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Pressure Analysis of Onshore and Offshore Shale Gas Reservoirs under Constant-Rate Condition Considering Thin Sandstone Layer and Interlayer Cross-Flow

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Abstract: The extraction of shale gas from onshore and offshore shale gas reservoirs will play an important role in meeting China's future energy needs, which will not only help alleviate the energy crisis but also contribute to climate change mitigation. As for the target shale formation enriched by thin sandstone layers in typical basins, an analytical calculation method is proposed to perform pressure analysis for multi-layer shale gas reservoirs considering the adsorption–desorption characteristics of shale layer and the interlayer cross-flow. Firstly, the changes in storage capacity and flow resistance are obtained by using the distance of investigation equation. According to the electrical analogy, the equivalent total storage capacity and flow resistance can be calculated considering the sandstone-shale crossflow. Because production from one time step to the other causes depletion of the storage capacity, the reservoir pressure in different time steps can be calculated based on the material balance equation. Numerical models have been constructed based on three typical reservoir lithology combinations (sandstone-shale, shale-sandstone-shale and sandstone-shale-sandstone) to validate the accuracy of the proposed analytical calculation method. Furthermore, three important factors (porosity, the ratio of horizontal/vertical permeability (k_h/k_v) and the layer thickness) have been selected for the sensitivity analysis to verify the stability. The comparative results indicate that the proposed analytical calculation method is suitable for pressure analysis in shale gas reservoirs containing thin sandstone layers. It will provide theoretical support for the further enhancement of the production of this type of gas reservoirs.

Keywords: shale gas; thin sandstone layer; interlayer crossflow; analytical method; pressure analysis

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1. Introduction

The global demand for hydrocarbon resources has risen rapidly in recent decades. However, conventional oil/gas resources are decreasing worldwide, it is essential to develop unconventional resources in onshore and offshore reservoirs in order to cover our shortages in energy [1].

The Energy Information Administration (EIA) has reported that the contributions from offshore oilfields has reached nearly 30% of global oil/gas production [2]. In the development of onshore and offshore unconventional resources, especially for some shale gas reservoirs that are rich in thin sandstone layers, the existed thin sandstone layers and interlayer crossflow have a significant impact on gas production [3,4].

As one of the representatives of China's terrestrial sedimentary basins, the Ordos Basin is rich in shale gas resources and is therefore promising for commercial development [5]. By analyzing the logging data from over 300 shale gas wells in the Ordos Basin, the target shale layers of these shale gas wells are generally interbedded with multilayered sandstones, coal seams and so on. Many layers around the target formation will be penetrated during drilling the horizontal well and hydraulic fracturing, while fluid flow exists between layers. Production characteristics of shale gas wells containing thin sandstone interbeds are different compared to conventional shale gas reservoirs [5,6]. Therefore, pressure analysis and performance prediction of shale gas reservoirs with thin interbedded sandstones is challenging and crucial, and have attracted the attention of many scholars at home and abroad [7–12].

Scholars around the world have conducted a lot of research on performance prediction as well as pressure analysis of multi-layer oil/gas reservoirs. Bruce [13] made use of the theory of capacitance and resistance and built a "power grid" to simulate the fluid flow in multi-layer oil reservoirs. Civan [14] proposed a cylindrical tank model to describe the flow within the reservoir drainage area of wells considering the non-darcy flow and derived some simplified analytical solutions to determine reservoir permeability and thickness. Villanueva-Triana and Civan [15] presented a commingled well production model in multilayer reservoir considering formation cross-flow and external boundary effects. Yousef et al. [16] presented a new procedure to quantify communication between vertical wells in a reservoir on the basis of fluctuations in production and injection rates. A more complicated model that includes capacitance, as well as resistive effects, were adopted in their model. Sayarpour et al. [17] introduced analytical solutions for fundamental differential equation of the capacitance model, which applied for rapid assessment at different levels of a field study, from a single well, to a group of wells, and to an entire field. Shahamat et al. [18] deployed a capacitance–resistance model to perform production analysis in single and two-layer sandstone reservoirs under constant-rate and constant-pressure conditions, considering the case of interlayer with and without crossflow. The above methods are more suitable for describing high permeability oil/gas reservoirs, but have limited application in unconventional reservoirs with low permeability and significant unsteady flow. Particularly, the sandstone-shale combination is not considered.

In this paper, we propose the analytical method to perform pressure analysis in multilayer shale gas reservoirs, contained three typical lithological combinations, the adsorption–desorption characteristics of shale gas, and the interlayer crossflow. The fluid flow, gas adsorption/desorption and interlayer crossflow presented in sandstone-shale combination are firstly described mathematically and the analytical calculation flowchart is made up. Furthermore, numerical models have been constructed based on three typical lithological combinations to validate the accuracy and stability of the proposed analytical calculation method. This will help to quickly and accurately assess pressure variations during shale gas production, so as to optimize the production scheme and enhance the total production of the shale gas reservoirs.

2. Model Development

As for the production and pressure analysis of multi-layered oil reservoirs, Shahamat et al. [18] proposed the "tank" model for a two-layered oil reservoir. In his model, many "tanks" with different reservoir characteristics are used to represent different layers based on the model assumption that each layer is homogeneous, so that the flow of fluid in each layer is separate, and thus, can be used to consider the presence or absence of interlayer cross-flow. In this paper, we improve his model and apply the new method to a multilayered shale gas reservoir with interlayer crossflow, which is shown in Figure 1. The analytical calculation process is constructed under the following assumptions:

(1) The shale layer and sandstone layer are both homogeneous, isopachous and isothermal.
(2) Flow is single gas phase.
(3) The gas rate is constant.
(4) The impact of gravity and capillary force is neglected.
(5) Gas desorption meets the Langmuir isotherm adsorption equation.

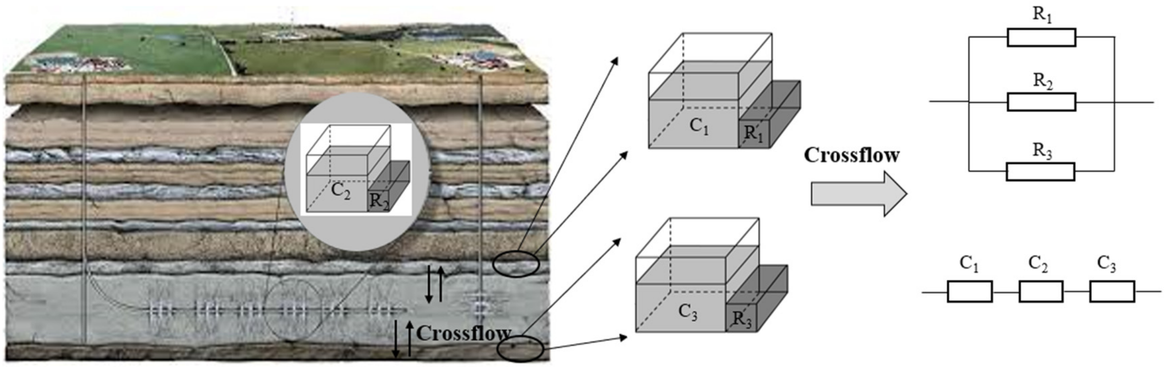


Figure 1. Schematic diagram of the "tank" model considering cross-flow in multi-layer shale gas reservoir.

When there is interlayer cross-flow, pressure is transferred between layers. The total production of the gas reservoir cannot be calculated simply by adding up the production from each layer. Based on the similarity between electrical resistance and flow resistance, the flow resistance of each layer of the reservoir can be simplified to be similar to electrical resistance as shown in Figure 1. Hence, for the case of inter-layer cross-flow, the flow resistance within a single layer is comparable to resistance of parallel resistors in the theory of electricity, subsequently, we can calculate the equivalent flow resistance R_e of the entire reservoir. Due to the different physical parameters of each layer, the flow resistance varies from layer to layer. Using the analogy of parallel resistors, the expression of R_e is as follows:

$$R_e = \frac{1}{\sum\limits_{i=1}^{N} \frac{1}{R_i}} \; (i = 1, 2, 3, \cdots, N) \tag{1}$$

$$R_i = \frac{\beta_1 B \mu \alpha_2 y_{e,i}}{2\pi k_i h_i x_f} \; (i = 1, 2, 3, \cdots, N) \tag{2}$$

where $\alpha_2 = \pi/6$ which is a constant depends on the criterion used for defining the distance of investigation and $\beta_1 = 2\pi \times 141.2$ which represents international unit conversion [19]. k_i is the permeability of i-th layer. $y_{e,i}$ is the distance of investigation. μ is the viscosity. h_i is the thickness of i-th layer. φ is porosity. c is the compressibility. B is formation volume factor.

Similarly, the total storage capacity of a multi-layered gas reservoir with inter-layer cross-flow can be likened to the superposition of the currents in each branch of a parallel circuit, and the expression for the equivalent total storage capacity C_e is as follows:

$$C_e = \sum_{i=1}^{N} C_i \; (i = 1, 2, 3, \cdots, N) \tag{3}$$

$$C_i = \frac{4 x_f c h_i \phi y_{e,i}}{5.625 B} \; (i = 1, 2, 3, \cdots, N) \tag{4}$$

where $y_{e,i}$ is the distance of investigation. μ is the viscosity. h_i is the thickness of i-th layer. φ is porosity. c is the compressibility. B is formation volume factor. x_f is the fracture half-length.

As for the constant-rate condition, the capacity/resistance ratio (CRR) is defined to analyze the constant-rate linear flow, which can be represented by combinations of parameters for determining the pressure change with production time [20,21].

$$CRR = \frac{8\pi\phi {x_f}^2}{5.615 \beta_1 \mu B} \sum_{i=1}^{N} (k_i h_i) \sum_{i=1}^{N} (c_i h_i) \; (N = 1, 2, 3, \ldots) \tag{5}$$

where β_1 = 2π × 141.2 which represents international unit conversion. k_i is the permeability of i-th layer. μ is the viscosity. h_i is the thickness of i-th layer. B is formation volume factor. φ is porosity. x_f is the fracture half-length.

In order to analyze the pressure propagation in formations, the distance of investigation (y_e) should be calculated to determine the distance affected by the pressure wave. Then the duration of fluid flow can also represented by the time of arrival at the boundary (t_{BDF}). The two parameters can be written as [18,19],

$$y_e = \alpha_1 \sqrt{\frac{\beta_2 kt}{\phi \mu c}} \tag{6}$$

$$t_{BDF} = \left(\frac{y_e}{\alpha_1}\right)^2 \frac{\phi \mu}{\beta_2} \frac{\sum_{i=1}^{N} c_i h_i}{\sum_{i=1}^{N} k_i h_i} (N = 1, 2, 3, \ldots) \tag{7}$$

where α_1 = 2.55 and β_2 = 0.00633. k_i is the permeability of i-th layer. μ is the viscosity. H_i is the thickness of i-th layer. Φ is porosity. C is the compressibility.

As the important feature in shale gas reservoir, gas adsorption in the shale layer must be considered, which can be calculated by the Langmuir isotherm adsorption equation [22] and its mathematical expression can be written as:

$$V = V_L \frac{P}{P_L + P} \tag{8}$$

where V_L and P_L represent Langmuir volume and pressure, respectively, and V and P is the volume and pressure of the adsorbed gas. When the gas adsorption is taken into account, the compressibility equation should be modified as [23]:

$$C_t{}^* = C_f + C_w S_w + C_g (1 - S_w) + C_{gd} \tag{9}$$

$$C_{gd} = \frac{0.031214 \rho_m V_L \overline{B_g} P_L}{\phi (\overline{p} + P_L)^2} \tag{10}$$

where C_f, C_w, C_g and C_{gd} is rock compressibility, water compressibility, free gas compressibility and adsorbed gas compressibility, respectively. S_w is water saturation, ρ_m is matrix density, $\overline{B_g}$ is gas reservoir volume factor, and ϕ is the porosity.

Similarly, the compressibility factor should also be modified by King [24]:

$$z^* = \frac{z}{(1 - S_w) + 0.031214 \rho_m \frac{zT}{\phi} \left(\frac{p_{sc}}{T_{sc}}\right) V_L \frac{1}{p + p_L}} \tag{11}$$

where P_{sc} and T_{sc} is the standard condition pressure and temperature, respectively. z is compressibility factor and T is the reservoir temperature.

Considering that many physical parameters of gas are pressure-dependent and the gas adsorption is essential [25], a general modified pseudo-pressure transformation is defined as:

$$m(p) = \frac{1}{k_i} \int_{p_b}^{p} k(p) \frac{p}{\mu(p) z^*(p)} dp \tag{12}$$

where $m(p)$ is the pseudo-pressure, k_i is the intrinsic permeability, $z^*(p)$ is the pressure-dependent compressibility factor.

Based on the above definitions and equations, a new analytical calculation method can be obtained to perform pressure analysis of onshore and offshore shale gas reservoirs under constant-rate condition considering interlayer cross-flow and thin sandstone layer.

Firstly, the capacity/resistance ratio (CRR) and the time of arrival at the boundary (t_{BDF}) can be calculated by Equations (5) and (7). Meanwhile, the given constant rate is q_c to be this value. The initial pseudo-pressure $m(p_{wf})_1$ of the reservoir is also calculated by Equation (12). Secondly, we can obtain the equivalent storage capacity C_e and equivalent resistance Re of the multi-layered gas reservoir. Then, the average pseudo-pressure of the gas reservoir corresponding to each time step can be calculated. Finally, using the definition of pseudo-pressure, we infer the real reservoir pressure in the gas reservoir, which is suitable for analyzing pressure variations during production. The specific flow chart is shown in Figure 2.

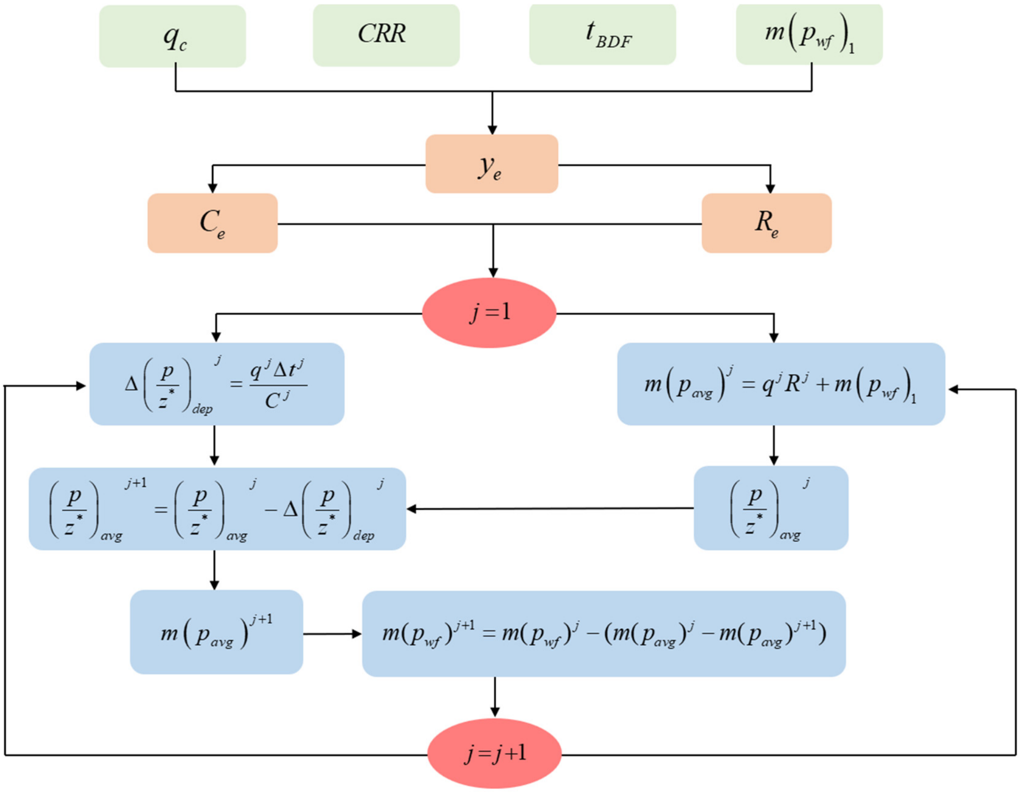


Figure 2. The analytical calculation flowchart for reservoir pressure in shale gas reservoir under constant-rate condition considering inter-layer cross-flow ("j" represents the time step, day).

3. Comparison and Validation

For the three typical lithological combinations of onshore and offshore shale gas reservoirs considering the thin sandstone layer (sandstone-shale, shale-sandstone-shale, and sandstone-shale-sandstone), a total of three numerical models built by Eclipse with inter-layer crossflow are built to compare and validate against the analytical method for pressure analysis. The main input parameters are summarized in Table 1.

Table 1. Input parameters for three numerical models.

Parameters	Value	Parameters	Value
Number of model grids	21 × 51 × 2	Initial Pressure (Psi)	2500
Model dimension (ft × ft × ft)	300 × 500 × 60	Porosity	0.1
Temperature (K)	318.15	Fracture permeability (mD)	500
Langmuir pressure (Psi)	650	Langmuir volume (Mscf/ton)	0.096

Table 1. *Cont.*

Parameters	Value	Parameters	Value
	Case 1		
Parameters	Shale Layer		Sandstone Layer
Horizontal permeability (mD)	0.0005		0.2
Vertical permeability (mD)	1		1
Thickness (ft)	50		10
Compressibility (Psi^{-1})	7.5×10^{-6}		1.1×10^{-5}
	Case 2		
Parameters	Shale Layer 1	Sandstone Layer	Shale Layer 2
Permeability (mD)	0.2	0.2	0.0002
Vertical permeability (mD)	1	1	1
Thickness (ft)	30	10	50
Compressibility (Psi^{-1})	7.5×10^{-6}	1.1×10^{-5}	7×10^{-6}
	Case 3		
Parameters	Sandstone layer 1	Shale layer	Sandstone layer 2
Permeability (mD)	0.1	0.0005	0.05
Vertical permeability (mD)	1	1	1
Thickness (ft)	10	50	15
Compressibility (Psi^{-1})	1.1×10^{-5}	7.1×10^{-6}	1.5×10^{-5}

3.1. Sandstone-Shale Combination

A two-layered numerical model was set up to represent the sandstone layer (the gray region) and shale layer (the yellow region), which is shown in Figure 3. The first row of the model represents the horizontal wellbore, which is directly connected to the sandstone and shale layers. The parameters of Case 1 are summarized in Table 1.

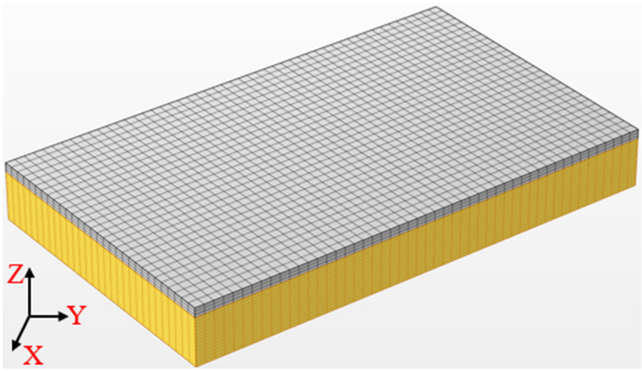


Figure 3. Schematic diagram of the numerical model of sandstone-shale combination.

According to the statistical results from actual production history, the production time of some wells has reached or has even exceeded 6 years. Therefore, the numerical model is set to produce for 2000 days under constant rate of 20 Mscf/d, under a fixed-yield condition, where inter-layer cross-flow is also considered. Based on the analytical calculation flowchart shown in Figure 2, the input values in this case are 15.68 for CRR, 90.92 for t_{BDF}, and 20 for q_c. The initial pseudo-pressure $m(p_{wf})_1$ is 9.53 accordingly. Then, the pseudo-pressures of the gas reservoirs corresponding to different time steps can be calculated step by step. From Figure 4, it can be seen that the pressure change with time calculated by the analytical method is in perfect agreement with the result calculated by the numerical model. Meanwhile, the results between pressure and production time presented in the log–log plot also demonstrates that the proposed analytical calculation method is suitable for analyzing the production pattern of transient linear flow and boundary-dominated flow in multi-layered gas reservoirs.

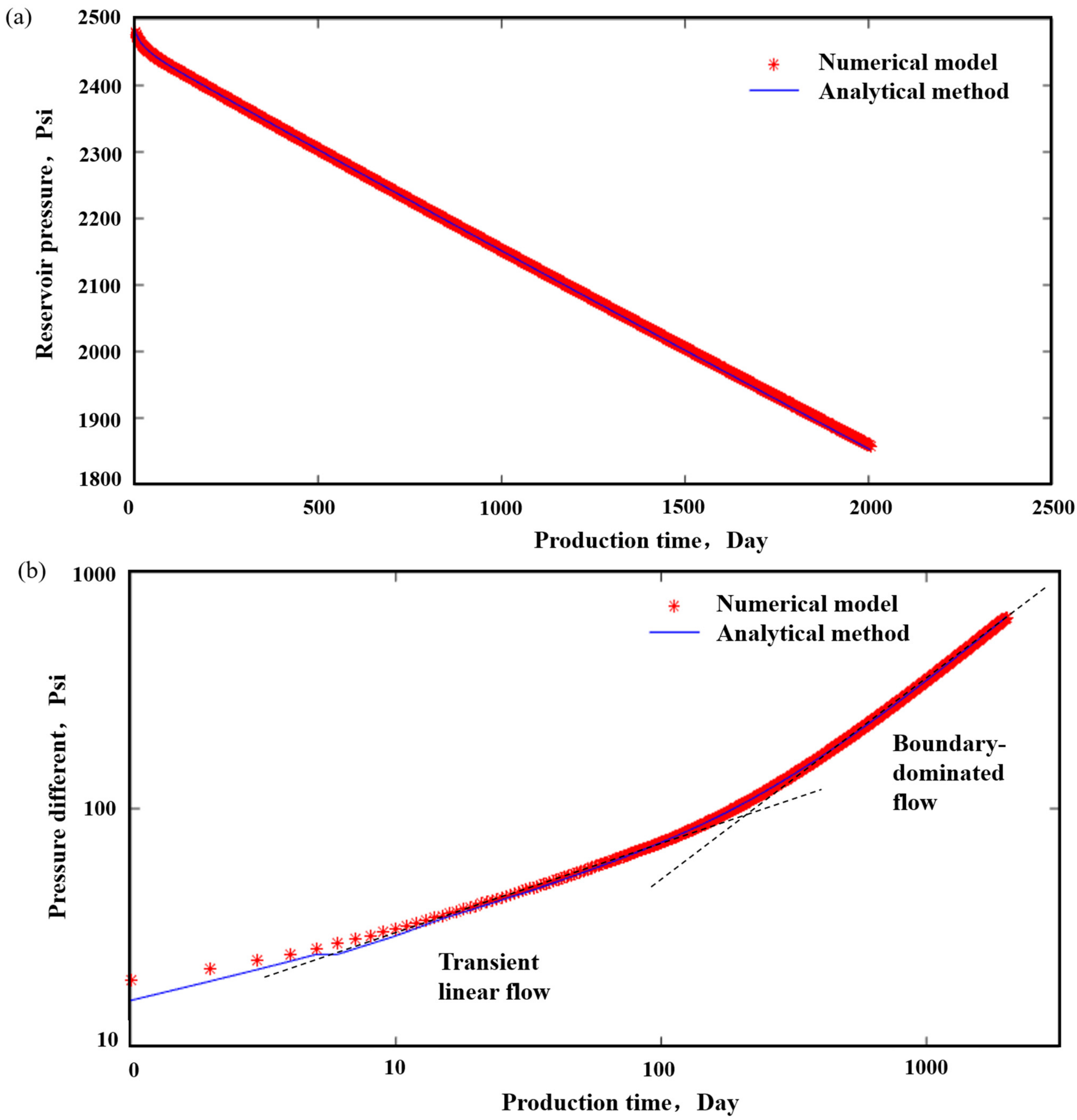


Figure 4. Comparison of reservoir pressure variation with time under constant-rate conditions considering inter-layer crossflow base on sandstone-shale combination (**a**) Cartesian coordinates; (**b**) Log-log coordinates.

3.2. Shale-Sandstone-Shale Combination

A three-layered numerical model was set up to represent shale layer 1 (the yellow region), the sandstone layer (the gray region) and shale layer 2 (the yellow region), as shown in Figure 5. The basic input parameters of Case 2 are also summarized in Table 1. Similarly, each layer is connected to the wellbore through the first row of the grids.

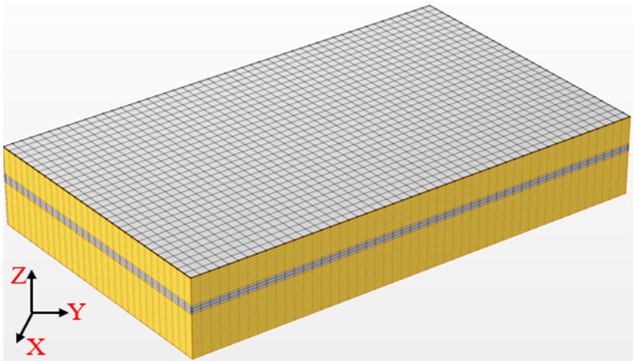


Figure 5. Schematic diagram of the numerical model of shale-sandstone-shale combination.

In order to study the production in most cases where a thin sandstone layer exists in the target shale formation, we set up a sandstone layer with a thickness of 10 ft between two shale layers and ensured that the fluid could flow among the layers. According to the vast majority of historical production data, the numerical model is also set to produce for 2000 days under a constant rate of 20 Mscf/d. Based on Equations (4) and (5), we can calculate the values of CRR and t_{BDF}, respectively, as 23.6 and 142.93. The input values of q_c and $m(p_{wf})_1$ can also obtained as 20 and 9.56. Subsequently, the pseudo-pressure of the gas reservoir at different time steps can be calculated step by step based on the flowchart in Figure 6. The results in Figure 6a show that the reservoir pressure calculated by the analytical method is in perfect agreement with the pressure calculated by the numerical model, which indicates that the pressure change of a multi-layered shale gas reservoir containing thin sandstone interbed can be accurately calculated by the proposed analytical calculation method. The log–log plot of pressure difference versus production time exhibits in Figure 6b. The transient flow can last for 300 days and while the pressure difference throughout the process is approximately 100 Psi. It further suggests the existence of two flow regimes (transient flow and boundary flow) in the production process of gas reservoir.

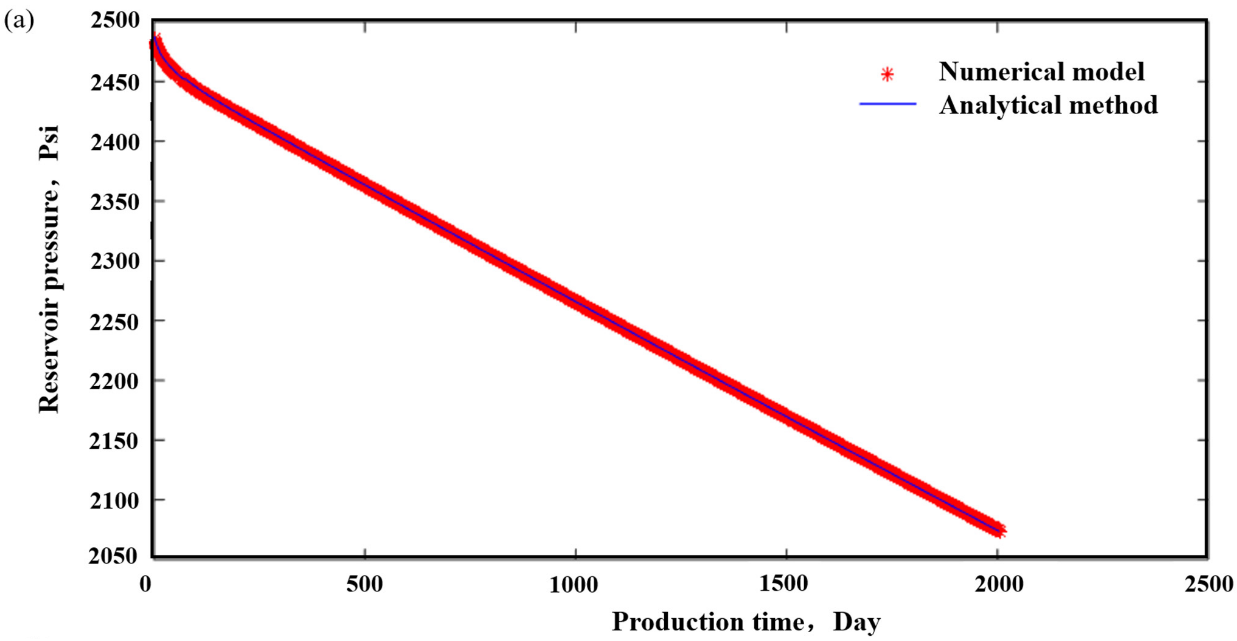


Figure 6. *Cont.*

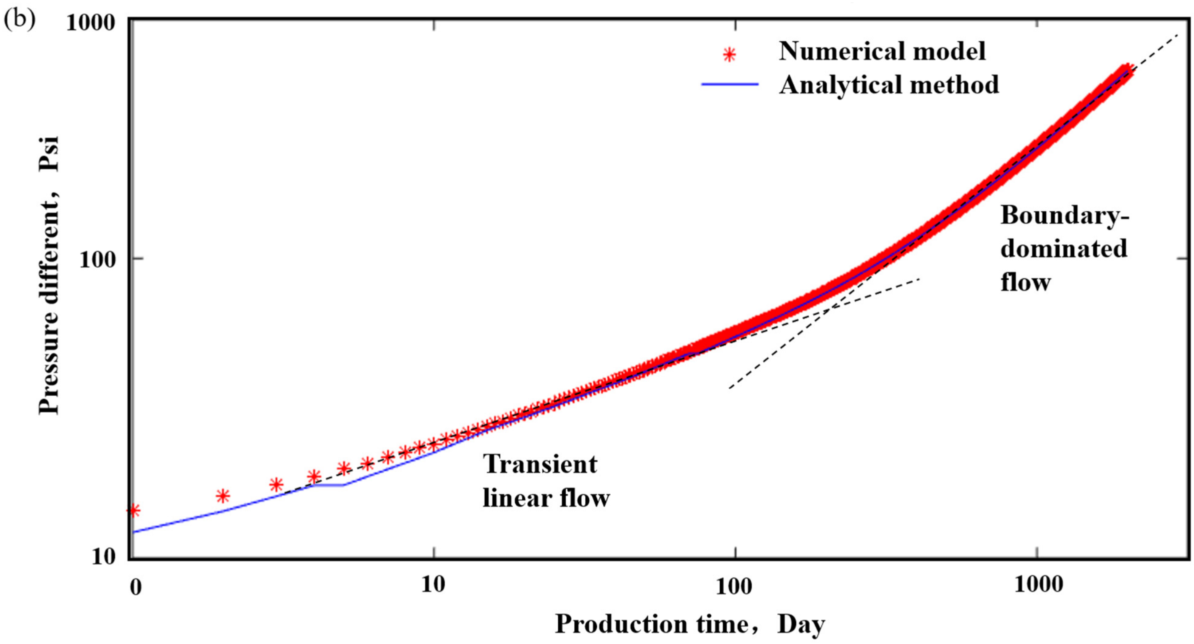


Figure 6. Comparison of reservoir pressure variation with time under constant-rate condition considering inter-layer crossflow base on shale-sandstone-shale combination. (**a**) Cartesian coordinates; (**b**) Log–log coordinates.

3.3. Sandstone-Shale-Sandstone Combination

As for the case where sandstone layers are enriched around the shale layer, we also set up a three-layer numerical model to compare with the analytical calculation method. From top to bottom, each layer represents sandstone layer 1 (the gray region), shale layer (the yellow region) and sandstone layer 2 (the gray region), as shown in Figure 7. The main input parameters of Case 3 are listed in Table 1.

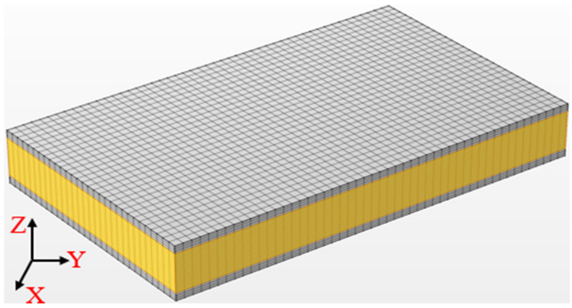


Figure 7. Schematic diagram of the numerical model of the sandstone-shale-sandstone combination.

Similarly, we set this sandstone-shale-sandstone model as constant-rate condition where inter-layer cross-flow is present. The production rate and time are set to be equal to the previous two models, i.e., the numerical model produces 2000 days at a constant rate of 20 Mscf/d. The input parameters can also be obtained as 16.99 for CRR, 133.97 for t_{BDF} and 9.56 for $m(p_{wf})_1$. Based on the flowchart in Figure 2, the reservoir pressure can be calculated for different time steps within the production time. The comparison results from numerical model as well as analytical method is presented in Figure 8a, which is in perfect agreement. It indicates that the pressure change of a multi-layered shale gas reservoir containing thin sandstone interbeds can be accurately calculated by the proposed analytical method. As is shown in Figure 8b, the transient flow can last for 200 days and while the pressure difference throughout the process is approximately 80 Psi. By comparing with Figure 6b, the shale gas reservoir enriched with sandstone layers have shorter transient flow times, suggesting that such target formations are easier to develop and can achieve higher gas production in the early stage. It also shows that the thin sandstone layer in the shale formation has a significant impact on production.

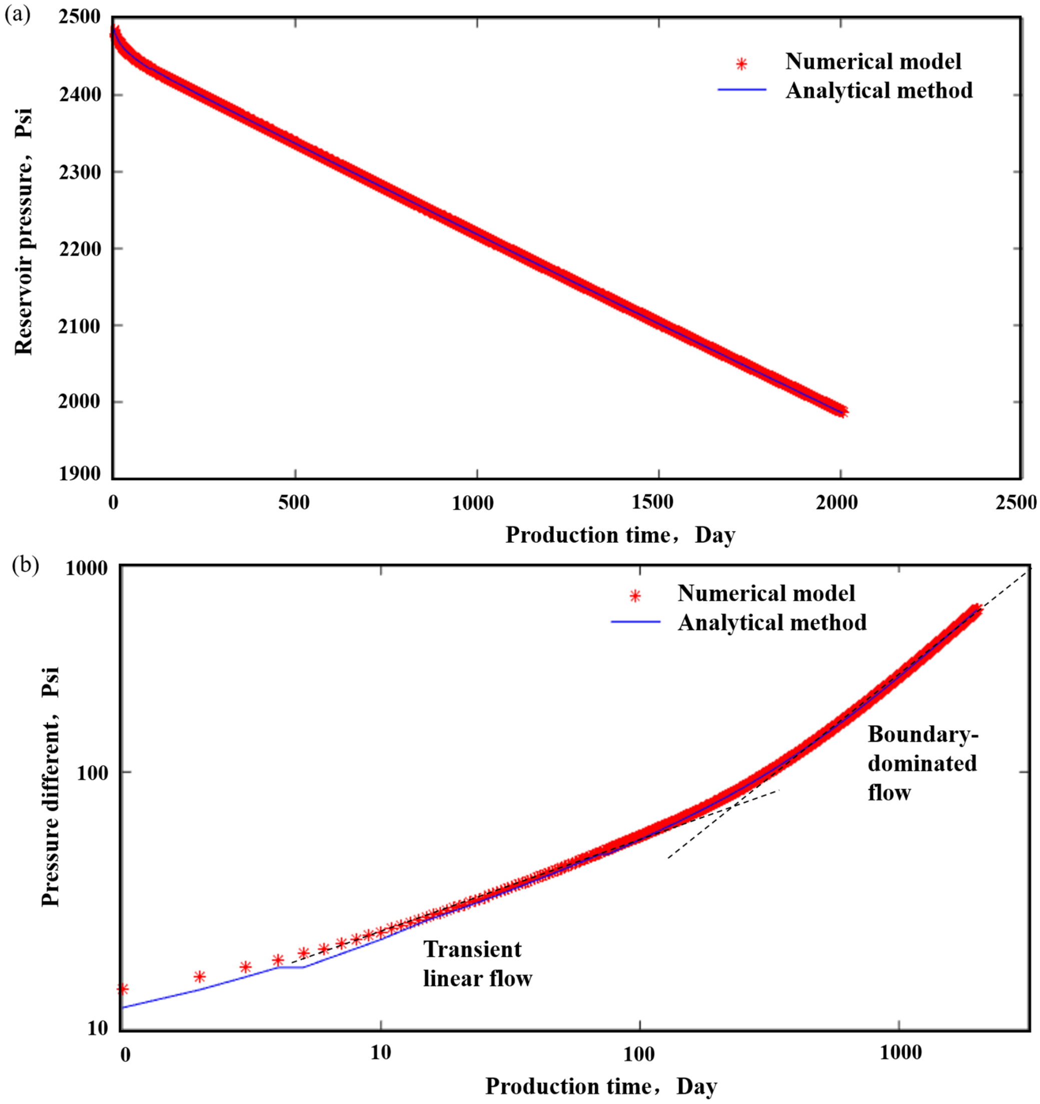


Figure 8. Comparison of reservoir pressure variation with time under constant-rate condition under inter-layer crossflow base on sandstone-shale-sandstone combination (**a**) Cartesian coordinates; (**b**) Log-log coordinates.

3.4. Multi-Stage Fractured Horizontal Well in Multi-Layer Shale Gas Reservoir

In order to further evaluate the proposed analytical method against numerical simulation in more complex problems, the numerical case of multi-stage fractured horizontal well in multi-layer shale gas reservoir is designed, as shown in Figure 9. There is a 10 ft sandstone layer contained in a shale formation with a total thickness of 60 ft. The length of the horizontal well is 300 ft with four hydraulic fractures equally spaced along the x-direction. The relevant parameters of sandstone layer and shale layer are also listed in Table 1. According to the vast majority of actual production situations, the numerical model is set to produce for 100 days under a constant rate of 217 m^3/d. Based on Equations (4) and (5), we can calculate the values of CRR and t_{BDF}, respectively, as 31.4 and 64.58. The input values of q_c and $m(p_{wf})_1$ can also obtained as 217 and 14.25. Subsequently, the pseudo-pressure of the gas reservoir at different time steps can be calculated step by step based on the flowchart in Figure 6. From the results in Figure 10, it can be clearly seen that the reservoir pressure calculated by the proposed analytical method matches very well with the pressure calculated by the numerical model, which further indicates that the proposed analytical method has a good application in real reservoir conditions.

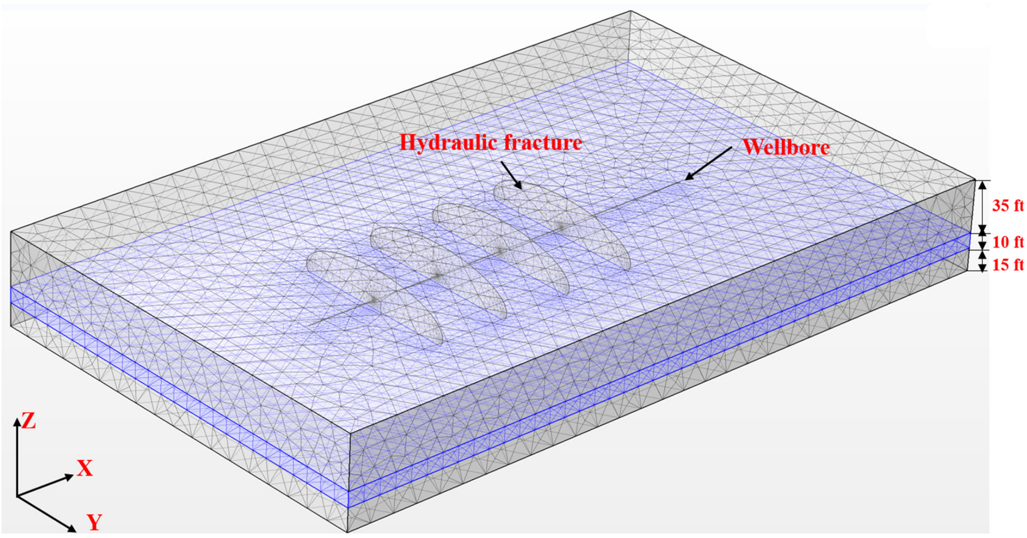


Figure 9. Schematic diagram of the numerical model of multi-stage fractured horizontal well in multi-layer shale gas reservoir.

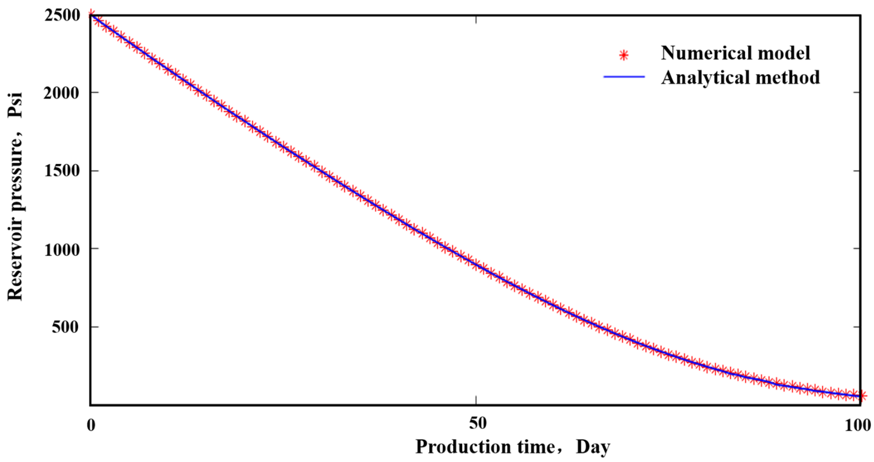


Figure 10. Comparison of reservoir pressure variation with time under constant-rate condition under inter-layer crossflow base on multi-stage fractured horizontal well in multi-layer shale gas reservoir.

4. Sensitivity Analysis

By comparing the numerical models and analytical method for three typical lithological combinations, it is demonstrated that the analytical calculation method is suitable for production prediction of shale gas reservoirs considering interlayer cross-flow and thin sandstone layers. Considering that interlayer cross-flow exists in both shale layer(*sh*) and sandstone layer(*sa*) in actual production, we must further verify the stability of the analytical calculation method. In this section, we optimize three important factors (porosity, the ratio of horizontal/vertical permeability (k_h/k_v) and the layer thickness) for the sensitivity analysis while keeping the other input parameters constant.

4.1. Analysis of the Porosity

We set the porosity of each layer based on the three typical lithological combinations (sandstone-shale, shale-sandstone-shale and sandstone-shale-sandstone combination), and the parameter values are mainly taken from the core analysis data of shale gas wells in the Ordos Basin. The comparative results of the analytical method and the numerical models are shown in Figure 11. It can be seen that the results are almost fully consistent with each other, which further demonstrated the stability of the analytical method. From the results in Figure 11a, we conclude that pressure difference decreases with increasing porosity and while the transient linear flow time is prolonged. Namely, the gas reservoirs with greater porosity will have longer production times and therefore higher total gas production. The porosity of sandstone and shale layers in Figure 11b are set to different values, we can find that the greater the porosity of a sandstone layer in the multilayer shale gas reservoir, the smaller the pressure difference will be. The reason may be that sandstones of a greater porosity contribute more free gas during the early stage of production. Similarly, the greater the porosity of a shale layer in the multilayer shale gas reservoir, the smaller the pressure difference will be, which is shown in Figure 11c. The reason is also because shale formations with larger porosity can produce more free gas at lower pressure difference. Through comparative analysis, the analytical method is applicable to the production prediction analysis of three typical lithology combinations considering different porosities.

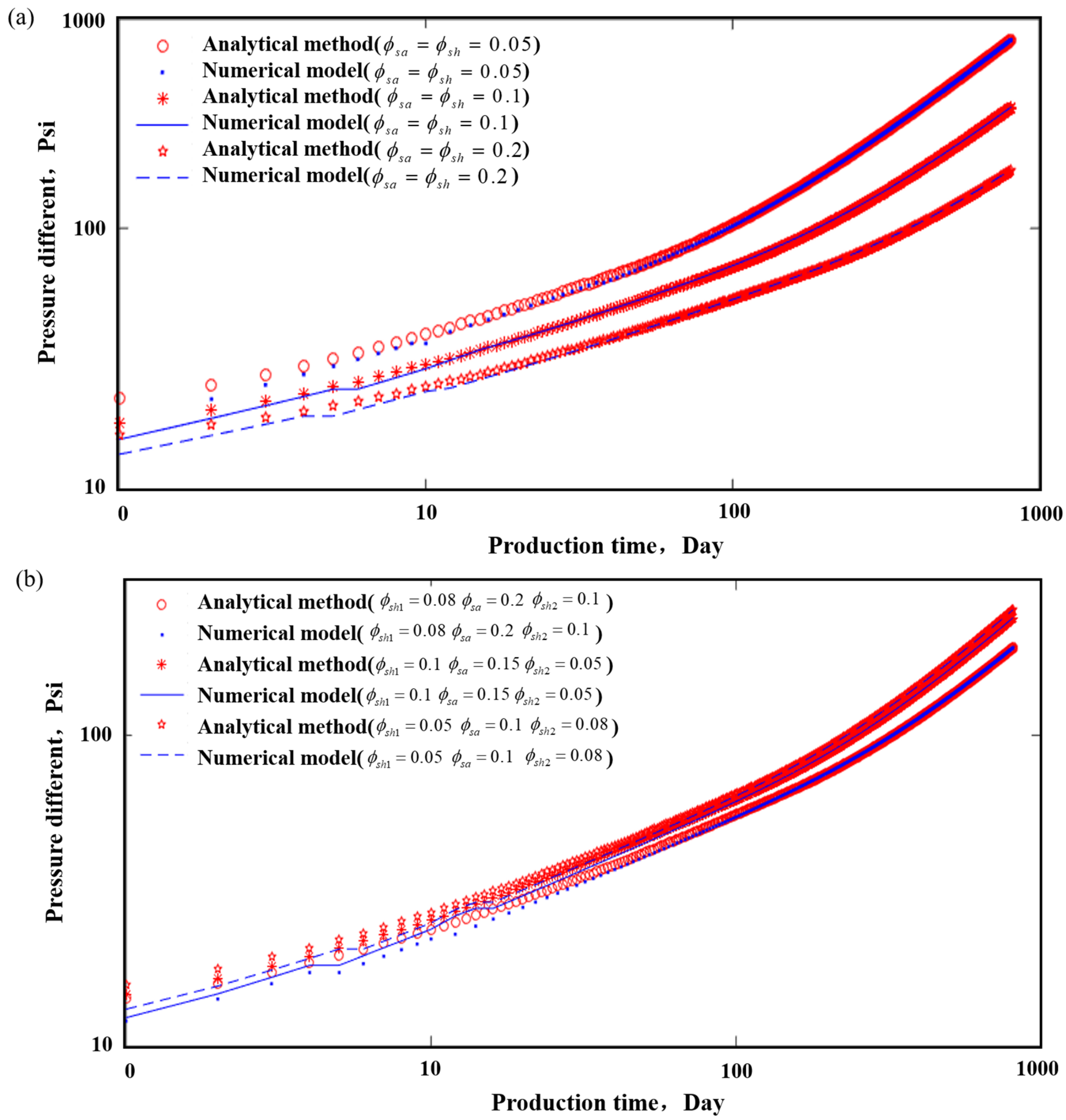


Figure 11. *Cont.*

(c)

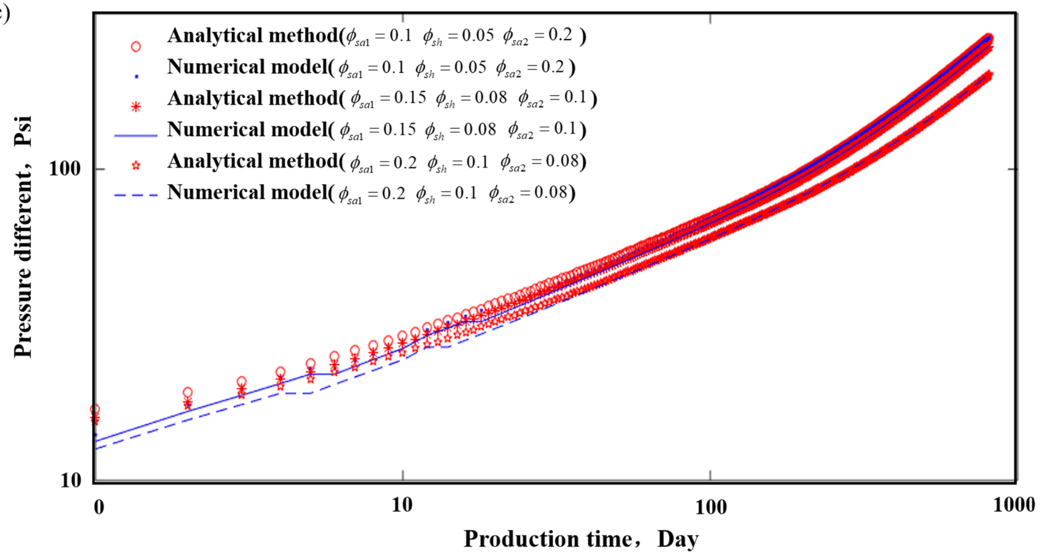


Figure 11. Comparison results for three typical lithological combination with different porosities considering inter-layer cross-flow. (**a**) sandstone-shale combination; (**b**) shale-sandstone-shale combination; (**c**) sandstone-shale-sandstone combination.

4.2. Analysis of the Ratio of Horizontal/Vertical Permeability

Combined with actual core data, we set different horizontal/vertical permeability ratios (k_h/k_v) for each layer of numerical models based on the three typical lithology combinations. As shown in Figure 12, the results calculated by analytical method and numerical models present a great match. For the sandstone-shale combination, the greater the horizontal/vertical permeability ratio in the sandstone, the smaller the pressure difference as shown in Figure 12a. Namely, the greater the vertical permeability of the sandstone, the better the connectivity between the shale and the sandstone is proved, and when the pressure is lowered to a certain degree, the free gas inside the sandstone can flow to the shale layer, so that the pressure difference of the whole reservoir is smaller under the constant-rate condition. As for the shale-sandstone-shale combination, the pressure drop in the reservoir is almost the same even though the horizontal/vertical permeability ratio is not the same in each layer in Figure 12b. The reason may be that the permeability of the shale layer is too low, while the sandstone in the two thick shale layers, although playing a good role in connectivity, has a small impact on the production, and the reservoir production almost originates from the shale layer, so even if different horizontal/vertical permeability ratios are given, the trend of the pressure difference under constant production condition is almost the same. For the results shown in Figure 12c, the thick shale formation with low permeability plays a greater role in production, whereas sandstones mainly play a connecting role and have less influence on pressure change. Certainly, the accurate fit between the results obtained from the analytical method and the numerical models further illustrates that the analytical method is also suitable for the production prediction analysis of three typical lithology combinations considering different horizontal/vertical permeability ratios.

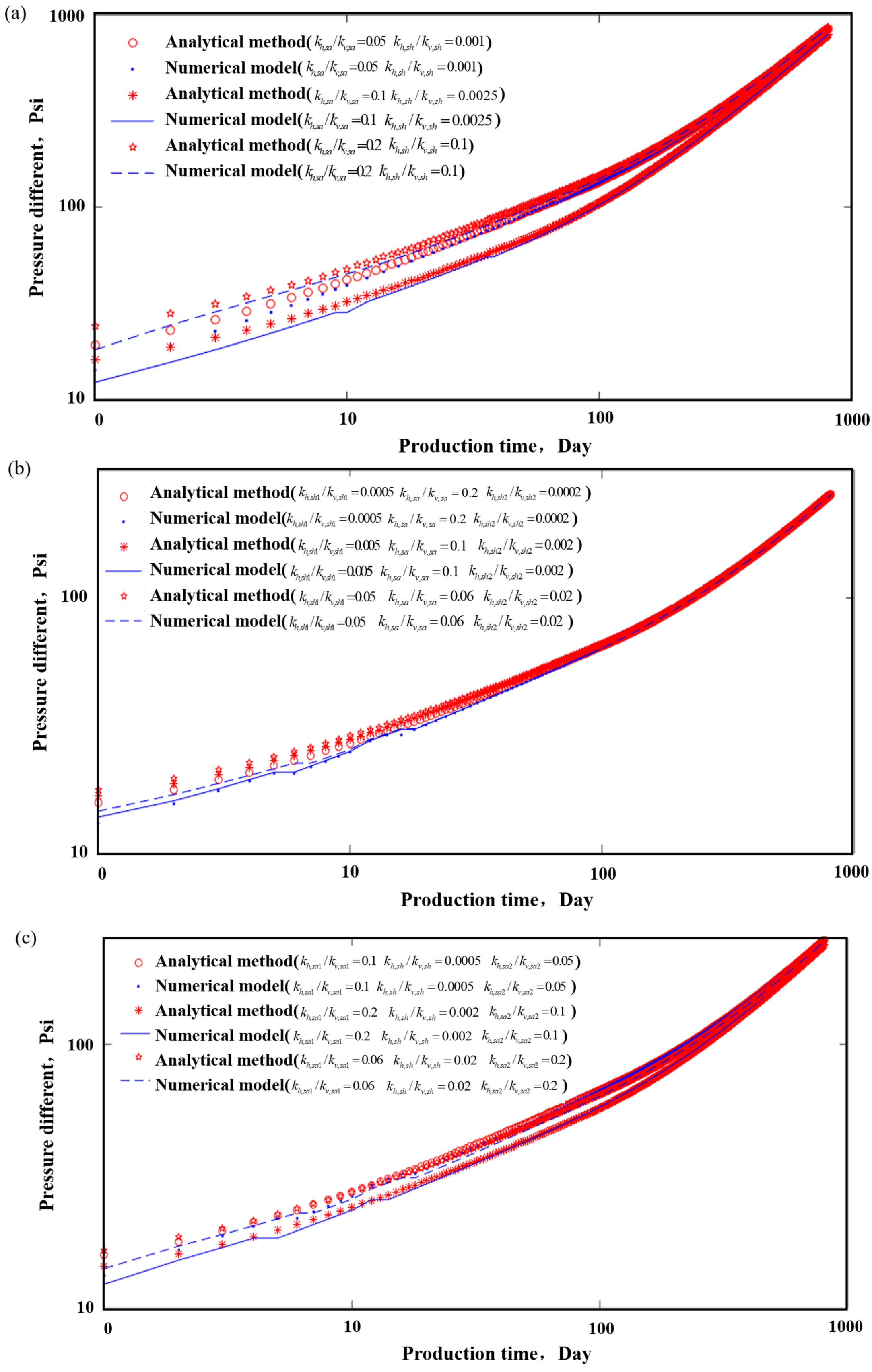


Figure 12. Comparison results for three typical lithological combination with different k_h/k_v considering inter-layer cross-flow: (**a**) sandstone-shale combination; (**b**) shale-sandstone-shale combination; (**c**) sandstone-shale-sandstone combination.

4.3. Analysis of the Thickness

In this case, we first set the different thickness in each layer for the three numerical models. Then, the results of the analytical method and numerical models were compared considering inter-layer crossflow under constant-rate condition, as demonstrated in Figure 13. The results of the three numerical models, corresponding to different thickness combinations of thin sandstone layers, are all in good agreement with the results of the analytical method. According to the results presented in Figure 13, it can be seen that the thickness of the sandstone layer has a relatively large effect on the reservoir pressure difference, due to the permeability and porosity of the sandstone is greater compared to the shale. The greater the thickness of the sandstone, the greater the gas that can be extracted from the reservoir with a smaller pressure drop. Meanwhile, we can find that the thicker the sandstone layer, the longer the duration of transient linear flow in the reservoir, which further suggests that the reservoir is producing for a longer period of time and the cumulative gas production is higher under constant production conditions. Furthermore, the perfect fit of the calculation results of the two methods under different thicknesses further indicates that the analytical calculation method has good stability.

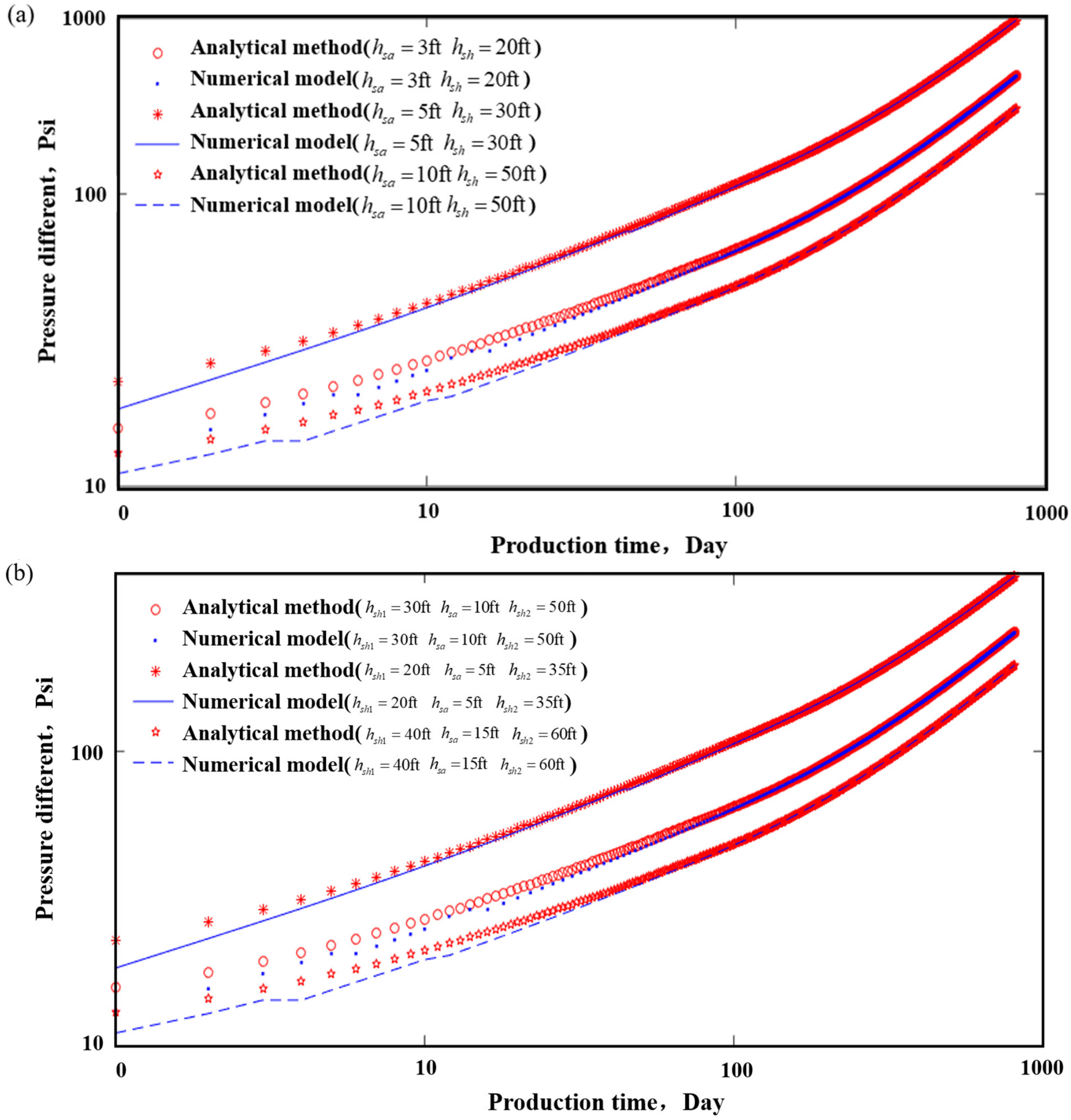


Figure 13. *Cont.*

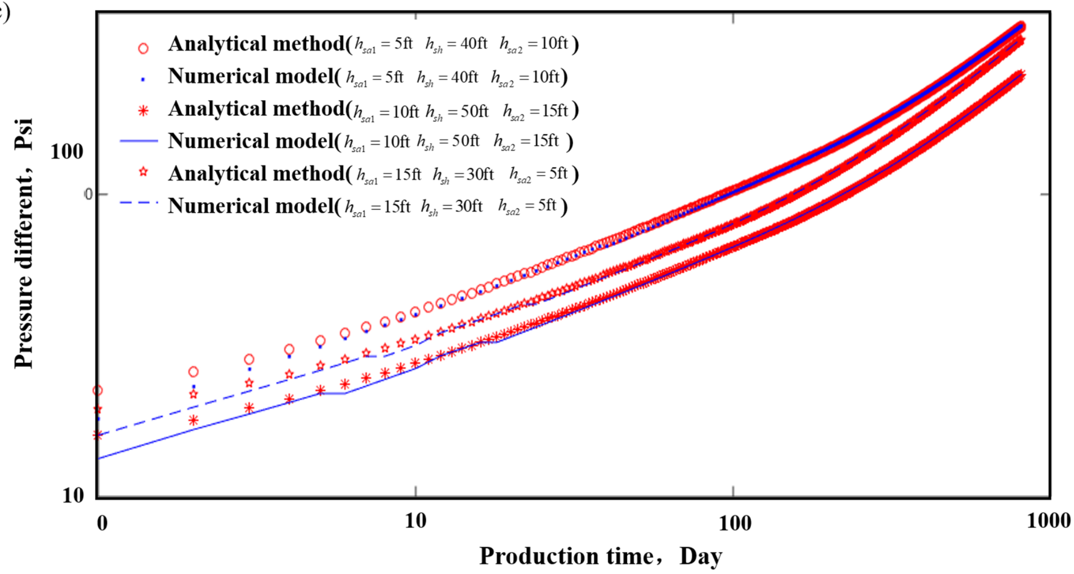
(c)


Figure 13. Comparison results for three typical lithological combination with different thickness considering inter-layer cross-flow. (**a**) sandstone-shale combination; (**b**) shale-sandstone-shale combination; (**c**) sandstone-shale-sandstone combination.

5. Conclusions

In this study, we proposed an analytical calculation method to perform pressure analysis of onshore and offshore shale gas reservoirs under constant-rate condition considering interlayer cross-flow and thin sandstone layer. Numerical models have been constructed based on three typical lithological combinations to validate the accuracy of the analytical method. The stability of the analytical method is also verified by conducting sensitivity analysis. The main conclusions can be drawn as follows:

(1) By comparing with the numerical models, the analytical calculation method proposed in this paper is accurate and stable. The analytical calculation method takes into account the desorption/adsorption properties of shale gas as well as the modified pseudo-pressure transformation function, which ensures more accurate reservoir pressure analysis.

(2) For three typical lithological combinations (sandstone-shale, shale-sandstone-shale and sandstone-shale-sandstone) at the actual site, sensitivity analysis show that for thick shale formation with porosity in the range of 0.05~0.1 and the thickness reaching 50 ft or above, the existence of thin sandstone layers can effectively increase gas reservoir production.

(3) When there is interlayer crossflow, the thin sandstone layers enriched by the shale formation plays a good role in promoting interlayer flow and the pressure wave propagates more widely in the reservoirs. As a result, the gas wells have longer stable production time and higher cumulative production under constant rate condition.

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