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Article

Socio-Thermodynamics—Evolutionary Potentials in a Population of Hawks and Doves

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Abstract: The socio-thermodynamics of a population of two competing species exhibits strong analogies with the thermodynamics of solutions and alloys of two constituents. In particular we may construct *strategy diagrams* akin to the phase diagrams of chemical thermodynamics, complete with regions of homogeneous mixing and miscibility gaps.

Keywords: thermodynamics; sociology

1. Introduction

Random thermal motion and forces of interaction compete to determine the behaviour of thermodynamic systems of molecules. The former finds its formal expression in the entropy s, which probabilistically tends to a maximum, and the latter in the energy e, which deterministically tends to a minimum. The relative strength of the entropic and energetic effects is determined by the temperature T, which reflects the strength of the random motion. Thus a thermodynamic system tends to a minimum by making the free energy e-Ts minimal.

Analogous dichotomies of entropic and energetic contributions to the overall comportment may be identified in other fields of study, like economics, ecology, biology and sociology, provided that their models are suitably simplified. In the present paper a sociological analogy is described and exploited. The *system* is a population of two species of birds, who compete for the same resource. The random element is introduced by the stochastic nature of inheritance between successive generations, while the deterministic element stems from the striving of the birds toward higher gain; this dictates evolutionary changes over the generations. The relative significance of the two traits is determined by the price of the resource.

The thermodynamic treatment of the population suggests that the species segregate when the price of the resource is high, while they form a homogeneously mixed society for a small price level. There exists a close analogy between the segregation of species into colonies at large price and the separation of constituents of a solution or an alloy into nearly pure phases at small temperatures.

The contest strategy of the birds is adapted from a model of game theory, which was invented by Maynard-Smith and Price [1] in order to show that two species can coexist in a population, even when they compete for the same resource (see also Straffin [2] and Dawkins [3]). I have used that game before in [4–6]. However, the present work differs from my previous efforts in that here I adopt the statistical definition of entropy rather than the entropy of phenomenological thermodynamics based on the second law. The present version has the advantage of simplicity and enhanced plausibility. The suitability and effectiveness of phase diagrams for the segregation and integration of human populations donsisting of different ethnic or religious groups has previously been pointed out by Mimkes [7].

2. Evolutionary Equilibria

2.1. Evolutionary Entropic Drift toward an Equi-Distribution

We consider a population of N birds, $N_H = z_H N$ hawks and $N_D = (1 - z_H)N$ doves. Generally the distribution $\{N_H, N_D\}$ changes from one generation to the next one, and we assume *a priori* that—by the stochastic character of inheritance—every one of the $\frac{N!}{N_H!N_D!}$ realizations of the new distribution is equally probable [8]. The maximum number of realizations occurs with the distribution $\{N_H, N_D\} = \{½,½\}$ so that statistically it is inevitable that any given initial distribution will be shifted toward that equi-distribution. The shift occurs unless, of course, there is a bias for a particular value of z_H other than ½. The shift may be expressed as the tendency of the population entropy S to grow [9]:

$$S = \ln \frac{N!}{N_H!N_D!} \approx -N(z_H \ln z_H + (1 - z_H)\ln(1 - z_H)) \tag{1}$$

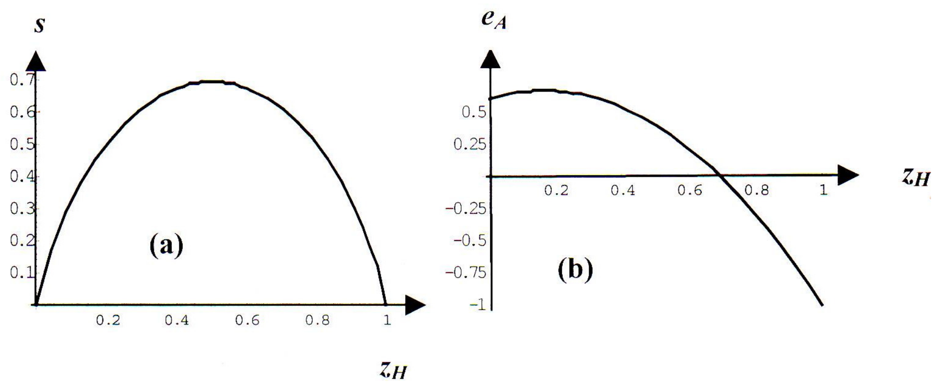
The growth ends when the maximum of S is reached at $z_H = ½$. Figure 1a provides a plot of this function.

Analysis shows that the slopes of the function at $z_H = 0$ and at $z_H = 1$ are infinite. Therefore the probability for a shift away from a pure population of either doves or hawks is overwhelming. On the other hand, the equi-distribution $z_H = ½$ is probabilistically unchanged. Thus we may say that the population is pushed toward the equi-distribution with an entropic driving force which we define as:

$$\text{entropic driving force} = \frac{\partial s}{\partial z_H} = -ln \frac{z_H}{1 - z_H} \tag{2}$$

where $s = S/N$ is the specific population entropy.

Figure 1. (**a**) Specific entropy $s = \frac{S}{N}$ as function of z_H; (**b**) Specific gain expectation as function of z_H for $\tau = 2$.



2.2. Gain-Driven Drift toward an Evolutionarily Stable Distribution

However, a bias *may exist* for a hawk fraction different from $z_H = ½$, if hawks and doves compete for the same resource. We consider a particularly simple contest strategy which, in view of a subsequent alternative, we call strategy A.

Strategy A

If two hawks meet over the resource, they fight until one is injured. The winner gains the value τ, while the loser, being injured, needs time for healing his wounds. Let that time be such that the hawk must buy two resources, worth 2τ, to feed himself during convalescence. Two doves do not fight; they merely engage in a symbolic conflict, posturing and threatening, but not actually fighting. One of them will eventually win the resource—always with the value τ—but on average both lose time such that after every dove-dove encounter they need to catch up by buying part of a resource, worth 0.2τ. When a hawk meets a dove, the dove walks away, while the hawk wins the resource; there is no injury, nor is any time lost.

Assuming that winning and losing the fights or the posturing game is equally probable, we conclude that the elementary expectation values for the gain per encounter are given by the arithmetic mean values of the gains in winning and losing, *i.e.*,

$$e_A^{HH} = 0.5(\tau - 2\tau) = -0.5\tau$$

$$e_A^{HD} = \tau$$

$$e_A^{DH} = 0$$

$$e_A^{DD} = 0.5\tau - 0.2\tau = 0.3\tau \tag{3}$$

for the four possible encounters HH,HD,DH, and DD [10]. The price is out of control for the population, but it has to adjust to it.

Thus the gain expectations e_A^H and e_A^D for a hawk or dove per encounter with any other bird are given by:

$$e_A^H = z_H e_A^{HH} + (1-z_H)e_A^{HD} \text{ and } e_A^D = z_H e_A^{DH} + (1-z_H)e_A^{DD} \tag{4}$$

The specific gain expectation e_A per bird—hawk *or* dove—and per encounter reads:

$$e_A = z_H e_A^H + (1-z_H)e_A^D \text{ or more explicitly, by (4)}$$

$$e_A = z_H{}^2(e_A^{HH} + e_A^{DD} - e_A^{HD} - e_A^{DH}) + z_H(e_A^{HD} + e_A^{DH} - 2e_A^{DD}) + e_A^{DD}\text{, or by (3)} \tag{5}$$

$$e_A = -1.2\,\tau\, z_H{}^2 + 0.4\,\tau\, z_H + 0.3\,\tau \tag{6}$$

This gain is graphically represented by a concave parabola as a function of z_H with a maximum at $z_H{}^{max} = 1/6$, see Figure 1b.

We assume that a population profits evolutionarily from a higher gain, so that—in the course of evolution, *i.e.*, over many generations—the distribution tends to the value $z_H{}^{max}$, because, in a manner of speaking, the population with that value of z_H is *fitter* than any other one. We may thus say that there is a gain-driven force toward $z_H{}^{max}$. It is defined as the increase of the gain with changing z_H:

$$\text{Gain-driven force} = \frac{\partial e_A}{\partial z_H} \tag{7}$$

2.3. Combined Entropic and Gain Driven Trends

Thus we have identified two evolutionary forces for an adjustment of the hawk fraction z_H: A probabilistic entropic one and a deterministic gain-driven one. Obviously in the interval $^1/_6 < z_H < ½$ the forces point into different directions. In general both forces are active, and we speak of an evolutionary equilibrium when they are equal in size and opposite to each other:

$$\text{Equilibrium condition } \frac{\partial s}{\partial z_H} = -\frac{\partial e_A}{\partial z_H} \text{ or } \frac{\partial(e_A + s)}{\partial z_H} = 0 \text{ or } \frac{\partial s}{\partial e_A} = -1 \tag{8}$$

We call $p_A = e_A + s$ the *evolutionary potential* and conclude that in equilibrium it has a maximum.

Figure 2 represents the evolutionary potential graphically for several prices τ. The maxima determine the hawk fractions of evolutionary equilibrium. Inspection shows that the equilibrium hawk fraction shifts from $^1/_6$ to $^1/_2$ for increasing values of τ.

It is useful for a comparison with thermodynamics to make the price explicit in the equilibrium condition by defining a unit gain, *i.e.*, a gain per price unit, *viz.*:

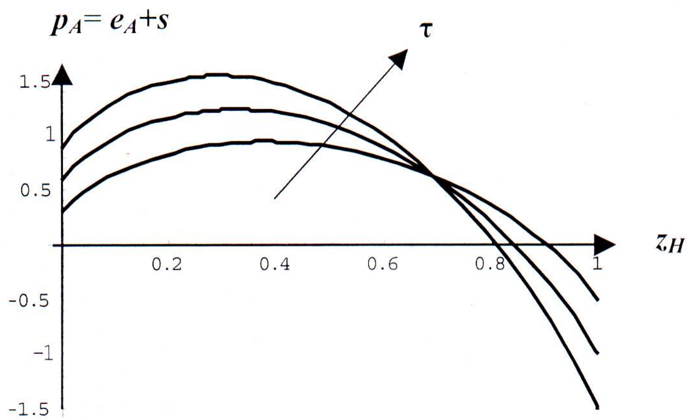
$$\text{unit gain} = \overline{e}_A = \frac{e_A}{\tau}.$$

Thus the equilibrium condition reads:

$$\frac{\partial s}{\partial z_H} = -\tau \frac{\partial \overline{e}_A}{\partial z_H}\text{, or } \frac{\partial(\tau\,\overline{e}_A + s)}{\partial z_H} = 0\text{, or } \frac{\partial s}{\partial \overline{e}_A} = -\tau \tag{9}$$

In terms of $\overline{e}_A$ the evolutionary potential reads $p_A = \tau\,\overline{e}_A + s$.

Figure 2. Evolutionary potential $p_A = e_A + s$ for increasing price τ = 1,2,3.



2.4. Analogy with Thermodynamics of Mixtures

In thermodynamic language we should consider the population of hawks and doves with hawk fraction z_H as a binary mixture of two constituents with the mol fraction X of constituent 1 (say). The entropy s would be the entropy of mixing of the constituents, while the price τ obviously is reciprocal to the thermodynamic temperature T; we have to set $\tau = 1/T$. The unit gain $\overline{e}_A$ represents the thermodynamic specific energy to within sign, and the potential $p_A = \tau\,\overline{e}_A + s$ in evolutionary thermodynamics plays the role of the thermodynamic free energy of a binary mixture,—again to within sign. Due to the sign difference the free energy of thermodynamics becomes minimal in equilibrium, while the evolutionary potential $\tau\,\overline{e}_A + s$ tends to a maximum.

Thus there is a far-reaching formal analogy between the present consideration of evolution in a population of hawks and doves and thermodynamics of binary mixtures. We proceed to show that the analogy can be expanded when we consider a different contest strategy of the birds and the possibility of segregation of hawks and doves into colonies.

3. Segregation of Hawks and Doves

3.1. Non-Convex Potentials. Concavification

Evolution is a slow process. It takes several generations to make its effect shown. During times which are short on an evolutionary time scale the hawk fraction in the population is constant and there is no way—in the short run—to increase the evolutionary potential $p_A = \tau\,\overline{e}_A + s$, at least not, if strategy A of Section 2.2 is employed by the birds. This is not an interesting case, and therefore we look at an alternative strategy, which is more interesting even without evolution.

Strategy B

The hawks adjust the severity of the fighting—and thus the gravity of an injury—to the prevailing price τ of the resource. If the price is higher than 1, they fight less, so that the mean time of convalescence in case of a defeat is shorter and the value to be bought during convalescence is reduced from 2 τ to $2\tau\,[1 - 0.2(\tau - 1)]$. Likewise the doves adjust the duration of the posturing, so that the payment for lost time is reduced from 0.2 τ to 0.2 $\tau[1 - 0.3(\tau - 1)]$ But that is not all: To be sure, in strategy B the doves will still not fight when they find themselves competing with a hawk, but they will try to grab the resource and run. Let them be successful 4 out of 10 times. However, if unsuccessful, they risk injury from the enraged hawk and may need a period of recovery at the cost of $2\tau[1 + 0.5(\tau - 1)]$ [11]. Thus the elementary expectation values for gains under strategy B may be written as:

$$e_B^{HH} = 0.5(\tau - 2\tau(1 - 0.2(\tau - 1))) = (0.2\tau - 0.7)\tau$$

$$e_B^{HD} = 0.6\tau$$

$$e_B^{DH} = 0.4\tau - 0.6 \cdot 2\tau(1 + 0.5(\tau - 1)) = -(0.6\tau + 0.2)\tau$$

$$e_B^{DD} = 0.5\tau - 0.2\tau(1 - 0.3(\tau - 1)) = (0.06\tau + 0.24)\tau \tag{10}$$

And therefore, by (5) the specific gain is given by:

$$e_B = 0.86\tau(\tau - 1)z_H^2 - (0.72\tau + 0.08)\tau z_H + (0.06\tau + 0.24)\tau \tag{11}$$

Obviously for $\tau > 1$ this function is convex. The entropy is unchanged from the previous case,—a concave function of z_H, see Figure 1a—and therefore the graph of the evolutionary potential $p_B = \tau\overline{e}_B + s$ may result—for a proper value of τ as a plot with variable curvature, see Figure 3a. Once again, if there is no evolution, z_H is fixed. There is a single value of p_B in that case and the population seems to have no choice: It should be stuck with that value [12].

However, there is an alternative. For a given $\overline{z}_H$ under the convex range of $p_B(z_H)$ the population can assume a higher potential than $p(\overline{z}_H)$ by forsaking homogeneity. Indeed, let the population segregate into dove-rich and hawk-rich colonies ′ and ″ with z'_H and z''_H respectively, and with $x = \frac{N'}{N}$ and $1 - x = \frac{N''}{N}$ as bird fractions in the respective colonies. Obviously, the population must then have the potential:

$$p_B(x, z'_H, z''_H) = xp_B(z'_H) + (1 - x)p_B(z''_H) \tag{12}$$

which is the weighed sum of the potentials in the colonies with weighing factors x and $1 - x$. This is a function of three variables, *viz.* x, z'_H, and z''_H which, however, are constrained by the condition:

$$\overline{z}_H = xz'_H + (1 - x)z''_H \tag{13}$$

Since the population strives for a maximal potential, it will effect the segregation in such a way that the function $p_B(x, z'_H, z''_H)$ has a maximum. Necessary condition for that to be the case is that the derivatives of the function:

$$xp_B(z'_H) + (1 - x)p_B(z''_H) + \lambda(\overline{z}_H - xz'_H - (1 - x)z''_H)$$

vanish. λ is the Lagrange multiplier that takes care of the constraint (13). Thus necessary conditions for a maximum may be written as:

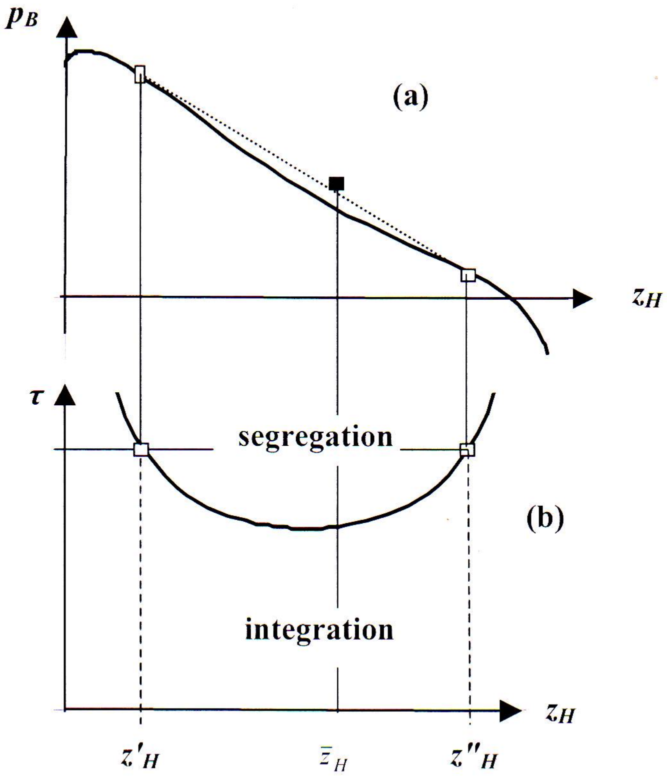
$$\lambda = \frac{\partial p_B(z'_H)}{\partial z'_H} = \frac{\partial p_B(z''_H)}{\partial z''_H} = \frac{p_B(z''_H) - p_B(z'_H)}{z''_H - z'_H} \tag{14}$$

These are two conditions for the calculation of z'_H, and z''_H which imply that z'_H, and z''_H are the abscissae of the points of contact of the common tangent to the concave parts of $p_B(z_H)$, see Figure 3a [13]. The colonial fractions of birds and the potential of the two-colony population follow from the constraint (13) and from (12). They read:

$$x = \frac{z''_H - \overline{z}_H}{z''_H - z'_H} \text{ and } p_B(x, z'_H, z''_H) = p_B(z''_H) + \frac{z''_H - \overline{z}_H}{z''_H - z'_H}(p_B(z'_H - p_B(z''_H)) \tag{15}$$

so that $p_B(x, z'_H, z''_H)$ is a linear function of $\overline{z}_H$. In Figure 3a the position of the colonies are marked by open rectangles and the position of the segregated population as a whole is marked by a black rectangle. The above-described graphical method for the determination of z'_H and z''_H may be called the *concavification* of the graph of the potential function $p_B(z_H)$.

Figure 3. (**a**) Evolutionary potential for strategy B; (**b**) Strategy diagram with areas of integration and segregation.



We conclude that a population which employs strategy B can segregate into colonies with definite but different hawk fraction and thereby achieve a higher potential than a homogeneously mixed population of hawks and doves. This possibility exists only for potentials with a convex branch. That branch vanishes for the model, if τ is too small, see Figure 3b. We may construct a *strategy diagram*, in which the area of segregation lies within the curve:

$$\tau = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{1.72 z_H (1 - z_H)}} \tag{16}$$

That diagram is shown in Figure 3b. We conclude that integration for all values of z_H is only possible for small prices. For high prices integration can only happen for values of z_H which are either close to 0 or close to 1: The conclusion is that *minorities can be integrated.*

3.2. Thermodynamic Analogy

In thermodynamics of mixtures there is an analogy to the above-described phenomenon of segregation in a population. Indeed, a solution like phenol (C_6H_5OH) in water has a *miscibility gap*, *i.e.*, it decompose into two *phases* at low temperature which are phenol-rich and water-rich respectively. By the analogy discussed in Section 2.4 high price corresponds to low temperature. Therefore phase diagrams are "upside-down" in comparison with strategy diagrams. In that case the equilibrium mol fractions X of the phases are obtained by *convexification* of the free energy function rather than concavification of the evolutionary potential.

4. Segregation in a Population with a Choice of Strategies

4.1. Intersecting Graphs p_A and p_B

We continue to consider the population of *N* hawks and doves. So far we have discussed the case that the birds could adopt only one contest strategy, either A or B. But now we let them choose: They may adopt either strategy depending on which one provides them with a higher value of the evolutionary potential at the extant price level. Therefore we must be interested in the tableau shown in Figure 4. The individual graphs shown in Figure 4a–e represent evolutionary potentials as functions of z_H appropriate to different values of τ. The dashed ones refer to Strategy A, while the solid ones refer to Strategy B.

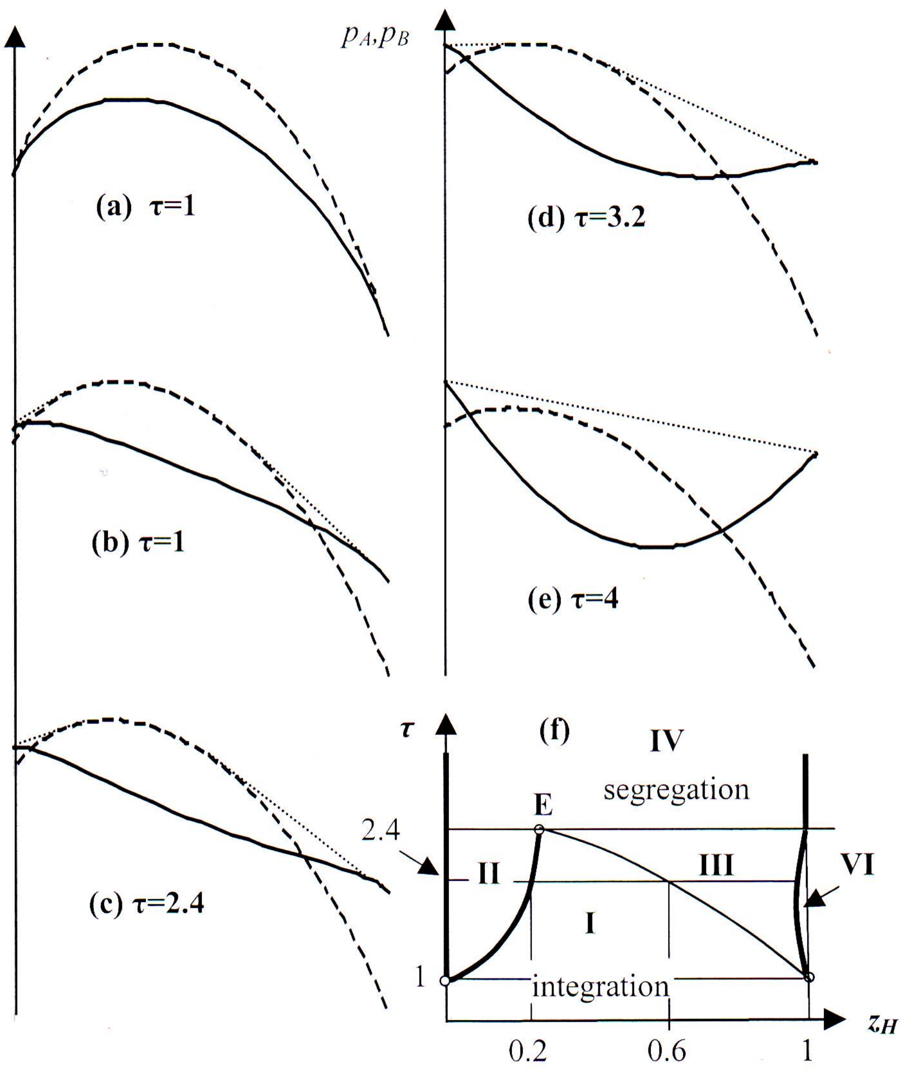
It is obvious that for $\tau = 1$ strategy A will be adopted by the birds, because it provides a higher potential and it does that for all values of z_H between 0 and 1. For higher values of τ the graphs of $p_A(z_H)$ and $p_B(z_H)$ intersect each other so that there is the possibility of concavification which was explained in Section 3.1. For $\tau = 2$, 2.4, and 3.2 there are *two* common tangents to concave parts of the potential functions, one each for small and large values of $z_{H,}$, see Figure 4. The states on those tangents represent populations that are segregated in colonies in which the birds employ different strategies. Thus for $\tau = 2.4$ and for $z_H \approx 0.1$ there will be a colony with $z_H << 1$ which employs strategy B and another colony with $z_H \approx 0.2$ which employs strategy A. On the other hand, again for $\tau = 2.4$, but for $z_H \approx 0.7$ there will be a hawk-rich colony with $z_H \approx 1$ employing strategy B and another colony with strategy A which has $z_H \approx 0.6$. Eventually, for $\tau > 3.6$ or thereabout the colonies will either be dove-rich or hawk-rich and both types employ strategy B. That is the high-price strategy.

Note that the graph $\max[p_A, p_B]$ is non-concave even for values of τ for which both p_A and p_B are concave. We may therefore use concavification of that graph as shown in Figure 4a–e and then construct a strategy diagram for the population by projecting the common tangents onto the appropriate horizontal lines in a (τ, z_H)-diagram, and by connecting the end points of those projections. In this manner we obtain Figure 4f in which six regions may be identified:

I. Homogeneous population employing Strategy A for all hawk fractions.
II. Segregated population with two types of colonies: Dove-rich colonies employing Strategy B and colonies with moderate hawk fractions and Strategy A.
III. Segregated population with two types of colonies: Hawk-rich ones with Strategy B and colonies with moderate hawk fractions and Strategy A.
IV. Segregated population with hawk-rich and dove-rich colonies both employing Strategy B.
V. Homogeneous dove-rich populations with Strategy B. (This region is not visible in Figure 4f, because on the scale of the figure it lies virtually on the τ-axis.)
VI. Homogeneous hawk-rich populations with Strategy B.

The horizontal line separating the regions IV from II and III is called the eutectic line and the point E, in which three types of colonies can exist, is called the eutectic point. Both namings are in analogy with thermodynamics of alloys, see below.

Figure 4. (**a**) through (**e**): Evolutionary potentials for strategies A (dashed) and B (solid) for different prices. Concavification by common tangents. 4f Strategy diagram.



4.2. Analogy with Thermodynamics of Solutions and Alloys

Chemical engineers and metallurgists are familiar with phase diagrams in thermodynamics of solutions and alloys that look much like the strategy diagram of Figure 4f, e.g., the phase diagram for a (Pb,Sb)-alloy, or a (Pb,Sn)-alloy; except that phase diagrams are (temperature$T$,mol-fractionX)-diagrams so that they are upside down compared to the (priceτ,hawk-fractionz_H)-diagram of the figure. This inversion is due to the fact that the price τ corresponds to the reciprocal of the temperature T, see Section 2.4.

The terms *eutectic line* and *eutectic point* used above come from metallurgy. Eutectic essentially means easy-melting and the eutectic point is *the* state (T,X) for which melting of an alloy is achieved at the lowest temperature. In the analogy to populations the eutectic line is the lower boundary of the region of segregation,—or miscibility gap. And the eutectic point marks the state (τ, z_H) where integration can be maintained up to the highest price level.

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8. We let the total number of N birds be unchanged from generation to generation.
9. Here and in the sequel we adopt the Stirling formula in order to replace factorials of large numbers: $\ln a! \approx a \ln a - a$.
10. This strategy was invented by the biologists J. Maynard-Smith and G.R. Price [1] and Dawkins [3] in order to show that a mixed population of two competing species may be evolutionarily stable. Here I modify the strategy of those biologists in a trivial manner by introducing the price τ, which I consider as dimensionless. In anticipation of misunderstandings or criticism I say this: Dawkins does not consider e_A in (6) as relevant, because he refuses the practicality of pacts or conspiracies which favour the gain for the population as a whole rather than the gain of the selfish individual. The relevance of e_A requires enforcement of social measures to make the gain *of the population* a criterion for behaviour. We assume that such measures are agreed upon in the population.

11. $\tau = 1$ is a reference price in which both strategies coincide, except for the grab-and-run feature of strategy B. Penalties for either fighting or posturing should never turn into rewards for whatever permissible value of τ. This condition imposes an upper bound on τ: $0 < \tau < 4.33$. That constraint could be avoided, if we allowed non-linear penalty reductions which we do not do for the sake of simplicity.
12. If there *were* evolution, the phase fraction would shift to the abscissa of the maximum of p_B over subsequent generations.
13. It is fortunate that (14) lends itself to such a simple graphical interpretation, since an analytic solution is impossible because of the ln-terms in the entropic part of $p_B(z_H)$. Actually, however, in Figure 3 the influence of entropy is arbitrarily enhanced by a factor 2, because otherwise the concave parts of $p_B(z_H)$ would not be sufficiently well pronounced to make the construction clear.