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A Note on the Moody Diagram

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Abstract: In this work, we underscore the significance of selecting an appropriate scaling to derive dimensionless quantities that accurately reflect their dimensional counterparts, thereby enhancing the comprehension of the underlying physics. For the loss of head in a pipe flow, we argue that employing inertial force (or kinetic energy) to non-dimensionalized pressure force (or mechanical energy loss) lacks physical justification. As a result, an anomalous trend emerges for the classical friction factor: it decreases as the dimensionless flow rate (Reynolds number) increases, contrary to the behavior observed in the corresponding dimensional quantities. Conversely, by non-dimensionalizing the pressure force with the viscous force, a novel friction factor arises. In laminar flow, it is constant, while in turbulent flow, it is a monotonically increasing function of the Reynolds number, mirroring the behavior observed in the dimensional problem.

Keywords: Moody diagram; friction factor; inertia in pipe flow

1. Introduction

The steady flow of Newtonian fluids through tubes appears in a plethora of applications, including various fields of engineering, physics, and biology, to name a few. For this reason, it was the topic of several studies in the nineteenth and twentieth centuries.

For laminar flow, G. H. L. Hagen (1797–1884) and J. L. M. Poiseuille (1797–1869) independently performed analytical and experimental studies that led to the same relation between the flow rate and the pressure drop, the so-called Hagen–Poiseuille equation.

For turbulent flow in tubes, prominent scientists like L. Prandtl, Th. von Kármán, J. Nikuradse, H. Darcy, H. Basin, C. Colebrook, and others performed experiments for wide ranges of Reynolds number and relative wall roughness. Colebrook [1,2] developed an equation for the friction factor (i.e., dimensionless head loss) as a function of the Reynolds number and relative roughness.

Since this equation is transcendental, it was not very practical for routine usage. In a celebrated work, Moody [3] developed a diagram plotting both (a dimensionless version of) the Hagen–Poiseuille and the Colebrook equations. Moody's diagram became a basic and indispensable tool that—since shortly after its publication nearly eighty years ago—has been routinely used by engineers and scientists worldwide in the design of a wide range of hydraulic systems.

Other friction factor expressions for the turbulent flow in pipes have been developed over the years, all of them explicit, with the goal of avoiding the iterative procedure required while using the Colebrook equation [4–10]. Brkić [4] proposed an approximation of the Colebrook equation based on the Lambert W-function. Ćojbašić and Brkić [5] developed—with a basis on previously existing models that were improved via genetic algorithms—two explicit alternatives to the Colebrook equation that presented negligible error.

Offor and Alabi [6] applied artificial intelligence for the prediction of the friction factor. They used a network having a 2-30-30-1 topology trained with the Levenberg–Marquardt back propagation algorithm fed by 60,000 datasets of Reynolds number and relative roughness, obtaining negligible deviation from the Colebrook equation prediction. Offor and Al-

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abi [7] proposed an explicit non-linear regression model with two logarithmic functions that reproduces with negligible error the Colebrook equation, with far less computational time.

Minhoni et al. [8] examined the performance of six explicit equations for calculating the friction factor by comparing their predictions with the ones of the implicit Colebrook equation. Based on the results, they observed that the equations of Vatankhah [9] and Offor and Alabi [6] provided predictions closest to the ones of the Colebrook equation.

In this work, we propose an alternate scaling for the head loss whose characteristics render more clear the role of inertia in this flow and ensure that the trends of the relationship between dimensionless quantities are the same ones observed in the dimensional problem.

2. Analysis

A simple force balance in a fluid element in the steady, laminar, isochoric flow of a Newtonian fluid through a horizontal tube of constant cross sectional area ultimately leads to the well-known Hagen–Poiseuille equation, which gives the volumetric flow rate Q as a function of the pressure gradient $\Delta p/L$ (Δp is the pressure difference between two axial positions separated by a distance L):

$$Q = \frac{\pi D^4}{128\mu}\frac{\Delta p}{L} \tag{1}$$

where D is the tube's inner diameter and μ is the viscosity of the fluid. Since, for the flow in a horizontal tube, all fluid elements move at constant kinetic and potential energies, the pressure drop is directly related to the total mechanical energy loss that a Lagrangian particle of unit mass experiences as it travels along the tube length L. Therefore, we can easily adapt Equation (1) to render it applicable to straight tubes of any orientation with respect to gravity, just by replacing Δp by $\rho g h_f$, where ρ is the fluid's mass density, g is the acceleration due to gravity, and h_f is the loss of mechanical energy (or head) due to friction, in length units. The result is as follows:

$$h_f = \frac{128\mu QL}{\pi D^4 \rho g} \tag{2}$$

The most common dimensionless version of this equation is as follows:

$$f = \frac{64}{Re} \tag{3}$$

where f is the so-called friction factor, and Re is the Reynolds number. These dimensionless quantities are defined as

$$f := \frac{h_f}{\frac{L}{D}\frac{V^2}{2g}} \tag{4}$$

and

$$Re := \frac{\rho V D}{\mu} \tag{5}$$

where V is the average axial velocity.

For turbulent flows, the also-famous Colebrook transcendental equation [1] is employed, instead of Equation (3):

$$f = \frac{1}{\left\{2\log_{10}\left(\frac{e/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)\right\}^2} \tag{6}$$

In this equation, e is the average rugosity of the tube's inner wall. The transcendental nature of Equation (6) requires iterations to obtain f as a function of Re and e/D. However, the convergence is quite fast (2–4 iterations) and very weakly dependent on the initial guess for f due to the appearance of f on the RHS of Equation (6) within a square root which, in turn, is part of the argument of a logarithm. Nevertheless, its transcendental nature posed practical problems to users in the last century who did not have access to efficient computer codes and spreadsheets. To address this problem, Moody [3] plotted Equations (3) and (6) in a diagram like the one shown in Figure 1.

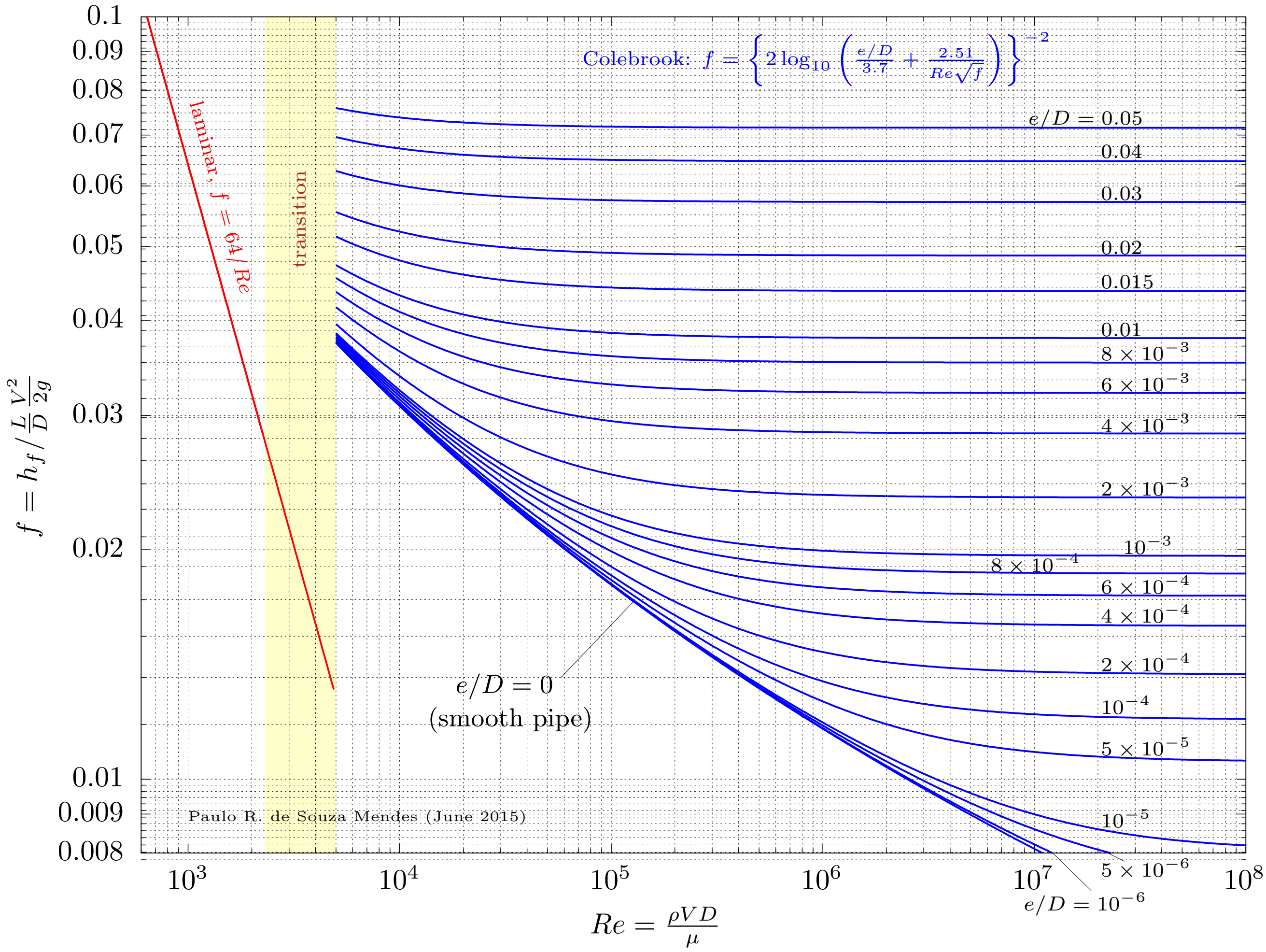

Figure 1. The classical Moody diagram.

3. Discussion

A perhaps puzzling feature of the Moody diagram is the fact that the dimensionless mechanical energy loss f *decreases* as the dimensionless flow rate *Re* is increased, both in the laminar and in the turbulent flow regimes.

The reason for this counter-intuitive behavior is the fact that, in Equation (4), an inertial force is used to non-dimensionalize the pressure force. However, in a steady laminar pipe flow, all material particles flow at a constant velocity, so that there is no inertial force involved, the pressure force being exactly balanced by the viscous force. In turbulent flow, inertia plays an indirect role only, which is due to the velocity fluctuations around average constant values experienced by the material particles.

Therefore, it seems more appropriate to use, in the non-dimensionalization of the pressure force, a characteristic *viscous force* rather than an inertial force. The result is a modified friction factor f^*, defined as

$$f^* := \frac{\rho g h_f}{32\mu \frac{V}{D}} \frac{D}{L} \tag{7}$$

Combining Equations (2) and (7), we obtain, for laminar flow,

$$f^* = 1 \tag{8}$$

Regarding turbulent flow, it is not difficult to re-write the Colebrook Equation (Equation (6)) in terms of the modified friction factor:

$$f^* = \frac{Re}{\left\{16 \log_{10}\left(\frac{e/D}{3.7} + \frac{0.314}{\sqrt{Ref^*}}\right)\right\}^2} \tag{9}$$

We now represent Equations (8) and (9) in a modified version of the Moody diagram, shown in Figure 2. In this figure, we observe that the friction factor is constant for laminar flow and increases monotonically with the Reynolds number for turbulent flow.

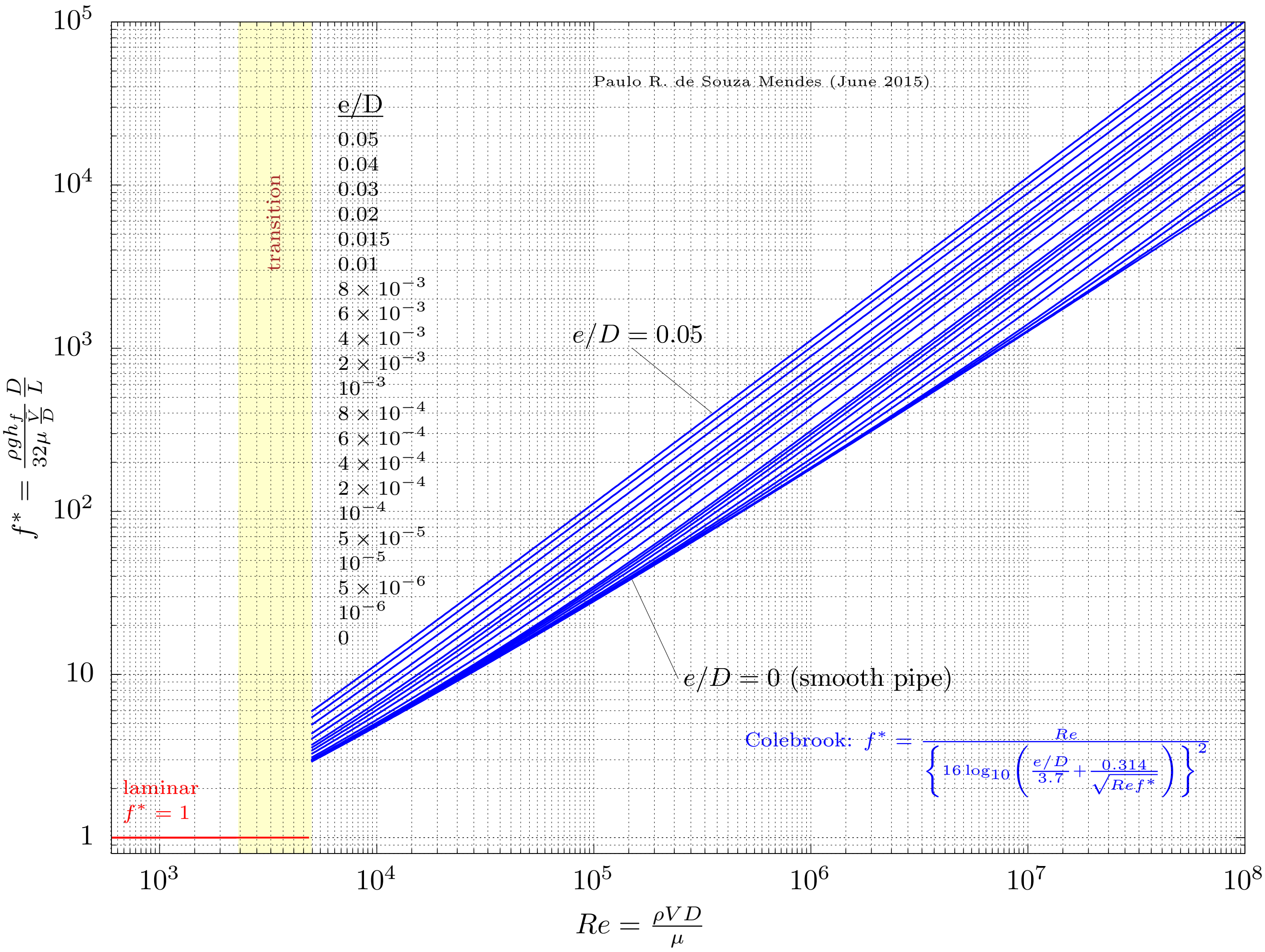

Figure 2. The modified Moody diagram.

This behavior observed for the non-dimensional quantities is in agreement with the one expected for the dimensional quantities because the new scaling is more consistent with the physics involved in this problem.

Moreover, for large-enough values of the Reynolds number Re, the modified friction factor f^* becomes a linear function of the Reynolds number, as can be easily inferred upon an inspection of Equation (9). It is easy to see that this region of linear dependence on Re in the modified Moody diagram corresponds to the so-called "fully rough zone" of the original Moody diagram, where the Moody friction factor is independent of the Reynolds number.

4. Concluding Remarks

In this brief note, we exemplify the importance of choosing the appropriate scaling to obtain dimensionless quantities that are faithful to their dimensional counterparts, thus aiding the understanding of the physics involved.

Specifically, we argue that using the inertial force (or kinetic energy) to non-dimensionalize the pressure force (or mechanical energy loss) is not physically justifiable because inertia is irrelevant in laminar flow, and for turbulent flow, it is solely associated with the velocity fluctuations.

As a result, an abnormal behavior is obtained for the friction factor, namely, it decreases as the dimensionless flow rate (the Reynolds number) is increased, both in laminar and turbulent flows, in contrast to what is observed for the corresponding dimensional quantities.

On the other hand, if we choose to non-dimensionalize the pressure force using the viscous force, we obtain a new friction factor that is constant for laminar flow and a monotonically increasing function of the Reynolds number, following the behavior observed in the dimensional problem.

Of course, this alternate non-dimensionalization does not greatly change the procedure of the solution of practical problems. This is so because the solution of engineering problems basically involves dimensional quantities, namely, the loss of mechanical energy h_f as a function of the flow rate Q (or of the average velocity V), which is independent of the choice of non-dimensionalization and can be evaluated either by Equation (4) or by Equation (7).

In summary, the main contribution of the non-dimensionalization presented in this paper resides in the fact that it clearly exposes the true physics involved in the flow of Newtonian fluids through straight conduits of a circular cross-section, a fluid mechanics problem of paramount importance in a wide range of fields of knowledge.

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