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Article

A Model of Nonsingular Universe

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Abstract: In the background of Friedmann–Robertson–Walker Universe, there exists Hawking radiation which comes from the cosmic apparent horizon due to quantum effect. Although the Hawking radiation on the late time evolution of the universe could be safely neglected, it plays an important role in the very early stage of the universe. In view of this point, we identify the temperature in the scalar field potential with the Hawking temperature of cosmic apparent horizon. Then we find a nonsingular universe sourced by the temperature-dependent scalar field. We find that the universe could be created from a de Sitter phase which has the Planck energy density. Thus the Big-Bang singularity is avoided.

Keywords: Hawking temperature; scalar field; nonsingular universe

1. Introduction

The standard model of cosmology claims that the primordial Universe was in a hot, dense, and highly curved state which is very close to the Big-Bang singularity [1]. The cosmic energy density, the temperature and other physical quantities become infinite at the singularity. Thus the presence of infinity of physical quantities indicates that the Einstein's General Relativity breaks down at the singularity. General Relativity must be modified at high densities. In other words, some other aspects of physics should be taken into account in the gravity theory.

It is usually believed that the Big-Bang singularity might be regularized in the quantum version of gravity theory. Actually, the absence of the Big-Bang singularity in a quantum setting could be expected on qualitative grounds although a completely self-consistent quantum theory of gravity is still not yet available so far. As is well known, there are only four fundamental constants in the theories

describing space, time, matter and thermodynamics. They are the Newton's constant G (related to General Relativity), the speed of light c (related to Special Relativity), the Planck's constant $\hbar$ (related to quantum mechanics) and the Boltzmann's constant k (related to thermodynamics). Consequently, one could have only four quantities that can be constructed from these four fundamental constants. They are Planck time t_p, Planck length l_p, Planck density ρ_p and Planck temperature T_p [2],

$$\begin{aligned} t_p &= \sqrt{\frac{\hbar G}{c^5}} = 5.4 \times 10^{-44}\ \mathrm{s} \\ l_p &= \sqrt{\frac{\hbar G}{c^3}} = 1.6 \times 10^{-35}\ \mathrm{m} \\ \rho_p &= \frac{c^5}{\hbar G^2} = 5.2 \times 10^{96}\ \mathrm{kg \cdot m^{-3}} \\ T_p &= \frac{l_p c^4}{Gk} = 1.4 \times 10^{32}\ \mathrm{K} \end{aligned} \tag{1}$$

These quantities are expected to play important role in the quantum theory of gravity. It is generally assumed that they set the scale for the quantum gravity effects. The time, length, energy density and temperature beyond these scales are highly impossible. In view of this point, the quantum effect should be considered in General Relativity and the Universe should not be singular.

The first attempt of taking quantum effect into account in gravity theory is the elegant work of Hawking [3]. According to General Relativity, the temperature of a black hole is absolutely zero. This violates the third law of thermodynamics which states that one can never approach the temperature of zero with finite operations. However, Hawking found that a black hole behaves like a black body, emitting thermal radiation, with a temperature inversely proportional to its mass M,

$$T_H = \frac{1}{8\pi M} \cdot \frac{\hbar c^3}{Gk} \tag{2}$$

In the formulae, four theories of physics, Special Relativity (c), General Relativity (G), quantum mechanics ($\hbar$) and thermodynamics (k) are present. Then the third law of thermodynamics is not violated. Motivated by this point, one expect the Big-Bang singularity may be eliminated by taking into account the quantum effect. On the other hand, in the background of Friedmann–Robertson–Walker (FRW) Universe, it is also found that the universe is filled with the Hawking radiation which comes from the apparent horizon due to quantum effect. The temperature of Hawking radiation is give by [4–7]

$$T_H = \frac{1}{2\pi r_A} \cdot \frac{\hbar c}{k} \tag{3}$$

where r_A is the radius of cosmic apparent horizon. This temperature could be observed by the comoving observers in the universe. We see that not only Special relativity and thermodynamics but also quantum mechanics are present in the formulae.

The purpose of this paper is to investigate whether the Big-Bang singularity could be smoothed by simply taking account of the cosmic Hawking radiation. The answer is yes. We find that the universe could be created from a de Sitter phase which has the Planck energy density. Thus the Big-Bang singularity is avoided.

Actually, many solutions of nonsingular universe have been presented in the literature. These solutions are based on various approaches to quantum gravity such as modified gravity models [8–11],

Lagrangian multiplier gravity actions (see e.g., [12–14]), non-relativistic gravitational actions [15–17], brane world scenarios [18,19] an so on. Here we shall not produce an exhaustive list of references, but we prefer the readers to read the nice review paper by Novello and Bergliaffa [20] and the references therein.

The paper is organized as follows. In Section 2, we shall derive the Friedmann equation and the acceleration equation sourced by the temperature-dependent scalar field. In Section 3, we present a nonsingular universe by simply considering the scalar potential for the oscillators. Section 4 gives the conclusion and discussion. In the following, we shall use the system of units in which $G = c = \hbar = k = 1$ and the metric signature $(-, +, +, +)$ until the end of the paper.

2. Temperature-dependent Scalar Field

We consider the Lagrangian density as follows

$$L = \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + V(\phi, T) \tag{4}$$

where $V(\phi, T)$ is the temperature-dependent scalar potential. The theory has been widely studied in the phase transitions of very early universe [21]. In the background of Friedmann–Robertson–Walker (FRW) Universe, T could be taken as the temperature of cosmic microwave background (CMB) radiation. The evolution of the temperature is given by

$$T_{CMB} \propto \frac{1}{a} \tag{5}$$

where a is the scale factor of the universe. The observations of CMB gives the present-day cosmic temperature [22]

$$T_{CMB} \simeq 2.73 \text{ K} \tag{6}$$

On the other hand, in the background of the spatially-flat FRW Universe, it is found that the universe is filled with the Hawking radiation which comes from the apparent horizon due to quantum effect. The temperature of Hawking radiation is given by [4–7]

$$T_H = \frac{H}{2\pi} \tag{7}$$

where H is the Hubble parameter. This temperature could be observed by the comoving observers in the universe. For the present-day universe, it is approximately

$$T_H \simeq 10^{-29} \text{ K} \tag{8}$$

It is very much smaller than the temperature of CMB. So it is very safe to neglect the quantum effect for the present-day Universe. Using the Friedmann equation, the temperature of CMB at the radiation dominated epoch can be rewritten as

$$T_{CMB} \propto \sqrt{H} \tag{9}$$

Thus, with the increasing of redshifts, the Hawking temperature T_H increases faster than the CMB temperature T_{CMB}. In other words, the quantum effect would become significant in the very early universe. In particular, the Hawking temperature at the Planck time may be as high as

$$T_H \simeq 10^{32} \text{ K} \tag{10}$$

Since the Hawking temperature becomes very important in the very early universe, we identify the temperature in the Lagrangian, Equation (4), with not the CMB temperature but the Hawking temperature.

Taking into account gravity, we have the action as follows

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi} + \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + V\left(\phi,\ T\right)\right] \tag{11}$$

where R is the Ricci scalar. The metric for the spatially flat FRW Universe is given by

$$ds^2 = -dt^2 + a\left(t\right)^2\left(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2\right) \tag{12}$$

where $a(t)$ is the scale factor of the Universe. Then the action becomes

$$\begin{aligned} S &= \int 4\pi r^2 dr \int dt\left[\frac{1}{16\pi}\left(-6\ddot{a}a^2 - 6\dot{a}^2a\right)\right. \\ &\left. -\frac{1}{2}a^3\dot{\phi}^2 + a^3V\left(\phi,\ H\right)\right] \end{aligned} \tag{13}$$

where dot denotes the derivative with respect to cosmic time t. Here we have replaced the Hawking temperature with the Hubble parameter in the scalar potential. Variation of the action with respect to the scale factor a gives the acceleration equation

$$2\dot{H} + 3H^2 = -8\pi\left[\frac{1}{2}\dot{\phi}^2 - V + H\frac{\partial V}{\partial H} + \frac{1}{3}\left(\frac{\partial V}{\partial H}\right)^{\cdot}\right] \tag{14}$$

On the other hand, variation of the action with respect to the scalar field ϕ gives the equation of motion for ϕ

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial\phi} = 0 \tag{15}$$

Equation (14) tells us the scalar field contributes a pressure as the following

$$p = \frac{1}{2}\dot{\phi}^2 - V + H\frac{\partial V}{\partial H} + \frac{1}{3}\left(\frac{\partial V}{\partial H}\right)^{\cdot} \tag{16}$$

Compared to the pressure of quintessence,

$$p = \frac{1}{2}\dot{\phi}^2 - V \tag{17}$$

the last two terms on the right hand side of Equation (16) come from the variation of scalar potential with respect to the Hawking temperature.

In order to find the energy density ρ contributed by the scalar field, we put

$$\rho = \frac{1}{2}\dot{\phi}^2 + V + F \tag{18}$$

where F is a function to be determined. Substituting Equation (16) and Equation (18) into the energy conservation equation

$$\dot{\rho} + 3H\left(\rho + p\right) = 0 \tag{19}$$

and taking into account Equation (15), we obtain

$$\left(F + H\frac{\partial V}{\partial H}\right)^{\cdot} + 3H\left(F + H\frac{\partial V}{\partial H}\right) = 0 \tag{20}$$

So we get

$$F = -H\frac{\partial V}{\partial H} + \frac{F_0}{a^3} \tag{21}$$

where F_0 is an integration constant. Since there is no constant in the Lagrangian density, Equation (4), we expect F_0 should be zero. So the energy density is given by

$$\rho = \frac{1}{2}\dot{\phi}^2 + V - H\frac{\partial V}{\partial H} \tag{22}$$

The last term comes from the variation of the scalar potential with respect to the Hawking temperature.

Now we can write the Friedmann equation as the following

$$3H^2 = 8\pi\left(\frac{1}{2}\dot{\phi}^2 + V - H\frac{\partial V}{\partial H}\right) \tag{23}$$

Equations (14), (15) and (23) constitute the main equations which govern the evolution of the universe. Among the three equations, only two of them are independent. But we have three unknown functions, $a(t)$, $\phi(t)$ and $V(\phi,\ T)$. So we are left with one degree of freedom. For simplicity, we may fix the expression of the scalar potential.

3. A Nonsingular Universe

We consider one of the simplest scalar potentials given by

$$V\left(\phi,\ T\right) = 2\pi^2\eta^2T^2\phi^2 \tag{24}$$

with η a positive dimensionless constant. This form of temperature-dependent potential comes from the high-temperature expansion of the *finite-temperature effective potential* [23]:

$$V_T\left(\phi,\ T\right) = V\left(\phi\right) + \frac{\lambda}{8}T^2\phi^2 - \frac{\pi^2}{90}T^4 + \cdots \tag{25}$$

where λ is a constant. For simplicity and to the first order of temperature corrections, we are only interested in the second term in the right hand of the equation.

Using the formulae of temperature, Equation (7), we can rewrite the scalar potential as follows

$$V\left(\phi,\ T\right) = \frac{1}{2}\eta^2H^2\phi^2 \tag{26}$$

Now the Friedmann equation, Equation (23), is reduced to

$$3 = 4\pi\left[\left(\frac{d\phi}{dx}\right)^2 - \eta^2\phi^2\right] \tag{27}$$

where x is defined by

$$x \equiv \ln a \tag{28}$$

Solving the differential equation, we find

$$\phi = \sqrt{\frac{3}{4\pi}} \frac{a^{\eta} - a^{-\eta}}{2\eta} \tag{29}$$

Without the loss of the generality, we set the integration constant to be zero.

Substituting it into the equation of motion, Equation (15), and using Equation (22) we obtain the energy density

$$\rho = \frac{\rho_p a^{4\eta}}{a^6 \left(1 + a^{2\eta}\right)^4} \tag{30}$$

where ρ_p is the integration constant. We will shortly find that it is the Planck energy density. Substituting the energy density into the energy conservation equation, Equation (19) we obtain the equation of state w

$$w \equiv \frac{p}{\rho} = 1 - \frac{4}{3}\eta \cdot \frac{1 - a^{2\eta}}{1 + a^{2\eta}} \tag{31}$$

In order that the energy density is not divergent when $a = 0$, we should require that

$$\eta \geq \frac{3}{2} \tag{32}$$

On the other hand, in order that the equation of state parameter is not less than minus one when $a = 0$, we should require that

$$\eta \leq \frac{3}{2} \tag{33}$$

So η is forced to be

$$\eta = \frac{3}{2} \tag{34}$$

Then the energy density is given by

$$\rho = \frac{\rho_p}{\left(1 + a^3\right)^4} \tag{35}$$

So the energy density increases with the increasing of redshifts. When $a = 0$, we have the maximum energy density

$$\rho = \rho_p \tag{36}$$

It is generally believed that the Planck energy density may be the maximum density in the universe. So the integration constant ρ_p is actually the Planck density. The evolution of the scale factor is found to be

$$\ln a + \frac{1}{6}a^6 + \frac{2}{3}a^3 = \sqrt{\frac{8\pi}{3}} \cdot \frac{t}{t_p} \tag{37}$$

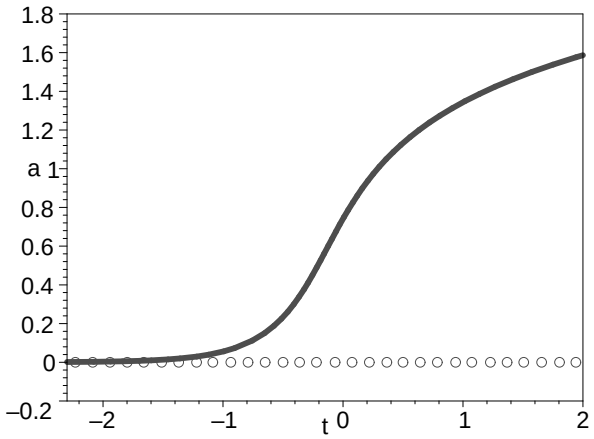
where t_p is the Planck time.

In Figure 1, we plot the evolution of the scale factor with respect to the cosmic time. It shows that the scale factor approaches zero when $t = -\infty$. The scale factor is actually physically meaningless. So

we do not worry about its vanishing at $t = -\infty$. It is the radius s of Hubble horizon that describes the physical size of the universe

$$s \equiv \frac{1}{H} \tag{38}$$

Figure 1. The evolution of the scale factor a with respect to the cosmic time t. It shows that the scale factor approaches zero when $t = -\infty$. The unite of time is the Planck time t_p.



In Figure 2, we plot the evolution of the Hubble horizon s with respect to the cosmic time t. It shows that the Hubble horizon approaches a finite constant when $t = -\infty$. Therefore, the universe is nonsingular. In Figure 3, we plot the evolution of the energy density ρ with respect to the cosmic time t. It shows that the energy density approaches the Planck density when $t = -\infty$. Since the universe is created with the finite Hubble scale and finite energy density, it is a nonsingular universe. We note that the cosmic time t is real and physical. So the minus infinity $t = -\infty$ is real and geodesic one.

Figure 2. The evolution of the Hubble radius s with respect to the cosmic time t. It shows that the Hubble horizon approaches a finite constant when $t = -\infty$. In other words, the physical size of the universe is bounded below. The unit of time and length are the Planck time t_p and Planck length l_p, respectively.

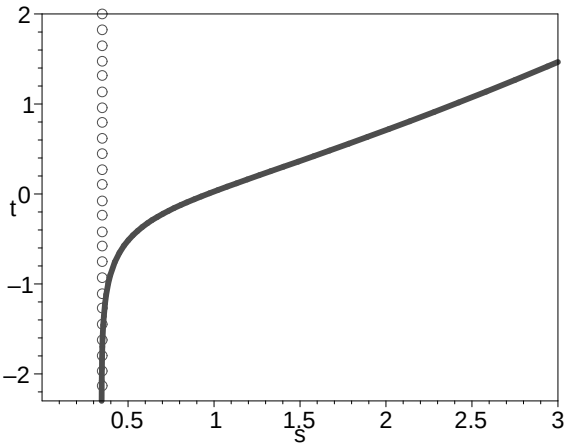
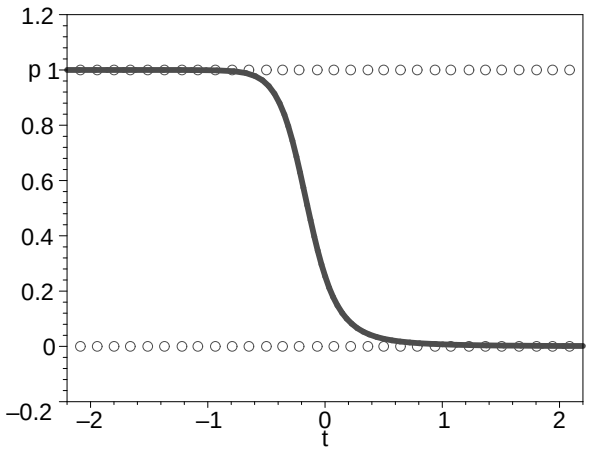


Figure 3. The evolution of energy density ρ with respect to the cosmic time t. It shows that the energy density approaches the Planck density when $t = -\infty$. The unit of time and energy density are the Planck time t_p and the Planck density ρ_p, respectively.



4. Conclusion and Discussion

Although the Hawking radiation on the late time evolution of the universe could be safely neglected, it plays an important role in the very early stage of the universe. In view of this point, we identify the temperature in the scalar field potential with the Hawking temperature of cosmic apparent horizon. Then we find a nonsingular universe sourced by the Hawking-temperature-dependent scalar field. We had better point out that the analysis of singularity here needs more attentions. For example, we need to modify gravity in a non-perturbation fashion to really make sense [24,25].

The extension of the idea to the general scalar-tensor theories is straightforward:

$$L = f\left(R,\ \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi,\ \phi,\ T\right) \tag{39}$$

with f being an arbitrary function of the variables. But we note that the whole approach of this paper can be applied only to the FRW universe, because for a general geometry the Hawking temperature has to be defined separately. As another example, we may take ϕ as a constant and take the potential as follows

$$V \propto T^4 \tag{40}$$

Then we would have the Friedmann equation as follows

$$3H^2 = 16\pi\rho_p\left(1 - \sqrt{1 - \frac{\rho}{\rho_p}}\right) \tag{41}$$

where ρ may be the energy density of radiation, dark matter and dark energy. It is apparent the maximum of ρ is the Planck density. It reveals a nonsingular universe. When $\rho \ll \rho_p$, it goes back the usual Friedmann equation.

On the other hand, if we take the potential as

$$V \propto \frac{1}{T^2} \tag{42}$$

we would have the Friedmann equation as follows

$$3H^2 = 4\pi\rho\left(1+\sqrt{1-\frac{\rho_X^2}{\rho^2}}\right) \tag{43}$$

where ρ_X is some constant. It is apparent the minimum of ρ is ρ_X. It reveals a nonsingular but future universe. When $\rho_X \leq \rho$, it restores to the usual Friedmann equation. Of course, when $T = const$, it is the quintessence.

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