviation
1	I consider the manual method to be more appropriate than CME for learning the concepts related to roundness. (C and M)	4.3	0.67
2	I consider that it is not necessary to know what the concepts of roundness and cylindricity represent, as there is equipment that gives me the results directly. (C)	1.1	0.32
3	Only the manual method used in the laboratory would have been sufficient to understand and apply roundness and cylindricity measurements in the industrial environment. (C and M)	1.6	0.70
4	The practice of measuring roundness and cylindricity with the CME is sufficient to acquire the necessary skills in their industrial application. (S)	2.5	1.08
5	The analytical procedure for the cylindricity evaluation is based on fundamental concepts of analytical geometry studied in my degree. (C)	3.7	1.25
6	I find that the application of the procedure to cylindricity calculation is very interesting. (M and S)	3.9	0.57
7	The explanation of the analytical procedure has allowed me to better understand the concept of cylindricity. (C and M)	4.0	1.33
8	I consider that the calculations involved are not complex. (C and S)	2.8	1.03
9	I consider that it is appropriate to implement the procedure in a computer application (i.e., Microsoft Excel) to obtain the results because a manual procedure for that can be difficult or unreasonable. (M)	4.3	0.82
10	In general, I consider that the practice carried out is well planned.	4.8	0.42

The survey was anonymous, and the level of satisfaction was indicated from 0 (completely disagree or dissatisfied) to 5 (completely agree or satisfied).

The results of the survey in Table 2 show that the students rated the activity as highly satisfactory (question 10), with an average of 4.8 points and a standard deviation of 0.42. From the survey questions, it can be drawn that there is a general perception that the manual method is more suitable for acquiring the concepts involved than the direct measurement on the coordinates measuring machine. In addition, it is thought that the use of the measuring machine, without first performing the manual method, would not be enough to get the necessary skills related to the industrial environment. Regarding the analytical methodology developed herein for the calculation of the cylindricity tolerance, it is believed that it is in line with the mathematical fundamentals usually covered in previous learning courses, although the calculations are too complex to be carried out without being helped by software application.

The results of the activity demonstrated that it is appropriate to the level of the students selected and the objectives were addressed, and that it guarantees the understanding of the concepts treated.

Moreover, taking into consideration the methodologies researched and the feedback experimented in this paper, two proposals to strengthen the acquisition of competencies by users can be established. Firstly, the relevance of the manual method suggests acting under this perspective in order to improve the data acquisition system. In this way, a higher number of points should be registered. With this objective in mind, an angle division system is being developed along with the use of a digital dial comparator, and data acquisition software. With this new system, the manual measurements will be carried out at higher speed and more points per section will be registered, allowing more realistic results to be obtained. This improvement line will definitely permit us to apply an intermediate procedure within both presented herein, which will lead to a better accuracy of MM.

On the other hand, from a conceptual viewpoint, and taking into consideration the difficulty of programming an application to manage the cylindricity concept, making it more visual, a new spreadsheet should be developed. This application is expected to allow the user to input the manual data and build a 3D sketch of the complete cylinder. The authors bear in mind these two goals for future research.

The advantages and disadvantages of both methods used to address roundness and cylindricity tolerances are summarized in Table 3.

Table 3. A comparison of MM and CME methodologies related to roundness and cylindricity tolerances.

	Advantages	Disadvantages
Manual methodology	Better acquisition of the theoretical concepts involved	Need for prior knowledge of the procedure
	More economical equipment	Need for assembly
	Possibility of on-machine measurements	High accuracy is difficult to obtain
	Approximate evaluation of the roundness tolerance by recording a few profile points	Punctual irregularities in the profile may be undetected.
	Possibility to obtain an approximate measure of cylindricity tolerance	Accuracy depends on the operator's skill
CME methodology	No need to know the theoretical concepts involved in the measurement of roundness and cylindricity	Theoretical concepts put into practice may not be acquired
	High measurement accuracy	High cost of equipment
	Automated and rapid measurement process	Measuring capacity is limited to the length range of the machine
	Possibility to evaluate cylindricity tolerance with high accuracy	Complex pre-calibration process
	Ability to visualise results and singular irregularities	On-machine measurements are not available

5. Conclusions

This article proposes a practical activity for the evaluation of roundness and cylindricity tolerances that improves their understanding based on three approaches: mathematical analysis of the concepts involved, self-experimentation and manual measurement of the tolerances, and verification of the results with an industrial roundness measuring machine. Furthermore, an analytical procedure for the calculation of the cylindricity tolerance has been proposed from the least squares cylinder. In addition, this method has never been seen before in the literature and is easily programmable in Excel. This analytical procedure is based on the fundamentals of geometry, according to the concepts of cylindricity tolerance, hence it shows a high educational value. Therefore, it is possible to calculate the cylindricity tolerance with a manual procedure without the need for complex mathematical algorithms.

The practical activity has been tested with some Mechanical Engineering students. By means of a satisfaction survey, the adequacy of the proposed methodology has been verified and users have reported a better understanding of the concepts and working procedures proposed.

The authors believe that this practice-based approach can enhance the self-learning of the concepts of roundness and cylindricity through the rehearsal with simple metrological instruments and reachable programming, i.e., Excel, and can provide users with the necessary skills for the industrial environment. If a cylindricity measuring equipment is not available the manual measurement procedure may be successfully applied. Small companies and, particularly, job-shop industries, do not dispose of very modern and sophisticated media and they should ask for external services to control the shapes of the products they are producing. External dependency makes decision making and parts quality control more difficult. Furthermore, although the roundness and cylindricity control was subcontracted, some technical knowledge is necessary to adopt the correction lines and decisions, if necessary. In this paper, it has been demonstrated that a system composed of two V-blocks on a table and a dial indicator is adequate to obtain good results if enough points are measured and good practice rules are carried out. This system can be applied in situ and for very different part sizes, in length and diameter. The concepts to be applied are not very complex and some elemental software may be pre-set for being used by medium-level technicians. These points establish the strength of the manual methodology.

Author Contributions: Conceptualization, E.G.-M., N.G.-G., V.M. and M.C.M.; methodology, E.G.-M. and M.C.M.; software, E.G.-M.; validation, E.G.-M., N.G.-G., M.C.M. and V.M.; formal analysis, E.G.-M., V.M. and M.C.M.; investigation, E.G.-M., N.G.-G. and M.C.M.; resources, V.M.; data curation, E.G.-M. and N.G.-G.; writing—original draft preparation, E.G.-M., V.M. and M.C.M.; writing—review and editing, V.M. and M.C.M.; visualization, E.G.-M. and M.C.M.; supervision, M.C.M.; project administration, V.M.; funding acquisition, V.M. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data used to support the findings of this study can be shared under request.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

As it is defined in Section 2.1.1, Equation (3) expresses the relationship between the distance from each point of the profile to the origin, r_i, the LSC center, and the radial error between the considered point and LSC, e_i. By trigonometry, using the angle difference formula, Equation (A1) is obtained.

$$r_i = R + c\cos\theta_i\cos\alpha + c\sin\theta_i\sin\alpha + e_i \tag{A1}$$

Considering that $a = c\cos\alpha$, $b = c\sin\alpha$, the final expression for r_i is derived, Equation (A2).

$$r_i = R + a\cos\theta_i + b\sin\theta_i + e_i \tag{A2}$$

By rearranging the terms and taking e_i off, the expression for S, is defined by Equation (4) of Section 2.1.1.

According to its definition, LSC is the circle which minimizes S, being necessary to equal zero the partial derivatives of S with respect to a, b, and R, Equation (A3).

$$\begin{cases} \frac{\partial S_i}{\partial R} = \sum\limits_{i=1}^{n} 2(R + a\cos\theta_i + b\mathrm{sen}\theta_i - r_i) = 0 \\ \frac{\partial S_i}{\partial a} = \sum\limits_{i=1}^{n} 2(R + a\cos\theta_i + b\mathrm{sen}\theta_i - r_i)(-\mathrm{sen}\theta_i) = 0 \\ \frac{\partial S_i}{\partial b} = \sum\limits_{i=1}^{n} 2(R + a\cos\theta_i + b\mathrm{sen}\theta_i - r_i)\cos\theta_i = 0 \end{cases} \tag{A3}$$

By rearranging the terms and separating the sums, this leads to the following system, Equation (A4).

$$\begin{cases} nR + a\sum\limits_{i=1}^{n}\cos\theta_i + b\sum\limits_{i=1}^{n}\text{sen}\theta_i = \sum\limits_{i=1}^{n} r_i \\ R\sum\limits_{i=1}^{n}\text{sen}\theta_i + a\sum\limits_{i=1}^{n}\text{sen}\theta_i\cos\theta_i + b\sum\limits_{i=1}^{n}\text{sen}^2\theta_i = \sum\limits_{i=1}^{n} r_i\sin\theta_i \\ R\sum\limits_{i=1}^{n}\cos\theta_i + a\sum\limits_{i=1}^{n}\cos^2\theta_i + b\sum\limits_{i=1}^{n}\text{sen}\theta_i\cos\theta_i = \sum\limits_{i=1}^{n} r_i\cos\theta_i \end{cases} \tag{A4}$$

In the particular case that the points Pi are equidistant, Equation (A5) is satisfied.

$$\begin{cases} \sum\limits_{i=1}^{n}\sin\theta_i = \sum\limits_{i=1}^{n}\cos\theta_i = \sum\limits_{i=1}^{n}\sin\theta_i\cos\theta_i = 0 \\ \sum\limits_{i=1}^{n}\sin^2\theta_i = \sum\limits_{i=1}^{n}\cos^2\theta_i = \frac{n}{2} \end{cases} \tag{A5}$$

Applying Equations (A4) and (A5) in combination, the expressions for a, b, and R, are obtained, Equation (A6).

$$\begin{cases} R = \frac{\sum\limits_{i=1}^{n} r_i}{n} = \frac{\sum\limits_{i=1}^{n}\sqrt{x_i^2+y_i^2}}{n} \\ a = \frac{2\sum\limits_{i=1}^{n} r_i\cos\theta_i}{n} = 2\overline{x} \\ b = \frac{2\sum\limits_{i=1}^{n} r_i\sin\theta_i}{n} = 2\overline{y} \end{cases} \tag{A6}$$

Appendix B

In this appendix it is shown that the point $\overline{X}(\overline{x}, \overline{y}, \overline{z})$ belongs to the least-squares line that best fits a cloud of points in space $P_i(x_i, y_i, z_i)$. As is well known, the distance from a point to a straight line is given by Equation (A7).

$$dis(P_i, r) = \frac{\left|\overrightarrow{PP_i} \times \overrightarrow{V}\right|}{\left|\overrightarrow{V}\right|} \tag{A7}$$

If $\overrightarrow{V}$ is a unity modulus vector, the distance is obtained from the Equation (A8).

$$d_i = dis(P_i, r) = \sqrt{(v(z_i - z_0) - w(x_i - x_0))^2 + (w(x_i - x_0) - u(y_i - y_0))^2 + (u(y_i - y_0) - v(x_i - x_0))^2} \tag{A8}$$

The least squares line is defined as the line that minimizes the sum of the distances to all points, ε. Equation (A9) corresponds to ε.

$$\varepsilon = \sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n}\left[(v(z_i - z_0) - w(y_i - y_0))^2 + (w(x_i - x_0) - u(z_i - z_0))^2 + (u(y_i - y_0) - v(x_i - x_0))^2\right] \tag{A9}$$

Taking the partial derivatives with respect to x_0 and y_0, Equations (A10) and (A11) are obtained.

$$\frac{\partial\varepsilon}{\partial x_0} = \sum_{i=1}^{n} 2(w(x_i - x_0) - u(z_i - z_0)) \cdot (-w) + 2(u(y_i - y_0) - v(x_i - x_0)) \cdot v \tag{A10}$$

$$\frac{\partial\varepsilon}{\partial y_0} = \sum_{i=1}^{n} 2(v(z_i - z_0) - w(y_i - y_0)) \cdot w + 2(u(y_i - y_0) - v(x_i - x_0)) \cdot (-u) \tag{A11}$$

Taking into account Equation (A12),

$$\frac{1}{n}\sum_{i=1}^{n}(x_i - x_0) = \overline{x} - x_0 \tag{A12}$$

Equation (A10) can be rewritten and set equal to zero, obtaining Equation (A13).

$$\frac{1}{n}\frac{\partial \varepsilon}{\partial x_0} = 0 \Leftrightarrow -w^2(\overline{x} - x_0) + u \cdot w(\overline{z} - z_0) + u \cdot v(\overline{y} - y_0) - v^2(\overline{x} - x_0) = 0 \tag{A13}$$

Rearranging the terms and operating afterwards, Equation (A14) can be established.

$$w(\overline{z} - z_0) + v(\overline{y} - y_0) = \left(w^2 + v^2\right)\frac{\overline{x} - x_0}{u} \tag{A14}$$

Proceeding in a similar way, after combining Equations (A11) and (A12), Equation (A15) is finally obtained.

$$\frac{1}{n}\frac{\partial \varepsilon}{\partial y_0} = 0 \Leftrightarrow u(\overline{x} - x_0) + w(\overline{z} - z_0) = \left(u^2 + w^2\right)\frac{\overline{y} - y_0}{u} \tag{A15}$$

Since $w(\overline{z} - z_0)$ appears in both Equations (A14) and (A15), expression of Equation (A16) can be obtained as a combination of them.

$$\left(u^2 + w^2\right)\frac{\overline{y} - y_0}{v} - u(\overline{x} - x_0) + v(\overline{y} - y_0) = \left(v^2 + w^2\right)\frac{\overline{x} - x_0}{u} \tag{A16}$$

Finally, Equation (A16) can be expressed as Equation (A17).

$$\frac{\overline{x} - x_0}{u} = \frac{\overline{y} - y_0}{v} \tag{A17}$$

If this procedure is repeated by taking the partial derivative $\frac{\partial \varepsilon}{\partial z_0}$, Equation (A18) is concluded in an analogous way, which shows that the center of gravity $\overline{X}(\overline{x}, \overline{y}, \overline{z})$ of the points $P_i(x_i, y_i, z_i)$ belongs to the least-squares line.

$$\frac{\overline{x} - x_0}{u} = \frac{\overline{y} - y_0}{v} = \frac{\overline{z} - z_0}{w} \tag{A18}$$

Appendix C

Table A1. Detail of the mathematical calculus developed for the MM procedure in Section 1.

Section 1	**Dial Comparator Reference, dc_{ref} = 0.5 mm**			
Angle, θ	**Dial Comp. Register, dc**	$x=\left(dc_{ref} + dc\right) \times cos\theta$	$y=\left(dc_{ref} + dc\right) \times sin\theta$	**Distance to LCS Center** $R=\sqrt{(x+a)^2 + (y+b)^2}$
90	0.03	0.00	0.53	0.59
45	−0.08	0.30	0.30	0.52
0	−0.08	0.42	0.00	0.49
−45	0.09	0.42	−0.42	0.61
−90	0.095	0.00	−0.60	0.54
−135	0.1	−0.42	−0.42	0.50
180	0.11	−0.61	0.00	0.54
135	0.03	−0.37	0.37	0.52
90	0.03	0.00	0.53	0.59
LSC center		$a = 2\overline{x}$	$b = 2\overline{y}$	$T_{roundness}$ $T_{roundness} = R_{\max} - R_{\min}$
		−0.07	−0.06	0.114

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