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Nonlinear Approach to Jouguet Detonation in Perpendicular Magnetic Fields

Andriy A. Avramenko [1], Igor V. Shevchuk [2,*], Margarita M. Kovetskaya [1], Yulia Y. Kovetska [1] and Andrii I. Tyrinov [1]

[1] Institute of Engineering Thermophysics, National Academy of Sciences, 03057 Kiev, Ukraine; tgetu.ittf@gmail.com (A.A.A.); kovetskamargarita@ukr.net (M.M.K.); kovetskajulia@ukr.net (Y.Y.K.); tyran@ukr.net (A.I.T.)
[2] Faculty of Computer Science and Engineering Science, TH Köln—University of Applied Sciences, 51643 Gummersbach, Germany
* Correspondence: igor_v.shevchuk@th-koeln.de

Abstract: The focus of this paper was Jouguet detonation in an ideal gas flow in a magnetic field. A modified Hugoniot detonation equation has been obtained, taking into account the influence of the magnetic field on the detonation process and the parameters of the detonation wave. It was shown that, under the influence of a magnetic field, combustion products move away from the detonation front at supersonic speed. As the magnetic field strength increases, the speed of the detonation products also increases. A dependence has been obtained that allows us to evaluate the influence of heat release on detonation parameters.

Keywords: detonation; heat release; shock wave; magnetic field

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1. Introduction

Studies of the influence of magnetic fields on combustion and explosion processes have aroused great interest from a wide range of technical applications, from energy to aerospace engineering [1–3]. Review [1] analyzes studies of the influence of the magnetic field on the intensity of the explosion, engine parameters, and flame characteristics.

Plane detonation waves under the influence of a magnetic field in free-flow pipes were studied in [3]. It was found that the magnetic field has virtually no effect on the detonation pressure behind the magnetic field, but has an obvious effect on the structure of the shock wave front, in cases where a magnetic field already exists. It is shown that the magnetic field affects the structure of the shock wave and increases the speed of the combustion front.

A magnetic field with a strength of 0.4 to 0.5 Tesla was shown to have a strong influence on the propagation of the combustion front [3].

Electromagnetic waves from the explosion of Trinitrotoluene (TNT) were experimentally studied in [4]. Experimental observations have shown that electromagnetic radiation occurs after the detonation of powerful explosive charges. The expansion of the detonation products caused a strong impact on the surrounding air. This caused an intense heat wave (T~11,000 K) lasting ~20 microseconds. Such temperatures created significant ionization of the air and induced electric and magnetic fields.

A mechanism for the generated magnetic disturbance was proposed in [5], based on a thermodynamic equilibrium ionization model and a magnetohydrodynamic (MHD) model, taking into account magnetic diffusion. The magnetic disturbance caused by the electromagnetic wave generated by the explosion was simulated and compared with experimental data.

A qualitative analysis of the structure of the electromagnetic field during the detonation of a condensed explosive in a magnetic field was performed in [6]. It was shown that the magnitude of the electric current increases under the influence of a detonation wave. The mechanism of the explosive disturbance of the natural magnetic field was considered in [7]. These authors numerically simulated the magnetic disturbance caused by an electromagnetic wave. It was shown that a clear magnetic disturbance can be observed only in the region of high conductivity. The authors of [8] present a study of the influence of a constant magnetic field and explosion-extinguishing materials on the explosion reaction of a C_3H_8/air mixture. It has been shown that ferromagnetic materials have better explosion-suppression properties and a magnetic field increases the ability of explosion-extinguishing materials to suppress an explosion. The effect of a magnetic field on the explosion of alkanes was studied in [9]. Here, the influence of the magnetic field on the maximum explosion pressure, the rate of pressure rise, and the speed of flame propagation was experimentally studied. It turned out that the magnetic field reduces the maximum explosion pressure, the rate of pressure rise, and the speed of flame propagation of the alkane gas, which reduces the intensity of the explosion. The authors of [10] present the results of a study of the influence of an electromagnetic field on an explosion of 9.5% methane in air. As a result, an increase in the combustion rate under the influence of an electromagnetic field was revealed.

The authors of [11,12] analyze the propagation of converging cylindrical detonation waves in an ideal [11] and non-ideal [12] gas with different initial densities and a variable azimuthally magnetic field. Three cases are considered, (1) when the gas is weakly ionized, (2) when the gas is strongly ionized, and (3) when the non-ionized gas undergoes intense ionization as a result of the passage of the detonation front. It was shown that the azimuthal magnetic field has a damping effect on the approach of the detonation front.

The studies of the propagation of shock waves in an ideal and non-ideal gas were performed in [13–16]. The effects of flow turbulence [13,16], the presence of nanoparticles in the gas [15], and the non-ideal parameters of the van der Waals gas [14] on the intensity of heat and mass transfer during the passage of the shock wave front have been revealed and discussed.

Studies of heat and mass transfer during the propagation of shock waves in microchannels were carried out in [17,18]. A comprehensive study [17] of the propagation of detonation in micro- and macro-channels showed that the flame acceleration regimes in these channels are different. In micro-channels, the main influence on the interaction of the flow with the wall is the propagation of the detonation wave, which leads to a loss of momentum and, as a consequence, to a decrease in the detonation velocity. The mechanism of detonation breakdown in macroscale and microscale channels was experimentally studied in [18]. It was shown that the mechanism of detonation propagation in microchannels is mainly influenced by boundary conditions.

The propagation of a detonation wave resulting from the release of energy during an exothermic chemical reaction in the presence of magnetic and electric fields was studied in [19]. Evidence is given for the existence of weak and strong detonation waves in a transverse magnetic field for a single-stage exothermic reaction. Stable and unstable manifolds of the resulting system are studied to prove the existence of weak and strong detonation waves.

The process of flame propagation in a closed pipe during the explosion of a propane/air gas mixture in a gradient electromagnetic field was experimentally studied in [20]. The experiments showed that the gradient electromagnetic field slowed down the combustion process and suppressed the increase in explosion pressure and the speed of flame propagation. A review of the influence of a magnetic field on the combustion and hydrocarbon emissions of magnetically conditioned hydrocarbon fuels was performed in [21]. Discussed here are studies of the influence of magnetic fields on the structure of hydrocarbons, flame behavior, and the performance of internal combustion engines. Most studies have shown that engine performance can be improved. However, conflicting results have been obtained regarding pollutant emissions, which require further in-depth studies.

Combustion and magnetohydrodynamic processes in improved rocket engines with pulsed detonation are considered in [22]. Research is focused on exploring potential rocket engine systems that involve magnetohydrodynamic (MHD) phenomena. In 1998, Dr. Jean-Luc Cambier proposed a new combined cycle propulsion concept, the pulsed detonation magnetohydrodynamic ejector (PDRIME). PDRIME is one of many promising MHD thrust boosting ideas for modern powertrain applications. A global study of the PDRIME propulsion concept and a detailed study of the underlying physics of detonation and MHD interactions was conducted. Control methods have been identified to improve system performance.

Despite the fact that studies of detonation waves in a magnetic field have been carried out for a long time [23], data on the influence of a magnetic field on the characteristics of a detonation wave have turned out to be contradictory [21] and require further investigation. Thus, the results of the study in [3] showed that the magnetic field did not affect the detonation pressure, but affected the structure of the shock wave and increased the speed of the pressure front. The results of experimental studies [9,20] showed that the electromagnetic field slows down the combustion process and suppresses the speed of flame propagation. An increase in the speed of a detonation wave under the influence of an electromagnetic field was discovered in [10].

As the above review shows, the strong physicochemical coupling produced by the interaction of magnetic fields and detonation phenomena has many potential applications, from detonation engines and power generation to aerospace applications and nuclear explosions. Research focuses, in particular, on the magnetic field influence on the combustion process of air/propane mixtures in high-speed flows. Also, a wide range of geophysical problems are associated with the study of electromagnetic effects during seismic activity, in the processes of deformation, and in the destruction of rocks in mines. Research into the interaction of an electromagnetic field with a shock wave is associated with the need to control nuclear tests.

For this reason, the objective of this work is to theoretically study the influence of a magnetic field on the Jouguet detonation process in non-ideal gases.

This will include an investigation into the effect of a magnetic field on the process of Jouguet detonation [24,25].

2. Mathematical Model

The schematic of the process of detonation in the magnetic field of an ideal gas, which passes through a plane detonation wave, considered in the present study is outlined in Figure 1.

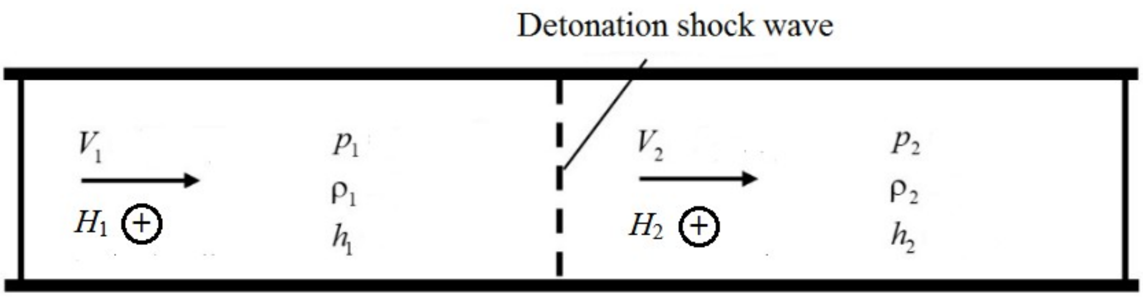

Figure 1. Schematic outline of the shock wave geometry.

A steady-state gas flow is described using the following system of mass, momentum, and energy conservation equations [26]

$$\rho_1 V_1 = \rho_2 V_2 \tag{1}$$

$$p_1 + \rho_1 V_1^2 + \frac{\mu_e H_1^2}{2} = p_2 + \rho_2 V_2^2 + \frac{\mu_e H_2^2}{2} \tag{2}$$

$$\rho_1 V_1\left[\left(h_1+\frac{V_1^2}{2}\right)+q\right]+\mu_e V_1 H_1^2=\rho_2 V_2\left(h_2+\frac{V_2^2}{2}\right)+\mu_e V_2 H_2^2 \tag{3}$$

$$V_1 H_1=V_2 H_2 \tag{4}$$

Here, V is velocity, ρ is density of a gas, p is pressure, h is enthalpy, μ_e is the magnetic permeability, H is the magnetic field strength, and q is the heat removed ($q < 0$) or released ($q > 0$) per unit mass by the corresponding process undergoing across the shock. Here, the subscript "1" denotes the parameters before the shock wave and the subscript "2" denotes the parameters after the shock wave.

Systems (1)–(4) are a modified system of Rankine–Hugoniot equations [27,28]. The Rankine–Hugoniot conditions (or jump conditions), also called the Rankine–Hugoniot relations, represent the relation between the states before and after a shock wave or a combustion wave (deflagration or detonation) in a one-dimensional fluid flow or under one-dimensional deformation in solids.

Systems (1)–(4) are closed using the equation of state for an ideal gas, as follows:

$$p=\rho\Re T \tag{5}$$

where T is the temperature and $\Re$ is the individual (specific) gas constant, together with the Mayer's relation:

$$\Re=\frac{k-1}{k} \tag{6}$$

Here, $k = c_p/c_v$ is the specific heat ratio, c_p is specific heat capacity at constant pressure, and c_v is the specific isochoric heat capacity.

Textbooks on technical thermodynamics indicate that the ideal gas equation for air is valid at pressures up to 10 bar and temperatures up to 800 °C. In principle, the whole theory of detonation proposed by Jouguet [24,25] is based on the use of the ideal gas equation. The authors, in this sense, follow the classical approach of Jouguet [24,25]. It was shown in [26] that the ideal gas equation can be used to simulate shock waves in plasma. Therefore, in this work, we use the ideal gas law to study detonation. The influence of the properties of the van der Waals gas on the characteristics of shock waves was studied in our work [14].

As a result, Equation (3) can be re-written in the following form:

$$\frac{k}{k-1}\frac{p_1}{\rho_1}+\frac{V_1^2}{2}+q+\frac{\mu_e H_1^2}{\rho_1}=\frac{k}{k-1}\frac{p_2}{\rho_2}+\frac{V_2^2}{2}+\frac{\mu_e H_2^2}{\rho_2} \tag{7}$$

Here, we used the following equation for the enthalpy:

$$h=c_p T \tag{8}$$

3. Modified Hugoniot Equation

Keeping Equations (1) and (4) in mind, we can recast Equation (2) as follows:

$$\begin{gathered}p_1-p_2=\rho_1 V_1(V_2-V_1)+\tfrac{\mu_e}{2}\left(H_2^2-H_1^2\right)=\rho_1 V_1(V_2-V_1)+\tfrac{\mu_e V_1^2 H_1^2}{2}\left(V_2^{-2}-V_1^{-2}\right)= \\ \rho_1 V_1(V_2-V_1)+\tfrac{\mu_e V_1^2 H_1^2}{2}\left(\tfrac{\rho_2^2}{\rho_2^2 V_2^2}-\tfrac{\rho_1^2}{\rho_1^2 V_1^2}\right)=\rho_1 V_1(V_2-V_1)+\tfrac{\mu_e H_1^2}{2\rho_1^2}\left(\rho_2^2-\rho_1^2\right)\end{gathered} \tag{9}$$

Multiplying Equation (9) by the following factor:

$$\frac{V_2+V_1}{\rho_1 V_1}=\frac{1}{\rho_1}+\frac{1}{\rho_2} \tag{10}$$

one can obtain the following:

$$(p_1-p_2)\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}\right)=V_2^2-V_1^2+\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}\right)\frac{\mu_e H_1^2}{2\rho_1^2}\left(\rho_2^2-\rho_1^2\right) \tag{11}$$

It follows from Equation (8) that:

$$V_2^2 - V_1^2 = \frac{2k}{k-1}\left(\frac{p_1}{\rho_1} - \frac{p_2}{\rho_2}\right) + 2\mu_e\left(\frac{H_1^2}{\rho_1} - \frac{H_2^2}{\rho_2}\right) + 2q = \frac{2k}{k-1}\left(\frac{p_1}{\rho_1} - \frac{p_2}{\rho_2}\right) + 2\frac{\mu_e H_1^2}{\rho_1^2}(\rho_1 - \rho_2) + 2q \tag{12}$$

A comparison of Equations (11) and (12) yields:

$$\left(1 - \frac{p_2}{p_1}\right)\left(1 + \frac{\rho_1}{\rho_2}\right) - \left(1 + \frac{\rho_1}{\rho_2}\right)\frac{\mu_e H_1^2}{2p_1}\left(\frac{\rho_2^2}{\rho_1^2} - 1\right) = \frac{2k}{k-1}\left(1 - \frac{p_2\rho_1}{\rho_2 p_1}\right) + 4\frac{\mu_e H_1^2}{2p_1}\left(1 - \frac{\rho_2}{\rho_1}\right) + \frac{2q\rho_1}{p_1} \tag{13}$$

or

$$\left(1 - \frac{p_2}{p_1}\right)\left(1 + \frac{\rho_1}{\rho_2}\right) = \frac{2k}{k-1}\left(1 - \frac{\rho_1}{\rho_2}\frac{p_2}{p_1}\right) + \frac{\mu_e H_1^2}{2p_1}\left(\frac{\rho_2}{\rho_1} - 1\right)^3 / \frac{\rho_2}{\rho_1} + \frac{2q\rho_1}{p_1} \tag{14}$$

Solving Equation (14), with respect to p_2/p_1, one can derive the following:

$$\frac{\overline{p}_2}{\overline{p}_1} = \frac{\frac{(k+1)}{(k-1)} - \frac{\rho_1}{\rho_2} + Q + \frac{(R-1)^3}{R}M}{\frac{(k+1)}{(k-1)}\frac{\rho_1}{\rho_2} - 1} \tag{15}$$

In a dimensionless form, this equation looks as follows:

$$P = \frac{\gamma - R^{-1} + Q + \frac{(R-1)^3}{R}M}{\gamma R^{-1} - 1} \tag{16}$$

where

$$P = \frac{\overline{p}_2}{\overline{p}_1}, \qquad R = \frac{\rho_2}{\rho_1}, \qquad \gamma = \frac{k+1}{k-1} \tag{17}$$

$$Q = 2q\frac{\rho_1}{\overline{p}_1} = 2kq\frac{\rho_1}{k\overline{p}_1} = 2\frac{kq}{a_1^2} \tag{18}$$

is the normalized heat release, a_1 is the speed of sound, and

$$M = \frac{\mu_e H_1^2}{2p_1} \tag{19}$$

is the dimensionless magnetic field.

The dimensionless parameters Q and M are obtained by nondimensionalizing the Hugoniot equation. The parameter Q characterizes the effect of heat release on the detonation process, while the parameter M characterizes the intensity of the magnetic field.

Equations (15) and (16) describe the modified Rankine–Hugoniot equation for detonation.

For flow without turbulence and without heat release or removal, Equation (16) reduces to the following:

$$P = \frac{\gamma - R^{-1}}{\gamma R^{-1} - 1} = \frac{\gamma - S}{\gamma S - 1} \tag{20}$$

which is the well-established Hugoniot adiabatic for pure ordinary gases [28,29]. Here, $R^{-1} = S$.

Equation (16) exhibits an asymptote for ρ_2/ρ_1, expressed as the following:

$$R = \gamma \tag{21}$$

Equation (21) is a classical relation for the maximum degree of gas compression when passing through a shock wave. It shows that, for an ideal gas, the maximum compression ratio cannot exceed six.

For the condition expressed as Equation (21), the pressure jump (16) becomes infinite.

The limiting condition (21) holds for detonation only in unsteady, strongly pinched waves. For a detonation propagating spontaneously at a constant speed, the limiting density, at a speed tending to infinity, is written differently. This relation will be given below.

Let us analyze the behavior of Rankine–Hugoniot equation.

The results of the calculations, according to Equation (16), are shown in Figures 2 and 3. The classical Poisson isentrope is also plotted in these Figures (curve 4),

$$P = R^k = S^{-k} \tag{22}$$

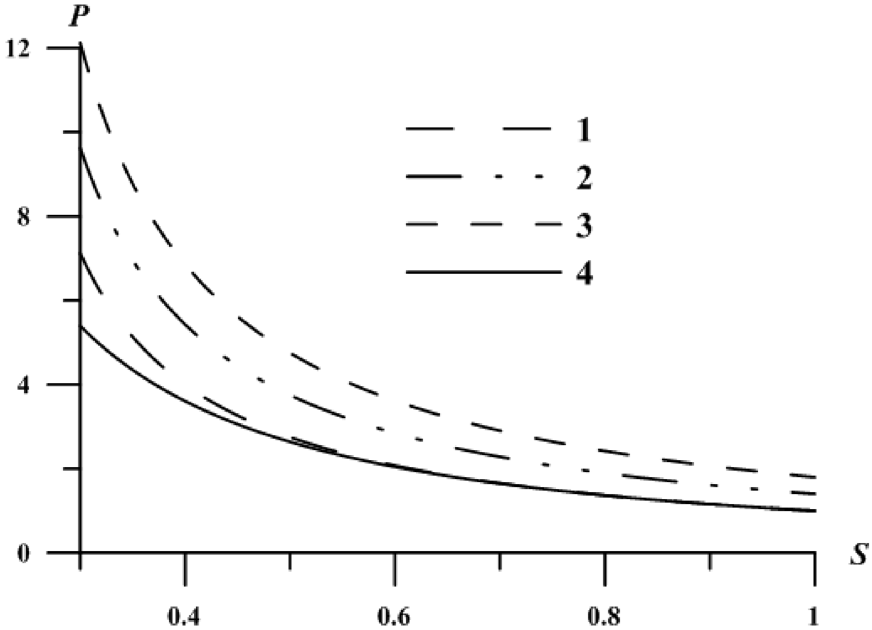

Figure 2. Effect of heat release, Q, on the Hugoniot curve at $M = 0$; 1—$Q = 0$; 2—$Q = 2$; 3—$Q = 4$; 4—the Poisson isentrope.

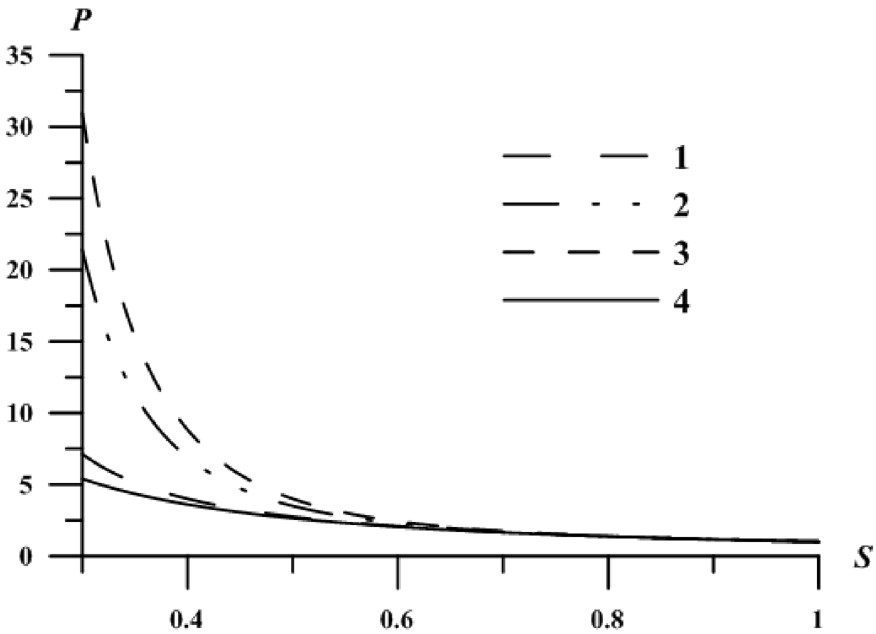

Figure 3. Effect of intensity of the magnetic field, M, on the Hugoniot curve at $Q = 0$; 1—$M = 0$; 2—$M = 3$; 3—$M = 5$; 4—the Poisson isentrope.

It can be seen that an increase in heat release, Q, leads to an equidistant rise in the Hugoniot curve compared to the case of $Q = 0$. An increase in the intensity of the magnetic field, M, leads to an increase in the left side of the Hugoniot curve ($S < 1$). However, all curves converge at a point, $S = 1$, if there is no heat release, $Q = 0$. The Poisson isentrope also approaches this point.

Such an increase in the pressure jump is explained by the fact that the flow with a higher level of magnetic field possesses a larger amount of kinetic energy, which transforms into a pressure jump during the passage of a shock wave.

The results of calculations using Equation (16), taking into account the joint influence of the factors Jo and M, are shown in Figures 4 and 5. Curve 5 in Figures 4 and 5 corresponds to the Poisson isentrope.

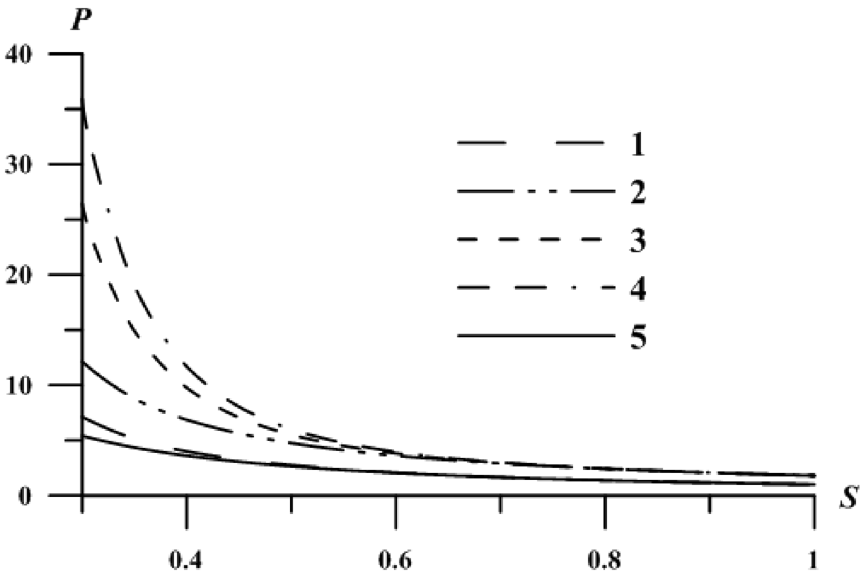

Figure 4. Effect of Q and M on the Hugoniot curve: 1—M = 0 Q = 0; 2—M = 0, Q = 4; 3—M =3, Q = 4; 4—M = 5, Q = 4; 5—the Poisson isentrope.

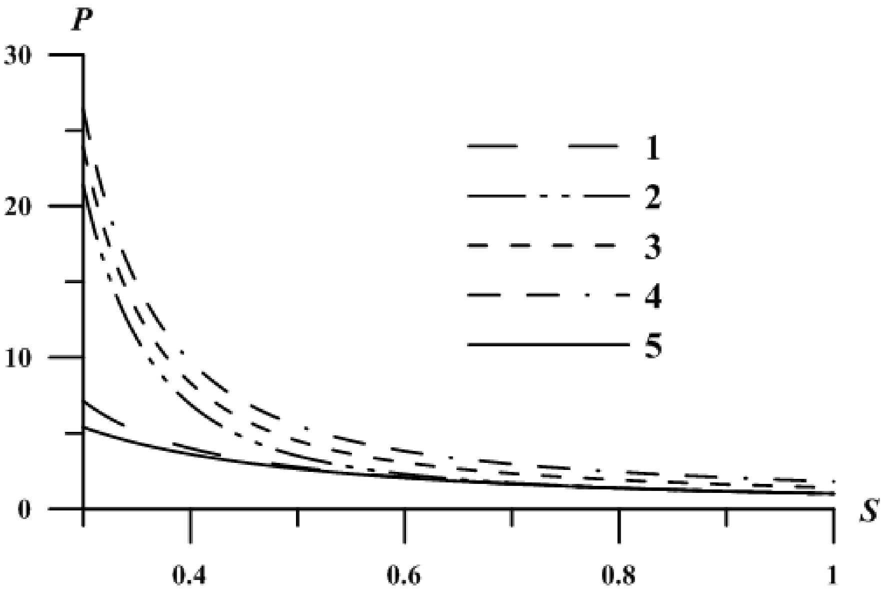

Figure 5. Effect of Q and M on the Hugoniot curve: 1—M = 0, Jo = 0; 2—M = 3, Jo = 0; 3—M = 3, Jo = 2; 4—M = 3, Jo = 4; 5—the Poisson isentrope.

It can be seen (Figure 4) that, under the condition Q = idem, the Hugoniot curves at different levels of the magnetic field converge at one point, the position of which is determined by the thermal effect, Q. Under the condition M = idem, the Hugoniot curves rise equidistantly with increasing of the Q, just as in the absence of a magnetic field (Figure 5).

4. Ultimate Compression and Speed Characteristics

To determine which point on the Hugoniot curve corresponds to the stable, normal detonation with a minimum velocity, we use the Jouguet selection rule [24,25]. This point corresponds to a point on the Hugoniot curve, D, through which the tangent passes, which also passes through the point $(P, S) = (1, 1)$ (Figure 6).

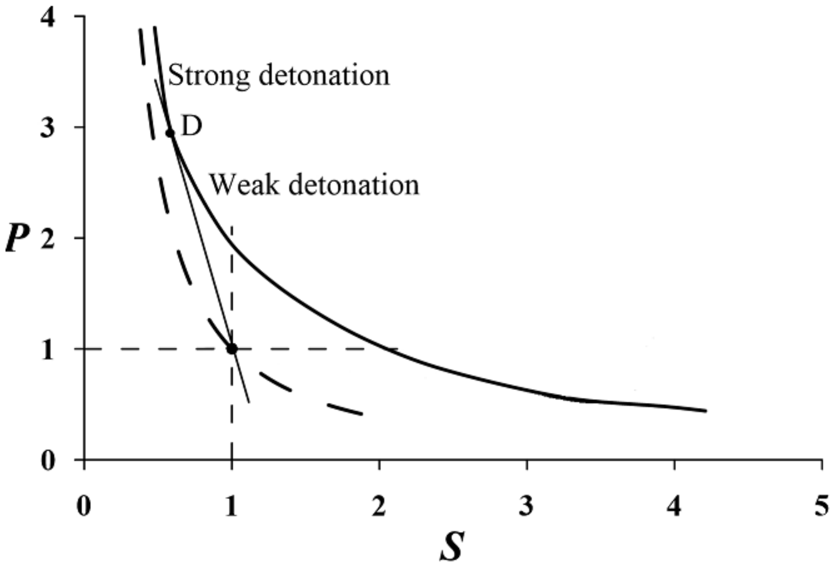

Figure 6. Jouguet selection rule.

As can be seen from Figure 6, the tangent equation is determined using the following condition:

$$\frac{P-1}{1-S} = \frac{dP}{dS} \tag{23}$$

From (16), we find that

$$\frac{P-1}{1-S} = \frac{dP}{dS} = \frac{1+\gamma P}{1-\gamma S} + \frac{(2+S)(1-S)^2}{S^3(1-\gamma S)}M \tag{24}$$

Let us determine the tangent of slope to this curve. To do this, we can, find from Equations (1) and (2), the following:

$$V_1 = \sqrt{\frac{p_2 - p_1 + \frac{\mu_e}{2}\left(H_2^2 - H_1^2\right)}{\rho_2 - \rho_1}\frac{\rho_2}{\rho_1}} = \sqrt{\frac{p_2 - p_1 + \frac{\mu_e}{2}H_1^2\left(\frac{H_2^2 V_2^2}{H_1^2 V_1^2}\frac{V_1^2 \rho_1^2}{V_2^2 \rho_2^2}\frac{\rho_2^2}{\rho_1^2} - 1\right)}{\rho_2 - \rho_1}\frac{\rho_2}{\rho_1}} = \sqrt{\frac{P-1+(R^2-1)M}{1-S}\frac{p_1}{\rho_1}} \tag{25}$$

$$V_2 = S\sqrt{\frac{P-1+(R^2-1)M}{1-S}\frac{p_1}{\rho_1}} \tag{26}$$

From Equation (25), we find the tangent at point D, as follows:

$$\tan(\alpha) = -\frac{P_D - 1}{1 - S_D} = -k\mathrm{Ma}_1^2 + \frac{R_D^2 - 1}{1 - S_D}M \tag{27}$$

On the other hand, the same tangent is determined using Equation (24), as follows:

$$\tan(\alpha) = \left(\frac{dP}{dS}\right)_D = \frac{1+\gamma P_D}{1-\gamma S_D} + \frac{(2+S_D)(1-S_D)^2}{S_D^3(1-\gamma S_D)}M \tag{28}$$

The solution of the system of Equations (27) and (28) enables finding the coordinates of the point D on the Hugoniot curve. This solution is very cumbersome and is, therefore, not presented here. In the absence of a magnetic field (M = 0), this solution goes over to the well-known Jouguet relations, as follows:

$$P_D = \frac{1+k\mathrm{Ma}_1^2}{1+k} \tag{29}$$

$$S_D = \frac{1+k\mathrm{Ma}_1^2}{(1+k)\mathrm{Ma}_1^2} \tag{30}$$

The limit of the solution for S_D, at a speed tending to infinity (Ma→∞), is independent of the intensity of the magnetic field and can be expressed as follows:

$$R_D = \left(\frac{\rho_2}{\rho_1}\right)_D = \frac{1+k}{k} \tag{31}$$

This equation determines the ultimate compression in detonation, which corresponds to the Jouguet condition.

For detonation engines, a very important characteristic is the speed of detonation products. To determine the velocity of detonation products, Equation (24) can be recast as follows:

$$\frac{P-1}{1-S} = \underbrace{\frac{1+\gamma P}{1-\gamma S}}_{H} + \frac{(2+S)(1-S)^2}{S^3(1-\gamma S)}M = \underbrace{k\frac{P}{S}}_{I} + \frac{(2+S)(1-S)^2}{S^3(1-\gamma S)}M \tag{32}$$

Here, the fact is taken into account that, at point D, the derivatives of the Poisson isentrope (denoted by the letter I in Equation (32)) and the Hugoniot Equation (16) in the absence of a magnetic field (denoted by the letter H in Equation (32)) are equal [24,25]. Let us multiply Equation (32) by S^2. This yields the following:

$$S^2\frac{P-1}{1-S} = kPS + \frac{(2+S)(1-S)^2}{S(1-\gamma S)}M = a_2^2\frac{\rho_1}{p_1} + \frac{(2+S)(1-S)^2}{S(1-\gamma S)}M \tag{33}$$

Now, we transform Equation (26) for the velocity of detonation products, as follows:

$$V_2^2 = S^2\frac{P-1+(R^2-1)M}{1-S}\frac{p_1}{\rho_1} = S^2\frac{P-1}{1-S}\frac{p_1}{\rho_1} + S^2\frac{(R^2-1)M}{1-S}\frac{p_1}{\rho_1} \tag{34}$$

Taking this into account, we can rewrite Equation (34) as follows:

$$S^2\frac{P-1}{1-S}\frac{p_1}{\rho_1} + S^2\frac{(R^2-1)M}{1-S}\frac{p_1}{\rho_1} = a_2^2 + \frac{(2+S)(1-S)^2M}{S(1-\gamma S)}\frac{p_1}{\rho_1} + S^2\frac{(R^2-1)M}{1-S}\frac{p_1}{\rho_1} \tag{35}$$

A comparison of Equations (34) and (35) allows us to find the equation for the velocity of detonation products, as follows:

$$V_2^2 = a_2^2 + \frac{a_2^2}{PS}\left(\frac{(2+S)(1-S)^2}{S(1-\gamma S)} + S^2\frac{(R^2-1)}{1-S}\right)\frac{M}{k} \tag{36}$$

or in a dimensionless form:

$$\mathrm{Ma}_2^2 = 1 + \left(1 + S + \frac{2 - 3S + S^3}{S(1-\gamma S)}\right)\frac{M}{kPS} \tag{37}$$

In Equations (36) and (37), the parameter P can be eliminated using the modified Hugoniot Equation (16).

Equations (36) and (37) show that, in a zero magnetic field, combustion products flow from the detonation front at a critical (sonic) velocity. As the intensity of the magnetic field increases, the velocity of the detonation products increases. Obviously, the energy of the magnetic field increases the kinetic energy of the averaged flow behind the detonation wave, due to the Lorentz force.

The results obtained above are in qualitative agreement with the results of the work in [30]. This study demonstrated the influence of a magnetic field on the propagation of a detonation wave in a gaseous explosive mixture. As the magnetic field increases from 0 to 5T, the speed of the detonation wave increases from 2800 m/s to 6608 m/s.

If we substitute the limiting solution (31) into Equation (37), then we obtain the following:

$$\mathrm{Ma}_2^2 = 1 + \frac{2\left(k^3 + k + 1\right)M}{k^3(1 + 3k + (k^2-1)Q) + k(k-1)M} \tag{38}$$

For monatomic gases at $k = 5/3$, Equation (38) can be transformed into the following relation:

$$\mathrm{Ma}_2^2 = 1 + \frac{1773M}{5(675 + 27\,M + 200\,Q)} \tag{39}$$

For diatomic gases at $k = 7/5$, we have the following:

$$\mathrm{Ma}_2^2 = 1 + \frac{16,075\,M}{22,295 + 875\,M + 4116\,Q} \tag{40}$$

It follows from these equations that the thermal effect weakens the effect of the magnetic field. This follows from the generalized equation of the first and second laws of thermodynamics, as follows:

$$dq = d\varepsilon - \frac{p}{\rho}d\rho - Hdj \tag{41}$$

where ε is the internal energy and j is the magnetization.

In the detonation, the velocity in front of the shock wave is not an independent quantity, but it is determined using the thermal effect. Let us define this dependence. To do this, we exclude pressure from Equations (16) and (25). As a result, we have the following:

$$\mathrm{Ma}_1^2 = 2\frac{S(Q^* + k(1-S)) + M(2 - k(1-S))}{kS(1-S)(1 + S - k(1-S))} \tag{42}$$

where

$$Q^* = Q\frac{k-1}{2} \tag{43}$$

It follows from Equation (42) that both the thermal effect and the magnetic field lead to the acceleration of the flow ahead of the shock wave.

5. Conclusions

In the present study, the process of detonation in an ideal gas flow in a magnetic field is considered.

The novelty of the study and its results consists of the following: In contrast to previously published works [19,27], in this article, the Hugoniot equation is obtained with allowance for the thermal effect. We demonstrated the influence of the thermal effect and the magnetic field on the Jouguet point. A differential Equation (21) was obtained, which includes the effects of the magnetic field. We have also shown that the relations for pressure and density do not depend on the magnetic field. Calculations indicated that the speed behind the detonation wave exceeds the speed of sound; the speed before the shock wave depends on the thermal effect and the magnetic field. The influence of the magnetic field behind the wave for diatomic and monatomic gases was also revealed.

The modified detonation Hugoniot equation considers the influence of the magnetic field on the detonation. We revealed the combined effect of heat release and magnetic field intensity on the position of the Hugoniot curve. On the Hugoniot curve, the coordinates of the point corresponding to stable normal detonation with a minimum velocity were determined.

An equation was obtained for the velocity of detonation products. At zero magnetic field, combustion products flow away from the detonation front at a critical (sonic) velocity. As the intensity of the magnetic field increases, the velocity of the detonation products increases. The energy of the magnetic field increases the kinetic energy of the averaged flow behind the detonation wave, due to the Lorentz force.

A relation has been obtained that enables estimating the influence of heat release on detonation parameters.

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