

Review

Conformal Gravity: Dark Matter and Dark Energy

Robert K. Nesbet

IBM Almaden Research Center, 650 Harry Road, San Jose, CA 95120, USA;

E-Mail: rkn@almaden.ibm.com

Received: 28 August 2012; in revised form: 17 December 2012 / Accepted: 7 January 2013 /

Published: 9 January 2013

Abstract: This short review examines recent progress in understanding dark matter, dark energy, and galactic halos using theory that departs minimally from standard particle physics and cosmology. Strict conformal symmetry (local Weyl scaling covariance), postulated for all elementary massless fields, retains standard fermion and gauge boson theory but modifies Einstein–Hilbert general relativity and the Higgs scalar field model, with no new physical fields. Subgalactic phenomenology is retained. Without invoking dark matter, conformal gravity and a conformal Higgs model fit empirical data on galactic rotational velocities, galactic halos, and Hubble expansion including dark energy.

Keywords: dark matter; dark energy; galactic halos

1. Introduction

The current consensus paradigm for cosmology is the Λ CDM model [1]. Here Λ refers to dark energy, whose existence is inferred from accelerating Hubble expansion of the cosmos, while CDM refers to cold dark matter, observable to date only through its gravitational effects. The underlying assumption is that general relativity, as originally formulated by Einstein and verified by observations in our solar system, is correct without modification on the vastly larger scale of galaxies. Extrapolating back in time, initial big-bang cosmic inflation is an independent postulate.

Dark energy, dark matter, and the big-bang concept are reconciled only with some difficulty to some of the principles deduced from traditional laboratory and terrestrial physics. In particular, it is not obvious that traditional thermodynamics can be assumed for studying extreme situations such as cosmic inflation and the collapse of matter into black holes.

In the interest of reducing such uncertainties, the present review considers recent evidence supporting a theory, with minimal deviation from well-established theory of fields and particles, that fits the

same cosmological data that motivates Λ CDM, while explaining dark energy, motivating early cosmic expansion, and removing the need for dark matter.

In the theory considered here the simple postulate of universal conformal symmetry for all elementary (massless) fields combines conformal gravity [2–4] with a conformal scalar field model [3], introducing no new fundamental fields [5]. While other modified gravitational theories have been shown to account for aspects of empirical cosmology, the postulate of universal conformal symmetry proposed here is a minimalist baseline requiring extension only if found to be in conflict with observation.

Accepted theories of massless fermion and gauge boson fields exhibit strict conformal symmetry [6], defined by invariance of an action integral under local Weyl scaling [2], such that $g_{\mu\nu}(x) \rightarrow g_{\mu\nu}(x)\Omega^2(x)$ for fixed coordinate values. For a scalar field, $\Phi(x) \rightarrow \Phi(x)\Omega^{-1}(x)$. Standard general relativity and the electroweak Higgs model are not conformal. Since a conformal energy-momentum tensor must be traceless, this suggests a fundamental inconsistency [3,4]. The gravitational field equation equates the Einstein tensor, with nonvanishing trace, to the traceless tensors of quantum fields. Dynamical interactions which produce elementary particle mass redistribute energy terms among the interacting fields, while the total energy-momentum tensor remains traceless [7]. In the conformal Higgs model, a dynamical term breaks symmetry and determines dark energy while preserving the conformal trace condition [5,8].

Postulating conformal symmetry for all elementary fields modifies both gravitational and electroweak theory [3,5]. Conformal gravity [4] retains the logical structure of general relativity, but replaces the Einstein–Hilbert Lagrangian density, proportional to the Ricci curvature scalar, by a quadratic contraction of the conformal Weyl tensor [2]. This removes the inconsistency of the gravitational field equation. Mannheim and Kazanas [9] showed that this preserves subgalactic phenomenology, modifying gravitation only on a galactic scale. Formal objections to this conclusion [10] have been refuted in detail by Mannheim [11].

Conformal theory, not invoking dark matter, was shown some time ago to fit observed excessive rotation velocities outside galactic cores for eleven typical galaxies, using only two universal constants [4,12]. More recently, rotation velocities for 138 dwarf and spiral galaxies whose orbital velocities are known outside the optical disk have been fitted to conformal gravity [13–15]. The data determine a third parameter that counteracts an otherwise increasing velocity at very large radii.

The Higgs mechanism for gauge boson mass determines a nonvanishing scalar field amplitude that breaks both conformal and SU(2) gauge symmetries. The Higgs mechanism is preserved in conformal theory, but the tachyonic mass parameter w^2 of the Higgs model is required to be of dynamical origin. In uniform, isotropic geometry, conformal gravitational and Higgs scalar fields imply a modified Friedmann cosmic evolution equation [5]. Parameter w^2 determines a cosmological constant (dark energy) [8].

The modified Friedmann equation has been parametrized to fit relevant cosmological data within empirical error limits, including dark energy but not invoking dark matter [5]. The integrated Friedmann scale parameter indicates that mass and radiation density drive cosmic expansion in the early universe, while cosmic acceleration is always positive. The cosmological time dependence of nominally constant parameters of the conformal Higgs model couples scalar and gauge fields and determines parameter w^2 . The implied cosmological constant, an unanticipated consequence of the standard model Higgs mechanism, is in order-of-magnitude agreement with its empirical Λ CDM value [5,8].

Conformal theory is consistent with a model of galactic halos that does not require unobservable dark matter [16]. Hence conformal theory removes the need for dark matter except possibly for galactic clusters. As shown below, the postulate of universal conformal symmetry significantly alters theory relevant to galaxy and cluster formation. The implications have not yet been incorporated into a dynamical model.

2. Postulates and Formalism

Conformal gravity theory has recently been reviewed by Mannheim [4]. Conventions used by Mannheim are modified here in some details to agree with electroweak theory references, in particular as applied to the Higgs scalar field [17,18]. Sign changes can arise from the use here of flat-space diagonal metric $\{1, -1, -1, -1\}$ for contravariant coordinates $x^\mu = \{t, x, y, z\}$. Natural units are assumed with $c = \hbar = 1$.

Variational theory for fields in general relativity is a straightforward generalization of classical field theory [19]. Given Riemannian scalar Lagrangian density $\mathcal{L}$, action integral $I = \int d^4x \sqrt{-g} \mathcal{L}$ is required to be stationary for all differentiable field variations, subject to appropriate boundary conditions. The determinant of metric tensor $g_{\mu\nu}$ is denoted here by g . Gravitational field equations are determined by metric functional derivative $X^{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta I}{\delta g_{\mu\nu}}$. Any scalar $\mathcal{L}_a$ determines energy-momentum tensor $\Theta_a^{\mu\nu} = -2X_a^{\mu\nu}$, evaluated for a solution of the coupled field equations. Generalized Einstein equation $\sum_a X_a^{\mu\nu} = 0$ is expressed as $X_g^{\mu\nu} = \frac{1}{2} \sum_{a \neq g} \Theta_a^{\mu\nu}$. Hence summed trace $\sum_a g_{\mu\nu} X_a^{\mu\nu}$ vanishes for exact field solutions. Trace $g_{\mu\nu} X_a^{\mu\nu} = 0$ for a bare conformal field [4].

Weyl tensor $C_\lambda^{\mu\kappa\nu}$, a traceless projection of the Riemann tensor [2,4], defines a conformally invariant action integral, with Lagrangian density $\mathcal{L}_W = -\alpha_g C_\lambda^{\mu\kappa\nu} C_{\mu\kappa\nu}^\lambda$. Removing a 4-divergence [4],

$$\mathcal{L}_g = -2\alpha_g (R^{\mu\nu} R_{\mu\nu} - \frac{1}{3} R^2) \quad (1)$$

Here Ricci tensor $R^{\mu\nu}$, a symmetric contraction of the Riemann tensor, defines Ricci scalar $R = g_{\mu\nu} R^{\mu\nu}$. The relative coefficient of the two quadratic terms in $\mathcal{L}_g$ is fixed by conformal symmetry [4].

The metric tensor in quadratic line element $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ is determined by gravitational field equations. Outside a bounded spherical source density, the field equations implied by conformal $\mathcal{L}_g$ have an exact solution [9] given by static exterior Schwarzschild (ES) metric

$$ds_{ES}^2 = B(r) dt^2 - \frac{dr^2}{B(r)} - r^2 d\omega^2 \quad (2)$$

where $d\omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$. Gravitational potential $B(r) = 1 - 2\beta/r + \gamma r - \kappa r^2$, with constants of integration β, γ, κ . These constants extend Birkhoff's theorem [20], which implies constant β for standard general relativity, to conformal gravity.

A uniform, isotropic cosmos with Hubble expansion is described by the Robertson–Walker (RW) metric

$$ds_{RW}^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\omega^2 \right) \quad (3)$$

where k is a curvature constant.

A conformally invariant action integral is defined for complex scalar field Φ by Lagrangian density [4,5,8]

$$\mathcal{L}_\Phi = (\partial_\mu \Phi)^\dagger \partial^\mu \Phi - \frac{1}{6} R \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 \quad (4)$$

where R is the Ricci scalar. The Higgs mechanism [18] postulates incremental Lagrangian density $\Delta \mathcal{L}_\Phi = w^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2$, replacing $-\lambda (\Phi^\dagger \Phi)^2$. Because term $w^2 \Phi^\dagger \Phi$ breaks conformal symmetry, universal conformal symmetry requires it to be produced dynamically. The scalar field equation, including $\Delta \mathcal{L}_\Phi$, is

$$\partial_\mu \partial^\mu \Phi = \left(-\frac{1}{6} R + w^2 - 2\lambda \Phi^\dagger \Phi\right) \Phi \quad (5)$$

If derivatives in $\partial_\mu \partial^\mu \Phi$ can be neglected, this equation has an exact solution $\Phi^\dagger \Phi = \phi_0^2 = (w^2 - \frac{1}{6} R) / 2\lambda$ [5,18].

Cosmic Hubble expansion is conventionally considered in uniform, isotropic geometry, using the Robertson–Walker metric, for which the conformal action integral of Weyl vanishes identically [4]. Conformal gravity is inactive in this context, but a conformal scalar field affects gravity through the Ricci scalar term in its Lagrangian density. In conformal theory, Hubble expansion is determined by a scalar field [4]. Since a Higgs scalar field must exist, to produce gauge boson masses in electroweak theory, the simplest way to account for both established cosmology and electroweak physics is to equate the cosmological and Higgs scalar fields [8]. Postulating universal conformal symmetry, this requires no otherwise unknown fields or particles.

3. Dark Matter: Galactic Rotation Velocities

The concept of dark matter originates from dynamical studies [21,22] which indicate that a spiral galaxy such as our own would lack long-term stability if not augmented by an additional gravitational field from some unseen source. This concept is supported by other cosmological data [23]. Anomalous excess centripetal acceleration, observed in orbital rotation velocities [24,25] and gravitational lensing [26–28], is attributed to a dark matter galactic halo [29,30]. The parametrized Friedmann cosmic expansion equation of standard theory requires a large dark matter mass density [23]. A central conclusion of standard Λ CDM cosmology is that the inferred dark matter significantly outweighs observed baryonic matter [1,23].

In standard Λ CDM theory [1], phenomena and data that appear to conflict with Einstein general relativity are attributed to dark matter [23], assumed to be essentially unobservable because of negligible direct interaction with radiation or baryonic matter. An important logical point is that if dark matter is identified only by its gravitational field, it is not really an independent entity. Any otherwise unexplained gravitational field, entered into Poisson's equation, implies a source density, which can conveniently be labelled as dark matter. Attributing physical properties to dark matter, other than this pragmatic definition as a field source, may be an empty exercise.

An alternative strategy is to treat non-Einsteinian phenomena as evidence for failure or inadequacy of the theory. The MOND (modified newtonian dynamics) model of Milgrom [31], motivated by anomalous velocities v observed for dust or hydrogen gas in outer galactic circular orbits, has been very successful

in fitting empirical data [23,32]. Observed velocities are constant or increasing at large radius r , while Keplerian v^2 would drop off as $1/r$. MOND models this effect by modifying Newton's second law for low acceleration $a \leq a_0$, a universal constant not defined by standard relativity.

MOND replaces acceleration a by $a\mu(a/a_0) \rightarrow a^2/a_0$ as $a/a_0 \rightarrow 0$ in Newton's law $F/m = a$. For a Keplerian circular orbit, $F/m = GM/r^2$. Then $a = v^2/r \rightarrow v^4/(a_0 r^2)$ implies $v^4 = a_0 GM$, explicitly the Tully–Fisher (TF) relation [33], where M is galactic baryonic mass [34]. The empirical TF relation is inferred from observed galactic orbital rotation velocities [23].

This empirical v^4 law appears to be valid in particular for largely gaseous galaxies [34], whose baryonic mass is well-defined. It does not follow readily from the Λ CDM model of a dark matter galactic halo [23,32]. Gravitational theory can be revised specifically to agree with TF [35], at the cost of postulating otherwise unknown scalar and vector fields, but this does not necessarily describe other phenomena such as Hubble expansion. The relativistic theory of Moffat [36] has been parametrized to fit rotation velocities for a large set of galaxies [37]. A massive vector field is introduced and nominal constants are treated as variable scalar fields. The theory describes other aspects of cosmology including gravitational lensing [38].

In conformal theory [9] the most general spherically symmetric static exterior Schwarzschild metric outside a source density defines a relativistic gravitational potential $B(r) = 1 - 2\beta/r + \gamma r - \kappa r^2$. A circular orbit with velocity v is stable if $v^2 = \frac{1}{2}r dB/dr = \beta/r + \gamma r/2 - \kappa r^2$. If dark matter is omitted, parameter $\beta = GM$, proportional to total galactic baryonic mass M . Defining N^* as total visible plus gaseous mass in solar units, and neglecting κ , Mannheim [12] determined two universal parameters such that $\gamma = \gamma^* N^* + \gamma_0$ fits rotational data for eleven typical galaxies, not invoking dark matter [4,12]. Parameter γ_0 , independent of galactic mass, implies an isotropic cosmological source. Hence the parametrized gravitational field forms a spherical halo. A consistency test, if adequate data are available, is that the same field should account for gravitational lensing.

Constant of integration κ determines a radius at which incremental radial acceleration vanishes. This removes the objection that γ by itself would imply indefinitely increasing velocities. The fit of conformal gravity to rotational data [12] has recently been extended, including parameter κ , to 111 spiral galaxies whose orbital velocities are known outside the optical disk [13]. κ is treated as a global constant, not dependent on mass or on a specific boundary condition.

The fit of mass-independent γ_0 to observed data implies a significant effect of the cosmic background, external to a baryonic galactic core [12]. Parameter γ_0 is equivalent to cosmic background curvature [4,9]. Attributed to a galactic halo, this is a direct measurement of a centripetal effect. In the conformal halo model [16], discussed below, total galactic mass M determines halo radius r_H , so that $\kappa_H = \gamma_0/2r_H$ is a function of M . It would be informative to fit rotation data using parameters κ_H and explicitly mass-dependent $\kappa^* N^*$. The conformal halo model is consistent with conformal theory of both anomalous rotation and the Hubble expansion [5].

Conformal gravity has been shown to be consistent with the TF relation [4]. This argument is supported by the conformal model of galactic halos, described below [16]. Outside the galactic core, but for $r \ll r_H$, conformal velocity function $v^2(r) = GM/r + \gamma r/2$ has a broad local minimum at $r_x^2 = 2GM/\gamma$. Evaluated at r_x , $\gamma r_x/2 = GM/r_x$, such that $v^4(r_x) = 4(\gamma r_x/2)(GM/r_x) = 2\gamma GM$. If $\gamma^* N^* \ll \gamma_0$ and dark matter is omitted, this is an exact baryonic Tully–Fisher relation, as inferred from

recent analysis of galactic data [34]. Centripetal acceleration at r_x determines MOND parameter [23] $a_0 = 2\gamma$.

4. Dark Matter: Hubble Expansion

It was first recognized by Hubble [39] that galaxies visible from our own exhibit a very regular centrifugal motion, characterized as uniform expansion of the cosmos [1]. Redshift z , a measure of relative velocity, is nearly proportional to a measure of distance deduced from observed luminosity. Refining the observed data by selecting Type Ia supernovae as “standard candles”, cosmic expansion has been found to be accelerating in the current epoch [40,41].

That Einstein’s equations can imply expansion of a uniform, isotropic universe was first shown by Friedmann [42,43] and LeMaître [44]. This is described by the Friedmann equations, which determine cosmic scale parameter $a(t)$ and deceleration parameter $q(t)$.

In conformal gravitational theory the Einstein–Hilbert Lagrangian density is replaced by a uniquely determined quadratic contraction of the Weyl tensor, which vanishes identically in uniform, isotropic Robertson–Walker (RW) geometry. Vanishing of the metric functional derivative of action integral I_g [4], for the RW metric, can be verified by direct evaluation.

The conformal gravitational action integral replaces the standard Einstein–Hilbert action integral, but in the uniform model of cosmology its functional derivative drops out completely from the gravitational field equations. The observed Hubble expansion requires an alternative gravitational mechanism. This is supplied by a postulated conformal scalar field [4]. A nonvanishing conformal scalar field determines gravitational field equations that differ from Einstein–Hilbert theory. The Newton–Einstein gravitational constant G is not relevant. As shown by Mannheim [45], the gravitational constant determined by a scalar field is inherently negative.

The conformal Higgs model [5] differs from Mannheim because scalar field Lagrangian terms proportional to $\Phi^\dagger\Phi$ in Higgs and conformal theory have opposite algebraic signs. A consistent theory must include both terms and solve interacting gravitational and scalar field equations. This determines a modified field equation in which Einstein tensor $R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R$, where $R^{\mu\nu}$ is the Ricci tensor and $R = g_{\mu\nu}R^{\mu\nu}$, is replaced by tensor $R^{\mu\nu} - \frac{1}{4}g^{\mu\nu}R$, traceless as required by conformal theory [4]. In uniform RW geometry this determines a modified Friedmann evolution equation [5] that differs from the standard equation used in all previous work, including that of Mannheim [45].

In the standard Einstein equation, $R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} + \Lambda g^{\mu\nu} = -8\pi G\Theta_m^{\mu\nu}$, $\Theta_m^{\mu\nu}$ is the energy-momentum tensor due to matter and radiation. Radiation energy density can be neglected in the current epoch. Cosmological constant Λ must be determined empirically.

For uniform cosmic mass-energy density ρ_m , in RW geometry the R^{00} Einstein equation reduces to standard Friedmann equation $\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{1}{3}(\kappa\rho_m + \Lambda)$. Here $\dot{a}/a = h(t)$ is defined in Hubble units such that at present time t_0 , $h(t_0) = 1$, $a(t_0) = 1$ and coefficient $\kappa = 8\pi G/H_0^2$, where H_0 is the Hubble constant. This implies sum rule $\Omega_m + \Omega_k + \Omega_\Lambda = 1$, usually presented as a pie-chart for the energy budget of the universe. The dimensionless weight functions are $\Omega_m(t) = \frac{\kappa\rho_m(t)}{3h^2(t)}$, $\Omega_k(t) = \frac{-k}{a^2(t)h^2(t)}$, $\Omega_\Lambda(t) = \frac{\Lambda}{3h^2(t)}$. The second Friedmann equation determines acceleration weight $\Omega_q = -q = \frac{\ddot{a}a}{\dot{a}^2}$.

In standard Λ CDM, curvature parameter $\Omega_k(t_0)$ is negligible while dark energy $\Omega_\Lambda(t_0) = 0.73$ and mass $\Omega_m(t_0) = 0.27$ [46,47]. This empirical value of Ω_m is much larger than implied by the verifiable density of baryonic matter, providing a strong argument for abundant dark matter [23].

Mannheim [45] showed that Type Ia supernovae data for redshifts $z \leq 1$ could be fitted equally well with $\Omega_q(t_0) = -q = 0.37$ and $\Omega_\Lambda(t_0) = 0.37$, assuming $\Omega_m = 0$. This argues against the need for dark matter. However, for $\Omega_m = 0$, the standard Friedmann sum rule reduces to $\Omega_k + \Omega_\Lambda = 1$. This would imply current curvature weight $\Omega_k(t_0) = 0.63$, much larger than its consensus empirical value [47]. The modified Friedmann equation derived from conformal Higgs theory [5] avoids this problem. Fitted parameters, without dark matter, are consistent with current cosmological data [47]. Anomalous imaginary-mass term w^2 in the Higgs scalar field Lagrangian becomes a cosmological constant (dark energy) in the modified Friedmann equation [8]. Dark energy dominates the current epoch.

5. Dark Energy

Coupled scalar and gauge boson fields produce gauge boson mass through the Higgs mechanism [18], starting in the electroweak transition epoch. The universal conformal symmetry postulate requires Higgs parameter w^2 , which breaks conformal symmetry, to be a dynamical consequence of the theory. Conformal symmetry extends the Higgs model to include the metric tensor field [8]. The modified Friedmann equation determines cosmological time variation of Ricci scalar R , present in the Lagrangian density of the bare conformal scalar field. This in turn induces a neutral gauge current density that dresses the scalar field with an induced gauge field [8]. This determines parameter w^2 , which becomes dark energy in the modified Friedmann equation, preserving the Higgs mechanism for gauge boson masses and the trace condition for the coupled field equations.

In conformal Higgs theory [5], the conformal trace condition removes the second Friedmann equation and the sum rule becomes $\Omega_m + \Omega_k + \Omega_\Lambda + \Omega_q = 1$. Higgs scalar field constants ϕ_0, w^2 [5,18] define effective gravitational parameters $\bar{\kappa} = -3/\phi_0^2$ and $\bar{\Lambda} = \frac{3}{2}w^2$. This results in dimensionless weight functions $\Omega_m(t) = \frac{2\bar{\kappa}\rho_m(t)}{3h^2(t)}$, $\Omega_k(t) = -\frac{k}{a^2(t)h^2(t)}$, $\Omega_\Lambda(t) = \frac{w^2}{h^2(t)}$, $\Omega_q(t) = \frac{\ddot{a}(t)}{a(t)h^2(t)}$. Solving the modified Friedmann equation with $\Omega_m = \Omega_k = 0$, a fit to Type Ia supernovae magnitude data for redshifts $z \leq 1$ finds $\Omega_\Lambda(t_0) = 0.732$ [5], in agreement with consensus empirical value $\Omega_\Lambda(t_0) = 0.726 \pm 0.015$ [47]. The computed acceleration weight is $\Omega_q(t_0) = 0.268$. Note that only one effective independent parameter is involved in fitting the modified Friedmann equation to $z \leq 1$ redshift data.

Fitting conformal gravitation to galactic rotation data, the Schwarzschild gravitational potential $B(r)$ contains a universal nonclassical term $\gamma_0 r$ [9]. Coefficient γ_0 , independent of galactic luminous mass, must be attributed to the background Hubble flow [45]. On converting the local Schwarzschild metric to conformal RW form, this produces a curvature parameter $k = -\frac{1}{4}\gamma_0^2$ [9] which is small and negative, consistent with other empirical data. This supports the argument for modifying the standard Friedmann equation.

The modified Friedmann equation determines scale parameter $a(t)$ and Hubble function $h(t) = \frac{\dot{a}}{a}(t)$, for redshift $z(t) = 1/a(t) - 1$. A numerical solution from $t = 0$ to current $t = t_0$ is determined by four fixed parameters [5]. Adjusted to fit two dimensionless ratios characterizing CMB acoustic peak structure [48], as well as $z \leq 1$ Type Ia supernovae magnitudes, implied parameter values $\Omega_a(t_0)$ to three

decimals are: $\Omega_\Lambda = 0.717, \Omega_k = 0.012, \Omega_m = 0.000, \Omega_r = 0.000$, with computed acceleration weight $\Omega_q = 0.271$ [5]. Consensus empirical values are $\Omega_\Lambda = 0.725 \pm 0.016, \Omega_k = -0.002 \pm 0.011$ [47].

In the current epoch, dark energy and acceleration terms are of comparable magnitude, the curvature term is small, and other terms are negligible. The negative effective gravitational constant implies energy-driven rapid inflation of the early universe. Hubble function $h(t)$ rises from zero to a maximum at $z = 1371$, prior to the CMB epoch, then descends as $t \rightarrow \infty$ to a finite asymptotic value determined by the cosmological constant [5]. Acceleration weight $\ddot{a}a/\dot{a}^2$ is always positive. Although deduced from the same data fitted by standard Λ CDM, the implied behavior of the early universe is significantly different. Whether this is consistent with a big-bang singularity at $t = 0$ is at present difficult to assess, since the time-dependence of nominally constant Higgs model parameters is not yet known.

The standard Higgs mechanism, responsible for gauge field mass, can be derived using classical U(1) and SU(2) gauge fields, coupled to Higgs SU(2) doublet scalar field Φ by covariant derivatives [18]. In the conformal Higgs model, dark energy occurs as a property of the finite Higgs scalar field produced by this symmetry-breaking mechanism [5,8]. Ricci scalar R in the conformal scalar field Lagrangian density requires extending the Higgs model to include the classical relativistic metric tensor. If derivatives of Φ can be neglected, scalar field Equation (5) has an exact solution given by

$$\Phi^\dagger \Phi = \phi_0^2 = (w^2 - \frac{1}{6}R)/2\lambda \quad (6)$$

The phase is arbitrary, so ϕ_0 can be a real constant. Its experimental value is $\phi_0 = 180 \text{ GeV}$ [18]. Consistent with the modified Friedmann equation of conformal theory, empirical dark energy weight $\Omega_\Lambda = w^2 = 0.717$ [5], in Hubble units. Hence $w = 0.847\hbar H_0 = 1.273 \times 10^{-33} \text{ eV}$.

In conformal theory, dark energy appears in the energy-momentum tensor of the scalar field required by the Higgs mechanism to produce gauge boson masses. The implied cosmological constant can be computed as the self-interaction of the Higgs scalar field due to induction of an accompanying gauge boson field [8]. The required transition amplitude depends on the cosmological time derivative of the dressed scalar field.

From the scalar field equation, $\phi_0^2 = -\zeta/2\lambda$, where $\zeta = \frac{1}{6}R - w^2$. Computed from the integrated modified Friedmann equation, $\zeta(t_0) = 1.224 \times 10^{-66} \text{ eV}^2$ [8]. Given $\phi_0 = 180 \text{ GeV}$, the empirical value of dimensionless Higgs parameter $\lambda = -\frac{1}{2}\zeta/\phi_0^2$ is -0.189×10^{-88} [8]. For $\lambda < 0$ the conformal Higgs scalar field does not have a stable fluctuation [49], required to define a massive Higgs particle. The recent observation of a particle or resonance at 125 GeV is consistent with such a Higgs boson, but may prove to be an entirely new entity when more definitive secondary properties are established [50]. Because the conformal Higgs field retains the finite constant field amplitude essential to gauge boson and fermion mass, while accounting for empirically established dark energy, an alternative explanation of the recent 125 GeV resonance might avoid a severe conflict with observed cosmology.

Expressed in Hubble weights for the modified Friedmann equation, the RW metric Ricci scalar is $R = 6\frac{\dot{a}^2}{a^2}(1 - \Omega_k + \Omega_q)$, which depends on $a(t)$. $\phi_0^2 = (w^2 - \frac{1}{6}R)/2\lambda$ is not strictly constant, but varies in time on a cosmological scale ($\sim 10^{10} \text{ yrs}$). Numerical solution of the modified Friedmann equation [5], with fixed w^2 and λ , implies logarithmic time derivative $\frac{\dot{\phi}_0}{\phi_0}(t_0) = -2.651H_0$.

This cosmological time derivative defines a very small scale parameter that drives dynamical coupling of scalar and gauge fields, in turn determining Higgs parameter w^2 [8]. This offers an explanation, unique

to conformal theory, of the huge disparity in magnitude between parameters relevant to cosmological and elementary-particle phenomena.

Solving the coupled field equations for $g_{\mu\nu}$, Φ , and induced neutral gauge field Z_μ , using computed time derivative $\frac{\dot{\phi}_0}{\phi_0}(t_0)$, gives $w \simeq 2.651\hbar H_0 = 3.984 \times 10^{-33} \text{eV}$ [8]. A more accurate calculation should include charged fields $W_\mu^\pm$ and the presently unknown time dependence of Higgs parameter λ . The approximate calculation [8] agrees in magnitude with the value implied by dark energy Hubble weight $\Omega_\Lambda(t_0) = 0.717$: $w = 1.273 \times 10^{-33} \text{eV}$. These numbers justify the conclusion that conformal theory explains both existence and magnitude of dark energy.

6. Galactic Halos

A galaxy forms by condensation of matter from uniform, isotropic background density ρ_m into observed galactic density ρ_g . Conservation of mass and energy requires that total galactic mass M must be missing from a surrounding depleted background. Since this is uniform and isotropic, it can be modeled by a depleted sphere of radius r_H , such that $4\pi\rho_m r_H^3/3 = M$. In particular, the integral of $\rho_g - \rho_m$ must vanish. Any gravitational effect due to this depleted background could be attributed to a spherical halo of dark matter surrounding a galaxy. This is the current consensus model of galactic halos [1,23,30]. Conformal theory provides an alternative interpretation of observed effects, including lensing and anomalous galactic rotation, as gravitational effects of this depleted background [16]. This halo model accounts for the otherwise remarkable fact that galaxies of all shapes are embedded in essentially spherical halos.

What, if any, would be the gravitational effect of a depleted background density? An analogy, in well-known physics, is vacancy scattering of electrons in conductors. In a complex material with a regular periodic lattice independent electron waves are by no means trivial functions, but they propagate without contributing to scattering or resistivity unless there is some lattice irregularity, such as a vacancy. Impurity scattering depends on the difference between impurity and host atomic T-matrices [51]. Similarly, a photon or isolated mass particle follows a geodesic in the cosmic background unless there is some disturbance of the uniform density ρ_m . Both the condensed galactic density ρ_g and the extended subtracted density $-\rho_m$ must contribute to the deflection of background geodesics. Such effects would be observed as gravitational lensing of photons and as radial acceleration of orbiting mass particles, following the basic concepts of general relativity.

Conformal analysis of galactic rotation, not assuming dark matter [4,13], fits observed velocities consistent with empirical regularities. Excessive centripetal radial acceleration independent of galactic mass is associated with an extragalactic source [12]. The conformal Higgs model [5], not invoking dark matter, infers positive (centrifugal) acceleration weight Ω_q [5] due to the cosmic background. In the current epoch this is dominated by dark energy, due to the universal Higgs mechanism [8], which is not affected by galaxy formation. In conformal theory, Ω_m is negative for positive mass because gravitational coefficient $\bar{\kappa}$ is negative for a scalar field [4,5]. Hubble weight Ω_m , negative and currently small, contributes to positive acceleration Ω_q . Reduction of Ω_m by removal of mass in a depleted sphere implies a decrease of Ω_q relative to the cosmic background [16]. This is consistent with observed centripetal acceleration attributed to a galactic halo.

The effect of subtracted density $-\rho_m$ in standard Einsteinian gravity would be centrifugal radial acceleration, contrary to what is observed. The challenge to Λ CDM is to incorporate or explain away the gravitational effect of missing matter of total mass M that is drawn into an observed galaxy. It seems unlikely that a net mass $-M$ can simply be ignored. A similar problem occurs for MOND, which postulates standard gravity, but scales the implied acceleration by factor $\mu(a/a_0)$ without changing its sign.

Conformal gravity resolves this sign conflict in a fundamental but quite idiosyncratic manner. Uniform, isotropic source density eliminates the conformal Weyl tensor and its resulting gravitational effects. In the conformal Higgs model, this leaves a modified gravitational field equation due to the scalar field Lagrangian density. The effective gravitational constant differs in sign and magnitude from standard theory. Hence the effect of a depleted halo should be centripetal, as observed. Analysis based on Newton–Einstein constant G is inappropriate for uniform, isotropic geometry, including the use of Planck-scale units for the early universe.

The depleted halo model removes a particular conceptual problem affecting analysis of anomalous galactic rotation in conformal gravity theory [4,12,13]. In empirical parameter $\gamma = \gamma^* N^* + \gamma_0$, γ_0 does not depend on galactic mass, so must be due to the surrounding cosmos [12]. Mannheim considers this to represent the net effect of distant matter, integrated out to infinity [4]. Divergent effects may not be a problem, since external effects would be cut off by integration constants κ , as in recent fits to orbital velocity data [13]. However, since the corresponding interior term, coefficient $\gamma^* N^*$, is centripetal, one might expect the exterior term to describe attraction toward an exterior source, hence a net centrifugal effect. However, if coefficient γ_0 is due to a subtractive halo, the implied sign change predicts net centripetal acceleration, in agreement with observation.

Integration parameter κ , included in fitting rotation data [13], acts to cut off gravitational acceleration at a boundary radius. In the halo model [16], κ is determined by the boundary condition of continuous acceleration field at halo radius r_H , determined by galactic mass, except for the nonclassical linear potential term to due to the baryonic galactic core. Three independent terms in effective gravitational potential $B(r)$, each including a κr^2 cutoff, contribute to orbital velocity v [16]:

$$v_{core}^2 = \frac{GM}{r}(1 - r^3/r_H^3) \quad (7)$$

$$v_{halo}^2 = \frac{1}{2}\gamma_0 r(1 - r/r_H) \quad (8)$$

$$v_{ext}^2 = \frac{1}{2}N^*\gamma^* r(1 - r/r_*) \quad (9)$$

for r between galactic radius r_g and halo radius r_H . Parameters $\kappa_{core} = GM/r_H^3$ and $\kappa_{halo} = \gamma_0/2r_H$ cut off the acceleration field at r_H . Geodesic deflection within halo radius r_H is caused by the difference between gravitational acceleration due to ρ_g and that due to ρ_m [16]. Because mass density $\Delta\rho = \rho_g - \rho_m$ inside r_H integrates to zero, the Keplerian core term terminates at r_H . $\kappa^* = \gamma^*/2r_*$ may depend on galactic cluster environment.

The conformal gravitational field equation is

$$X_g^{\mu\nu} + X_\Phi^{\mu\nu} = \frac{1}{2}\Theta_m^{\mu\nu} \quad (10)$$

which has an exact solution in the depleted halo, where $\Theta_m^{\mu\nu} = 0$. Outside r_g , the source-free solution of $X_g^{\mu\nu} = 0$ in the ES metric [9] determines parameters proportional to galactic mass. $X_\Phi^{\mu\nu} = 0$ is solved in the RW metric as a modified Friedmann equation without ρ_m . This determines $\Omega_q(halo)$, which differs from $\Omega_q(cosmos)$ determined by $X_\Phi^{\mu\nu} = \frac{1}{2}\Theta_m^{\mu\nu}(\rho_m)$. These two equations establish a relation between $\Delta\Omega_q = \Omega_q(halo) - \Omega_q(cosmos)$ and $\Delta\rho = \rho_g - \rho_m$, which reduces to $-\rho_m$ in the halo.

Geodesic deflection in the halo is due to net gravitational acceleration $\Delta\Omega_q$, caused by $\Delta\rho$. Because the metric tensor is common to all three equations, the otherwise free parameter γ_0 in equation $X_g^{\mu\nu} = 0$ must be compatible with the X_Φ equations. This can be approximated in the halo (where $X_g^{\mu\nu} = 0$) by solving equation $\Delta X_\Phi^{\mu\nu} = \frac{1}{2}\Delta\Theta_m^{\mu\nu}$, where $\Delta X_\Phi = X_\Phi(halo) - X_\Phi(cosmos)$ and $\Delta\Theta_m = \Theta_m(halo) - \Theta_m(cosmos) = -\Theta_m(\rho_m)$. Integration constant γ_0 is determined by conformal transformation to the ES metric. Note that if the subtracted $X_\Phi(cosmos)$ were omitted, this would imply $\gamma_0 = 0$.

From the modified Friedmann equation, acceleration weight $\Omega_q(cosmos) = 1 - \Omega_\Lambda - \Omega_k - \Omega_m$. Assuming a gravitationally flat true vacuum, $\Omega_m = 0$ implies $\Omega_k = 0$ in the halo. Equation $X_\Phi^{\mu\nu}(halo) = 0$ implies $\Omega_q(halo) = 1 - \Omega_\Lambda$, so that $\Delta\Omega_q = \Omega_k + \Omega_m$. If $|\Delta\Omega_q| \ll \Omega_q$, dark energy weight Ω_Λ cancels out, as does any vacuum value of k independent of ρ_m . Because both Ω_k and Ω_m contain negative coefficients, if ρ_m implies positive k in the cosmic background, $\Delta\Omega_q$ is negative, producing centripetal acceleration. Thus positive γ_0 , deduced from galactic rotation, is determined by the cosmic background, as anticipated by Mannheim [4].

To summarize the logic of the present derivation, Equation (10) has an exact solution for $r_g \leq r \leq r_H$, outside the observable galaxy but inside its halo, assumed to be a true vacuum with $\Theta_m^{\mu\nu} = 0$ because all matter has been absorbed into the galactic core. For an isolated galaxy, Ω_q is nonzero, dominated at present time t_0 by dark energy Ω_Λ . Observed effects due to deflection of background geodesics measure difference function $\Delta\Omega_q = \Omega_k + \Omega_m$, inferred from the inhomogeneous cosmic Friedmann equation in the RW metric. Observable γ_0 is determined by transformation into the ES metric.

ES and RW metrics are related by a conformal transformation such that $|k| = \frac{1}{4}\gamma_0^2$ [9], subject to analytic condition $k\gamma_0 < 0$ [16]. This relates solutions of the field equations. At present time t_0 , with $a(t_0) = 1$ and $h(t_0) = 1$, $\gamma_0^2 = -4\Omega_k = -4\Delta\Omega_q$ in Hubble units H_0^2/c^2 , if Ω_m can be neglected. Empirical coefficient $\gamma_0 = 3.06 \times 10^{-30} cm^{-1}$, deduced from anomalous galactic rotation velocities [12,13], implies $\Omega_k = -0.403 \times 10^{-3}$, consistent with consensus empirical value $\Omega_k = -0.002 \pm 0.011$ [47].

The depleted conformal halo model implies that a galaxy of mass M produces a halo of exactly equal and opposite mass deficit. Hence the ratio of radii r_H/r_g should be very large, the cube root of the mass-density ratio ρ_g/ρ_m . Thus if the latter ratio is 10^5 a galaxy of radius 10 kpc would be accompanied by a halo of radius $10 \times 10^{5/3} = 464$ kpc. Equivalence of galactic and displaced halo mass resolves the paradox for Λ CDM that despite any interaction other than gravity, the amount of dark matter inferred for a galactic halo is strongly correlated with the galactic luminosity or baryonic mass [30,52]. The skew-tensor theory of Moffat [37] avoids this problem by an additional long-range field generated by the baryonic galaxy. Renormalization group flow of parameters models the MOND postulate of modified Newtonian acceleration.

7. Open Questions

The conformal Higgs model accounts for scalar field parameter w^2 , which becomes universal dark energy. Conformal symmetry does not preclude nonzero λ in the bare scalar field Lagrangian density. The possible time dependence of λ is not known. Empirical value $\lambda = -0.189 \times 10^{-88}$ follows from well-established values of Hubble dark-energy weight Ω_Λ and Higgs scalar field amplitude ϕ_0 , but is not determined by theory limited to neutral gauge field Z_μ . Analysis of the coupled field equations incorporating charged gauge fields $W_\mu^\pm$ involves conceptual difficulties, not yet resolved, regarding self-interaction and an electrically charged vacuum.

Although conformal theory implies an initial epoch of rapid, inflationary Hubble expansion, this cannot be treated in detail until the time-dependence of several nominally constant parameters is known. The modified Friedmann equation determines the time variation of Ricci scalar R on a cosmological scale (Hubble time unit $1/H_0 = 4.38 \times 10^{17} s$). Implied rate scale $\dot{\phi}_0/\phi_0$ affects other parameters. Whether or not conformal theory can explain empirical data relevant to the “big-bang” model, such as relative deuterium abundance and nucleosynthesis in general, cannot be tested until the early time dependence of parameters is known.

The conformal halo model apparently eliminates the need for dark matter for an isolated galaxy. The implications for galactic clusters have not been explored. Individual halo mass is only part of the dark matter inventory for clusters [53]. The conformal long-range interaction between galaxies whose halos do not overlap determines Equation (9). Analysis of the implications for a galactic cluster has not yet been carried out.

Crucially, the classical Newtonian virial theorem is not valid, so observed high thermal energy within a galactic cluster cannot be used without entirely new dynamical analysis to estimate the balance between baryonic matter, radiative energy, and hypothetical dark matter. These remarks apply directly to models of galaxy formation. In the conformal halo model, any growing galaxy is stabilized by the net gravitational effect of its accompanying depleted halo. A detailed dynamical model has not yet been worked out.

8. Conclusions

Conformal theory can explain the existence of galactic halos and the existence and magnitude of dark energy. Cosmological data including anomalous galactic rotation velocities and parameters relevant to Hubble expansion are fitted without invoking dark matter. Conformal gravity, the conformal Higgs model, and the depleted halo model are mutually consistent, removing several paradoxes or apparent logical contradictions in cosmology.

In uniform, isotropic (Robertson–Walker) geometry, the Weyl tensor basic to conformal gravity vanishes identically. Observed gravitational acceleration can be attributed to a background scalar field, identified here with a conformal Higgs field. The implied Hubble expansion agrees with supernovae redshift data and determines centrifugal acceleration in the early universe, as required for a spontaneous big-bang model. The tachyonic mass parameter in the conformal Higgs model is identified with dark energy, which is simply a secondary consequence of the SU(2) symmetry-breaking finite scalar field amplitude required to explain weak gauge boson masses. This tachyonic mass parameter is generated

by a new and very small scale parameter, the cosmological time derivative of the gravitational Ricci scalar. This removes a longstanding apparent conflict between magnitudes of elementary-particle and cosmological parameters.

References

1. Dodelson, S. *Modern Cosmology*; Academic Press: New York, NY, USA, 2003.
2. Weyl, H. Gravitation und elektrizität. *Sitzungber. Preuss. Akad. Wiss.* **1918**, 465–480.
3. Mannheim, P.D. Conformal cosmology with no cosmological constant. *Gen. Rel. Grav.* **1990**, *22*, 289–298.
4. Mannheim, P.D. Alternatives to dark matter and dark energy. *Prog. Part. Nucl. Phys.* **2006**, *56*, 340–445.
5. Nesbet, R.K. Cosmological implications of conformal field theory. *Mod. Phys. Lett. A* **2011**, *26*, 893–900.
6. DeWitt, B.S. *Relativity, Groups, and Topology*; DeWitt, C., DeWitt, B.S., Eds.; Gordon and Breach: New York, NY, USA, 1964.
7. Mannheim, P.D. Comprehensive solution to the cosmological constant, zero-point energy, and quantum gravity problems. *Gen. Rel. Grav.* **2011**, *43*, 703–750.
8. Nesbet, R.K. Conformal higgs model of dark energy. arXiv:1004.5097v2 [physics.gen-ph].
9. Mannheim, P.D.; Kazanas, D. Exact vacuum solution to conformal weyl gravity and galactic rotation curves. *Astrophys. J.* **1989**, *342*, 635–638.
10. Flanagan, E.E. Fourth order weyl gravity. *Phys. Rev. D* **2006**, *74*, 023002:1–023002:4.
11. Mannheim, P.D. Schwarzschild limit of conformal gravity in the presence of macroscopic scalar fields. *Phys. Rev. D* **2007**, *75*, 124006:1–124006:15.
12. Mannheim, P.D. Are galactic rotation curves really flat? *Astrophys. J.* **1997**, *479*, 659–664.
13. Mannheim, P.D.; O’Brien, J.G. Impact of a global quadratic potential on galactic rotation curves. *Phys. Rev. Lett.* **2011**, *106*, 121101:1–121101:4.
14. Mannheim, P.D.; O’Brien, J.G. Fitting galactic rotation curves with conformal gravity and a global quadratic potential. *Phys. Rev. D* **2012**, *85*, 124020:1–124020:26.
15. O’Brien, J.G.; Mannheim, P.D. Fitting dwarf galaxy rotation curves with conformal gravity. *Mon. Not. R. Astron. Soc.* **2012**, *421*, 1273–1282.
16. Nesbet, R.K. Proposed explanation of galactic halos. arXiv:1109.3626v3 [physics.gen-ph].
17. Peskin, M.E.; Schroeder, D.V. *Introduction to Quantum Field Theory*; Westview Press: Boulder, CO, USA, 1995.
18. Cottingham, W.N.; Greenwood, D.A. *An Introduction to the Standard Model of Particle Physics*; Cambridge University Press: New York, NY, USA, 1998.
19. Nesbet, R.K. *Variational Principles and Methods in Theoretical Physics and Chemistry*; Cambridge University Press: New York, NY, USA, 2003.
20. Deser, S.; Franklin, J. Schwarzschild and birkhoff a la weyl. *Am. J. Phys.* **2005**, *73*, 261–264.
21. Ostriker, J.P.; Peebles, P.J.E. A numerical study of the stability of flattened galaxies: Or, can cold galaxies survive? *Astrophys. J.* **1973**, *186*, 467–480.

22. Ostriker, J.P.; Peebles, P.J.E.; Yahil, A. The size and mass of galaxies and the mass of the universe. *Astrophys. J.* **1974**, *193*, L1–L4.
23. Sanders, R.H. *The Dark Matter Problem*; Cambridge University Press: New York, NY, USA, 2010.
24. Rogstad, D.H.; Shostak, G.S. Gross properties of five SCD galaxies. *Astrophys. J.* **1972**, *176*, 315–321.
25. Bosma, A. 21-cm line studies of spiral galaxies. *Astron. J.* **1981**, *86*, 1825–1846.
26. Walsh, D.; Carswell, R.F.; Weymann, R.J. Twin quasistellar objects or gravitational lens. *Nature* **1979**, *279*, 381–384.
27. Paczynski, B. Giant luminous arcs discovered in two clusters of galaxies. *Nature* **1987**, *325*, 572–573.
28. Bartelmann, M. Gravitational lensing. *Class. Quantum Gravity* **2010**, *27*, doi:10.1088/0264-9381/27/23/233001.
29. Persic, M.; Salucci, P.; Stel, F. The universal rotation curve of spiral galaxies. I. *Mon. Not. R. Astron. Soc.* **1996**, *281*, 27–48.
30. Salucci, P.; Lapi, A.; Tonini, C.; Gentile, G.; Yegorova, I.; Klein, U. The universal rotation curve of spiral galaxies. II. *Mon. Not. R. Astron. Soc.* **2007**, *378*, 41–51.
31. Milgrom, M. A modification of newtonian dynamics. *Astrophys. J.* **1983**, *270*, 365–370.
32. Sanders, R.H.; McGaugh, S.S. Modified newtonian dynamics as an alternative to dark matter. *Ann. Rev. Astron. Astrophys.* **2002**, *40*, 263–317.
33. Tully, R.B.; Fisher, J.R. A new method for determining the distances to galaxies. *Astron. Astrophys.* **1977**, *54*, 661–673.
34. McGaugh, S.S. Novel test of MOND with gas rich galaxies. *Phys. Rev. Lett.* **2011**, *106*, 121303:1–121303:4.
35. Bekenstein, J.D. Relativistic gravitation theory for the MOND paradigm. *Phys. Rev. D* **2004**, *70*, 083509:1–083509:28.
36. Moffat, J.W. Scalar-tensor-vector gravity theory. *J. Cosmol. Astropart. Phys.* **2006**, *2006*, 4–23.
37. Brownstein, J.R.; Moffat, J.W. Galaxy rotation curves without non-baryonic dark matter. *Astrophys. J.* **2006**, *636*, 721–761.
38. Moffat, J.W.; Rahvar, S.; Toth, V.T. Applying MOG to lensing: Einstein rings, abell 520 and the bullet cluster. arXiv:1204.2985v1[astro-ph.CO].
39. Hubble, E. A relation between distance and radial velocity among extra-galactic nebulae. *Proc. Nat. Acad. Sci. USA* **1929**, *15*, 168–173.
40. Riess, A.G.; Filippenko, A.V.; Challis, P.; Clocchiatti, A.; Diercks, A.; Garnavich, P.M.; Gilliland, R.L.; Hogan, C.J.; Jha, S.; Kirshner, R.P.; *et al.* Observational evidence from supernovae for an accelerating universe and a cosmological constant. *Astron. J.* **1998**, *116*, 1009–1038.
41. Perlmutter, S.; Aldering, G.; Goldhaber, G.; Knop, R.A.; Nugent, P.; Castro, P.G.; Deustua, S.; Fabbro, S.; Goobar, A.; Groom, D.E.; *et al.* Measurement of Ω and Λ from 42 high-redshift supernovae. *Astrophys. J.* **1999**, *517*, 565–586.
42. Friedmann, A. Über die Krümmung des raumes. *Zeits. Phys.* **1922**, *10*, 377–386.
43. Friedmann, A. Über die Möglichkeit einer welt mit konstanter negativer krümmung des raumes. *Zeits. Phys.* **1924**, *21*, 326–332.

44. LeMaître, G. Un univers homogène de masse constante et de rayon croissant. *Ann. Soc. Sci. Bruxelles* **1927**, *A47*, 49–59.
45. Mannheim, P.D. How recent is cosmic acceleration? *Int. J. Mod. Phys. D* **2003**, *12*, 893–904.
46. Komatsu, E.; Dunkley, J.; Nolte, M.R.; Bennett, C.L.; Gold, B.; Hinshaw, G.; Jarosik, N.; Larson, D.; Limon, M.; Page, L.; *et al.* Five-year WMAP Observations. *Astrophys. J. Supp.* **2009**, *180*, 330–381.
47. Komatsu, E.; Smith, K.M.; Dunkley, J.; Bennett, C.L.; Gold, B.; Hinshaw, G.; Jarosik, N.; Larson, D.; Nolte, M.R.; Page, L.; *et al.* Seven-year WMAP Observations. *Astrophys. J. Supp.* **2011**, *192*, 18–74.
48. Wang, Y.; Mukherjee, P. Observational constraints on dark energy and cosmic curvature. *Phys. Rev. D* **2007**, *76*, 103533:1–103533:11.
49. Nesbet, R.K. The higgs scalar field with no massive higgs particle. arXiv:1009.1372v3 [physics.gen-ph].
50. Cho, A. Higgs boson makes its debut after decades-long search. *Science* **2012**, *337*, 141–143.
51. Nesbet, R.K. Theory of spin-dependent conductivity in GMR materials. *IBM J. Res. Dev.* **1998**, *42*, 53–71.
52. McGaugh, S.S. Balance of dark and luminous mass in rotating galaxies. *Phys. Rev. Lett.* **2005**, *95*, 171302:1–171302:4.
53. Allen, S.W.; Evrard, A.E.; Mantz, A.B. Cosmological parameters from observations of galaxy clusters. *Ann. Rev. Astron. Astrophys.* **2011**, *49*, 409–470.

© 2013 by the author; licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution license (<http://creativecommons.org/licenses/by/3.0/>).