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Article

Finite-Time Chaos Suppression of Permanent Magnet Synchronous Motor Systems

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Abstract: This paper considers the problem of the chaos suppression for the Permanent Magnet Synchronous Motor (PMSM) system via the finite-time control. Based on Lyapunov stability theory and the finite-time controller are developed such that the chaos behaviors of PMSM system can be suppressed. The effectiveness and accuracy of the proposed methods are shown in numerical simulations.

Keywords: finite-time; chaos; PMSM; Lyapunov stability theory

1. Introduction

The PMSM system had been appealing to more and more industry engineers for many AC motor applications. Recently, there have been many researches presenting numerous kinds of control methods for PMSM systems [1–3]. The permanent magnet synchronous motor (PMSM) plays an important role in industrial applications due to its simple structure, high power density, low maintenance cost, and high efficiency [4–7]. In these two decades, the dynamic characteristics and stability analysis of PMSM had emerged as a new and attractive research field, such as bifurcation, chaos, and limit cycle dynamic behaviors [8–13], *etc.* Among chaos phenomenon which is a deterministic nonlinear dynamical system that has been generally developed over the past two decades, in fields such as engineering science, medical science, biological engineering, and secure communication. Chaotic systems are very complex, dynamic nonlinear systems and their response possesses intrinsic characteristics such as broadband noise-like waveforms, prediction difficulty, and sensitivity to initial condition variations, *etc.* Moreover, many profound theories and methodologies [14–18] have been developed to deal with this issue. For the chaos suppression of permanent magnet synchronous motor systems, some kinds of control design and determination of stability have been conferred [19–28]. Some control methods had been studied to stabilize PMSM systems, such as optimal Lyapunov exponents placement [19], passive control [20], fuzzy control [21–23], impulsive control [24,25], sensorless control [26,27], and the cascade adaptive approach [28]. Therefore, chaos suppression aimed at eliminating the undesired chaotic behavior has become an important issue in the field of nonlinear control. From the viewpoint of control theory, chaos suppression can be considered as a stability problem. Recently, secure communication has received a lot attention with regards to the internet and personal information. An increasing number of studies have addressed secure communication via the chaos theory. In past research reports, chaos systems have been an attractive topic for their potential applications in secure communication [29–32]. In recent years, digital chaos signals have been extensively used in commercial applications, such as multimedia systems, mobile and wireless communications [33–35]. In [36], by using XOR logic operation, a chaotic watermark was obtained between the binary watermark and the binary chaotic image, and then the chaotic watermark was embedded into an order less image of each block of the least significant bit.

On the other hand, based on its unique powerful advantages, a particular property of asymptotic stability, finite-time stability has received a lot attention recently. It is surely more useful for some problems or applications to obtain the finite-time stable element than convergence within infinity time. Moreover, the finite-time control techniques have demonstrated better robustness and disturbance rejection properties. Based on proposed fractional controllers, the finite-time stability and the settling time can be guaranteed and computed [37–49]. However, few studies have focused on the finite-time suppression chaos of permanent magnet synchronous motor (PMSM) systems.

Motivated by the above discussion, this paper aims to achieve the finite-time chaos suppression for the permanent magnet synchronous motor (PMSM) system by the proposed controllers. Based on finite-time stability theory, the chaos suppression of PMSM is analyzed. Finally, an example is given to illustrate the usefulness of the obtained results.

2. Preliminaries

In order to derive the main results, the following definition and lemma are needed:

Definition [46]: Consider the nonlinear dynamical system modeled by:

$$\dot{x} = g(x), \tag{1}$$

where the system state variable $x \in R^n$. If there exists a constant $T > 0$ (may depend on the initial system state $x(0)$), such that:

$$\lim_{t \to T} \| x(t) \| = 0, \tag{2}$$

and $\| x(t) \| \equiv 0$, if $t \geq T$, then system $\dot{x} = g(x)$ is finite-time stable.

Lemma [44]: Assume that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \leq -\rho V^{\lambda}(t),\ \forall t \geq t_0,\ V(t_0) \geq 0 \tag{3}$$

where $\rho > 0$ and $0 < \lambda < 1$ are two constants. Then, for any given t_0, $V(t)$ satisfies the following inequality:

$$V^{1-\lambda}(t) \le V^{1-\lambda}(t_0) - \rho(1-\lambda)(t-t_0),\ t_0 \le t \le t_r \tag{4}$$

$V(t) \equiv 0$, $\forall t \ge t_r$, with t_r given by:

$$t_r = t_0 + \frac{V^{1-\lambda}(t)}{\rho(1-\lambda)}. \tag{5}$$

3. Problem Formulation and Main Results

Based on d-q axis, the dynamic model of a permanent synchronous motor with a smooth air gap can be described by the following differential equation [8]:

$$\begin{aligned} \frac{di_d}{dt} &= -i_d + i_q w + \tilde{u}_d \\ \frac{di_q}{dt} &= -i_q - i_d w + \gamma w + \tilde{u}_q \\ \frac{dw}{dt} &= \sigma(i_q - w) - \tilde{T}_L \end{aligned} \tag{6}$$

where i_d, i_q, and w are state variables, which denote d, q axis stator currents, w is the motor angular speed, respectively. $\tilde{T}_L$, $\tilde{u}_d$, and $\tilde{u}_q$ are the external load torque, the direct- and quadrature axis stator voltage components of the motor, respectively. γ and σ are system operating parameters. In this paper, we only consider the case that the system is unforced. This case can be thought of as that, after an operating period of the system, the external inputs are set to zero, namely, $\tilde{T}_L = \tilde{u}_d = \tilde{u}_q = 0$. Then, the system (6) becomes:

$$\begin{aligned} \frac{di_d}{dt} &= -i_d + i_q w \\ \frac{di_q}{dt} &= -i_q - i_d w + \gamma w \\ \frac{dw}{dt} &= \sigma(i_q - w) \end{aligned} \tag{7}$$

or:

$$\begin{aligned} \dot{x}_1(t) &= -x_1(t) + x_2(t)x_3(t) \\ \dot{x}_2(t) &= -x_2(t) - x_1(t)x_3(t) + \gamma \cdot x_3(t) \\ \dot{x}_3(t) &= \sigma \cdot [x_2(t) - x_3(t)] \end{aligned} \tag{8}$$

where $x_1 = i_d$, $x_2 = i_q$, $x_3 = w$.

This paper aims at proposing a controller to suppression chaotic oscillation for PMAM in finite time, we add the single control $u(t)$ to system (8) and then the controlled PMSM system can be expressed by:

$$\begin{aligned} \dot{x}_1(t) &= -x_1(t) + x_2(t)x_3(t) \\ \dot{x}_2(t) &= -x_2(t) - x_1(t)x_3(t) + \gamma \cdot x_3(t) + u(t) \\ \dot{x}_3(t) &= \sigma \cdot [x_2(t) - x_3(t)] \end{aligned} \tag{9}$$

To achieve this aim, we propose the main results based on the finite-time stability theory (Definition). According to the Lyapunov stability theorem and the Lemma, if there is a feedback controller such that $V \le -\rho V^{\lambda}(t)$ where $V(t) = \frac{1}{2}(x_1^2(t) + x_2^2(t) + x_3^2(t))$ is the defined Lyapunov function and $\rho > 0$ and $0 < \lambda < 1$ are two real constants, the system's state variables converging to zero reaching in finite-time can be obtained. Therefore, the proposed controller $u(t)$ is designed as:

$$\begin{aligned} u(t) = x_2(t) - \rho|x_2(t)|^{\lambda}\operatorname{sgn}(x_2(t)) - (-x_1^2(t) + \gamma x_2(t)x_3(t) - \sigma x_3^2(t) + \sigma x_2(t)x_3(t) \\ + \rho|x_1(t)|^{\lambda+1} + \rho|x_3(t)|^{\lambda+1})\frac{\operatorname{sgn}(x_2(t))}{|x_2(t)|} \end{aligned} \tag{10}$$

where ρ is gain which is positive constant.

Theorem 1. Based on the proposed designed controller in Equation (10), the system's state variables in (9) will converge to zero in finite-time and the finite-time suppression chaotic oscillation for PMAM can be achieved.

Proof. Define the Lyapunov function:

$$V(t) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2), \tag{11}$$

where $V(t)$ is a legitimate Lyapunov function candidate, the time derivatives of $V(t)$, along the trajectories of system (9) with (10) satisfy:

$$\begin{aligned} \dot{V}(t) = x_1(t)(-x_1(t) + x_2(t)x_3(t)) + x_2(t)(-x_2(t) - x_1(t)x_3(t) + \gamma x_3(t) + u(t)) \\ + x_3(t)(\sigma x_2(t) - \sigma x_3(t)) \end{aligned}$$

By substituting the feedback controller (10), one can obtain:

$$\dot{V}(t) = -\rho|x_1(t)|^{\lambda+1} - \rho|e_y(t)|^{\lambda+1} - \rho|e_z(t)|^{\lambda+1} \tag{12}$$

From the above equation, we can obtain:

$$\dot{V}^{\frac{2}{\lambda+1}}(t) \le -2\rho^{\frac{2}{\lambda+1}}(\frac{1}{2}|x_1(t)|^2), \tag{13}$$

$$\dot{V}^{\frac{2}{\lambda+1}}(t) \le -2\rho^{\frac{2}{\lambda+1}}(\frac{1}{2}|x_2(t)|^2), \tag{14}$$

and:

$$\dot{V}^{\frac{2}{\lambda+1}}(t) \le -2\rho^{\frac{2}{\lambda+1}}(\frac{1}{2}|x_3(t)|^2) \tag{15}$$

It implies:

$$3\dot{V}^{\frac{2}{\lambda+1}}(t) \le -2\rho^{\frac{2}{\lambda+1}}\left(\frac{1}{2}|x_1(t)|^2\right) - 2\rho^{\frac{2}{\lambda+1}}(\frac{1}{2}|x_2(t)|^2) - 2\rho^{\frac{2}{\lambda+1}}(\frac{1}{2}|x_3(t)|^2) \tag{16}$$

$$\dot{V}^{\frac{2}{\lambda+1}}(t) \le -\frac{2}{3}\rho^{\frac{2}{\lambda+1}}(V(t)) \tag{17}$$

Therefore, we can get:

$$\dot{V}(t) \le -\left(\frac{2}{3}\right)^{\frac{\lambda+1}{2}} \rho\left(V(t)\right)^{\frac{\lambda+1}{2}} \tag{18}$$

From the Definition and Lemma, $x_1(t)$, $x_2(t)$ and $x_3(t)$ can converge to zero in finite-time. The finite-time suppression chaotic oscillation for PMAM is guaranteed, completing the proof.

Remark: In order to avoid chattering, $\mathrm{sgn}(x(t))$ is replaced by $\frac{x(t)}{|x(t)|+\upsilon}$ in the simulation, where υ is an appropriate minimal value.

4. Numerical Simulation and Analysis

In this section, a numerical example is presented to demonstrate and verify the performance of the proposed results. Finite-time chaos suppression on PMSM via the finite-time stability theory will be conducted. Typical chaotic attractors behavior of PMSM is shown in Figures 1 and 2 with parameters given by $\gamma = 20$, $\sigma = 5.46$ [8].

Figure 1. The state variables of the PMSM system.

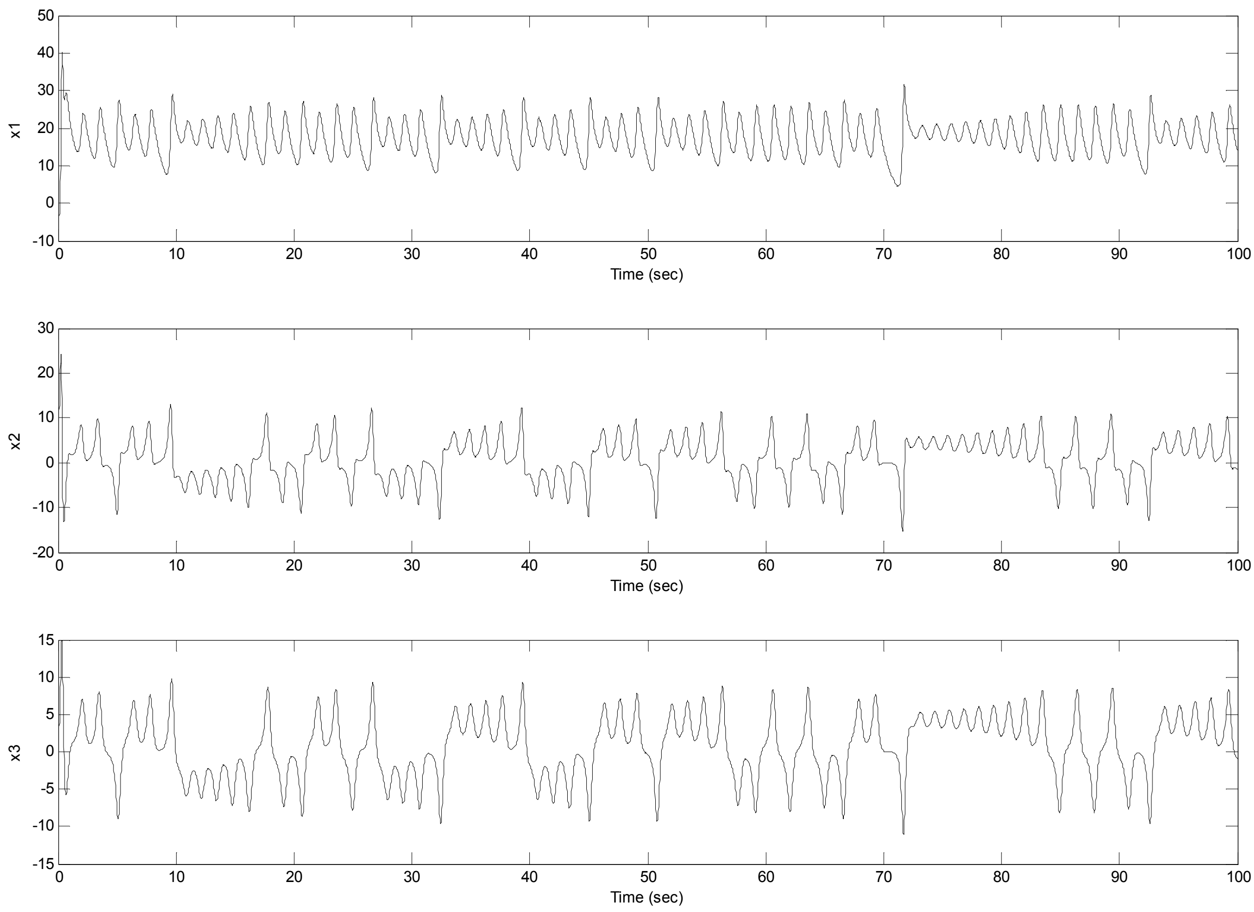
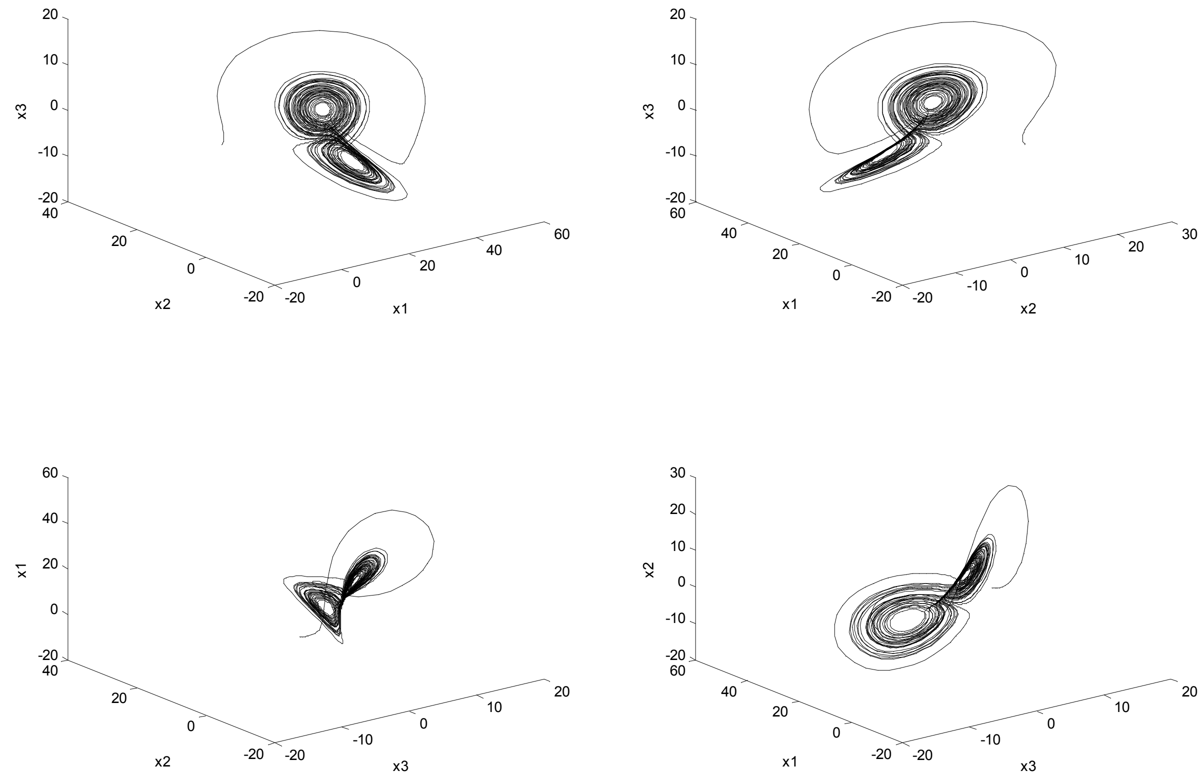


Figure 2. The chaotic attractor of the PMSM system.



The simulation was done with a four order Runge-Kutta integration algorithm in Matlab 7 with the control parameters are set as $\rho = 0.2$ and $\lambda = 0.7$; the initial conditions $x(0) = \begin{bmatrix} -5 & 10 & -1 \end{bmatrix}^T$. The system state responses trajectory of the controller design shown in Figure 3 depicts the time responses of the control input of $u(t)$.

Figure 3. The state responses of the controlled PMSM system.

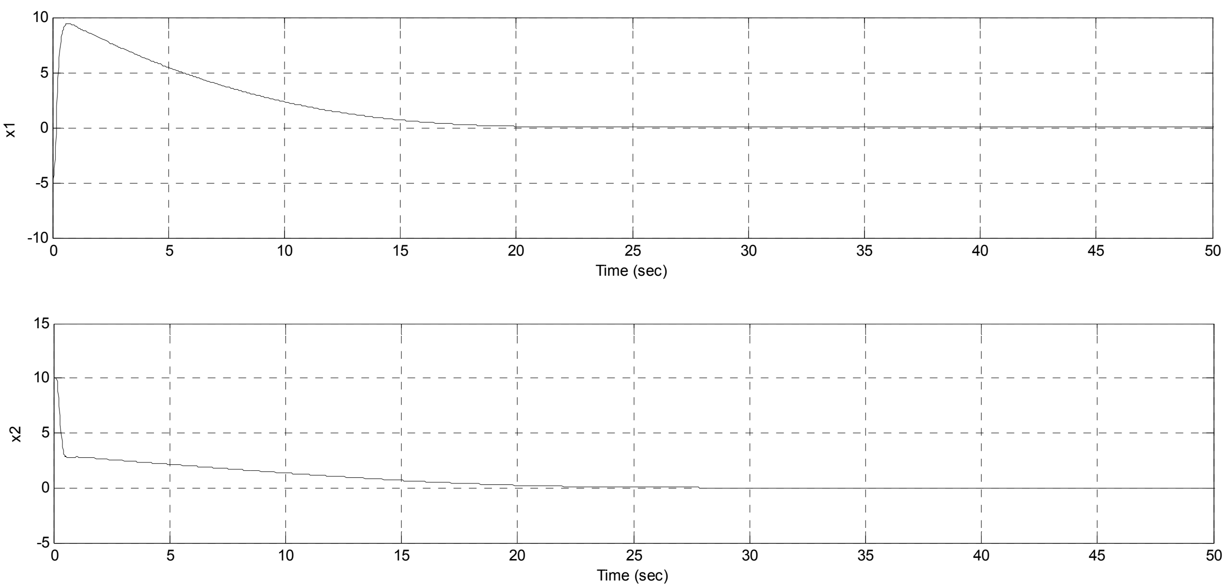
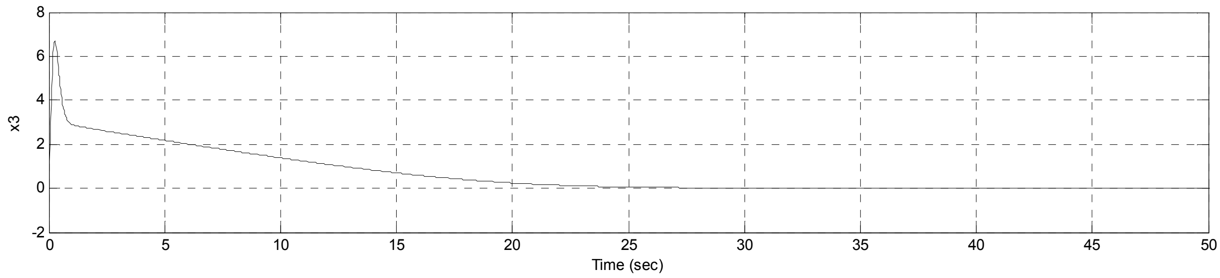


Figure 3. *Cont.*



It is seen clearly that the PMSM state reaches the desired goal in finite-time chaos suppression. From the simulation results, the effectiveness of the proposed method and the designed controller is shown.

5. Conclusions

In conclusion, the finite-time control of chaos suppression for PMSM systems was presented. Based on finite-time stability theory, the proposed control law is very effective according to the theoretical method and simulation results. The proposed controller is simple and easy to implement. This study should prove helpful to maintain industrial servo driven systems' secure operation and applies chaos control methods to the plant. Numerical simulation displayed the feasibility and usefulness of the central discussion.

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Conflicts of Interest

The author declares no conflict of interest.

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