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Exponential H_∞ Output Control for Switching Fuzzy Systems via Event-Triggered Mechanism and Logarithmic Quantization

Jiaojiao Ren [1,*], Can Zhao [1], Jianying Xiao [1], Renfu Luo [1] and Nanrong He [2]

[1] School of Electronic Information and Electrical Engineering, Chengdu University, Chengdu 610106, China; zhaocan@cdu.edu.cn (C.Z.); xiaojianying@cdu.edu.cn (J.X.); luorenfu@stu.cdu.edu.cn (R.L.)
[2] School of Economics and Management, Beihang University, Beijing 100191, China; nanronghe@buaa.edu.cn
* Correspondence: renjiaojiao@cdu.edu.cn

Abstract: This paper investigates the problem of exponential H_∞ output control for switching fuzzy systems, considering both impulse and non-impulse scenarios. Unlike previous research, where the average dwell time (ADT: τ_a) and the upper bound of inter-event intervals (IEIs: T) satisfy the condition $\tau_a \geq \frac{\ln \mu + (\alpha+\beta)T}{\alpha} = \frac{\ln \mu + \beta T}{\alpha} + T$, implying that frequent switching is difficult to achieve, this paper demonstrates that by adopting the mode-dependent event-triggered mechanism (ETM) and a switching law, frequent switching is indeed achieved. Moreover, the question of deriving the normal L_2 norm constraint is solved through the ADT method, although only a weighted L_2 norm constraint was obtained previously. Additionally, by constructing a controller-mode-dependent Lyapunov function and adopting logarithmic quantizers, the sufficient criteria of exponential H_∞ output control problem are presented. The validity of established results is demonstrated by a given numerical simulation.

Keywords: switching fuzzy systems; exponential H_∞ output control; event-triggered mechanism; logarithmic quantization; average dwell time; switching law

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1. Introduction

Switching systems [1–3] consist of a family of subsystems and a switching rule to orchestrate them, constituting a type of hybrid system. These systems find wide-ranging applications, including in transportation systems, communication systems, and biochemical processes [4,5]. Furthermore, since the Takagi–Sugeno fuzzy model (T-S fuzzy model) [6,7] can offer a highly accurate approach to nonlinear systems, it is possible to describe switching nonlinear systems using the T-S fuzzy model and study them using modern linear theory. Therefore, in this paper, a switching fuzzy model is adopted to depict complex switching nonlinear systems.

In reality, physical systems inevitably encounter disturbances or faults. Consequently, H_∞ control [8,9] for switching systems has garnered significant attention in recent decades. Additionally, many evolutionary systems undergo rapid changes at specific moments. In mathematical model, such processes can be represented by state jumps, neglecting the durations of the rapid changes. These processes are called systems with impulse effects [8–10]. Impulse systems represent a very important type of hybrid system. In addition, due to bandwidth constraints in practical network systems, proper quantizers [11,12] and controllers [13–18] should be jointly designed to achieve a given control task and reduce unnecessary information transmission. Research on quantized switching fuzzy systems using ETM is currently limited. Therefore, in this paper, a logarithmic quantizer and a mode-dependent event-triggered mechanism are employed to improve control efficiency and ensure smooth signal transmission.

However, applying the event-triggered mechanism instead of the time-triggered mechanism to switching systems is highly challenging due to their switching characteristics, as discussed in [19]. Initially, the problem of observer-based ET control for switched linear systems was studied in [20], where subsystems and controllers were synchronous. Furthermore, in 2018, considering the phenomenon of asynchronism and incorporating the concepts of minimum dwell time (MDT) and maximal asynchronous interval, sufficient conditions were presented in [21]. Nevertheless, in [21], at most once, system switching was allowed during an IEI. To reduce the conservatism of existing results, the authors in [17,18] adopted the ADT method without restricting the MDT, enabling frequent switching within an IEI. However, the condition (9) in [17] and (15) in [18], $\tau_a \geq \frac{\ln\mu+(\alpha+\beta)T}{\alpha} = \frac{\ln\mu+\beta T}{\alpha} + T$, means that ADT must be no less than upper bound of IEIs. In other words, less than one system switching can happen in each interevent interval in average. Therefore, dealing with the phenomenon of asynchronism and achieving frequent switching within an IEI in a true sense are still open questions, which have inspired the current study.

On the other hand, the L_2 norm bound constraint plays an essential role in areas such as H_∞ control, L_2 analysis and dissipativity-based filtering for switching systems. Researchers have pursued this point and made significant achievements in this field [22–27]. In [26], the authors aimed to prove that the integration of $\Gamma(s)$ from 0 to ∞ is not less than zero. However, the condition $\Gamma(s) \geq 0$ cannot be guaranteed. Therefore, the inequality (36) cannot hold. For this reason, a weighted L_2 norm bound constraint was proposed in continuous-time systems [23], discrete-time systems [28], and stochastic networked control systems [29]. Nevertheless, taking [23] as an example, due to the presence of $e^{-\alpha t}$, the error state $e(t)$ may tend toward infinity. To address this limitation, the concept of MMDT was developed in [27,30,31]; the normal L_2 norm bound constraint could be derived from the two-direction inequality. Nevertheless, in [27,30,31], the constructed Lyapunov function is a system mode-dependent function, and the MMDT method can be employed directly. However, if the Lyapunov function is a controller-mode-dependent function, the left side of the MMDT method cannot be adopted directly. Therefore, in this paper, by applying the ADT method [32] and utilizing some mathematical techniques, proving the normal L_2 norm bound constraint inequality constitutes a primary contribution of our work.

Motivated by the above discussions, the exponential H_∞ output control problem of switching fuzzy systems with time delay will be studied both with and without impulses. The main contributions are as follows: (1) by introducing a new mode-dependent ETM and a switching law, frequent switching in an IEI is indeed achieved, removing the restriction $\tau_a \geq \frac{\ln\mu+(\alpha+\beta)T}{\alpha}$; (2) the normal L_2 norm bound constraint inequality is derived using the ADT method along with some mathematical techniques; (3) the mode-dependent event generator and the logarithmic quantizer are jointly designed to enhance control efficiency and minimize unnecessary data transmission; (4) in references [17,18], the continuity of $V(\xi(t))$ and $V(t)$ at $t = t_{q+1}$ was imprecisely addressed, particularly when t_{q+1} is a potential event-triggered instant. This issue is effectively addressed in our paper. Then, by constructing a controller-mode-dependent Lyapunov function, less conservative conditions are established for the exponential H_∞ output control of quantized switching fuzzy systems, whether with or without impulses. Finally, an example will be provided to confirm the validity of the proposed method.

This paper is organized as follows: In Section 2, the switching fuzzy model, ETM, and logarithmic quantizer are described and some preliminaries are introduced. The exponential H_∞ output control problem for constructed model is investigated for both impulse and non-impulse scenarios in Section 3. In Section 4, an example is provided to demonstrate the effectiveness of the proposed methods. Conclusions are drawn in Section 5.

2. Problem Statement and Preliminaries

We consider the following switching fuzzy model [1,33]:

Region Rule j: If $\varrho(t)$ is $N_j(\varrho(t))$, then
Local Plant Rule i: If $\varrho_1(t)$ is $M_{ji1}(\varrho_1(t))$ and $\ldots$ $\varrho_p(t)$ is $M_{jip}(\varrho_p(t))$, then

$$\begin{cases} \dot{x}(t) = A_{ji1}x(t) + B_{ji1}f(x(t)) + B_{ji2}f(x(t-\tau(t))) + C_{ji}u(t) + D_{ji1}w(t), t \in [t_k, t_{k+1}) \\ \Delta x = x(t_k) - x(t_k^-) = I_{ji}x(t_k^-), \quad t = t_k \\ y(t) = A_{ji2}x(t), \\ z(t) = A_{ji3}x(t), \\ x_0 = x(\theta) = \phi(\theta), \quad \theta \in [-\tau, 0], \end{cases} \tag{1}$$

where $i = 1,2,\ldots,r, j = 1,2,\ldots,s$; $N_j(\varrho(t)) = \begin{cases} 1 & \varrho(t) \in Region \quad j \\ 0 & otherwise \end{cases}$ $(j = 1,2,\ldots,s)$ is a classical set, which means that the system, over interval $[t_k, t_{k+1})$, only belongs to a certain Region j. That is, the switching subsystem j is active over $[t_k, t_{k+1})$. Regions j are the mutually disjoint regions. The union of all regions forms the universe of discourse; each Region j corresponds to each switching subsystem j. All switching subsystems constitute a switching system; region rules describe how the system switches, and local plant rules describe the fuzzy plant rule under each switching subsystem; $\varrho(t) = [\varrho_1(t), \varrho_2(t), \ldots, \varrho_p(t)]^T$; $\varrho_l(t)$ and $M_{jil}(\varrho_l(t)) \in [0,1]$ are premise variables and the membership functions, respectively; $x(t)$ represents the state of system; $y(t)$ represents the network measurement; $z(t)$ represents the estimated signal; $f(x(t))$ represents the nonlinear activation function; $w(t) \in \mathcal{L}_2[0,\infty]$ represents the noise input; $\phi(\theta)$ represents the initial function on $[-\tau, 0]$. $A_{ji1} \in \mathbb{R}^{n\times n}$, $A_{ji2} \in \mathbb{R}^{m\times n}$, $A_{ji3} \in \mathbb{R}^{q\times n}$, $B_{ji1} \in \mathbb{R}^{n\times n}$, $B_{ji2} \in \mathbb{R}^{n\times n}$, $C_{ji} \in \mathbb{R}^{n\times p}$, $D_{ji1} \in \mathbb{R}^{n\times q}$, and $I_{ji} \in \mathbb{R}^{n\times n}$ are constant matrices; $\tau(t)$ represents delay, the condition $0 \leq \tau(t) \leq \tau$ and $\dot{\tau}(t) \leq \tilde{\mu}$ hold, where τ and $\tilde{\mu}$ are constant scalars; the switching instant t_k satisfies $t_i < t_j, i < j \in \{0,1,2,\ldots,\infty\}$, $\lim_{k\to\infty} t_k = \infty$.

Remark 1. *Unlike existing ones [2,3,13,14], this paper focuses on studying switching fuzzy systems that combine the characteristics of T-S fuzzy systems and switching systems. With the aid of classical sets, the switching characteristics of the system are also expressed in a form similar to fuzzy rules, thereby achieving uniformity in expression format. Until now, switching fuzzy systems have not fully been investigated due to their complex nature.*

Assumption 1 ([30]). *The nonlinear activation function $f_i(\cdot)$ is continuous. For all $x_1, x_2 \in R^n$, there exist two constant matrices L^- and L^+ such that the following holds:*

$$[f(x_1) - f(x_2) - L^-(x_1 - x_2)]^T[f(x_1) - f(x_2) - L^+(x_1 - x_2)] \leq 0, \tag{2}$$

Remark 2. *To relax the restriction $\tilde{\mu} < 1$ in [22,23,31,32], a parameter ρ is introduced, which should satisfy $0 < \rho < min\{1, \frac{1}{\tilde{\mu}}\}$.*

The defuzzification is carried out by

$$\begin{cases} \dot{x}(t) = \sum_{j=1}^{s}\sum_{i=1}^{r} N_j(\varrho(t))h_{ji}(\varrho(t))(A_{ji1}x(t) + B_{ji1}f(x(t)) + B_{ji2}f(x(t-\tau(t))) \\ \qquad + C_{ji}u(t) + D_{ji1}w(t)), \quad t \neq t_k \\ \Delta x = x(t_k) - x(t_k^-) = \sum_{j=1}^{s}\sum_{i=1}^{r} N_j(\varrho(t))h_{ji}(\varrho(t))I_{ji}x(t_k^-), \quad k = 1,2,\ldots \\ y(t) = \sum_{j=1}^{s}\sum_{i=1}^{r} N_j(\varrho(t))h_{ji}(\varrho(t))A_{ji2}x(t), \\ z(t) = \sum_{j=1}^{s}\sum_{i=1}^{r} N_j(\varrho(t))h_{ji}(\varrho(t))A_{ji3}x(t), \\ x_0 = x(\theta) = \phi(\theta), \quad \theta \in [-\tau, 0], \end{cases} \tag{3}$$

let $\wp(t) = \prod_{l=1}^{p} M_{jil}(\varrho_l(t))$; then, $h_{ji}(\varrho(t)) = \frac{\wp(t)}{\sum_{i=1}^{r}\wp(t)}$. Here, we suppose $M_{jil}(\varrho_l(t)) \geq 0$; thus, $h_{ji}(\varrho(t)) \geq 0$ and $\sum_{i=1}^{r} h_{ji}(\varrho(t)) = 1$.

In this paper, we employ a combination of an event generator and the logarithmic quantizer $q_{ji}(.)$, effectively reducing the communication burden of the network. At each discrete time $t_k = kh$, both the system state and mode are sampled, where h represents the fixed sampling period. The sequence of event-triggered instants is denoted as $\{t_{s_k}\}_{k\geq 0}$ with $t_{s_0} = 0$. Here, a mode-dependent ET transmission scheme is proposed:

$$t_{s_{k+1}} = \min\{\min_{t_\varsigma > t_{s_k}} \{t_\varsigma | \varphi(e(t_\varsigma), x(t_{s_k})) \geq 0\}, t_{s_k} + Hh\}, k \geq 0, \tag{4}$$

where $t_\varsigma(t_\varsigma > t_{s_k})$ is a new sampling instant, $e(t_\varsigma) = x(t_\varsigma) - x(t_{s_k})$ denotes the sampled-data error, $\varphi(e(t_\varsigma), x(t_{s_k})) = \sum_{j'=1}^{s}\sum_{i'=1}^{r} N_{j'}(\varrho(t_\varsigma))h_{j'i'}(\varrho(t_\varsigma))(e^T(t_\varsigma)\Phi_{j'i'}e(t_\varsigma) - \vartheta_{j'i'}x^T(t_{s_k})$ $\Phi_{j'i'}x(t_{s_k}))$, and H is a positive constant which limits the upper bound of IEIs.

Remark 3. *To enhance the efficiency of data transmission, this paper adopts an event-triggered mechanism instead of a time-triggered mechanism [19]. Furthermore, by combining the proposed ETM and a switching law, frequency system mode switching during an IEI will be achieved without constraining the MDT of each subsystem. However, in the existing results [17,18], frequent switching cannot be realized. The condition (9) in [17] and (15) in [18], represented as* $\tau_a \geq \frac{\ln\mu + (\alpha+\beta)T}{\alpha}$*, implies that the ADT must be no less than the upper bound of IEIs. Therefore, these conditions do not allow for frequent switching to occur.*

Then, a fuzzy output feedback controller with quantized input and ETM is considered. The flowchart of exponential H_∞ output control for quantized switching fuzzy systems is shown in Figure 1.

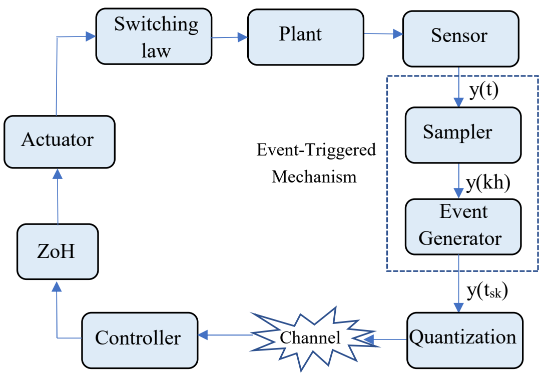

Figure 1. The structure of H_∞ control for quantized switching fuzzy systems with ETM.

Controller Region Rule j': If $\varrho(t_{s_k})$ is $N_{j'}(\varrho(t_{s_k}))$, then
Controller Local Plant Rule i': If $\varrho_1(t_{s_k})$ is $M_{j'i'1}(\varrho_1(t_{s_k}))$ and $\ldots \varrho_p(t_{s_k})$ is $M_{j'i'p}(\varrho_p(t_{s_k}))$, then

$$u(t) = \sum_{j'=1}^{s}\sum_{i'=1}^{r} N_{j'}(\varrho(t_{s_k}))h_{j'i'}(\varrho(t_{s_k}))K_{j'i'}q_{j'i'}(y(t_{s_k})), t \in [t_{s_k}, t_{s_{k+1}}), \tag{5}$$

where $K_{j'i'}$ and $q_{j'i'}(.)$ are the controller gain and logarithmic quantizer, respectively. The latter $q_{j'i'}(.)$ satisfies

$$q_{j'i'}(y(t_{s_k})) = [q^1_{j'i'}(y_1(t_{s_k})), q^2_{j'i'}(y_2(t_{s_k})), \ldots, q^m_{j'i'}(y_m(t_{s_k}))]^T, i' = 1,2,\ldots,r, j' = 1,2,\ldots,s,$$

and

$$q^\iota_{j'i'}(y_\iota(t_{s_k})) = -q^\iota_{j'i'}(-y_\iota(t_{s_k})), \quad \iota = 1,2,\ldots,m.$$

To phrase the quantized level set of $q^{\iota}_{j'i'}(.)$, the Q_ι is defined as follows:

$$
\begin{aligned}
Q_\iota =& \{\pm Q_d^{(j',i',\iota)} | Q_d^{(j',i',\iota)} = (\rho^{\iota}_{j',i'})^d Q_0^{(j',i',\iota)}, d = \pm 1, \dots\} \\
& \cup \{\pm Q_0^{(j',i',\iota)}\} \cup \{0\}, \rho^{\iota}_{j',i'} \in (0,1), Q_0^{(j',i',\iota)} > 0
\end{aligned}
$$

where $\rho^{\iota}_{j',i'}$ and $Q_0^{(j',i',\iota)}$ denote the quantizer density and the initial quantization values of $q^{\iota}_{j'i'}(.)$, respectively. Then, the quantizer $q^{\iota}_{j'i'}(.)$ is given as

$$
q^{\iota}_{j'i'}(y_\iota(t_{s_k})) = \begin{cases} Q_d^{(j',i',\iota)}, & y_\iota(t_{s_k}) \in (\frac{Q_d^{(j',i',\iota)}}{1+\sigma^{\iota}_{j'i'}}, \frac{Q_d^{(j',i',\iota)}}{1-\sigma^{\iota}_{j'i'}}] \\ 0, & y_\iota(t_{s_k}) = 0 \\ -q^{\iota}_{j'i'}(-y_\iota(t_{s_k})), & y_\iota(t_{s_k}) < 0 \end{cases} \tag{6}
$$

where $\sigma^{\iota}_{j'i'} = (1 - \rho^{\iota}_{j',i'})/(1 + \rho^{\iota}_{j',i'})$, which means $0 < \sigma^{\iota}_{j'i'} < 1$.

From Equation (6), in situation $y_\iota(t_{s_k}) \in (\frac{Q_d^{(j',i',\iota)}}{1+\sigma^{\iota}_{j'i'}}, \frac{Q_d^{(j',i',\iota)}}{1-\sigma^{\iota}_{j'i'}}]$, due to $q^{\iota}_{j'i'}(y_\iota(t_{s_k})) = Q_d^{(j',i',\iota)}$, one has

$$
\frac{q^{\iota}_{j'i'}(y_\iota(t_{s_k}))}{1+\sigma^{\iota}_{j'i'}} \le y_\iota(t_{s_k}) \le \frac{q^{\iota}_{j'i'}(y_\iota(t_{s_k}))}{1-\sigma^{\iota}_{j'i'}},
$$

then

$$
q^{\iota}_{j'i'}(y_\iota(t_{s_k})) \le (1+\sigma^{\iota}_{j'i'}) y_\iota(t_{s_k}), \quad q^{\iota}_{j'i'}(y_\iota(t_{s_k})) \ge (1-\sigma^{\iota}_{j'i'}) y_\iota(t_{s_k}).
$$

Thus, Equation (6) is also rephrased by a sector expression as

$$
(1-\sigma^{\iota}_{j'i'}) y_\iota^2(t_{s_k}) \le q^{\iota}_{j'i'}(y_\iota(t_{s_k})) y_\iota(t_{s_k}) < (1+\sigma^{\iota}_{j'i'}) y_\iota^2(t_{s_k}). \tag{7}
$$

Obviously, the inequality (7) also holds for $y_\iota(t_{s_k}) = 0$ and $y_\iota(t_{s_k}) < 0$.
According to the inequality (7), the following inequalities hold:

$$
\begin{aligned}
& (q^{\iota}_{j'i'}(y_\iota(t_{s_k})) - (1-\sigma^{\iota}_{j'i'}) y_\iota(t_{s_k})) y_\iota(t_{s_k}) \ge 0, \\
& (q^{\iota}_{j'i'}(y_\iota(t_{s_k})) - (1+\sigma^{\iota}_{j'i'}) y_\iota(t_{s_k})) y_\iota(t_{s_k}) \\
& = (q^{\iota}_{j'i'}(y_\iota(t_{s_k})) - (1-\sigma^{\iota}_{j'i'}) y_\iota(t_{s_k}) - 2\sigma^{\iota}_{j'i'} y_\iota(t_{s_k})) y_\iota(t_{s_k}) < 0
\end{aligned}
$$

Let $\Theta_{j'i'} = diag\{\sigma^1_{j'i'}, \sigma^2_{j'i'}, \dots, \sigma^m_{j'i'}\}$; (7) could be rewritten as

$$
q_{j'i'}(y(t_{s_k})) = G_{j'i'} y(t_{s_k}) + q^g_{j'i'}(y(t_{s_k})) \tag{8}
$$

and

$$
(q^g_{j'i'}(y(t_{s_k})))^T (q^g_{j'i'}(y(t_{s_k})) - 2\Theta_{j'i'} y(t_{s_k})) \le 0, \tag{9}
$$

where $G_{j'i'} = I - \Theta_{j'i'}$, I is the identity matrix.

By combining (3), (5), and (8), one has

$$\begin{cases} \dot{x}(t) = \sum_{j=1}^{s}\sum_{i=1}^{r}\sum_{j'=1}^{s}\sum_{i'=1}^{r}\sum_{i''=1}^{r} N_j(\varrho(t))h_{ji}(\varrho(t))N_{j'}(\varrho(t_{s_k}))h_{j'i'}(\varrho(t_{s_k}))h_{j'i''}(\varrho(t_{s_k}))\times \\ \qquad ((A_{ji1}+C_{ji}K_{j'i'}G_{j'i'}A_{j'i''2})x(t)+B_{ji1}f(x(t))+B_{ji2}f(x(t-\tau(t))) \\ \qquad -C_{ji}K_{j'i'}G_{j'i'}A_{j'i''2}e(t)+C_{ji}K_{j'i'}q^{g}_{j'i'}(y(t_{s_k}))+D_{ji1}w(t)), \quad t\neq t_k \\ \Delta x = x(t_k)-x(t_k^-) = \sum_{j=1}^{s}\sum_{i=1}^{r} N_j(\varrho(t))h_{ji}(\varrho(t))I_{ji}x(t_k^-), k=1,2,\dots \\ y(t) = \sum_{j=1}^{s}\sum_{i=1}^{r} N_j(\varrho(t))h_{ji}(\varrho(t))A_{ji2}x(t), \\ z(t) = \sum_{j=1}^{s} N_j(\varrho(t))h_{ji}(\varrho(t))A_{ji3}x(t), \\ x_0 = x(\theta)=\phi(\theta), \quad \theta\in[-\tau,0], \end{cases} \tag{10}$$

where, if we assume $t \in [t_k, t_{k+1})$, $t_{s_k} < t_k$, $N_j(\varrho(t)) = 1$, $N_{j_1}(\varrho(t)) = 0, (j_1 \neq j)$, $N_{j'}(\varrho(t_{s_k})) = 1$, and $N_{j_2}(\varrho(t_{s_k})) = 0, (j_2 \neq j')$, it means that Region j and Region j' are activated in $[t_k, t_{k+1})$ and at the moment t_{s_k}, respectively. In other words, the system mode and controller mode are j and j' over the interval $[t_k, t_{k+1})$ and $[t_{s_k}, t_{s_{k+1}})$, respectively.

When $t \in [t_k, t_{k+1})$, the following two cases will be discussed:

Case 1: if there is no event-triggered instant in this interval, i.e., $t_{s_k} < t_k < t_{k+1} \leq t_{s_{k+1}}$,

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r}\sum_{i'=1}^{r}\sum_{i''=1}^{r} h_{ji}(\varrho(t))h_{j'i'}(\varrho(t_{s_k}))h_{j'i''}(\varrho(t_{s_k}))((A_{ji1}+C_{ji}K_{j'i'}G_{j'i'}A_{j'i''2})x(t) \\ \qquad +B_{ji1}f(x(t))+B_{ji2}f(x(t-\tau(t)))-C_{ji}K_{j'i'}G_{j'i'}A_{j'i''2}e(t) \\ \qquad +C_{ji}K_{j'i'}q^{g}_{j'i'}(y(t_{s_k}))+D_{ji1}w(t)), \quad t\neq t_k, \\ \Delta x = x(t_k)-x(t_k^-) = \sum_{i=1}^{r} h_{ji}(\varrho(t))I_{ji}x(t_k^-), k=1,2,\dots \end{cases} \tag{11}$$

where $e(t) = x(t) - x(t_{s_k})$. In this case, because the interval $[t_k, t_{k+1})$ is a subset of $[t_{s_k}, t_{s_{k+1}})$, the system mode and controller mode are j and j' over the interval $[t_k, t_{k+1})$, respectively. The terms $\sum_{j=1}^{s} v_j(\varrho(t))$ and $\sum_{j'=1}^{s} v'_j(\varrho(t_{s_k}))$ can be removed by specifying the subscripts of coefficient matrices as j and j', respectively.

Case 2: if there are $n (n \in N^+)$ event-triggered instants in this interval, i.e., $t_{s_k} < t_k \leq t_{s_{k+1}} < t_{s_{k+2}} < \ldots < t_{s_{k+n}} < t_{k+1} \leq t_{s_{k+n+1}}$,

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r}\sum_{i'=1}^{r}\sum_{i''=1}^{r} h_{ji}(\varrho(t))h_{j'i'}(\varrho(t_{s_k}))h_{j'i''}(\varrho(t_{s_k}))((A_{ji1}+C_{ji}K_{j'i'}G_{j'i'}A_{j'i''2})x(t) \\ \qquad +B_{ji1}f(x(t))+B_{ji2}f(x(t-\tau(t)))-C_{ji}K_{j'i'}G_{j'i'}A_{j'i''2}e(t) \\ \qquad +C_{ji}K_{j'i'}q^{g}_{j'i'}(y(t_{s_k}))+D_{ji1}w(t)), \quad t\in[t_k,t_{s_{k+1}}), \\ \dot{x}(t) = \sum_{i=1}^{r}\sum_{i'=1}^{r}\sum_{i''=1}^{r} h_{ji}(\varrho(t))h_{ji'}(\varrho(t_{s_k}))h_{ji''}(\varrho(t_{s_k}))((A_{ji1}+C_{ji}K_{ji'}G_{ji'}A_{ji''2})x(t) \\ \qquad +B_{ji1}f(x(t))+B_{ji2}f(x(t-\tau(t)))-C_{ji}K_{ji'}G_{ji'}A_{ji''2}e(t) \\ \qquad +C_{ji}K_{ji'}q^{g}_{ji'}(y(t))+D_{ji1}w(t)), \quad t\in[t_{s_{k+1}},t_{k+1}), \\ \Delta x = x(t_k)-x(t_k^-) = \sum_{i=1}^{r} h_{ji}(\varrho(t))I_{ji}x(t_k^-), \quad k=1,2,\dots \end{cases} \tag{12}$$

where $e(t) = \begin{cases} x(t) - x(t_{s_k}), t \in [t_k, t_{s_{k+1}}), \\ x(t) - x(t_{s_{k+1}}), t \in [t_{s_{k+1}}, t_{s_{k+2}}), \\ \dots \\ x(t) - x(t_{s_{k+n}}), t \in [t_{s_{k+n}}, t_{k+1}), \end{cases}$ and $q^g_{ji'}(y(t)) = \begin{cases} q^g_{ji'}(y(t_{s_{k+1}}))), t \in [t_{s_{k+1}}, t_{s_{k+2}}), \\ q^g_{ji'}(y(t_{s_{k+2}}))), t \in [t_{s_{k+2}}, t_{s_{k+3}}), \\ \dots \\ q^g_{ji'}(y(t_{s_{k+n}}))), t \in [t_{s_{k+n}}, t_{k+1}), \end{cases}$

In this case, the interval $[t_k, t_{k+1})$ should be divided into two subintervals: $[t_k, t_{s_k})$ and $[t_{s_{k+1}}, t_{k+1})$, for separate discussion. The system mode and controller mode are j and j' over the interval $[t_k, t_{s_k})$, respectively. The system mode and controller mode are both j over the interval $[t_{s_{k+1}}, t_{k+1})$, respectively. The terms $\sum_{j=1}^{s} N_j(\varrho(t))$ and $\sum_{j'=1}^{s} N_{j'}(\varrho(t_{s_k}))$ can also be removed by specifying the subscripts of coefficient matrices as their corresponding values. In $[t_k, t_{s_k})$, the subscripts of coefficient matrices should be j and j', respectively. In $[t_{s_{k+1}}, t_{k+1})$, the subscripts of coefficient matrices should all be j.

Remark 4. *To the authors' knowledge, two types of quantizers have been studied for quantized feedback control [11,12,34,35]. The first type is memoryless or static quantizers, such as logarithmic quantizers [11,12], characterized by an infinite number of quantization levels. The second type is dynamic quantizers [34,35], which feature a finite number of quantization levels. However, dynamic quantizers may be impractial for the following reasons: (1) the main focus of existing papers is stabilization rather than performance control, leading to typically poor transient responses; (2) when practical communication channels encounter noise, disturbances, and other factors, the results may not be valid. Therefore, in this paper, logarithmic quantizers have been adopted to study exponential H_∞ output control for switching fuzzy systems.*

Remark 5. *In this paper, the system is considered in two cases based on the number of event-triggered instants. However, in [17,18], the second case is inaccurately described. This inaccuracy can be found in line 37 on page 3121 of [17] and line 9 on page 254 of [18]. The reason for this discrepancy is that the interval $[t_q, t_{q+1})$ is semi-closed, meaning that the inequalities $s_{k+m} \leq t_{q+1} < s_{k+m+1}$ and $b_{r+m}h + \tau_{b_{r+m}} \leq t_{q+1} < b_{r+m+1}h + \tau_{b_{r+m+1}}$ indicate that the instants s_{k+m} and $b_{r+m}h + \tau_{b_{r+m}}$ cannot be triggered within $[t_q, t_{q+1})$. Therefore, in references [17,18], there are only $m - 1(\in N^+)$ rather than $m(\in N^+)$ triggered instants in $[t_q, t_{q+1})$.*

3. Main Results

In this section, firstly, by proposing the ETM and the logarithmic quantizer, employing a switching law, and utilizing the ADT method, sufficient conditions are provided for the exponential H_∞ output control problem of the system (10) without impulses. Secondly, the system (10) with impulses is considered. The non-weighted L_2 norm bound constraint inequality is derived. Moreover, frequent switching within an IEI is truly achieved.

Switching Law 1 ([27]). *For given scalars $\beta > 0$, $\alpha > 0$, and $0 < \alpha_* < \alpha$, $c_* > 0$ and a sequence of switching instants $t_0 < t_1 < t_2 < \ldots < t_k < \ldots$, $\lim_{k\to\infty} t_k = \infty$, such that the inequality $-\alpha T_\downarrow(t,\tau) + \beta T_\uparrow(t,\tau) \leq c_* - \alpha_*(t-\tau)$ holds, for any $t \geq \tau \geq t_0$, where $T_\uparrow(t_{k+1}, t_k)$ and $T_\downarrow(t_{k+1}, t_k)$ are the total active time of the system mode and the controller mode, respectively, which are asynchronous and synchronous over $[t_k, t_{k+1})$.*

Theorem 1. *Given scalars $\alpha > 0$, $\beta > 0$, $0 < \rho < min\{1, \frac{1}{\tilde{\mu}}\}$, τ, $\tilde{\mu}$, γ, and $\mu \geq 1$ and the matrices L^- and L^+, under switching law 3.1, if there exist matrices $P_j > 0$, $Q_j > 0$, $R_j > 0$, $Z_j > 0$, $F_j > 0$, N, $\tilde{G}_{jj'ii'}$, and $\tilde{G}_{jii'}$ and scalars $\lambda_{wj} > 0$ $(w = 1,2,3,4)$, for any $i, i', i'' = 1,2,\ldots,r$ and $j, j' = 1,2,\ldots,s$ such that*

$$P_j \le \mu P_{j'}, Q_j \le \mu Q_{j'}, R_j \le \mu R_{j'}, Z_j \le \mu Z_{j'}, \tag{13}$$

$$F_j \le \mu F_{j'}, \quad j \ne j' \tag{14}$$

$$\Omega_{jj'}^{ii'i''} < 0, \quad j \ne j' \tag{15}$$

$$\Omega_{jj}^{ii'i''} < 0, \tag{16}$$

$$\alpha_* > \frac{\ln \mu}{T_a}. \tag{17}$$

Then, the system (10), without impulses, is globally exponentially stable (GES) with an H_∞ performance index (HPI) γ where

$$
\begin{aligned}
&(\Omega_{jj'}^{ii'i''})_{1,1} = -\beta P_{j'} + Q_{j'} - e^{-\alpha\tau} R_{j'} + A_{ji3}^T A_{ji3} + \vartheta_{j'i'} \Phi_{j'i'} - \frac{L^{-T}L^{+} + L^{+T}L^{-}}{2}\lambda_{1j'},\\
&(\Omega_{jj'}^{ii'i''})_{1,4} = e^{-\alpha\tau} R_{j'},\ (\Omega_{jj'}^{ii'i''})_{1,5} = A_{ji1}^T N^T + A_{j'i''2}^T G_{j'i'}^T \tilde{G}_{jj'ii'}^T + P_{j'}^T,\\
&(\Omega_{jj'}^{ii'i''})_{1,6} = \frac{L^{-T} + L^{+T}}{2}\lambda_{1j'},\ (\Omega_{jj'}^{ii'i''})_{1,10} = A_{ji2}^T \Theta_{j'i'}^T,\ (\Omega_{jj'}^{ii'i''})_{1,11} = -\vartheta_{j'i'} \Phi_{j'i'},\\
&(\Omega_{jj'}^{ii'i''})_{2,2} = -\frac{L^{-T}L^{+} + L^{+T}L^{-}}{2}\lambda_{2j'} - (1-\rho\mu)e^{-\alpha\rho\tau} Q_{j'},\ (\Omega_{jj'}^{ii'i''})_{2,7} = \frac{L^{-T} + L^{+T}}{2}\lambda_{2j'},\\
&(\Omega_{jj'}^{ii'i''})_{3,3} = -\frac{L^{-T}L^{+} + L^{+T}L^{-}}{2}\lambda_{3j'},\ (\Omega_{jj'}^{ii'i''})_{3,8} = \frac{L^{-T} + L^{+T}}{2}\lambda_{3j'},\ (\Omega_{jj'}^{ii'i''})_{4,9} = -\lambda_{4j'} I,\\
&(\Omega_{jj'}^{ii'i''})_{4,4} = -\frac{L^{-T}L^{+} + L^{+T}L^{-}}{2}\lambda_{4j'} - e^{-\alpha\tau} R_{j'},\ (\Omega_{jj'}^{ii'i''})_{5,5} = -N - N^T + \tau^2 R_{j'},\\
&(\Omega_{jj'}^{ii'i''})_{5,6} = N B_{ji1},\ (\Omega_{jj'}^{ii'i''})_{5,8} = N B_{ji2},\ (\Omega_{jj'}^{ii'i''})_{5,10} = N C_{ji} K_{j'i'},\\
&(\Omega_{jj'}^{ii'i''})_{5,11} = \tilde{G}_{jj'ii'} G_{j'i'} A_{j'i''2},\ (\Omega_{jj'}^{ii'i''})_{5,12} = N D_{ji1},\ (\Omega_{jj'}^{ii'i''})_{6,6} = -\lambda_{1j'} I + Z_{j'} + F_{j'},\\
&(\Omega_{jj'}^{ii'i''})_{7,7} = -\lambda_{2j'} I - (1-\rho\mu)e^{-\alpha\rho\mu} Z_{j'},\ (\Omega_{jj'}^{ii'i''})_{8,8} = -\lambda_{3j'} I,\\
&(\Omega_{jj'}^{ii'i''})_{9,9} = -\lambda_{4j'} I - e^{-\alpha\tau} F_{j'},\ (\Omega_{jj'}^{ii'i''})_{10,10} = -I,\ (\Omega_{jj'}^{ii'i''})_{10,11} = -\Theta_{j'i'} A_{ji2},\\
&(\Omega_{jj'}^{ii'i''})_{11,11} = \vartheta_{j'i'} \Phi_{j'i'} - \Phi_{j'i'}, (\Omega_{jj'}^{ii'i''})_{12,12} = -\tilde{\gamma}^2 I,\ \tilde{\gamma} = \gamma\sqrt{\frac{(\alpha_* - \ln\mu/T_a)}{\eta\alpha e^{c_*}}},
\end{aligned}
$$

and

$$
\begin{aligned}
&(\Omega_{jj}^{ii'i''})_{1,1} = \alpha P_j + Q_j - e^{-\alpha\tau} R_j + A_{ji3}^T A_{ji3} + \vartheta_{ji'} \Phi_{ji'} - \frac{L^{-T}L^{+} + L^{+T}L^{-}}{2}\lambda_{1j},\\
&(\Omega_{jj}^{ii'i''})_{1,4} = e^{-\alpha\tau} R_j,\ (\Omega_{jj}^{ii'i''})_{1,5} = A_{ji1}^T N^T + A_{ji''2}^T G_{ji'}^T \tilde{G}_{jii'}^T + P_j^T,\\
&(\Omega_{jj}^{ii'i''})_{1,6} = \frac{L^{-T} + L^{+T}}{2}\lambda_{1j},\ (\Omega_{jj}^{ii'i''})_{1,10} = A_{ji2}^T \Theta_{ji'}^T,\ (\Omega_{jj}^{ii'i''})_{1,11} = -\vartheta_{ji'} \Phi_{ji'},\\
&(\Omega_{jj}^{ii'i''})_{2,2} = -\frac{L^{-T}L^{+} + L^{+T}L^{-}}{2}\lambda_{2j} - (1-\rho\mu)e^{-\alpha\rho\tau} Q_j,\ (\Omega_{jj}^{ii'i''})_{2,7} = \frac{L^{-T} + L^{+T}}{2}\lambda_{2j},\\
&(\Omega_{jj}^{ii'i''})_{3,3} = -\frac{L^{-T}L^{+} + L^{+T}L^{-}}{2}\lambda_{3j},\ (\Omega_{jj}^{ii'i''})_{3,8} = \frac{L^{-T} + L^{+T}}{2}\lambda_{3j},\ (\Omega_{jj}^{ii'i''})_{4,9} = -\lambda_{4j} I,\\
&(\Omega_{jj}^{ii'i''})_{4,4} = -\frac{L^{-T}L^{+} + L^{+T}L^{-}}{2}\lambda_{4j} - e^{-\alpha\tau} R_j,\ (\Omega_{jj}^{ii'i''})_{5,5} = -N - N^T + \tau^2 R_j,
\end{aligned}
$$

$$(\Omega_{jj}^{ii'i''})_{5,6} = NB_{ji1},\ (\Omega_{jj}^{ii'i''})_{5,8} = NB_{ji2},\ (\Omega_{jj}^{ii'i''})_{5,10} = NC_{ji}K_{ji'},$$
$$(\Omega_{jj}^{ii'i''})_{5,11} = \tilde{G}_{jii'}G_{ji'}A_{ji''2},\ (\Omega_{jj}^{ii'i''})_{5,12} = ND_{ji1},\ (\Omega_{jj}^{ii'i''})_{6,6} = -\lambda_{1j}I + Z_j + F_j,$$
$$(\Omega_{jj}^{ii'i''})_{7,7} = -\lambda_{2j}I - (1-\rho\mu)e^{-\alpha\rho\mu}Z_j,\ (\Omega_{jj}^{ii'i''})_{8,8} = -\lambda_{3j}I,$$
$$(\Omega_{jj}^{ii'i''})_{9,9} = -\lambda_{4j}I - e^{-\alpha\tau}F_j,\ (\Omega_{jj}^{ii'i''})_{10,10} = -I,\ (\Omega_{jj}^{ii'i''})_{10,11} = -\Theta_{ji'}A_{ji2},$$
$$(\Omega_{jj}^{ii'i''})_{11,11} = \vartheta_{ji'}\Phi_{ji'} - \Phi_{ji'},\ (\Omega_{jj}^{ii'i''})_{12,12} = -\tilde{\gamma}^2 I,\ \tilde{\gamma} = \gamma\sqrt{\frac{(\alpha_* - \ln\mu/T_a)}{\eta\alpha e^{c_*}}}.$$

with other elements $(\Omega_{jj'}^{ii'i''})_{a,b} = 0$ *and* $(\Omega_{jj}^{ii'i''})_{a,b} = 0$.

Moreover, the controller gain is given by

$$K_{j'i'} = C_{ji}^{-1}N^{-1}\tilde{G}_{jj'ii'} \quad \text{and} \quad K_{ji'} = C_{ji}^{-1}N^{-1}\tilde{G}_{jii'}.$$

Proof. According to the Definition 2.1 in reference [30], firstly, when $w(t) \neq 0, w(t) \in \mathcal{L}_2[0,\infty)$ and $\phi(\theta) = 0$, we will prove that the inequality $\|z\|_2 < \gamma\|w\|_2$ holds. We consider

$$V(t, x_t, v(t)) = \sum_{i=1}^{3} V_i(t, x_t, v(t)), \tag{18}$$

with

$$V_1(t, x_t, v(t)) = \sum_{j=1}^{s} v_j(\varrho(t))x^T(t)P_{v(t_{s_k})}x(t),$$

$$\begin{aligned}
V_2(t, x_t, v(t)) = &\sum_{j=1}^{s} v_j(\varrho(t))(\int_{t-\rho\tau(t)}^{t} e^{-\alpha(t-s)}x^T(s)Q_{v(t_{s_k})}x(s)ds \\
&+ \tau\int_{t-\tau}^{t} e^{-\alpha(t-s)}(s - t + \tau)\dot{x}^T(s)R_{v(t_{s_k})}\dot{x}(s)ds), \\
V_3(t, x_t, v(t)) = &\sum_{j=1}^{s} v_j(\varrho(t))(\int_{t-\rho\tau(t)}^{t} e^{-\alpha(t-s)}f^T(x(s))Z_{v(t_{s_k})}f(x(s))ds \\
&+ \int_{t-\tau}^{t} e^{-\alpha(t-s)}f^T(x(s))F_{v(t_{s_k})}f(x(s))ds),
\end{aligned}$$

where, $v(t)$ is the switching signal that determines the system mode or controller mode at instant t. The constructed Lyapunov function is dependent on both the controller mode and system mode. The modes of Lyapunov matrices $P_{v(t_{s_k})}$, $Q_{v(t_{s_k})}$, $R_{v(t_{s_k})}$, $Z_{v(t_{s_k})}$, and $F_{v(t_{s_k})}$ are determined by the controller, specifically by the ETM. The system state $x(t)$ depends on system mode $v(t)$; $v(t) = j, t \in [t_k, t_{k+1})$ means that Region j is activated in $[t_k, t_{k+1})$, and $N_j(\varrho(t)) = 1$ and $N_{j_1}(\varrho(t)) = 0$ $(j_1 \neq j)$. In the following, we simplified $V(t, x_t, v(t))$ and $V_i(t, x_t, v(t))$ as $V(t)$ and $V_i(t)$, respectively.

Case 1: if no instant is triggered in $[t_k, t_{k+1})$, then the controller mode is $v(t_{s_k}) = j'$, the system mode is $v(t) = j$, and we have

$$\dot{V}_1(t) = \dot{x}^T(t)P_{j'}x(t) + x^T(t)P_{j'}\dot{x}(t), \tag{19}$$
$$\begin{aligned}
\dot{V}_2(t) = &x^T(t)Q_{j'}x(t) + \tau^2\dot{x}^T(t)R_{j'}\dot{x}(t) - e^{-\alpha\rho\tau(t)}(1 - \rho\dot{\tau}(t))x^T(t - \rho\tau(t))Q_{j'}x(t - \rho\tau(t)) \\
&- \tau\int_{t-\tau}^{t} e^{-\alpha(t-s)}\dot{x}^T(s)R_{j'}\dot{x}(s)ds - \alpha V_2(t, x_t, j).
\end{aligned} \tag{20}$$

Due to $0 < \rho < min\{1, \frac{1}{\tilde{\mu}}\}$, one has

$$\dot{V}_2(t) \leq x^T(t)Q_{j'}x(t) + \tau^2\dot{x}^T(t)R_{j'}\dot{x}(t) - e^{-\alpha\rho\tau}(1-\rho\tilde{\mu})x^T(t-\rho\tau(t))Q_{j'}x(t-\rho\tau(t)) - \tau e^{-\alpha\tau}\int_{t-\tau}^{t}\dot{x}^T(s)R_{j'}\dot{x}(s)ds - \alpha V_2(t, x_t, j). \tag{21}$$

Furthermore, by adopting Jensen's inequality, one has

$$-\tau e^{-\alpha\tau}\int_{t-\tau}^{t}\dot{x}^T(s)R_{j'}\dot{x}(s)ds \leq -e^{-\alpha\tau}\int_{t-\tau}^{t}\dot{x}^T(s)dsR_{j'}\int_{t-\tau}^{t}\dot{x}(s)ds = -e^{-\alpha\tau}(x(t)-x(t-\tau))^TR_{j'}(x(t)-x(t-\tau)). \tag{22}$$

Furthermore,

$$\begin{aligned}\dot{V}_3(t) =& f^T(x(t))(Z_{j'}+F_{j'})f(x(t)) - e^{-\alpha\rho\tau(t)}(1-\rho\dot{\tau}(t))f^T(x(t-\rho\tau(t)))Z_{j'}f(x(t-\rho\tau(t)))\\ &- e^{-\alpha\tau}f^T(x(t-\tau))F_{j'}f(x(t-\tau)) - \alpha V_3(t),\\ \leq& f^T(x(t))(Z_{j'}+F_{j'})f(x(t)) - e^{-\alpha\rho\tau}(1-\rho\mu)f^T(x(t-\rho\tau(t)))Z_{j'}f(x(t-\rho\tau(t)))\\ &- e^{-\alpha\tau}f^T(x(t-\tau))F_{j'}f(x(t-\tau)) - \alpha V_3(t).\end{aligned} \tag{23}$$

From (2), and for any positive scalars $\lambda_{1j'}$, $\lambda_{2j'}$, $\lambda_{3j'}$, and $\lambda_{4j'}$, we have

$$\begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^T \begin{bmatrix} -\frac{L^{-T}L^{+}+L^{+T}L^{-}}{2}\lambda_{1j'} & \frac{L^{-T}+L^{+T}}{2}\lambda_{1j'} \\ * & -\lambda_{1j'}I \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} \geq 0, \tag{24}$$

$$\begin{bmatrix} x(\tilde{t}) \\ f(x(\tilde{t})) \end{bmatrix}^T \begin{bmatrix} -\frac{L^{-T}L^{+}+L^{+T}L^{-}}{2}\lambda_{2j'} & \frac{L^{-T}+L^{+T}}{2}\lambda_{2j'} \\ * & -\lambda_{2j'}I \end{bmatrix} \begin{bmatrix} x(\tilde{t}) \\ f(x(\tilde{t})) \end{bmatrix} \geq 0, \tag{25}$$

$$\begin{bmatrix} x(\hat{t}) \\ f(x(\hat{t})) \end{bmatrix}^T \begin{bmatrix} -\frac{L^{-T}L^{+}+L^{+T}L^{-}}{2}\lambda_{3j'} & \frac{L^{-T}+L^{+T}}{2}\lambda_{3j'} \\ * & -\lambda_{3j'}I \end{bmatrix} \begin{bmatrix} x(\hat{t}) \\ f(x(\hat{t})) \end{bmatrix} \geq 0, \tag{26}$$

$$\begin{bmatrix} x(\check{t}) \\ f(x(\check{t})) \end{bmatrix}^T \begin{bmatrix} -\frac{L^{-T}L^{+}+L^{+T}L^{-}}{2}\lambda_{4j'} & \frac{L^{-T}+L^{+T}}{2}\lambda_{4j'} \\ * & -\lambda_{4j'}I \end{bmatrix} \begin{bmatrix} x(\check{t}) \\ f(x(\check{t})) \end{bmatrix} \geq 0, \tag{27}$$

where $\tilde{t}$, $\hat{t}$, and $\check{t}$ represent $t-\rho\tau(t)$, $t-\tau(t)$, and $t-\tau$, respectively.

From the system (11), for any appropriately dimensioned matrix N, we have

$$\begin{aligned}0 =& 2\sum_{j=1}^{s}\sum_{j'=1}^{s}\sum_{i=1}^{r}\sum_{i'=1}^{r}\sum_{i''=1}^{r} N_j(\varrho(t))N_{j'}(\varrho(t_{s_k}))h_{ji}(\varrho(t))h_{j'i'}(\varrho(t_{s_k}))h_{j'i''}(\varrho(t_{s_k}))\dot{x}^T(t)N\times\\ &[-\dot{x}(t) + (A_{ji1} + C_{ji}K_{j'i'}G_{j'i'}A_{j'i''2})x(t) + B_{ji1}f(x(t)) + B_{ji2}f(x(t-\tau(t)))\\ &- C_{ji}K_{j'i'}G_{j'i'}A_{j'i''2}e(t) + C_{ji}K_{j'i'}q^{g}_{j'i'}(y(t_{s_k})) + D_{ji1}w(t)].\end{aligned} \tag{28}$$

When no ET happens, the inequality $e^T(t_\varsigma)\Phi_{j'i'}e(t_\varsigma) - \vartheta_{j'i'}x^T(t_{s_k})\Phi_{j'i'}x(t_{s_k}) < 0$ holds, that is, $\vartheta_{j'i'}(x(t)-e(t))^T\Phi_{j'i'}(x(t)-e(t)) - e^T(t)\Phi_{j'i'}e(t) > 0$. By denoting $\Psi(t) = z^T(t)z(t) - \tilde{\gamma}^2w^T(t)w(t) = x^T(t)A^T_{ji3}A_{ji3}x(t) - \tilde{\gamma}^2w^T(t)w(t)$, $\xi(t) = [x^T(t), x^T(t-\rho\tau(t)), x^T(t-\tau(t)), x^T(t-\tau), \dot{x}^T(t), f^T(x(t)), f^T(x(t-\rho\tau(t))), f^T(x(t-\tau(t))), f^T(x(t-\tau)), {q^{g}_{j'i'}}^T(y(t_{s_k})), e^T(t), w^T(t)]^T$, from (9), (18)–(28), and condition (15), we can obtain

$$\dot{V}(t) - \beta V_1(t) + \alpha(V_2(t) + V_3(t)) + \Psi(t)$$
$$\leq \sum_{j=1}^{s}\sum_{j'=1}^{s}\sum_{i=1}^{r}\sum_{i'=1}^{r}\sum_{i''=1}^{r} N_j(\varrho(t))N_{j'}(\varrho(t_{s_k}))h_{ji}(\varrho(t))h_{j'i'}(\varrho(t_{s_k}))h_{j'i''}(\varrho(t_{s_k}))\xi^T(t)\Omega_{jj'}^{ii'i''}\xi(t) < 0. \quad (29)$$

Obviously,

$$\dot{V}(t) - \beta V(t) + \Psi(t) < 0 \quad (30)$$

We multiply both sides of the inequality (30) by the term $e^{-\beta t}$

$$e^{-\beta t}\dot{V}(t) - e^{-\beta t}\beta V(t) + e^{-\beta t}\Psi(t) < 0. \quad (31)$$

Then, integrating the inequality (31) from t_k to t_{k+1}^- yields

$$\int_{t_k}^{t_{k+1}^-} (e^{-\beta s}V(s))' ds \leq -\int_{t_k}^{t_{k+1}^-} e^{-\beta s}\Psi(s) ds, \quad (32)$$

thus

$$e^{-\beta t_{k+1}}V(t_{k+1}^-) - e^{-\beta t_k}V(t_k) \leq -\int_{t_k}^{t_{k+1}} e^{-\beta s}\Psi(s) ds, \quad (33)$$

and

$$V(t_{k+1}^-) \leq e^{\beta(t_{k+1}-t_k)}V(t_k) - \int_{t_k}^{t_{k+1}} e^{\beta(t_{k+1}-s)}\Psi(s) ds. \quad (34)$$

If t_{k+1} is a triggered instant, $V(t)$ will be switching at t_{k+1}. From the condition (13) and the inequality (34), we have that

$$V(t_{k+1}) \leq \mu V(t_{k+1}^-) \leq \mu e^{\beta(t_{k+1}-t_k)}V(t_k) - \mu \int_{t_k}^{t_{k+1}} e^{\beta(t_{k+1}-s)}\Psi(s) ds. \quad (35)$$

If t_{k+1} is not a triggered instant, $V(t)$ is continuous at t_{k+1}, similarly, from the inequality (30),

$$V(t_{k+1}) = V(t_{k+1}^-) \leq e^{\beta(t_{k+1}-t_k)}V(t_k) - \int_{t_k}^{t_{k+1}} e^{\beta(t_{k+1}-s)}\Psi(s) ds. \quad (36)$$

Case 2: if there are n event-triggered instants within the interval $[t_k, t_{k+1})$, where $n \in N^+$, we assume that t_k is not one of the triggered instants. Therefore, during the interval $[t_k, t_{s_{k+1}})$, the system is in mode j and the controller is in mode j'. Subsequently, in the interval $[t_{s_{k+1}}, t_{k+1})$, both the system and the controller are in mode j.

For $t \in [t_k, t_{s_{k+1}})$, similar to Case 1, and since $t_{s_{k+1}}$ is a triggered instant, one can obtain

$$V(t_{s_{k+1}}) \leq \mu V(t_{s_{k+1}}^-) \leq \mu e^{\beta(t_{s_{k+1}}-t_k)}V(t_k) - \mu\int_{t_k}^{t_{s_{k+1}}} e^{\beta(t_{s_{k+1}}-s)}\Psi(s) ds. \quad (37)$$

For $t \in [t_{s_{k+1}}, t_{k+1})$, the mode-dependent matrices $P_{v(t_{s_{k+1}})}$, $Q_{v(t_{s_{k+1}})}$, $R_{v(t_{s_{k+1}})}$, $Z_{v(t_{s_{k+1}})}$, and $F_{v(t_{s_{k+1}})}$, and free-weighting matrices $\lambda_{1v(t_{s_{k+1}})}$, $\lambda_{2v(t_{s_{k+1}})}$, $\lambda_{3v(t_{s_{k+1}})}$, and $\lambda_{4v(t_{s_{k+1}})}$ are P_j, Q_j, R_j, Z_j, and F_j, and λ_{1j}, λ_{2j}, λ_{3j}, and λ_{4j}, respectively.

From the system (12), for any appropriately dimensioned matrix N, we have

$$0 = 2\sum_{j=1}^{s}\sum_{i=1}^{r}\sum_{i'=1}^{r}\sum_{i''=1}^{r} N_j(\varrho(t))h_{ji}(\varrho(t))h_{ji'}(\varrho(t_{s_k}))h_{ji''}(\varrho(t_{s_k}))\dot{x}^T(t)N\times$$
$$[-\dot{x}(t) + (A_{ji1} + C_{ji}K_{ji'}G_{ji'}A_{ji''2})x(t) + B_{ji1}f(x(t)) + B_{ji2}f(x(t-\tau(t)))$$
$$- C_{ji}K_{ji'}G_{ji'}A_{ji''2}e(t) + C_{ji}K_{ji'}q^g_{ji'}(y(t_{s_{k+1}})) + D_{ji1}w(t)]. \tag{38}$$

When no ET happens, the inequality $e^T(t_\varsigma)\Phi_{ji'}e(t_\varsigma) - \vartheta_{ji'}x^T(t_{s_k})\Phi_{ji'}x(t_{s_k}) < 0$ holds, that is, $\vartheta_{ji'}(x(t)-e(t))^T\Phi_{ji'}(x(t)-e(t)) - e^T(t)\Phi_{ji'}e(t) > 0$. By denoting $\Psi(t) = z^T(t)z(t) - \tilde{\gamma}^2 w^T(t)w(t) = x^T(t)A^T_{ji3}A_{ji3}x(t) - \tilde{\gamma}^2 w^T(t)w(t)$, $\xi_1(t) = [x^T(t), x^T(t-\rho\tau(t)), x^T(t-\tau(t)), x^T(t-\tau), \dot{x}^T(t), f^T(x(t)), f^T(x(t-\rho\tau(t))), f^T(x(t-\tau(t))), f^T(x(t-\tau)), q^{g\,T}_{ji'}(y(t_{s_{k+1}})), e^T(t), w^T(t)]^T$, combining (9), (16), (38), and the event-triggered mode change from j' to j, we can obtain

$$\dot{V}(t) + \alpha V(t) + \Psi(t)$$
$$\leq \sum_{j=1}^{s}\sum_{i=1}^{r}\sum_{i'=1}^{r}\sum_{i''=1}^{r} N_j(\varrho(t))h_{ji}(\varrho(t))h_{ji'}(\varrho(t_{s_k}))h_{ji''}(\varrho(t_{s_k}))\xi_1^T(t)\Omega^{ii'i''}_{jj}\xi_1(t) < 0, \tag{39}$$

Similarly, if t_{k+1} is a triggered instant, and integrating the inequality (39) from $t_{s_{k+1}}$ to t_{k+1}, then

$$V(t_{k+1}) \leq \mu V(t^-_{k+1}) \leq \mu e^{-\alpha(t_{k+1}-t_{s_{k+1}})}V(t_{s_{k+1}}) - \mu\int_{t_{s_{k+1}}}^{t_{k+1}} e^{-\alpha(t_{k+1}-s)}\Psi(s)ds. \tag{40}$$

If t_{k+1} is not a triggered instant, then

$$V(t_{k+1}) = V(t^-_{k+1}) \leq e^{-\alpha(t_{k+1}-t_{s_{k+1}})}V(t_{s_{k+1}}) - \int_{t_{s_{k+1}}}^{t_{k+1}} e^{-\alpha(t_{k+1}-s)}\Psi(s)ds. \tag{41}$$

It can be concluded from (37) and (40) that

$$\begin{aligned}
V(t_{k+1}) \leq& \mu^2 e^{-\alpha(t_{k+1}-t_{s_{k+1}})+\beta(t_{s_{k+1}}-t_k)}V(t_k) - \mu^2 e^{-\alpha(t_{k+1}-t_{s_{k+1}})}\int_{t_k}^{t_{s_{k+1}}} e^{\beta(t_{s_{k+1}}-s)}\Psi(s)ds \\
&- \mu\int_{t_{s_{k+1}}}^{t_{k+1}} e^{-\alpha(t_{k+1}-s)}\Psi(s)ds \\
=& \mu^{\tilde{N}_c(t^+_{k+1},t_k)}e^{-\alpha T_\downarrow(t_{k+1},t_k)+\beta T_\uparrow(t_{k+1},t_k)}V(t_k) \\
&- \int_{t_k}^{t_{s_{k+1}}} \mu^{\tilde{N}_c(t^+_{k+1},s)}e^{-\alpha T_\downarrow(t_{k+1},s)+\beta T_\uparrow(t_{k+1},s)}\Psi(s)ds \\
&- \int_{t_{s_{k+1}}}^{t_{k+1}} \mu^{\tilde{N}_c(t^+_{k+1},s)}e^{-\alpha T_\downarrow(t_{k+1},s)+\beta T_\uparrow(t_{k+1},s)}\Psi(s)ds \\
=& \mu^{\tilde{N}_c(t^+_{k+1},t_k)}e^{-\alpha T_\downarrow(t_{k+1},t_k)+\beta T_\uparrow(t_{k+1},t_k)}V(t_k) \\
&- \int_{t_k}^{t_{k+1}} \mu^{\tilde{N}_c(t^+_{k+1},s)}e^{-\alpha T_\downarrow(t_{k+1},s)+\beta T_\uparrow(t_{k+1},s)}\Psi(s)ds,
\end{aligned} \tag{42}$$

where $\tilde{N}_c(t^+_{k+1}, t_k)$ means the number of the controller switching over interval $(t_k, t_{k+1}]$. $\tilde{N}_c(t^+_{k+1}, t_k) = 2$, $T_\downarrow(t_{k+1}, t_k) = t_{k+1} - t_{s_{k+1}}$, $T_\uparrow(t_{k+1}, t_k) = t_{s_{k+1}} - t_k$; $\tilde{N}_c(t^+_{k+1}, s) = 2$ if $s \in [t_k, t_{s_{k+1}})$; $\tilde{N}_c(t^+_{k+1}, s) = 1$ if $s \in [t_{s_{k+1}}, t_{k+1})$.

It can also be concluded from (37) and (41) that

$$\begin{aligned}V(t_{k+1}) \leq &\mu e^{-\alpha(t_{k+1}-t_{s_{k+1}})+\beta(t_{s_{k+1}}-t_k)} V(t_k) - \mu e^{-\alpha(t_{k+1}-t_{s_{k+1}})} \int_{t_k}^{t_{s_{k+1}}} e^{\beta(t_{s_{k+1}}-s)}\Psi(s)ds \\ &- \int_{t_{s_{k+1}}}^{t_{k+1}} e^{-\alpha(t_{k+1}-s)}\Psi(s)ds \\ = &\mu^{\tilde{N}_c(t_{k+1}^+,t_k)} e^{-\alpha T_\downarrow(t_{k+1},t_k)+\beta T_\uparrow(t_{k+1},t_k)} V(t_k) \\ &- \int_{t_k}^{t_{k+1}} \mu^{\tilde{N}_c(t_{k+1}^+,s)} e^{-\alpha T_\downarrow(t_{k+1},s)+\beta T_\uparrow(t_{k+1},s)} \Psi(s)ds, \end{aligned} \tag{43}$$

where $\tilde{N}_c(t_{k+1}^+, t_k) = 1$, $T_\downarrow(t_{k+1}, t_k) = t_{k+1} - t_{s_{k+1}}$, $T_\uparrow(t_{k+1}, t_k) = t_{s_{k+1}} - t_k$; $\tilde{N}_c(t_{k+1}^+, s) = 1$ if $s \in [t_k, t_{s_{k+1}})$; $\tilde{N}_c(t_{k+1}^+, s) = 0$ if $s \in [t_{s_{k+1}}, t_{k+1})$.

Similarly, the inequalities (35) and (36) can be rewritten as

$$\begin{aligned}V(t_{k+1}) \leq &\mu^{\tilde{N}_c(t_{k+1}^+,t_k)} e^{-\alpha T_\downarrow(t_{k+1},t_k)+\beta T_\uparrow(t_{k+1},t_k)} V(t_k) \\ &- \int_{t_k}^{t_{k+1}} \mu^{\tilde{N}_c(t_{k+1}^+,s)} e^{-\alpha T_\downarrow(t_{k+1},s)+\beta T_\uparrow(t_{k+1},s)} \Psi(s)ds, \end{aligned} \tag{44}$$

when t_{k+1} is a triggered instant, $\tilde{N}_c(t_{k+1}^+, t_k) = 1$, $T_\downarrow(t_{k+1}, t_k) = 0$, $T_\uparrow(t_{k+1}, t_k) = t_{k+1} - t_k$. $\tilde{N}_c(t_{k+1}^+, s) = 1$, $s \in [t_k, t_{k+1})$. When t_{k+1} is not a triggered instant, $\tilde{N}_c(t_{k+1}^+, t_k) = 0$, $T_\downarrow(t_{k+1}, t_k) = 0$, $T_\uparrow(t_{k+1}, t_k) = t_{k+1} - t_k$. $\tilde{N}_c(t_{k+1}^+, s) = 0$, $s \in [t_k, t_{k+1})$.

In both Case 1 and Case 2, the relationship between $V(t_k)$ and $V(t_{k+1})$ can be expressed as in inequality (42)–(44), regardless of whether t_{k+1} is an event-triggered instant or not. Therefore, through repeated iterations, we have

$$\begin{aligned}V(t_{k+1}) \leq &\mu^{\tilde{N}_c(t_{k+1}^+,t_k)} e^{-\alpha T_\downarrow(t_{k+1},t_k)+\beta T_\uparrow(t_{k+1},t_k)} \big(\mu^{\tilde{N}_c(t_k^+,t_{k-1})} e^{-\alpha T_\downarrow(t_k,t_{k-1})+\beta T_\uparrow(t_k,t_{k-1})} V(t_{k-1}) \\ &- \int_{t_{k-1}}^{t_k} \mu^{\tilde{N}_c(t_k^+,s)} e^{-\alpha T_\downarrow(t_k,s)+\beta T_\uparrow(t_k,s)} \Psi(s)ds\big) \\ &- \int_{t_k}^{t_{k+1}} \mu^{\tilde{N}_c(t_{k+1}^+,s)} e^{-\alpha T_\downarrow(t_{k+1},s)+\beta T_\uparrow(t_{k+1},s)} \Psi(s)ds \\ = &\mu^{\tilde{N}_c(t_{k+1}^+,t_{k-1})} e^{-\alpha T_\downarrow(t_{k+1},t_{k-1})+\beta T_\uparrow(t_{k+1},t_{k-1})} V(t_{k-1}) \\ &- \int_{t_{k-1}}^{t_{k+1}} \mu^{\tilde{N}_c(t_{k+1}^+,s)} e^{-\alpha T_\downarrow(t_{k+1},s)+\beta T_\uparrow(t_{k+1},s)} \Psi(s)ds \\ \leq &\dots \\ \leq &\mu^{\tilde{N}_c(t_{k+1}^+,t_0)} e^{-\alpha T_\downarrow(t_{k+1},t_0)+\beta T_\uparrow(t_{k+1},t_0)} V(t_0) \\ &- \int_{t_0}^{t_{k+1}} \mu^{\tilde{N}_c(t_{k+1}^+,s)} e^{-\alpha T_\downarrow(t_{k+1},s)+\beta T_\uparrow(t_{k+1},s)} \Psi(s)ds \\ = &e^{-\alpha T_\downarrow(t_{k+1},0)+\beta T_\uparrow(t_{k+1},0)+\tilde{N}_c(t_{k+1}^+,0)\ln\mu} V(0) \\ &- \int_0^{t_{k+1}} e^{-\alpha T_\downarrow(t_{k+1},s)+\beta T_\uparrow(t_{k+1},s)+\tilde{N}_c(t_{k+1}^+,s)\ln\mu} \Psi(s)ds \end{aligned} \tag{45}$$

For any $t \in [0, \infty)$, the initial condition $\phi(\theta) = 0, \theta \in [-\tau, 0]$, then

$$\int_0^t e^{-\alpha T_\downarrow(t,s)+\beta T_\uparrow(t,s)+\tilde{N}_c(t^+,s)\ln\mu} \Psi(s)ds \leq -V(t) \leq 0. \tag{46}$$

It follows that

$$\int_0^t e^{-\alpha T_\downarrow(t,s)+\beta T_\uparrow(t,s)+\tilde{N}_c(t^+,s)\ln\mu} z^T(s)z(s)ds \leq \tilde{\gamma}^2 \int_0^t e^{-\alpha T_\downarrow(t,s)+\beta T_\uparrow(t,s)+\tilde{N}_c(t^+,s)\ln\mu} w^T(s)w(s)ds. \tag{47}$$

We note that $\tilde{N}_c(t^+, s)$ is less than or equal to the number of system switching $N_v(t^+, s)$. For more details regarding the reason, please refer to [17]. Therefore, $0 \leq \tilde{N}_c(t^+, s) \leq N_v(t^+, s) \leq \frac{t-s}{T_a} + N_0$. For $N_0 = 1$ and $\mu > 1$, from (47), one has

$$\int_0^t e^{-\alpha T_\downarrow(t,s)+\beta T_\uparrow(t,s)} z^T(s)z(s)ds \leq \tilde{\gamma}^2 \int_0^t e^{-\alpha T_\downarrow(t,s)+\beta T_\uparrow(t,s)+(\frac{t-s}{T_a}+1)\ln\mu} w^T(s)w(s)ds. \tag{48}$$

The left side of (48) may be reduced to be $-\alpha(t-s)$, combining the switching law 3.1, we have

$$\int_0^t e^{-\alpha(t-s)} z^T(s)z(s)ds \leq \tilde{\gamma}^2 \mu e^{c_*} \int_0^t e^{-(\alpha_* - \frac{\ln\mu}{T_a})(t-s))} w^T(s)w(s)ds, \tag{49}$$

where $\alpha_* - \frac{\ln\mu}{T_a} > 0$.

Integrating (49) from $t = 0$ to ∞,

$$\int_0^\infty \int_0^t e^{-\alpha(t-s)} z^T(s)z(s)dsdt \leq \tilde{\gamma}^2 \mu e^{c_*} \int_0^\infty \int_0^t e^{-(\alpha_* - \frac{\ln\mu}{T_a})(t-s)} w^T(s)w(s)dsdt. \tag{50}$$

Interchanging the order of integrals yields

$$\int_0^\infty e^{\alpha s} z^T(s)z(s) \int_s^\infty e^{-\alpha t}dtds \leq \tilde{\gamma}^2 \mu e^{c_*} \int_0^\infty e^{-(\alpha_* - \frac{\ln\mu}{T_a})(-s)} w^T(s)w(s) \int_s^\infty e^{-(\alpha_* - \frac{\ln\mu}{T_a})t}dtds. \tag{51}$$

Then, substituting $\tilde{\gamma} = \gamma\sqrt{\frac{(\alpha_* - \ln\mu/T_a)}{\eta\alpha e^{c_*}}}$, we have

$$\int_0^\infty z^T(s)z(s)ds \leq \gamma^2 \int_0^\infty w^T(s)w(s)ds. \tag{52}$$

It should be noted that that inequality (52) still holds when $\mu = 1$. Therefore, for all $\mu \geq 1$, inequality (52) remains valid. Secondly, when $w(t) = 0$, we will demonstrate that the system is GES if conditions (13) to (17) are satisfied. Let us consider Case 1 and condition (15),

$$\dot{V}(t) - \beta V(t) \leq 0. \tag{53}$$

Integrating (53) from t_k to t_{k+1}, and whether t_{k+1} is a triggered instant or not, we have

$$V(t_{k+1}) \leq \mu^{\tilde{N}_c(t_{k+1}^+, t_k)} e^{-\alpha T_\downarrow(t_{k+1},t_k)+\beta T_\uparrow(t_{k+1},t_k)} V(t_k). \tag{54}$$

We consider Case 2 and condition (16), for $t \in [t_k, t_{s_{k+1}})$; then,

$$\dot{V}(t) - \beta V(t) \leq 0. \tag{55}$$

For $t \in [t_{s_{k+1}}, t_{k+1})$, then

$$\dot{V}(t) + \alpha V(t) \leq 0. \tag{56}$$

We combine (55) and (56), integrating both side of these two inequalities from t_k to $t_{s_{k+1}}$, and $t_{s_{k+1}}$ to t_{k+1}, respectively, and, whether t_{k+1} is a triggered instant or not, we have

$$V(t_{k+1}) \leq \mu^{\tilde{N}_c(t_{k+1}^+, t_k)} e^{-\alpha T_\downarrow(t_{k+1},t_k)+\beta T_\uparrow(t_{k+1},t_k)} V(t_k). \tag{57}$$

Therefore, when $w(t) = 0$, the relationship between $V(t_k)$ and $V(t_{k+1})$ can also be expressed in the generalized form as (54) and (57). Following the similar process of (45), then

$$
\begin{aligned}
&V(t_{k+1}) \le \mu^{\tilde{N}_c(t_{k+1}^+,t_k)} e^{-\alpha T_\downarrow(t_{k+1},t_k)+\beta T_\uparrow(t_{k+1},t_k)} V(t_k) \\
&\le \mu^{\tilde{N}_c(t_{k+1}^+,t_k)} e^{-\alpha T_\downarrow(t_{k+1},t_k)+\beta T_\uparrow(t_{k+1},t_k)} \mu^{\tilde{N}_c(t_k^+,t_{k-1})} e^{-\alpha T_\downarrow(t_k,t_{k-1})+\beta T_\uparrow(t_k,t_{k-1})} V(t_{k-1}) \\
&\le \dots \\
&\le e^{-\alpha T_\downarrow(t_{k+1},0)+\beta T_\uparrow(t_{k+1},0)+\tilde{N}_c(t_{k+1}^+,0)\ln\mu} V(0).
\end{aligned} \tag{58}
$$

Under the switching law 3.1 and $0 \le \tilde{N}_c(t^+,s) \le N_v(t^+,s) \le \frac{t-s}{T_a}+1$, then

$$
V(t) \le e^{-\alpha T_\downarrow(t,0)+\beta T_\uparrow(t,0)+\tilde{N}_c(t^+,0)\ln\mu} V(0) \le \mu e^{c_*} e^{-(\alpha_*-\frac{\ln\mu}{T_a})t} V(0). \tag{59}
$$

Since $\sum_{j=1}^{s} N_j(\varrho(t)) x^T(t) P_{v(t_{s_k})} x(t) = V_1(t) \le V(t)$, we have

$$
\|x(t)\|_2^2 \le \frac{V(t)}{\inf\limits_{j\in\{1,2,\dots,s\}} \lambda_{min}(P_j)}, \tag{60}
$$

and since

$$
\begin{aligned}
V(0) \le & \sup_{j\in\{1,2,\dots,s\}} x^T(0)\lambda_{max}(P_j)x(0) + \int_{-\rho\tau(0)}^{0} e^{\alpha s} x^T(s) \sup_{j\in\{1,2,\dots,s\}} \lambda_{max}(Q_j)x(s)ds \\
& +\tau \int_{-\tau}^{0} e^{\alpha s}(s+\tau)\dot{x}^T(s) \sup_{j\in\{1,2,\dots,s\}} \lambda_{max}(R_j)\dot{x}(s)ds \\
& + \int_{-\rho\tau(0)}^{0} e^{\alpha s} f^T(x(s)) \sup_{j\in\{1,2,\dots,s\}} \lambda_{max}(Z_j) f(x(s))ds \\
& + \int_{-\tau}^{0} e^{\alpha s} f^T(x(s)) \sup_{j\in\{1,2,\dots,s\}} \lambda_{max}(F_j) f(x(s))ds \\
\le & \|x(0)\|_2^2 \sup_{j\in\{1,2,\dots,s\}} \lambda_{max}(P_j) + \sup_{-\tau\le\varsigma\le 0} \|x(\varsigma)\|_2^2 \rho\tau \sup_{j\in\{1,2,\dots,s\}} \lambda_{max}(Q_j) \\
& + \sup_{-\tau\le\varsigma\le 0} \|\dot{x}(\varsigma)\|_2^2 \tau^3 \sup_{j\in\{1,2,\dots,s\}} \lambda_{max}(R_j) + \sup_{-\tau\le\varsigma\le 0} \|x(\varsigma)\|_2^2 \rho\tau l^2 \sup_{j\in\{1,2,\dots,s\}} \lambda_{max}(Z_j) \\
& + \sup_{-\tau\le\varsigma\le 0} \|x(\varsigma)\|_2^2 \tau l^2 \sup_{j\in\{1,2,\dots,s\}} \lambda_{max}(F_j) \\
\le & \kappa \|\tilde{x}_0\|_\tau^2,
\end{aligned} \tag{61}
$$

where $\|\tilde{x}_0\|_\tau^2 = \sup\limits_{-\tau\le\varsigma\le 0} \|x(\varsigma)\|_2^2 + \sup\limits_{-\tau\le\varsigma\le 0} \|\dot{x}(\varsigma)\|_2^2$, $l^2 = \|L^+L^-\|^2$ and $\kappa = (\sup\limits_{j\in\{1,2,\dots,s\}} \lambda_{max}(P_j) + \rho\tau \sup\limits_{j\in\{1,2,\dots,s\}} \lambda_{max}(Q_j) + \tau^3 \sup\limits_{j\in\{1,2,\dots,s\}} \lambda_{max}(R_j) + \rho\tau l^2 \sup\limits_{j\in\{1,2,\dots,s\}} \lambda_{max}(Z_j) + \tau l^2 \sup\limits_{j\in\{1,2,\dots,s\}} \lambda_{max}(F_j)$.

It follows from (59)–(61) that

$$
\|x(t)\|_2^2 \le \frac{\mu e^{c_*}\kappa \|\tilde{x}_0\|_\tau^2}{\inf\limits_{j\in\{1,2,\dots,s\}} \lambda_{min}(P_j)} e^{-(\alpha_*-\frac{\ln\mu}{T_a})t}. \tag{62}
$$

Since $\alpha_* > \frac{\ln\mu}{T_a}$, when $w(t)=0$, the system is GES. □

Remark 6. *In this paper, $\tilde{N}_c(t^+,s)$ and $N_v(t^+,s)$ are the discontinuity numbers of $v(t)$ with respect to the controller and the system over the interval $(s,t]$, respectively. For example, in inequality (42), $\tilde{N}_c(t_{k+1}^+,t_k)=2$ indicates that controller mode switching occurs twice at instants*

$t_{s_{k+1}}$ and t_{k+1}; while in inequality (43), $\tilde{N}_c(t^+_{k+1}, t_k) = 1$, signifies that controller mode switching occurs only once at instant $t_{s_{k+1}}$.

Remark 7. *As mentioned in references [17,18], the continuity of $V(\xi(t))$ and $V(t)$ at $t = t_{q+1}$ is not correct. Since $V(\xi(t))$ is a controller-mode-dependent function and the instant t_{q+1} is possibly an ET instant, $V(\xi(t_{q+1}))$ is not necessarily equal to $V(\xi(t^-_{q+1}))$.*

Remark 8. *In [21], the asynchronous phenomenon is, addressed through the concepts of MDT, denoted as τ_d, and the maximal asynchronous period, denoted as τ_m; in [17,18], the asynchronous phenomenon, represented by the term $e^{-\alpha T_{\downarrow}(0,t)+\beta T_{\uparrow}(0,t)}$, is investigated through the upper bound of IEIs, denoted as T, and ADT, denoted as τ_a. However, $-\alpha T_{\downarrow}(0,t) + \beta T_{\uparrow}(0,t) = -\alpha t + (\alpha + \beta) T_{\uparrow}(0,t) \leq -\alpha t + (\alpha+\beta) T N_{\sigma}(0,t)$, where $T_{\uparrow}(0,t)$ has been enlarged as $TN_{\sigma}(0,t)$. It is evident that the obtained results are conservative. In this paper, the switching law 3.1 is adopted to address the asynchronous phenomenon and mitigate the conservatism of existing results.*

Remark 9. *The L_2 norm bound constraint for the switching system holds significant importance in various areas such as H_∞ synchronization, L_2 gain analysis, external stability, and more. However, in [26], due to the uncertainty in the sign of $\Gamma(s)$, the authors cannot guarantee that inequality (36) holds. To address this issue, a "weighted" L_2 norm bound constraint is proposed in [23]. Nevertheless, due to the existence of $e^{-\alpha t}$, $e(t)$ may tend toward infinity. Recently, the concept of MMDT was introduced in [27], and the term "weighted" was appropriately eliminated using a two-direction inequality. However, in this paper, the constructed Lyapunov function is a controller-mode-dependent function. We cannot conclude that $\max\{\frac{t-s}{T_{max}} - 1, 0\} \leq \tilde{N}_c(t^+, s) \leq N_v(t^+, s) \leq \frac{t-s}{T_{min}} + 1$. Only $0 \leq \tilde{N}_c(t^+, s) \leq N_v(t^+, s) \leq \frac{t-s}{T_{min}} + 1$ can be obtained, and only the condition about T_{min} will be given. Therefore, the authors still employed the ADT in this article. The ADT can also be expressed as a two-direction inequality, i.e., $0 \leq N_v(t^+, s) \leq \frac{t-s}{T_a} + N_0$. Moreover, the inequality $0 \leq N_v(t^+, s) \ln \mu \leq (\frac{t-s}{T_a} + N_0) \ln \mu$ holds. The term "weighted" has been successfully removed.*

Next, for the system (10), one has

Theorem 2. *Given scalars τ, $\tilde{\mu}$, γ, $\mu' \geq 1$, $\alpha > 0$, and $0 < \rho < min\{1, \frac{1}{\tilde{\mu}}\}$ and the matrices L^- and L^+, under switching law 3.1, if there exist $P_j > 0$, $Q_j > 0$, $R_j > 0$, $Z_j > 0$, and $F_j > 0$, scalars $\lambda_{wj} > 0$ $(w = 1,2,3,4)$, N, $\tilde{G}_{jj'ii'}$, and $\tilde{G}_{jii'}$, for any $i, i', i'' = 1,2,\ldots,r$ and $j, j', \acute{j}, \grave{j} = 1,2,\ldots,s$, such that*

$$\begin{bmatrix} -\mu' P_{\acute{j}} & (I+I_{ji})^T P_{\grave{j}} \\ * & -P_{\grave{j}} \end{bmatrix} \leq 0, Q_j \leq \mu' Q_{j'}, R_j \leq \mu' R_{j'}, \tag{63}$$

$$Z_j \leq \mu' Z_{j'}, F_j \leq \mu' F_{j'}, \quad j \neq j' \tag{64}$$

$$\Omega^{ii'i''}_{jj'} < 0, \qquad j \neq j' \tag{65}$$

$$\Omega^{ii'i''}_{jj} < 0, \tag{66}$$

$$\alpha_* > \frac{\ln \mu}{T_a}. \tag{67}$$

Then, the system (10) is GES with an HPI γ, where $\Omega^{ii'i''}_{jj'} = (\Omega^{ii'i''}_{jj'})_{12\times 12}$ and $\Omega^{ii'i''}_{jj} = (\Omega^{ii'i''}_{jj})_{12\times 12}$ are the same as Theorem 1.

Moreover, the controller gain of (5) is given by

$$K_{j'i'} = C_{ji}^{-1} N^{-1} \tilde{G}_{jj'ii'} \quad \text{and} \quad K_{ji'} = C_{ji}^{-1} N^{-1} \tilde{G}_{jii'}.$$

Proof. We select the same Lyapunov function as Equation (18), and then analyze the GES with an HPI γ according to Case 1 and Case 2, respectively. When $t \in [t_k, t_{k+1})$, one has

Case 1: if there is no triggered instant in $[t_k, t_{k+1})$ and t_{k+1} is a triggered instant, the system mode is j over the interval $[t_k, t_{k+1})$, and the controller mode at instants t^+_{k+1} and t^-_{k+1} is $\tilde{j}$ and j', respectively. For $\tilde{j} \neq j$ and $j' \neq j$,

$$\begin{aligned} V_1(t^+_{k+1}) - \mu' V_1(t^-_{k+1}) &= x^T(t^+_{k+1}) P_{\tilde{j}} x(t^+_{k+1}) - \mu' x^T(t^-_{k+1}) P_{j'} x(t^-_{k+1}) \\ &= \sum_{i=1}^{r} h_{ji}(\varrho(t)) x^T(t^-_{k+1})[(I+I_{ji})^T P_{\tilde{j}} (I+I_{ji}) - \mu' P_{j'}] x(t^-_{k+1}). \end{aligned} \tag{68}$$

If t_{k+1} is not a triggered instant, the system mode is j over the interval $[t_k, t_{k+1})$, and the controller mode at both instants t^+_{k+1} and t^-_{k+1} is j'. For $j' \neq j$,

$$\begin{aligned} V_1(t^+_{k+1}) - \mu' V_1(t^-_{k+1}) &= x^T(t^+_{k+1}) P_{j'} x(t^+_{k+1}) - \mu' x^T(t^-_{k+1}) P_{j'} x(t^-_{k+1}) \\ &= \sum_{i=1}^{r} h_{ji}(\varrho(t)) x^T(t^-_{k+1})[(I+I_{ji})^T P_{j'} (I+I_{ji}) - \mu' P_{j'}] x(t^-_{k+1}). \end{aligned} \tag{69}$$

Case 2: if there are $n(n \in N^+)$ triggered sampled instants in $[t_k, t_{k+1})$ and t_{k+1} is a triggered instant, the system mode is j over the interval $[t_k, t_{k+1})$, and the controller mode at instants t^+_{k+1} and t^-_{k+1} is $\hat{j}$ and j, respectively. For $\hat{j} \neq j$,

$$\begin{aligned} V_1(t^+_{k+1}) - \mu' V_1(t^-_{k+1}) &= x^T(t^+_{k+1}) P_{\hat{j}} x(t^+_{k+1}) - \mu' x^T(t^-_{k+1}) P_j x(t^-_{k+1}) \\ &= \sum_{i=1}^{r} h_{ji}(\varrho(t)) x^T(t^-_{k+1})[(I+I_{ji})^T P_{\hat{j}} (I+I_{ji}) - \mu' P_j] x(t^-_{k+1}). \end{aligned} \tag{70}$$

If t_{k+1} is not a triggered instant, the system mode over the interval $[t_k, t_{k+1})$, and the controller mode at both instants t^+_{k+1} and t^-_{k+1} is j.

$$\begin{aligned} V_1(t^+_{k+1}) - \mu' V_1(t^-_{k+1}) &= x^T(t^+_{k+1}) P_j x(t^+_{k+1}) - \mu' x^T(t^-_{k+1}) P_j x(t^-_{k+1}) \\ &= \sum_{i=1}^{r} h_{ji}(\varrho(t)) x^T(t^-_{k+1})[(I+I_{ji})^T P_j (I+I_{ji}) - \mu' P_j] x(t^-_{k+1}). \end{aligned} \tag{71}$$

From condition (63) and using the Schur complement, Equations (68)-(71) are shown to be less than or equal to 0. This implies that

$$V_1(t^+_{k+1}) \leq \mu' V_1(t^-_{k+1}). \tag{72}$$

For $V_2(t)$ and $V_3(t)$, if t_{k+1} is a triggered instant, we have

$$V_2(t^+_{k+1}) \leq \mu' V_2(t^-_{k+1}), \tag{73}$$

$$V_3(t^+_{k+1})) \leq \mu' V_3(t^-_{k+1}) \tag{74}$$

If t_{k+1} is not a triggered instant, then

$$V_2(t^+_{k+1}) = V_2(t^-_{k+1}), \tag{75}$$

$$V_3(t^+_{k+1}) = V_3(t^-_{k+1}) \tag{76}$$

Taking the same procedure as Theorem 1, we will complete the proof. □

4. Examples

In this section, a numerical example will be provided to demonstrate the effectiveness of the proposed method, which includes ETM, ADT, logarithmic quantizers, and the switching law. Table 1 presents a comparison of performance and features between the studies [17,18,21,28–30] and the method proposed in this paper. Our method has achieved frequent switching and a normal L_2 norm constraint, which are the main innovative points of this article. If the MMDT method is employed, only the minimum dwell time $T_{\min}$ condition is provided, not the average dwell time T_a. Furthermore, we construct a controller-mode-dependent Lyapunov function, enabling a more nuanced understanding of system behavior under different controller modes. This leads to more precise stability guarantees and potentially allows for adjustments to the event-triggered instants in a timely manner. The proposed method in this paper demonstrates its generality and superiority through a comparison with the methods employed and the control goals achieved in existing studies. The symbol $'-'$ represents that the corresponding references do not study this control objective.

Table 1. Comparative performance and features of the existing results and our results.

	ETM	ADT	MMDT	Switching Law	Logarithmic Quantizers	Frequent Switching	Normal L_2 Norm Constraint
[21]	Yes	Yes	No	No	No	No	-
[17]	Yes	Yes	No	No	No	No	-
[18]	Yes	Yes	No	No	No	No	-
[28]	No	Yes	No	No	Yes	-	No
[29]	Yes	Yes	No	No	No	-	No
[30]	No	No	Yes	No	No	-	Yes
Our method	Yes	Yes	No	Yes	Yes	Yes	Yes

Example 1. *We consider the systems (10) with $s = 2$ and $r = 2$ as shown below:*
Subsystem 1:

$$A_{111}=\begin{bmatrix}-1 & 0\\ 0 & -8.32\end{bmatrix}, A_{121}=\begin{bmatrix}-3.5 & 0\\ 0 & -2.5\end{bmatrix}, B_{111}=\begin{bmatrix}-1 & 0.4\\ 0 & -0.1\end{bmatrix}, B_{121}=\begin{bmatrix}-5 & 0.8\\ 0.4 & -0.5\end{bmatrix},$$
$$B_{112}=\begin{bmatrix}-1.7 & 0.6\\ 0.2 & -0.6\end{bmatrix}, B_{122}=\begin{bmatrix}-3.1 & 0\\ 0 & -2.5\end{bmatrix}, C_{11} = \begin{bmatrix}-2 & 3\\ 1 & -5\end{bmatrix}, C_{12} = \begin{bmatrix}-3 & 1\\ 0 & -4\end{bmatrix},$$
$$D_{111} = \begin{bmatrix}0.2\\ 0.3\end{bmatrix}, D_{121} = \begin{bmatrix}-0.13\\ 0.1\end{bmatrix}, A_{112} = [-0.2, 0.2], A_{122} = [0.3, -0.2],$$
$$A_{113} = \begin{bmatrix}-1 & 1\\ 0 & -1\end{bmatrix}, A_{123} = \begin{bmatrix}-2 & 1\\ 0 & -2\end{bmatrix}.$$

Subsystem 2:

$$A_{211}=\begin{bmatrix}-0.6 & 0\\ 0 & -0.5\end{bmatrix}, A_{221}=\begin{bmatrix}-0.4 & 0\\ 0 & -0.5\end{bmatrix}, B_{211}=\begin{bmatrix}0.3 & 0.15\\ 0.4 & -0.3\end{bmatrix}, B_{221}=\begin{bmatrix}0.2 & 0.1\\ 0.1 & -0.2\end{bmatrix},$$
$$B_{212}=\begin{bmatrix}-0.04 & 0.06\\ 0.01 & -0.15\end{bmatrix}, B_{222}=\begin{bmatrix}-0.12 & 0.1\\ 0.02 & -0.1\end{bmatrix}, C_{21} = \begin{bmatrix}-0.1 & 0.2\\ 0 & -4\end{bmatrix}, C_{22}=\begin{bmatrix}-1 & 0\\ 0.1 & -2\end{bmatrix},$$
$$D_{211}=\begin{bmatrix}0.2\\ 0.1\end{bmatrix}, C_{221} = \begin{bmatrix}0.1\\ 0.2\end{bmatrix}, A_{212} = [0.3, 0.3], A_{222} = [0.1, 0.1],$$
$$A_{213} = \begin{bmatrix}-0.3 & 0.2\\ 0.1 & -0.3\end{bmatrix}, A_{223} = \begin{bmatrix}-0.6 & 0\\ 0 & 1\end{bmatrix},$$

We let

$I_{11} = \begin{bmatrix} 0.1 & 2.1 \\ -0.6 & 0.1 \end{bmatrix}, I_{12} = \begin{bmatrix} -2.1 & 0.1 \\ 0.9 & -1.2 \end{bmatrix}, I_{21} = \begin{bmatrix} 2.3 & 1.8 \\ 0.2 & -2.0 \end{bmatrix}, I_{22} = \begin{bmatrix} -0.1 & 0 \\ 0 & -3.0 \end{bmatrix},$
$\tau(t) = 0.6 + 0.5cost, f(x(t)) = \begin{bmatrix} 0.1x_1(t) + tanh(0.1x_1(t)) + 0.1x_2(t) \\ -0.1x_2(t) - tanh(0.1x_2(t)) \end{bmatrix}, w(t) = e^{-0.005t}cost,$
$h_{11}(\varrho(t)) = sin^2(3x_1(t)), h_{12}(\varrho(t)) = cos^2(3x_1(t)), h_{21}(\varrho(t)) = sin^2(3x_2(t)),$
$h_{22}(\varrho(t)) = cos^2(3x_2(t)).$

Then, $L^- = \begin{bmatrix} 0.1 & 0.1 \\ 0 & -0.1 \end{bmatrix}$ and $L^+ = \begin{bmatrix} 0.2 & 0.1 \\ 0 & -0.2 \end{bmatrix}$, $\tau = 1.1$, and $\tilde{\mu} = 0.5$. We choose $\alpha = 0.6$, $\beta = 0.4$, $\rho = 0.5$, $\mu = 1.2$, $c_* = 2$, $\alpha_* = 0.5$, $\Phi_{j'i'} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, $\vartheta_{j'i'} = 0.6$, $\rho^1_{j',i'} = 0.4$, $\sigma^1_{j'i'} = 0.43$, $Q_0^{j',i',1} = 0.01$, and $\phi(\theta) = \theta, \theta \in [-1.1, 0]$. The switching signal and noise input $w(t) = e^{-0.005t}cost$ in this example are shown in Figure 2 and Figure 3, respectively.

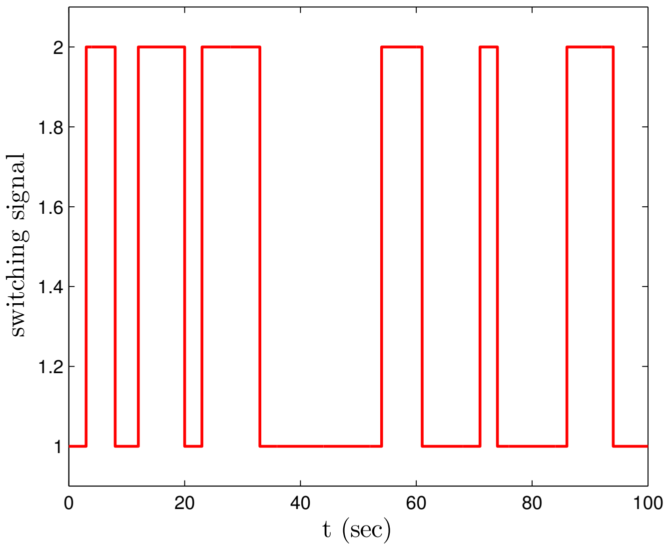

Figure 2. The switching signal.

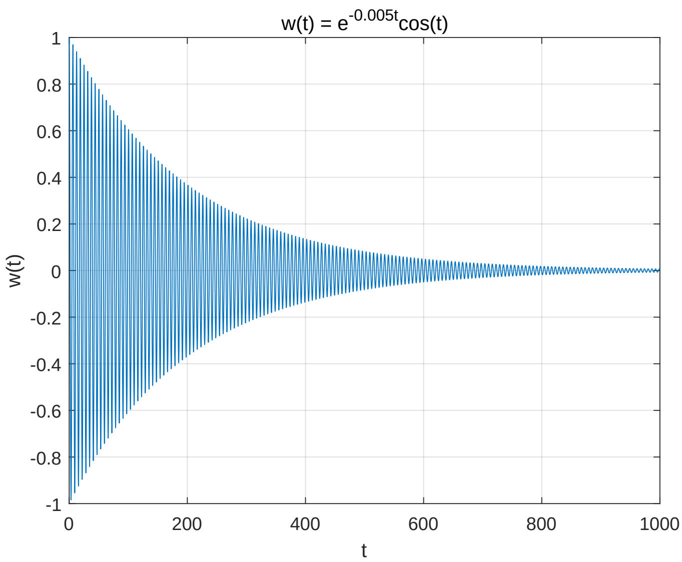

Figure 3. The noise input $w(t) = e^{-0.005t}cost$.

According to Theorem 2, we obtain the control gain matrices as

$$K_{11} = \begin{bmatrix} 0.4677 \\ 0.6953 \end{bmatrix}, \quad K_{12} = \begin{bmatrix} 0.2845 \\ 0.2263 \end{bmatrix},$$
$$K_{21} = \begin{bmatrix} -0.9645 \\ 0.5213 \end{bmatrix}, K_{22} = \begin{bmatrix} 0.4239 \\ -1.7852 \end{bmatrix}.$$

for the best $\gamma = 1.1568$, which is shown in Figure 4.

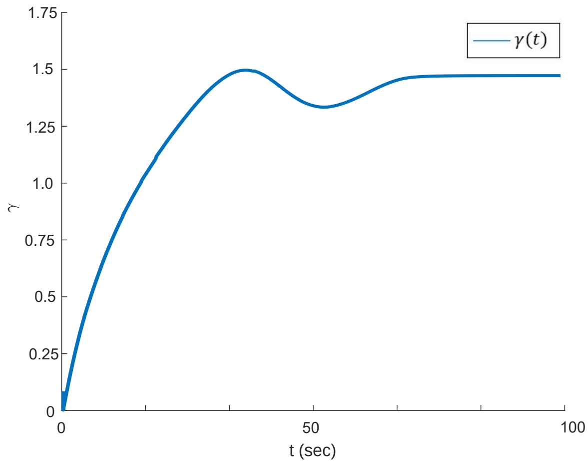

Figure 4. The best performance γ.

Figures 5 and 6 depict the state trajectories and phase of the system (10). Figure 7 shows the event triggering instants and triggering duration of the same system (10).

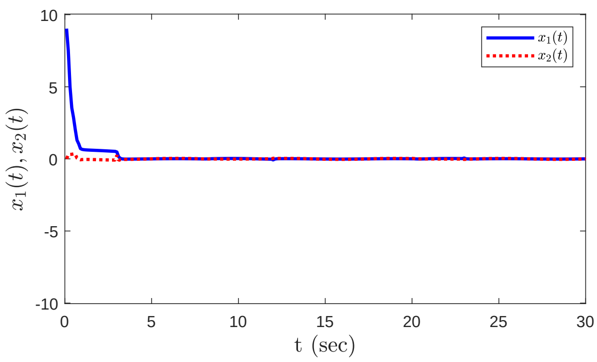

Figure 5. The state trajectories of the system (13).

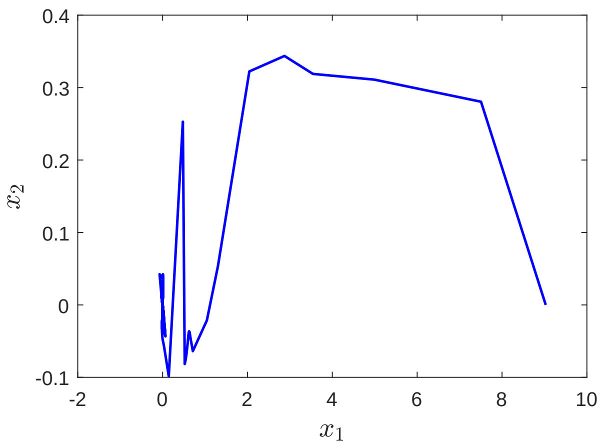

Figure 6. The phase of the system (13).

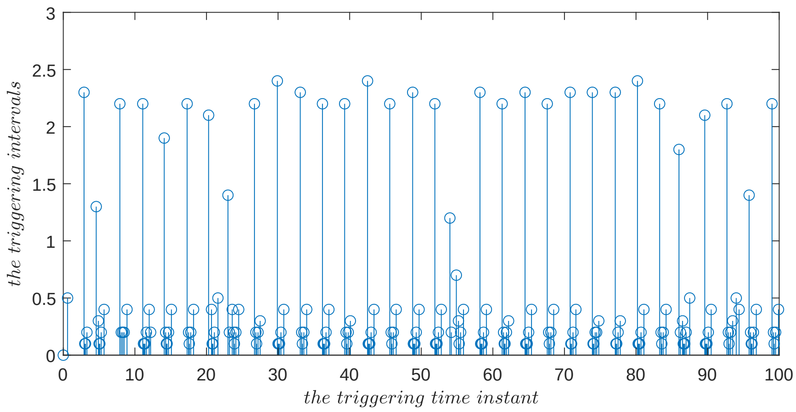

Figure 7. The event triggering instants and triggering duration of the system (13).

5. Conclusions

In this paper, the exponential H_∞ output control problem of delayed switching fuzzy systems was studied, with or without impulses. Frequent switching was indeed allowed to occur in an IEI by introducing the switching law and designing a mode-dependent ETM. Furthermore, a normal (non-weighted) L_2 norm constraint was derived using the ADT method and some mathematical techniques. By constructing a controller-mode-dependent Lyapunov function and adopting logarithmic quantizers, we have derived some new conditions to guarantee that the switching fuzzy system is GES with an HPI γ. An example was provided to demonstrate the feasibility of the proposed methods.

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