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Appendix

Section 2 of the paper identified the basic models integrated into the International Futures (IFs) system and listed some of the components within each. This appendix provides the key equations of the models related to the paper, explains the computational sequence, and gives basic information concerning the supporting database and parameterization of the models. Those users who wish additional information should turn to the Help system that is integrated with both the on-line and stand-alone versions of IFs (at http://www.ifs.du.edu/ifs/index.aspx) and explore the documents of the project, both working reports and publications (at http://www.ifs.du.edu/).

Each of the models within the IFs system is very large, generally comparable in character and structure to the most substantial models in their respective issue areas at institutions such as the United Nations Population Division (population forecasting), the World Bank (economic forecasting), the International Institute for Applied Systems Analysis (education forecasting), the World Health Organization (health forecasting), and so on. We therefore must be selective here with respect to documentation.

The model structure is recursive (sequential computation of each equation in every annual time step) rather than relying upon simultaneous or iterative within-year solution procedures. Much attention is paid to maintaining accounting identities, including (1) those around global production, consumption, and trade of food categories (crops and meat) and of energy types (oil, natural gas, coal, hydropower, nuclear power, and new renewables), both in physical and value terms; and (2) those involving inter-sectoral flows and inter-agent (households, firms, and governments) flows nationally and internationally, in value terms. Because the model's orientation is long-term forecasting, it is also important that it track stocks (accumulations such as the growth of atmospheric carbon and the decline of fossil fuel resource bases) as well as annual flows; yet the model structure is not systems dynamics in form, but rather a hybrid involving also many econometrically estimated specifications. Further, its long-term character and its integration of multiple issue areas means that specifications of algorithmic structures (such as endogenization of multifactor productivity driven by human, social, physical, and natural capital elements) can become as important as equations. All of this is to explain that the equations below are only a part of the overall system.

Sequencing of equations for recursive solution frequently involves moving out of one major model (e.g., population) into another (e.g., economics) and then later back again to earlier models. The sequencing is actually somewhat different in the first year of the model's computation, when many variables are initialized, than in all subsequent years. And we should point out that, prior to the first or base year of computation (currently 2010) the system relies on an extensive "pre-processor" of data for all its models, reconciling (again often with algorithms) physical and value estimates that are often incompatible and filling holes in data for the system's 183 countries (often using cross-sectional formulations tied to income levels). We focus here, however, only on the annual computations for years after the initialization.

Notation explanation. In the equations that follow we show variable names (explained in the text) in capital letters and parameters in lower case. We use bold face to represent values exogenous to the system, namely initial conditions of variables (from data) or parameters. The subscript "r" refers to geographic region, which in IFs is almost always a country (the model now represents 183 countries). Second subscripts represent additional dimensionality (s for economic sector, f for food type, e for energy type, g for government spending sector). The superscript "t" refers to the current time step; "t-1" to a variable computed in a previous time step and carried forward; and "t=1" to initial conditions.

Population and Economic foundations. The first calculations are of basic variables in what are essentially the two core models of the IFs system (see again Figure 1 of the article). In the demographic model we draw heavily upon the age-sex population distributions and other variables computed at the end of the previous time step to compute population (simply a sum across the age distribution), population growth, median age, HIV rate, AIDs deaths, calorie demands, sub-populations of importance (e.g., the size of the working-age population), and household-size. In the economic model, again using variables from previous years that we will explain below, we compute, inter alia, labor supply, female share of labor, exogenous technological growth, human capital, social capital, physical capital, knowledge capital, and productivity growth.

Agriculture. We then use such basic variables as important drivers for demand and supply sides in the physical models of the system, namely agriculture and energy, as well as a few infrastructure variables that we omit here because of more substantial treatment later (in interaction with variables in each of these models that also carry over from past years). Turning first to agricultural production, crop and meat/fish supply have very different bases and IFs determines them in separate procedures. Crop production depends on yields per hectare of land under cultivation and on the amount of land cultivated. Yield functions are almost invariably some kind of saturating exponential which represents decreasing marginal returns on inputs such as fertilizer or farm machinery. IFs also uses a saturating exponential, but imposes it on a Cobb-Douglas form. The Cobb-Douglas function is used in part to maintain symmetry with the economic submodel but more fundamentally to introduce labor as a factor of production along with capital. Especially in less developed countries (LDCs) where a rural labor surplus exists, there is little question that labor, and especially labor efficiency improvement, can be an important production factor. "Know-how" is also important in agriculture and there is therefore a technology term.

IFs computes yield in two stages. The first provides a basic yield (BYL) representing change in long-term factors such as capital and labor. The second stage uses this basic yield as an input and modifies it based on prices and therefore on the representation over time of the supply-demand equilibrium.

The basic yield (BYL) requires capital in agriculture (KAG), labor (LABS), technological advance (AGTECH), a scaling parameter (CD), and an exponent (CDALF). In addition a saturation coefficient (SATK) introduces the behavior of the saturating exponential. Interpret AGTECH as a factor-neutral technological progress coefficient.

$$BYL_r = CD_r * (1 + AGTECH_r)^{t-1}$$
$$* KAG_r^{\boldsymbol{CDALF}_{r,s=1}} * LABS_{r,s=1}^{(1-\boldsymbol{CDALF}_{r,s=1})} * SATK_r$$

where

$$CD_r = \frac{\boldsymbol{YL}_r^{t=1}}{\boldsymbol{KAG}_r^{t=1(\boldsymbol{CDAAG}_r)} * \boldsymbol{LABS}_{r,s=1}^{t=1\ (1-\boldsymbol{CDAAG}_r)}}$$

$$AGTECH_r = AGTECH_r^{t-1} * \left(1 + TECHGRO_{r,s=1}\right)$$

The saturation coefficient is a multiplier of the Cobb-Douglas function. It is the ratio of the gap between an exogenously specified maximum possible yield and the most recently computed yield to the gap between the maximum yield and the initial yield, raised to an exogenous yield exponent. With positive parameters the form produces decreasing marginal returns.

The basic yield represents the long-term tendency in yield but, because agricultural production levels are quite responsive to short–term factors such as fertilizer use levels and intensity of cultivation, the annual yield will vary significantly around that tendency. Those short-term factors under farmer control (therefore excluding weather) depend in turn on prices, or more specifically on the profit (FPROFITR) that the farmer expects. Because of computational sequence, we use food stocks as a proxy for profit level and adjust basic yield accordingly.

There are, however, additional factors that can influence agricultural yield. The one of importance to us here is global climate change. IFs therefore recomputes yield (YL), modifying it by two multipliers. The first summarizes the impact on yield of changes in precipitation and temperature resulting from global levels of atmospheric carbon (ENVYLCHG); we lag that variable from the previous time step and will see its computation near the end of this appendix. The second factor is a regional yield multiplier (ylm) that allows the model user to introduce assumptions about weather patterns and other uncertain elements in the agricultural system.

$$YL_r = BYL_r * \left(1 + ENVYLCHG_r^{t-1}\right) * \boldsymbol{ylm}_r$$

Finally, agricultural production (AGP) in the first or crop category is the product of yield and land devoted to crops (LD).

$$AGP_{r,f=1} = YL_r * LD_{r,l=1}$$

The production of fish has two components, ocean and mariculture. Total global ocean fish catch (OFSCTH) is set exogenously, as is each region's share in it (RFSSH) and the regional value of aquaculture (AQUACUL). Livestock production (AGPLV) is dependent on the herd size (LVHERD) and the slaughter rate (SLR). Total fish and livestock production, food category two, is the sum. Some food production will never make it to markets, but will be lost in the field or in distribution systems to pests, spoilage, *etc*. That loss (LOSS) is a function of GDP per capita in a table function that captures the tendency of loss to decrease with higher income levels. A loss multiplier (LOSSM) allows scenario introduction.

Energy. Basic total energy demand (BENDEM) for a given region or country is tied very closely to gross domestic product (GDP). IFs actually uses GDP from a previous time cycle (with an estimate of growth) because the recursive structure of IFs computes current GDP later.

The units of energy required for every unit of gross domestic product (ENDK) are a function of GDP per capita in purchasing power terms (GDPPCP), computed in a table function.

$$ENDK_r = \boldsymbol{TablFunc}(GDPPCP_r)$$

Initial data from countries/regions are unlikely to fall exactly on this table function initially. To reconcile computed energy demand (ENDEM) in the first year with empirical demand, IFs computes an internal adjustment multiplier (ENDM),which relies in turn on energy demand the first year; initial energy demand is apparent consumption computed from the sum across types of energy production (ENP) plus imports (ENM) minus exports (ENX).

$$BENDEM_r = GDP_r^{t-1} * (1 + GDPR_r^{t=1}) * ENDK_r * ENDM_r$$

$$where$$

$$ENDM_r = \frac{ENDEM_r^{t=1}}{GDP_r^{t=1} * \boldsymbol{ENDK}_r^{t=1}}$$

$$ENDEM_r^{t=1} = \sum^{E} \boldsymbol{ENP}_{r,e}^{t=1} + \boldsymbol{ENM}_r^{t=1} - \boldsymbol{ENX}_r^{t=1}$$

Final energy demand (ENDEM) is a price-responsive function of this basic energy demand. Possible tax on the consumer's price added by carbon taxes (cartaxenpriadd) is added to the basic market price. In an earlier version of the submodel, we used a smoothed or moving-average, regionally-specific energy price (SENPRI) relative to the initial price value (ENPRI). Because energy is a quite highly integrated global market, and in order to enhance behavioral stability, we have gone to using the world energy price (lagged one year) relative to initial price; prices affect demand through an elasticity (elasde). The user can force change in energy demand directly via an energy demand multiplier (endemm).

$$ENDEM_r = BENDEM_r *$$

$$\left(1 + \frac{\boldsymbol{cartaxenpriadd}_{r\,r} + WEP^{t\text{-}1} - \boldsymbol{ENPRI}_r^{t=1}}{\boldsymbol{ENPRI}_r^{t=1}} * \boldsymbol{elasde}_r\right) * \boldsymbol{endemm}_r$$

The basic computation of energy production (ENP) uses only capital as a factor or production. Energy production is the quotient of capital in each energy category (KEN) and the appropriate capital-to-output ratio (QE). The model user can modify a multiplier to this ratio (QEM) to represent changes in technology. The capital-to-output ratio is itself a function of resource availability. Known reserves (RESER) pose a direct constraint on production; they are constrained by ultimate resource assumptions in an important process not described here. The reserve-to-production ratio may not fall below a specified factor (PRODTF). In the case of oil and gas, for example, no more than about 10% of known reserves can be produced in a given year. Within the reserve constraint, the user can force increases or decreases in production via an energy production multiplier (ENPM). A capacity utilization factor (CPUTF) also affects the production level and is computed dynamically over time to help maintain market equilibrium (as are prices).

$$ENP_{r,e} = MIN \begin{cases} \dfrac{KEN_{r,e}}{QE_{r,e} * \boldsymbol{qem}_e} * \boldsymbol{enpm}_{r,e} * CPUTF_r \\ \dfrac{RESER_{r,e}}{\boldsymbol{prodtf}_{r,e}} * CPUTF_r \end{cases}$$

Return to the Economic Model and Production. The physical flows of the partial equilibrium models for energy and agriculture, along with the change over time in relative prices for those goods (computed in processes that equilibrate the global market but also represent changing production cost fundamentals), provide inputs to two of the six sectors in the economic model (those six being agriculture, energy, other raw materials, manufactures, energy, and information and communications technology). They can therefore next be integrated with more value-based computations for the other sectors in the important production side of the economic model.

A Cobb-Douglas function produces value added (VADD) as a function of capital (KS) and labor (LABS), a cumulative technological growth factor (TEF), and a scaling parameter (CDA) computed in the first time step. The capital exponent CDALFS) and its labor complement are endogenous, and the capital share declines with GDP per capita [1].

$$VADD_{r,s} = CDA_{r,s} * TEF_{r,s} * KS_{r,s}^{\boldsymbol{CDALFS}_{r,s}} * LABS_{r,s}^{(1-CDALFS_{r,s})}$$

$$\begin{aligned} &where \\ &TEF_{r,s} = TEF_{r,s}^{t-1} * \left(1 + MFPGRO_{r,s}\right) \end{aligned}$$

The annual growth rate in multifactor productivity (MFPGRO) requires, of course, further explanation. As discussed above, there is a base rate (MPRATE) linked to systemic technology advance and a convergence premium. Specifically, the base rate sums the exogenously specified rate of advance in the leader (mfpleadr) and the premium computed for convergence of each country/region (MFPPrem), a function of GDP per capita at purchasing power parity (GDPPCP).

$$\begin{aligned} &MFPGRO_{r,s} = MFPRATE_{r,s} \\ &+ HumanCapitalTerm_{r,s} + SocialCapitalTerm_{r,s} \\ &+ PhsyicalCapitalTerm_{r,s} + KnowledgeTerm_{r,s} \\ &+ MFPCOR_{r,s} \end{aligned}$$

$$\begin{aligned} &where \\ &MFPRATE_{r,s} = \boldsymbol{mfpleadr}_s + MFPPrem_r \\ &where \\ &MFPPrem_r = Func\left(GDPPCP_r\right) \end{aligned}$$

On top of the base rate, multiple (currently four) terms additively affect/shift growth over time, each comparing country performance with structural expectations [2]. The model computes an adjustment or correction factor (MFPCOR) in the first year so as to make the overall growth rate initially consistent with recent historical experience for the country.

Turning to the four clusters of drivers discussed above, we discuss the human capital term illustratively. The annual change in MFP attributable to education (CNGEDUC) is the sum of two terms. The first compares the endogenous computation of average years of education (EDYRSAG25) of the population at age 25 or older (responsive to all of the factors represented in the education module) minus the expected value of the same variable computed from a cross-sectional function (EXPECTEDEDYRSAG25). The second term similarly compares the portion of the GDP that government directs to education (g=EDUC) with the expected value of the same ratio. The contribution to the human capital from health is directly comparable. Four parameters from the literature (in bold face) convert differences from expected values into shifts of productivity growth.

$$HumanCapitalTerm_{r,s} = CNTEDUC_r + CNGHLTH_r$$
$$CNGEDUC_r = (EDYRSAG25_r - EXPECTEDYRSAG25_r) * \boldsymbol{mfpedyrs}$$
$$+ \left(\frac{GDS_{r,g=EDUC}}{GDP_r} - Expected \frac{GDS_{r,g=EDUC}}{GDP_r} \right) * \boldsymbol{mfpedspn}$$
$$CNGHLTH_r = (LIFEXP_r - ExpectedLIFEXP_r) * \boldsymbol{mfplife}$$
$$+ \left(\frac{GDS_{r,g=Health}}{GDP_r} - Exppected \frac{GDS_{r,g=Health}}{GDP_r} \right) * \boldsymbol{mfphlspn}$$

Often across the IFs system, we estimate our own parameters from our database of over 2,000 series across the multiple issue areas. But in many critical areas, especially those in which there are large literatures, we draw from those literatures so as to incorporate expertise that ranges far beyond our own. Hughes [3] described the parameterization of the production system, drawn from an extensive literature of estimations and stylized facts on productivity [4]. Illustratively, parameterization considered years of education and educational expenditures as a pair. Analyses in the literature include:

- Barro and Sala-i-Martin [5] reported that a 1 standard deviation increase in male secondary education raised economic growth by 1.1% per year, and a 1 standard deviation increase in male higher education raised it by 0.5%. Barro [6] reported that one extra year of male upper-level education raised growth by 1.2% per year.
- Chen and Dahlman [7] concluded that a rise of 20% in average years of schooling raises annual growth by 0.15 percent and that an increase in average years by 1 year raises growth by 0.11 percent.
- Jamison, Lau, and Wang [8] used the Barro-Lee measure of average years of school for males between 15 and 60, but concluded that the “effect was small”.
- Bosworth and Collins [9] argued that each year of additional education adds about 0.3% to annual growth.
- The OECD [10] found that one additional year of education (about a 10% rise in human capital) raised GDP/capita in the long run by 4–7%.
- Barro and Sala-i-Martin [5] concluded that increasing education spending as a portion of GDP by 1.5 points (one standard deviation) raised growth by 0.3%.

- Baldacci, Clements, Gupta, and Cui [11] found that raising education spending in developing countries by 1% a year and keeping it higher added about 0.5% per year to growth rates. They also found that 2/3 of the effect of higher spending is felt within 54 years but the full impact shows up only over 10–15 years.

Gross regional or domestic product (GDP) is simply the sum of value added across sectors, which would also equal the sum of production for final demand across sectors. And the GDP per capita (GDPPC) follows easily.

The basic GDP figures for the model are represented in dollars at official exchange rate values. It is important, however, to estimate the value of GDP and GDPPC at purchasing power parity levels as well (GDPP and GDPPCP). To do that we need to compute a purchasing power parity conversion value (PPPConV). Data sources provide the initial conversion value. IFs uses an analytic function based on GDP per capita to compute change in the conversion value over time.

$$GDPP_r = GDP_r * PPPConV_r$$

$$GDPPCP_r = GDPPC_r * PPPConV_r$$

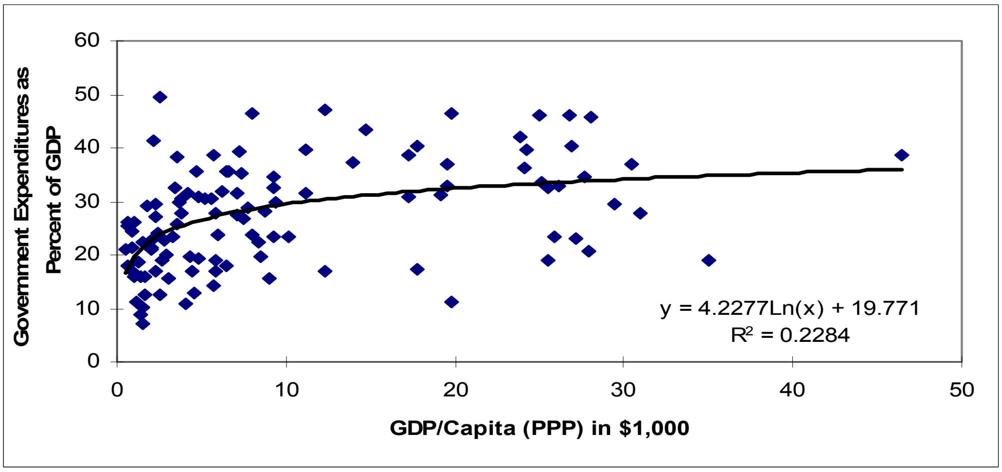
$$where$$

$$PPPConV_r = PPConV_r^{t=1} * \frac{AnalFunc(GDPPC_r)}{AnalFunc(GDPPC_r^{t=1})}$$

Broader Financial Flows and the Social Accounting Matrix. The computational flow moves next to financial flows, beginning with computations of assorted international flows, including foreign direct investment (maintaining stocks over time as well as flows), portfolio investment, IMF and World Bank credits and loans, and worker remittances. As in many areas of the model, we do not, of course, expect to be able to forecast these with any reasonable accuracy for 183 countries over the long run. But they are important variables for which we can provide basic relationships, thereby also adding handles for users undertaking scenario analysis.

Turning to the domestic side of financial flows, and beginning with expenditures, Figure A.1 shows the function estimated cross-sectionally in order to fill the relatively few holes in government expenditures as a portion of GDP (using data from the World Development Indicators).

Figure A.1. Government Expenditure Share as Function of GDP/capita (PPP).



Government expenditures consist of a combination of direct consumption/expenditure and transfer payments. As a general rule, transfer payments grow with GDP per capita more rapidly than does consumption. And within transfer payments, pension payments are growing especially rapidly in many countries, particularly in more-economically developed ones.

In future years the total of government expenditures is calculated from the sum of direct consumption and transfers. The two components, however, each require a moderately complex calculation that we do not elaborate here. Computation of government consumption (direct expenditures on the military, education, health, R&D, foreign aid, and other categories) begins with use of the function to compute an estimated government consumption (EstGovtConsum) as a portion of GDP, using GDP per capita (PPP) as the driver. The initialization discussion above showed the empirical base of that function. It carries a behavioral assumption of generally increasing expenditures with increases in GDP per capita.

The estimated value then enters a convergence calculation that IFs uses in a number of instances. In the first year a ratio term (GovConR) was computed that represented the degree to which a country's consumption/GDP differed from the estimated value. That ratio multiplies the estimated term in future years, allowing the function normally to increase consumption/GDP as GDP per capita rises. At the same time, such divergence from estimated functions is almost as often a matter of data inadequacy or of temporary factors for a country as it is of persistent idiosyncrasy. The convergence function allows the country/region's value to converge towards the functional calculation over a period of time (govfinconv), usually quite long. Such convergence also helps avoid ceiling effects (e.g., government consumption as 100% of GDP) as GDP per capita rises.

The second term in the equation below is called the Wagner term, after the discoverer of the long-term behavioral tendency for government consumption to rise as a share of GDP, even at stabile levels of GDP per capita. This is built into the consumption calculation through an exogenous parameter (wagnerc) that is multiplied by the number of the forecast year.

$$
\begin{aligned}
&GOVCON_r = Converge(EstGovtConsum_r * GovConR_r^{t=1}, EstGovtConsum_r, \boldsymbol{govfinconv}) \\
&* WagnerTerm * \boldsymbol{govexpm}_r * MulExp_r^{t-1} \\
&where \\
&WagnerTerm = 1 + t * \boldsymbol{wagnerc} \\
&EstGovtConsum_r = AnalFunc(GDPPCP_r^{t=1})
\end{aligned}
$$

Almost finally, government consumption is further modified by an exogenous multiplier of government expenditures, allowing the user to directly control it by country/region and by an endogenously computed multiplier on expenditures (MulExp) that reflects the balance or imbalance in government expenditures and the debt level. Finally, and not shown, there is a simple adjustment to reflect the effect that changing levels of foreign assistance receipts can have on consumption.

The division of government expenditures into target destination categories (GDS) is, of course, also a key agent-class behavior. We do not describe it in detail here, but it involves determining demand for military, health, education, R&D, infrastructure and a residual other category of expenditures from extended representations of the demand for all but R&D and the residual other category. Actual expenditures are normalized to total government consumption.

Governance. The IFs system represents a number of governance variables in the general categories of security, capacity, and inclusion. Here we illustrate just two. With respect to capacity, one of the most powerful measures of capacity (or more accurately, lack of capacity) may well be corruption. We rely in our analysis on the Transparency International measure of corruption perceptions, which in spite of the name they give it is actually a measure of transparency (higher values are more transparent or less corrupt). Note that the basic formulation in IFs for corruption/transparency (below) contains four drivers, all of which are significant, and which collectively explain nearly 80 percent of the cross-country variation in corruption in the most recent year of data for each variable. The first term, and the one that by itself explains the most variation, is a long-term development term, in this case GDP per capita (for some variables to be discussed below, such as democracy, that development variable is years of education).

Interestingly another very powerful term is the UNDP Gender Empowerment Measure (GEM), which, in spite of its high correlation with GDP per capita, makes its own contribution. A secondary term is the extent of democracy using the Polity scale (DEMOC). That this term makes an independent contribution to transparency suggests the power that inclusion may have to increase accountability and transparency, reducing corruption. An even-less-powerful but still-significant term is the dependence of the country on exports of energy (ENX) converted to value terms with prices (ENPRI)—in a few years, and in the aftermath of the Arab Spring beginning in 2011, it will be interesting to know if this term drops out of analyses of change in governance regime and character. A multiplier for scenario analysis is the only exogenous element added to the basic formulation (govcorruptm). This equation has an R-squared in 2010 of 0.76.

$$GOVCORRUPT_r^t = (1.576 + 0.1133 * GDPPCP_r^t + 2.270 * GEM_r^t + 0.02779 * DEMOC_r^t - 0.04566 * (\mathrm{ENX}_r^t * \mathrm{ENPRI}_r^t / \mathrm{GDP}_r^t)) * \boldsymbol{govcorruptm_r^t}$$

With respect to inclusion, we pay particular attention to regime type. As with capacity, the forecasting of regime type in IFs has multiple elements: (1) a basic statistical formulation tied to literature analysis and our own estimations; (2) a recognition of country-specific differences (tied in part to path dependencies); and (3) an algorithmic specification of a number of additional factors, including global waves and neighborhood effects.

Most analyses of democratization place much emphasis on a developmental variable such as GDP per capita. GDP per capita and adults' years of education are very highly correlated across countries, and we found that, although the correlation of GDP per capita and democracy level is slightly higher than that of education years and democracy, when we added the size of the youth bulge and the extent of dependence on energy exports, the better broad developmental driving variable proved to be years of adults' education. With additional exploration, however, we found a slight further advantage for the Gender Empowerment Measure, and so replaced the education variable with the GEM (which is, itself, strongly influenced by adults' education). In the equation below, the basic IFs formulation, all terms are significant with T-scores above 2.0 in absolute terms. In earlier work we also explored a linkage to the survival/self-expression dimension of the World Value Survey, but have found that other development variables statistically force it out of the relationship.

$$DEMOC_r^t = 13.39 + 11.37 * GEM_r^t - 9.734 * YTHBULGE_r^t - 0.2317 * (\mathrm{ENX}_r^t * \mathrm{ENPRI}_r^t / \mathrm{GDP}_r^t)) * \boldsymbol{democm_r^t}$$

IFs has the capability of doing an historical simulation between 1960 and 2010 so that we can compare our forecasts with data. Our forthcoming governance volume [12] documents our use of that in order to build a broader forecasting structure on top of the basic equation above, as well as documenting the rest of the governance model. Governance variables enter the economic model primarily via the production function described above.

Agricultural Demand. Sequentially it could have been computed earlier (many of the IFs sequential steps could be changed), but agricultural demand is dependent on estimates of income. Crop demand has three components: feed, industrial and food. These equations are important but do not greatly affect the dynamics that surround analysis in this article, so we do not document them here.

Back to the Economy: But looking forward with investment. The determination of investment by destination that will carry changes in capital stock to the next time period is a two-step procedure. First, IFs computes demand for investment by each sector (IFSDEM), responsive primarily to inventory (or stock) levels. This is a reasonably extensive process involving the use of what engineers term a PID controller to feed back information from inventories (the integral of disequilibrium and annual change of inventories (the derivative term in PID) to the demand for investment funds.

More generally, a variety of PID controller mechanisms help the model in the chasing of equilibrium over time. These mechanisms show up in all price calculations (food and energy prices in the physical models and relative prices of all other sectors in the economic model), in determinations of interest rates for balancing savings and investments, and in determinination of exchange rates for relative currency values. It is typical to talk of alternative "closures" in describing economic models, that is the use of hard specification of supply or demand side variables to determine equilibrium. Our more open method of search for it with signals back to the supply and demand sides allows both exogenous interventions on both sides (related to the kinds of scenario specifications described in the article) and more elaborate specifications of both supply and demand sides, including the multiple linkages across models that this appendix has been describing.

Building Infrastructure. Here we compute many infrastructure demand and access variables including Road Density, Paved Roads, Rural Roads Access Index, Cost of adding a lane km of paved road, Land Area Equipped for Irrigation, Per Hectare Cost of equipping land for irrigation, Fixed Telephone Line density, Cost of adding a fixed land line, Access to Electricity Grid, Electricity Consumption, Electricity Transmission Loss, Electricity Generation Capacity, Electricity Generation Capacity Cost, Computers per 100 people, Access to Sanitation facilities, Cost of Sanitation, Access to Safe Water, Cost of Safe Water. We illustrate this with only one, electricity access.

$$INFRAELECACC_{urban} = \frac{100}{1 + e^{-(1.144 - 4.858 * poverty\ level + 0.837 * GOVEFFECT)}}$$

$$INFRAELECACC_{rural} = \frac{100}{1 + e^{-(-0.500 - 6.925 * poverty\ level + 0.858 * GOVEFFECT)}}$$

where

> INFRAELECACC is the percentage of the urban or rural population with access to electricity, poverty level is the fraction of the total population that lives on less than $1.25 per day, and GOVEFFECT is a measure of governance effectiveness developed as part of the World Bank's World Governance Indicators project.

We recognize that there is a strong connection between the use of electricity and of solid fuels in the home. In general, as households move up the energy ladder, they increase their use of the former and decrease their use of the latter. We also include a link from access to electricity to the use of solid fuels in the home. This in turn enters the health model and affects the level of respiratory disease.

International Political Variables. We next compute a number of international political variables, including a power measure based on hard capabilities and an estimate of intra-dyadic threat. Those are not of great relevance to this article, so we do not elaborate them.

Population Dynamics. We are in a position at this point to compute a number of variables relevant to the dynamics of population over time. Although births, deaths, and migration all influence population dynamics, the most influential of the three is births. We therefore focus here on the critical variable, total fertility rate (TFR). IFs determines the TFR and then imposes that on the fertility distribution of the region/country.

Infant mortality (INFMOR), years of average education for those 15 and older (EDYRSAG15), and contraception use (CONTRUSE) are key drivers of fertility rates. In addition there is an exogenous multiplier on the rate (tfrm), and shift in that function with technological or cultural change (ttfrr).

$$TFR_r = (3.8812 + 0.0217 * INFMOR_r - 0.8327 * \ln(EDYRAG15_r) \\ - 0.0095 * CONTRUSE_r) * \boldsymbol{tfrm_r} * \left(1 + (t-1) * \boldsymbol{ttfrr}\right)$$

Total fertility rate is, however, unlikely to shift indefinitely toward zero. In fact, it requires a value of about 2.0 simply to maintain a steady population (unless life expectancies are growing). TFR is therefore bound by a minimum that responds to a global parameter (tfrmin) normally set at either 1.5 or 1.8.

Once we have computed the total fertility rate (TFR), the number of births in a given year is a simple function of the fertility distribution and the TFR. On the mortality side, mortality patterns determine life expectancy and affect the progression of each age category through time. IFs includes an entire health model, based on work from the Global Burden of Disease project of the World Health Organization, but we do not need to document that here. We also compute other demographic variables of importance at this point including contraceptive use, births, deaths, infant mortality, crude birth rate, crude death rate, calories per capita, and malnourished children.

Other Human Development Variables. At this point we turn to the education model of IFs and compute expenditures per student, gross enrollment demand, graduates per level, years of education for people over 25 and for people over 15, and literacy.

Having computed economic, health, and education variables, we are able to compute also the Human Development Index (HDI) in the standard equation of the United Nations Human Development Report Office.

Other Variables, Indicators, and Forward Linkages. At this stage there are further calls to many of the models in IFs, some of them repeatedly, in order to calculate a wide range of variables that carry over to the next time step and of indicators of interest to model users. These include health variables such as smoking prevalence; smoking impact; BMI; obesity; mortality by country, age, gender and disease type; life expectancy; deaths per disease type; infant mortality; crude death rate; population growth rate; years of life lost; and years lost to disability. They also include: income-related variables such as household income per capita, domestic Gini, population living with income under $1.25/day and $2/day, poverty gap, household savings, firm savings, and global Gini; environmental variables such as urban pollution measured with PM2.5 levels, annual carbon emissions from fossil fuels, advanced sustainability analysis, precipitation change, temperature change, and agricultural yield change; agricultural variables such as return ratio on land/yield investment, investment in agriculture, urban built-up land development, crop land development, and grazing land development; knowledge system variables such as knowledge system index, knowledge human capital index, knowledge ICT index, knowledge innovation index, and knowledge international transfer index.

To illustrate some of special importance to this article, consider carbon emissions and the stock of atmospheric carbon. The beginning point for examining the greenhouse effect is calculation of the atmospheric carbon dioxide in parts per million (C02PPM). The model calculates annual emissions of carbon from energy use (CARANN) and adds it to a cumulative tracking of carbon (SACARB), initialized exogenously for 2010 (carint). Emissions depend on global production (WENP) in the fossil fuel categories (oil, gas and coal), using fuel-specific coefficients representing tons of carbon generated per barrel of oil equivalent burned (carfuel). The oceans and other sinks annually absorb an exogenously specified amount of atmospheric carbon (carabr) and that retards the accumulation. Deforestation (or reforestation) has an impact via another parameter (carforst).

$$CARANN = WENP_{e=1} * \mathbf{carfuel1} + WENP_{e=2} * \mathbf{carfuel2} + WENP_{e=3} * \mathbf{carfuel3}$$

$$SACARB = SACARB^{t-1} + CARANN + (WFORST^{t-1} - WFORST) * \mathbf{carforst - carabr}$$

where

$$SACARB^{t=1} = \mathbf{carinit}$$

We use a table function (based on figures from the IPCC) to determine the average world temperature (WTEMP) in Centigrade from the atmospheric carbon dioxide level in parts per million.

$$WTEMP = TablFunc(CO2PPM)$$

Given forecasts of global temperature change over time we are able to compute temperature and precipitation changes post 1990 for each country (TEMPCHG and PRECHG) using data compiled for the MAGICC/SCENGEN climate model [13]. Building on work by Cline [14] and Rosenzweig and Igelesias [15] we then estimate a variable that combines the effects of those variables with carbon fertilization into a multiplier on agricultural yield resulting from environment (ENVYLCHG). We saw earlier the impact of this on yields.

Conclusion. As we indicated at the beginning of this appendix, the IFs modeling system is a compilation of many very large individual models. As a result, it is impossible to provide full detail here. We have tried, instead, to indicate the key equations related to this article, the overall dynamics of annual computations, the roots of the system in an extensive database, and the widespread reliance on the expertise of others to structure the models, their equations and our parameterization. We welcome inquiries for more information.

References and Notes

1. Estimation of the relationship for capital share uses Global Trade and Analysis Project (GTAP) data, as do a number of other aspects of the model. For instance, the input-output matrices and factor.
2. Not shown, there is also an exogenous additive parameter (mfpadd) allowing users to intervene and change growth paths for any country/region. The presentation of equations here omits a number of such "exogenous handle" parameters and terms not central to the exposition.
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