entropy

Article

Perturbation of Fractional Multi-Agent Systems in Cloud Entropy Computing

Rabha W. Ibrahim *, Hamid A. Jalab and Abdullah Gani

Received: 2 December 2015; Accepted: 6 January 2016; Published: 19 January 2016
Academic Editors: J. A. Tenreiro Machado and António M. Lopes

Faculty of Computer Science and Information Technology, University of Malaya, Kuala Lumpur 50603, Malaysia; hamidjalab@um.edu.my (H.A.J.); abdullah@um.edu.my (A.G.)
* Correspondence: rabhaibrahim@um.edu.my; Tel.: +60-1-2279-7153

Abstract: A perturbed multi-agent system is a scheme self-possessed of multiple networking agents within a location. This scheme can be used to discuss problems that are impossible or difficult for a specific agent to solve. Intelligence cloud entropy management systems involve functions, methods, procedural approaches, and algorithms. In this study, we introduce a new perturbed algorithm based on the fractional Poisson process. The discrete dynamics are suggested by using fractional entropy and fractional type Tsallis entropy. Moreover, we study the algorithm stability.

Keywords: fractional calculus; fractional differential equation; cloude entropy system; complexity

---

1. Introduction

Cloud entropy systems are currently increasing in number and developing. The increasing occurrence of clouds has led to a large number of unresolved tasks. Cloud computing recognizes the internet services delivered by generalized data centers in the form of hardware and software systems; it is also a distributed computing system that consists of a group of organized virtual machines [1]. The original technology in a cloud entropy system consists of long-established values of routing, visualization, code duplication, and so on. Overall, the system uses known computational methods. Recent network structure schemes involve the improvement of new leader lines for the future of computing procedures. Such networks present understandable improvements, such as applying feedback. This usage leads to the perturbation of cloud systems.

Another problem of cloud entropy systems is how to divide information without performance degradation. The effectiveness of standing transmission systems and databases in clouds should be analyzed. Developing measures for the system transmission efficiency is necessary to achieve improvements. A drawback system functioning in clouds is devoted to communication channels. The utility of perturbation offers the possibility of transfering to the dynamic control of a model, principles established for them, and conforming optimization methods. Control systems are applied to assess structural complexity because they have the ability to oversee all functioning units, the counting frequency through networks, and the present form of resources.

Multi-agent systems fundamentally appear in integer calculus by using ordinary differential equations. Various multi-agent systems cannot be repeated with the integer cases in a complex physical situation, but they are refined with fractional order (non-integer) differential equations [2]. Fractional multi-agent systems are considered and suggested by a collection of fractional differential operators, such as Riemann–Liouville, Caputo, and Jumarie [3–5]. Multi-agent systems have substantial documented research in the fields of military studies, economic systems, and control systems. An enormous number of these systems has been examined, such as distributed sensor networks, cloud computing systems composed of multiple servers, altitude control of satellites, and so on [6–10].

A perturbed multi-agent system is a scheme self-possessed of multiple networking agents within a location. This scheme can be utilized to address problems that are impossible or difficult for a specific agent to solve. Intelligent cloud-entropy management systems involve various functions, methods, procedural approaches, and algorithms. Overall, a dynamic system is necessary to improve a stable structure and its reliability. The decomposition of functional cloud components is conducted through certain methods for the system analysis of structural complexity. The dynamic explanations in the formal approximations of differential equations with control are employed in many works [11–13].

In this study, we establish a new perturbed multi-agent system for cloud entropy services based on the fractional Poisson process (see Figure 1). The discrete dynamics are proposed by using fractional entropy and fractional Tsallis entropy. Moreover, we examine the algorithm stability.

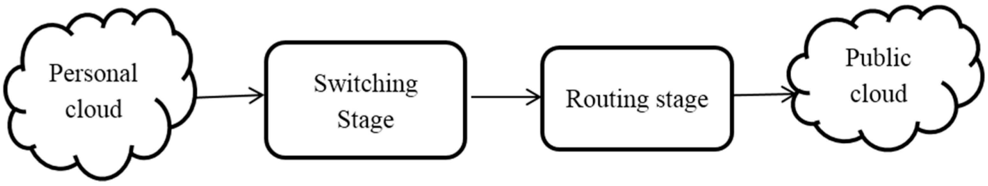

Figure 1. Structure of a perturbation cloud entropy system.

2. Setting

In this section, we consider the basic positions in which incidence establishes grounds for the control system theory and dynamic models of cloud service. We reflect a distributed database in clouds. Cloud databases comprise software or hardware routing. They supply data on the practical machine on which sections of data are deposited, the private cloud that determines the level of security in public, and other service information. A similar type of entering query is transformed in each incident to different situations and route maps and is influenced by the model structure and communication frequencies.

In such a setting, a continuum of agents that have nonhomogenous preferences pay a cost to move from one point to another in the state space. We suppose that n agents exist, whose communication network is a directed graph. Agent i has cost function $\Phi_i(t,\chi,u)$, which represents what an agent pays to have characteristics $\chi_i(t)$ (*i.e.*, the level of cloud computing at time t) under controller input $u_i(t)$. Thus, we may present the following fractional system:

$$D^{(\wp)}\chi_i(t) = \Phi_i(t,\chi,u), i = 1,...,n$$

$$(y_i(t) = \chi_i(t)),$$

where $y_i(t)$ is the outcome at time t, $D^{(\wp)}\chi_i(t)$ indicates the Riemann–Liouville calculus given by:

$$D^{(\wp)}\chi_i(t) = \frac{d}{dt}\int_a^t \frac{(t-\tau)^{-\wp}}{\Gamma(1-\wp)}\chi(\tau)\,d\tau,$$

which corresponds to the integral operator:

$$I_a^{\wp}\chi(t) = \int_a^t \frac{(t-\tau)^{\wp-1}}{\Gamma(\wp)}\chi(\tau)\,d\tau,$$

accordingly, $\wp \in (0,1)$. Saber *et al.* [14] presented the dynamics of a cloud through the following equation:

$$\begin{aligned}\chi_i{}'(t) &= \Phi_i(t,\chi,u), i = 1,...,n\\ &= \sum_{j\in\mathbb{N}} \alpha_{ij}\left(\chi_j(t) - \chi_i(t)\right),\end{aligned}$$

where $\alpha_{ij} > 0$ is the (i, j) (agent i can receive information from agent j; else, $\alpha_{ij} = 0$) element of the adjacency matrix $\chi(t) = (\chi_1(t), ..., \chi_n(t))^{\top}$, $y(t) = (y_1(t), ..., y_n(t))^{\top}$, $u(t) = (u_1(t), ..., u_n(t))^{\top}$. If $\chi_i(t) \to \chi_j(t)$ for all $t \in J = [0, T]$, $T \to \infty$, then the system is asymptotically stable (see [14]).

Our discussion is based on the linear perturbation system of the second type [15], that is:

$$D^{(\wp)}[\chi_i(t) + \phi_i(t, \chi, u)] = \varphi_i(t, \chi, u), i = 1, ..., n \tag{1}$$

$$(y_i(t) := \chi_i(t)),$$

where $\phi_i(t, \chi_i, u_i) = \varphi_i(t, \chi, u_i) - D^{(\wp)}\phi_i(t, \chi, u)$.

Function $\chi_i \in C[J, \mathbb{R}]$, which is the space of all continuous functions on J, is called a solution for problem (1) if and only if function χ_i is continuous for all $t \in J$ and χ_i fulfills Equation (1). Our objective is to establish the existence of a solution for Equation (1). The main significant benefit of the construction usage of fractional differential equations (ordinary and partial) in mathematical modeling is their nonlocal property. The integer order differential operator is a local, linear operator, whereas the fractional order differential operator is a nonlocal, nonlinear one. Thus, the following state of a system is influenced by its present and past states (this property is useful in cloud computing). Thus, fractional calculus has become increasingly popular in technical and industrial fields. In this study, a perturbation analysis model is studied to solve linear and nonlinear differential and integral equations. Various types of perturbation techniques and forms are applied depending on the actual situation and physical problems. A good outcome is obtained in a cloud entropy system by minimizing the time of utility and the cost of the system (see [16]).

3. Existence Results of A Perturbed System

We define a supremum norm $\| \, . \, \| \in C[J, \mathbb{R}]$ by $\| x \| := \sup_{t \in J} |x|$. Obviously, $C[J, \mathbb{R}]$ is a Banach space. We generate the following result, which can be found in [17]:

Lemma 3.1 *We let $\mathcal{B}$ be a closed convex and bounded subset of the Banach space $\mathbb{X}$ and $P : \mathbb{X} \to \mathbb{X}$ and $Q : \mathcal{B} \to \mathbb{X}$ be two operators, such that (a) P is nonlinear D-contraction, (b) Q is compact and continuous, and (c) $\chi = P\chi + Qy$ for all $\chi, y \in B$. The operator equation $P\chi + Q\chi = \chi$ then has a solution in $\mathcal{B}$.*

By applying certain properties of fractional operators, we obtain the following result:

Lemma 3.2 *We assume that $t \longmapsto \chi_i - \phi_i$ is injective (or increasing) in $\mathbb{R}$ and φ_i is bounded on $J \times \mathbb{R} \times \mathbb{R}$. Therefore, function χ_i is a solution of problem (1) if and only if it is a solution of the fractional integral equation*:

$$\chi_i(t) = \chi_i(t_0) - \phi_i(t_0, \chi(t_0), u(t_0)) + \phi_i(t, \chi, u) + \int_0^t \frac{(t-\tau)^{\wp-1}}{\Gamma(\wp)} \varphi_i(\tau, \chi(\tau), u(\tau)) \, d\tau.$$

Theorem 3.1 *We suppose that the assumptions of Lemma 3.2 hold. If constant $\ell > 0$ exists, such that*:

$$|\phi_i(t, \chi, u) - \phi_i(t, x, v)| \leqslant \ell\left(\frac{|\chi - x| + |u - v|}{\flat + |\chi - x| + |u - v|}\right), \ell \leqslant \flat, |\varphi_i| \leqslant \flat \tag{2}$$

then problem (1) has a solution on J.

Proof of Theorem 3.1 In view of Lemma 3.2, we have:

$$
\begin{aligned}
|\chi_i| = & \left|\chi_i(t_0) - \phi_i(t_0, \chi(t_0), u(t_0)) + \phi_i(t, \chi, u) + \int_0^t \frac{(t-\tau)^{\wp-1}}{\Gamma(\wp)} \varphi_i(\tau, \chi(\tau), u(\tau))\, d\tau\right| \\
\leqslant & |\chi_0| + \ell\left(\frac{|\chi-\chi_0| + |u-u_0|}{\flat + |\chi-\chi_0| + |u-u_0|}\right) + \int_0^t \frac{(t-\tau)^{\wp-1}}{\Gamma(\wp)} |\varphi_i(\tau, \chi(\tau), u(\tau))|\, d\tau \\
\leqslant & |\chi_0| + \ell\left(\frac{|\chi-\chi_0| + |u-u_0|}{\flat + |\chi-\chi_0| + |u-u_0|}\right) + \flat \int_0^t \frac{(t-\tau)^{\wp-1}}{\Gamma(\wp)} d\tau \\
= & |\chi_0| + \ell\left(\frac{|\chi-\chi_0| + |u-u_0|}{\flat + |\chi-\chi_0| + |u-u_0|}\right) + \frac{\flat T^{\wp}}{\Gamma(\wp+1)} := \mathcal{B}.
\end{aligned}
$$

$\mathcal{B}$ is clearly a closed, convex, and bounded subset of $\mathbb{X}$. We define the two operators $P : \mathbb{X} \to \mathbb{X}$ and $Q : \mathcal{B} \to \mathbb{X}$ as follows:

$$
(P)(\chi) := \phi_i(t, \chi, u), (Q)(\chi) := \chi_0 - \phi_i(t, \chi_0, u_0) + \int_0^t \frac{(t-\tau)^{\wp-1}}{\Gamma(\wp)} \varphi_i(\tau, \chi(\tau), u(\tau))\, d\tau.
$$

Thus, we have the following operator equation:

$$
(\chi)(t) = (P)(\chi)(t) + (Q)(\chi)(t) \tag{3}
$$

We aim to achieve the conditions of Lemma 3.1. First, we show that P is a nonlinear D-contraction on $\mathbb{X}$. Using (2), we obtain:

$$
\begin{aligned}
& |(P)(\chi, u)(t) - (P)(x, v)(t)| = |\phi_i(t, \chi, u) - \phi_i(t, x, v)| \\
& \leqslant \ell\left(\frac{|\chi - x| + |u - v|}{\flat + |\chi - x| + |u - v|}\right),
\end{aligned}
$$

By taking a supremum norm over J, we have:

$$
\| (P)(\chi) - (P)(x) \| \leqslant \ell\left(\frac{\| \chi - x \| + \| u - v \|}{\flat + \| \chi - x \| + \| u - v \|}\right).
$$

Thus, P is a D-contraction mapping defined by:

$$
D(\rho) := \frac{\ell\rho}{\flat + \rho}, \rho = \| \chi - x \| + \| u - v \| .
$$

Second, we aim to show that Q is a compact and continuous operator on $\mathcal{B}$ into $\mathbb{X}$. To prove that Q is continuous on $\mathcal{B}$, we let $\{\chi_m\}, \{u_m\}$ be two sequences in $\mathcal{B}$ converging to point $\chi, u \in \mathcal{B}$, respectively. By utilizing the dominated convergence theorem for integration, we have:

$$
\begin{aligned}
\lim_{m\to\infty} Q(\chi_m)(t) = & \lim_{m\to\infty} [\chi_0 - \phi_i(t, \chi_0, u_0) + \int_0^t \frac{(t-\tau)^{\wp-1}}{\Gamma(\wp)} \varphi_i(\tau, \chi_m(\tau), u_m(\tau))\, d\tau] \\
= & \chi_0 - \phi_i(t, \chi_0, u_0) + \lim_{m\to\infty} [\int_0^t \frac{(t-\tau)^{\wp-1}}{\Gamma(\wp)} \varphi_i(\tau, \chi_m(\tau), u_m(\tau))\, d\tau] \\
= & \chi_0 - \phi_i(t, \chi_0, u_0) + \lim_{m\to\infty} [\int_0^t \frac{(t-\tau)^{\wp-1}}{\Gamma(\wp)} \varphi_i(\tau, \chi(\tau), u(\tau))\, d\tau] \\
= & (Q)(\chi)(t).
\end{aligned}
$$

Q is accordingly continuous on $\mathcal{B}$ into $\mathbb{X}$. We proceed to show that Q is uniformly bounded:

$$\begin{aligned}|Q(\chi)(t)| &\leqslant |\chi_0 - \phi_i(t,\chi_0,u_0)| + [\int_0^t \frac{(t-\tau)^{\wp-1}}{\Gamma(\wp)} |\varphi_i(\tau,\chi(\tau),u(\tau))|\, d\tau] \\ &\leqslant |\chi_0 - \phi_i(t,\chi_0,u_0)| + \frac{\flat T^{\wp}}{\Gamma(\wp+1)}.\end{aligned}$$

Q is then proven to be uniformly bounded on $\mathcal{B}$. For $t_2 \geqslant t_1$, $t_1, t_2 \in J$, we obtain:

$$\begin{aligned}&|Q(\chi)(t_1) - Q(\chi)(t_2)| = |\int_0^{t_1} \frac{(t_1-\tau)^{\wp-1}}{\Gamma(\wp)} \varphi_i(\tau,\chi(\tau),u(\tau))\, d\tau] - \int_0^{t_2} \frac{(t_2-\tau)^{\wp-1}}{\Gamma(\wp)} \varphi_i(\tau,\chi(\tau),u(\tau))\, d\tau]| \\ &\leqslant |\int_0^{t_2} \frac{(t_2-\tau)^{\wp-1}}{\Gamma(\wp)} |\varphi_i(\tau,\chi(\tau),u(\tau))|\, |d\tau| \\ &\leqslant \frac{(t_2-t_1)^{\wp}\flat}{\Gamma(\wp+1)} := \epsilon,\end{aligned}$$

where $|t_2 - t_1| := \delta$. $Q(\mathcal{B})$ then becomes an equi-continuous set in $\mathbb{X}$. Thus, by virtue of the Arzelá–Ascoli theorem, we conclude that Q is compact. To achieve condition (c), we have:

$$\begin{aligned}|\chi| &= |P(\chi) + Q(\chi)| \\ &\leqslant |\chi_0| + \ell(\frac{|\chi-\chi_0| + |u-u_0|}{\flat + |\chi-\chi_0| + |u-u_0|}) + \int_0^t \frac{(t-\tau)^{\wp-1}}{\Gamma(\wp)} |\varphi_i(\tau,\chi(\tau),u(\tau))|\, d\tau \\ &\leqslant |\chi_0| + \ell(\frac{|\chi-\chi_0| + |u-u_0|}{\flat + |\chi-\chi_0| + |u-u_0|}) + \flat \int_0^t \frac{(t-\tau)^{\wp-1}}{\Gamma(\wp)} d\tau \\ &= |\chi_0| + \ell(\frac{|\chi-\chi_0| + |u-u_0|}{\flat + |\chi-\chi_0| + |u-u_0|}) + \frac{\flat T^{\wp}}{\Gamma(\wp+1)}\end{aligned}$$

Therefore, $\chi \in \mathcal{B}$. All the hypotheses of Lemma 3.1 are then demostrated; thus, the operator equation $P + Q = \chi$ has a solution in $\mathcal{B}$. As a result, system (1) has a solution defined on J. This outcome completes the proof. □

Remark 3.1 *For the special case in different forms, the equation:*

$$\chi(\boldsymbol{t}+1) = \boldsymbol{P}\chi(\boldsymbol{t}) + \boldsymbol{Qu}(\boldsymbol{t})$$

was considered by Pluzhnik et al. [18]. They requested control $\boldsymbol{u}$ *under constraint* $\boldsymbol{u} \in \mathcal{B} \subseteq \mathbb{X}$. *Theorem 3.1 shows that the constraint in [18] is valid. In the following section, we illustrate an algorithm to minimize the cost function by utilizing the fractional Poisson process incorporating the fractional Tsallis entropy.*

4. The Proposed Algorithm

To solve Equation (1), we need the following facts:

Objective function. Our primary objective is to investigate the optimal flat rate at which a company uses cloud computing. With the cost proposed in Section 2, all rational companies decide on the flat rate of switching to the cloud-computing pattern to minimize the estimated discount cost with respect to the effort cost. For this purpose, we define a suitable objective function as follows:

$$\hat{\Theta}(\chi) = \Theta(\chi) + \sum_{i=1}^{m} \rho(a_i, \phi_i(\chi)) + \sum_{i=1}^{n} [\rho(b_i, \varphi_i(\chi)) + \rho(b_i, -\varphi_i(\chi))] \tag{4}$$

Such a function satisfies the inequality constraints of the following equation:

$$(\phi_i(\chi) \geqslant \epsilon, |\varphi_i(\chi)| \leqslant \flat,\ \flat \to 0,\ \epsilon \geqslant \flat,\ \forall i),$$

which is a model of an optimization problem, where a_i and b_i are the positive constants; ρ is the penalty function, which modifies the original objective function; ϕ_i and φ_i are the constraints. The penalty strategy uses finite difference techniques as a fitness function (FF) in a cloud (see Figure 2).

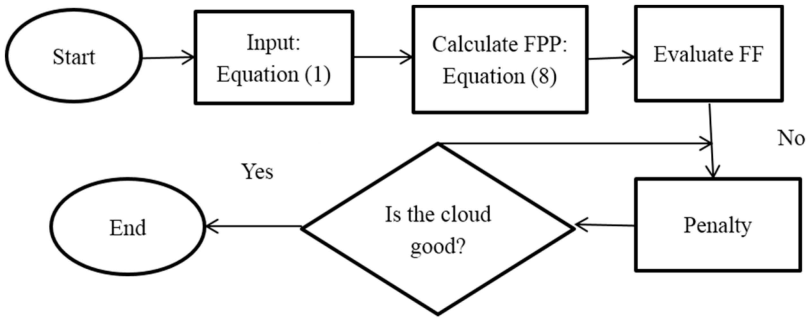

Figure 2. The proposed algorithm.

Fitness function (FF). The fitness function of a cloud between agents χ_i and χ_j is employed to control the process during the system evolution. Equation (1) can be reduced to minimize the problem as follows:

$$\hat{\Xi}_{\Omega}(\hat{\chi}) = \sum_{\chi_i \in \Omega} \left(\mathcal{D}(\chi_i) - \varphi_i(\chi_i)\right), \tag{5}$$

where $\mathcal{D}(\chi_i) := D^{(\wp)}\left[\chi_i(t) + \phi_i(t, \chi, u)\right]$, which satisfies the conditions:

$$\hat{\Xi}_{\partial\Omega}(\chi) = \sum_{\chi_i \in \partial\Omega} \left(\overline{\mathcal{D}}(\chi_i) - \overline{\varphi}(\chi_i)\right) = 0, \tag{6}$$

where $\hat{\chi}$ is the estimated value of χ. Problems (5) and (6) can be solved by letting the FF:

$$F(\chi) = \hat{\Xi}_{\Omega}(\hat{\chi}) + \varsigma \hat{\Xi}_{\partial\Omega}(\chi), \tag{7}$$

where ς is the penalty parameter.

Fractional entropy. In this study, we deal with a measure of entropy imposed by Tsallis [19] with the form:

$$\mathcal{T}_{\gamma}(\phi) = \frac{\int_x [\phi(x)]^{\gamma} dx - 1}{1 - \gamma}, \gamma \neq 1$$

or in the discrete form:

$$\mathcal{T}_{\gamma}(\phi) = \frac{1}{\gamma - 1}\left(1 - \sum_{k=1}^{m} \phi_i^{\gamma}\right), \gamma \neq 1$$

Several types of fractional entropy were recommended in [20–25]. At this stage, the appropriate amount of information based on observing the appearance of an event has probability P. The method is subject to the probability of extinction, which is described by the fractional Poisson process as follows [26]:

$$P_{\wp}(N, u) = \frac{(\sigma u)^N}{N!} \sum_{n=0}^{\infty} \frac{(n+N)!}{n!} \frac{(-\sigma u^{\wp})^n}{\Gamma(\wp(n+N)+1)}$$

where $\sigma \in \mathbb{R}$ is a physical coefficient, $\wp \in (0, 1]$, and u is the control in system (1). We let N be the number of agents and $\mathcal{I}$ be the average information. The source emits the symbols with probabilities $P_1, P_2, ..., P_N$, respectively, such that $P_i = P_{\wp}(i, u)$. Thus, we can calculate the total information by the following equation:

$$\mathcal{I} = \frac{1}{\gamma - 1}\left(1 - \sum_{i=1}^{m} P_i^{\gamma}\right), \gamma \neq 1 \tag{8}$$

Note that $0 < \sum_{i=1}^{m} P_i^{\gamma} < 1$. By applying Equation (8) to Equation (7), we conclude the following FFs:

$$\begin{aligned} F(\chi_i) &= \hat{\Xi}_{\Omega}(\hat{\chi}) + \varsigma\hat{\Xi}_{\partial\Omega}(\chi) \\ &\approx \sum_{\chi_i \in \Omega} D^{(\wp)}\chi_i(t) + (\mathcal{I} - \flat) + \varsigma[\sum_{\chi_i \in \partial\Omega} \overline{D}^{(\wp)}\chi_i(t) \\ &\quad + (\mathcal{I} - \flat)] \end{aligned} \tag{9}$$

The constraints of Equation (4) are met.

5. Stability of the Outcomes

In this section, we discuss the UH stability of Equation (1). We let $(\mathbb{X}, \parallel . \parallel)$ be a Banach space over $\mathbb{R}_+^n$ with a maximum norm. We have the following Hyers–Ulam stability (HUS):

Definition 5.1 We let ϵ be a nonnegative number. Equation (1) is then called stable in the Hyers-Ulam sense if $\delta > 0$ exists, such that for every $\mathcal{D}(\chi) \in C(\mathbb{R}_+^n, \mathbb{X})$ achieving Equtaion (5):

$$\parallel \mathcal{D}(\chi) - \varphi(\chi) \parallel \leqslant \epsilon\beta(\parallel \chi \parallel), \epsilon > 0, \tag{10}$$

where β is a positive function; for all $\chi \in \mathbb{R}_+^n$, an outcome $\eta \in \mathbb{R}_+^n$ exists with the property:

$$\parallel \chi(t) - \eta(t) \parallel \leqslant \delta \tag{11}$$

Theorem 5.1 *We suppose that $\mathcal{D}(\chi) \in C(\mathbb{R}_+^n, \mathbb{X})$, which satisfies Equation (1). If β is a linear function in $\parallel \chi \parallel$, then every solution is bounded and in HUS.*

Proof of Theorem 5.1 Let the condition (10) be achieved. Clearly, we obtain:

$$\begin{aligned} &\parallel \chi(t) - \eta(t) \parallel = \parallel u(\chi) - \eta(\chi) + \mathcal{D}(\chi) - \varphi(\chi) - \mathcal{D}(\chi) + \varphi(\chi) + \mathcal{D}(\eta) - \varphi(\eta) - \mathcal{D}(\eta) + \varphi(\eta) \parallel \\ &\leqslant \parallel \chi \parallel + \parallel \eta \parallel + 2 \parallel \mathcal{D}(\eta) - \varphi(\eta) \parallel + 2 \parallel \mathcal{D}(\chi) - \varphi(\chi) \parallel \\ &\leqslant \parallel \chi \parallel + \parallel \eta \parallel + \epsilon\beta(\parallel \chi \parallel) + \epsilon\beta(\parallel \eta \parallel) \\ &= \parallel \chi \parallel + \parallel \eta \parallel + \epsilon\beta(\parallel \chi \parallel + \parallel \eta \parallel) \\ &\leqslant b(1+\epsilon) := \delta, \end{aligned}$$

where $b := \max\{\parallel \chi \parallel + \parallel \eta \parallel, \beta(\parallel \chi \parallel + \parallel \eta \parallel)\}$. Hence, (11) is satisfied and this completes the proof. □

Theorem 5.2 *We suppose that $\mathcal{D}(\chi) \in C(\mathbb{R}_+^n, \mathbb{X})$, which satisfies Equation (1). We also let $\mathcal{D}(\chi)$ and φ be Lipschitz functions with Lipschitz constants λ_1 and λ_2,respectively. If contact $0 < \lambda_0 < 1$ exists, such that:*

$$\lambda := \lambda_0 + \lambda_1 + \lambda_2 < 1, 1 - \lambda_0 \leqslant \epsilon,$$

then every solution is bounded and HUS.

Proof of Theorem 5.2 Equation (10) is achieved and we obtain:

$$\begin{aligned} &\parallel \chi(t) - \eta(t) \parallel = \parallel \chi(t) - \eta(t) + \mathcal{D}(\chi) - \varphi(\chi) - \mathcal{D}(\chi) + \varphi(\chi) + \mathcal{D}(\eta) - \varphi(\eta) - \mathcal{D}(\eta) + \varphi(\eta) \parallel \\ &\leqslant \lambda_0 \parallel \chi - \eta \parallel + \parallel \mathcal{D}(\chi) - \varphi(\chi) \parallel + \parallel \mathcal{D}(\eta) - \varphi(\eta) \parallel + \parallel \varphi(\chi) - \mathcal{D}(\chi) \parallel + \parallel \varphi(\eta) - \mathcal{D}(\eta) \parallel \\ &\leqslant \lambda_0 \parallel \chi - \eta \parallel + \epsilon\beta(\parallel \chi \parallel) + \epsilon\beta(\parallel \eta \parallel) + \lambda_1 \parallel \chi - \eta \parallel + \lambda_2 \parallel \chi - \eta \parallel + (1 - \lambda_0) \parallel \chi - \eta \parallel \\ &\leqslant \lambda_0 \parallel \chi - \eta \parallel + \epsilon\beta(\parallel \chi \parallel) + \epsilon\beta(\parallel \eta \parallel) + \lambda_1 \parallel \chi - \eta \parallel + \lambda_2 \parallel \chi - \eta \parallel + \epsilon \parallel \chi - \eta \parallel \\ &:\leqslant \lambda \parallel \chi - \eta \parallel + \epsilon[\beta(\parallel \chi \parallel) + \beta(\parallel \eta \parallel) + \parallel \chi \parallel + \parallel \eta \parallel] \\ &\leqslant \lambda \parallel \chi - \eta \parallel + \epsilon\overline{\beta}, \end{aligned}$$

where $\overline{\beta} = \max\{\beta(\parallel \chi \parallel), \beta(\parallel \eta \parallel), \parallel \chi \parallel + \parallel \eta \parallel\}$. Thus, we conclude that:

$$\parallel \chi - \eta \parallel \leqslant \frac{\epsilon\overline{\beta}}{1 - \lambda} := \delta$$

Therefore, Eqution (11) is satisfied, and this completes the proof. □

6. Applications

Table 1 describes the experimental results for N = 10 agents and the different values of the fractional order $\wp$. We suppose the following parameters: $\gamma = 2$ and $\sigma = \varsigma = \rho = 1$, where γ is the dimension of the Tsallis entropy, σ is the physical coefficient of the Poisson process, ς is the penalty parameter, and ρ is the penalty function that is set as a constant function. The utility of the cloud entropy is evaluated at different times $t \in [1\text{–}5]$. The utility seems to increase with increasing time and decreasing the fractional Poisson process (FPP). The highest number appears at $t = 5$ and $\wp = 0.75$. Increasing the value of γ leads to increasing the utility of the cloud (consequently increasing the number of agents in the cloud [16]). The maximum utility goes to the fractional powers $\wp = 0.75$ and 0.5 with increasing time.

Table 1. Utility of the fractional cloud entropy system.

$\wp$	**Time**	$P_{\wp}(10)$	**Utility**
1	1	0.5380	0.48
	2	0.5281	0.49
	3	0.5139	0.491
	4	0.5128	0.492
	5	0.5117	0.5000
0.75	1	0.5800	0.5001
	2	0.5740	0.5005
	3	0.5585	0.5006
	4	0.5573	0.5007
	5	0.5561	0.5008
0.5	1	0.6113	0.4000
	2	0.6001	0.4001
	3	0.5839	0.50011
	4	0.5827	0.50012
	5	0.5814	0.5002

7. Conclusions

We have investigated a method for optimizing incomes and the arrangement originated by a system in cloud entropy computing. The method was supplementarily capable, surpassed mathematical software recommendations, and saved the overall cost and the task distribution. However, the objective function was engaged with the concept of the fractional differential equation. By utilizing this equation, we obtained an adaptation to the FF. This procedure was accessible to approximation by employing the fractional Tsallis entropy based on the fractional Poisson process. The stability illuminated the problems on the finite domain, which was suggested in the sense of the UH-stability strategy. The method offered two advantages, namely, it transformed the problem of constrained optimization into an unconstrained one and it had a suitable selection of the fractional order. The technique was a good approximation. Multiple connection could also be proposed by using the aforementioned technique. Furthermore, the method could be increased to a higher dimension when the number of agents in a multi-agent system became large.

Acknowledgments: The authors would like to thank the reviewers for their useful comments to improve the paper. This research is supported by Malaysian Ministry of Higher Education under the University Malaya High Impact Research Grant with reference: UM.C/625/1/HIR/MOE/FCSIT/03, and project number: RG312-14AFR from University of Malaya.

Author Contributions: Rabha W. Ibrahim designed the mathematical model; Hamid A. Jalab performed the experiments; Abdullah Gani wrote the paper; all the authors jointly worked on deriving the results. All authors have read and approved the final manuscript.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Kerrigan, B.; Chen, Y. A Study of Entropy Sources in Cloud Computers: Random Number Generation on Cloud Hosts. In *Computer Network Security*; Springer: Berlin/Heidelberg, Germany, 2012.
2. Tarasov, V.E. *Fractional Dynamics: Applications of Fractional Calculus to Dynamics of Particles, Fields and Media*; Springer: Berlin/Heidelberg, Germany, 2011.
3. Meerschaert, M.M.; Sikorskii, A. *Stochastic Models for Fractional Calculus*; Walter de Gruyter: Berlin, Germany, 2011.
4. Machado, J.T.; Mata, M.E. Pseudo phase plane and fractional calculus modeling of western global economic downturn. *Commun. Nonlinear Sci. Numer. Simul.* **2015**, *22*, 396–406. [CrossRef]
5. Machado, J.T.; Mata, M.E. A fractional perspective to the bond graph modelling of world economies. *Nonlinear Dyn.* **2014**, *80*, 1839–1852. [CrossRef]
6. Ghosh, S. *Distributed Systems: An Algorithmic Approach*, 2nd ed.; CRC press: Boca Raton, FL, USA, 2014.
7. Ren, W.; Beard, R.W. *Distributed Consensus in Multi-Vehicle Cooperative Control*; Springer: London, UK, 2008.
8. Li, D.; Liu, Q.; Wang, X. Distributed quantized consensus for agents on directed networks. *J. Syst. Sci. Complex.* **2013**, *26*, 489–511. [CrossRef]
9. Zeitz, M. Canonical forms for nonlinear systems. In Processdings of IFAC-Symposium, Capri, Italy, 14–16 June 1989; pp. 33–38.
10. Meng, D.; Jia, Y.; Du, J. Nonlinear finite-time bipartite consensus protocol for multi-agent systems associated with signed graphs. *Int. J. Control* **2015**, *88*, 1–24. [CrossRef]
11. Lu, C.; Wang, X.; Koutsoukos, X. Feedback utilization control in distributed real-time systems with end-to-end tasks. *IEEE Trans. Parallel Distrib. Syst.* **2005**, *16*, 550–561.
12. Diao, Y.; Gandhi, N.; Hellerstein, J.L.; Parekh, S.; Tilbury, D.M. Using mimo feedback control to enforce policies for interrelated metrics with application to the apache web server. In Proceedings of Network Operations and Management Symposium, 2002 (NOMS 2002), Florence, Italy, 15–19 April 2002; pp. 219–234.
13. Xu, W.; Zhu, X.; Singhal, S.; Wang, Z. Predictive control for dynamic resource allocation in enterprise data centers. In Proceedings of Network Operations and Management Symposium, 2006 (NOMS 2006), Vancouver, BC, Canada, 3–7 April 2006; pp. 115–126.
14. Olfati-Saber, R.; Fax, A.; Murray, R.M. Consensus and cooperation in networked multi-agent systems. *Proc. IEEE* **2007**, *95*, 215–233. [CrossRef]
15. Dhage, B.C. Approximation methods in the theory of hybrid differential equations with linear perturbations of second type. *Tamkang J. Math.* **2014**, *45*, 39–61. [CrossRef]
16. Ibrahim, R.W.; Jalab, H.A.; Gani, A. Cloud entropy management system involving a fractional power. *Entropy* **2016**, *18*. [CrossRef]
17. Dhage, B.; O'Regan, D. A fixed point theorem in banach algebras with applications to functional integral equations. *Funct. Differ. Equ.* **2004**, *7*, 259–267.
18. Pluzhnik, E.; Nikulchev, E.; Payain, S. Concept of feedback in future computing models to cloud systems. **2014**. arXiv:1402.4663.
19. Tsallis, C. *Introduction to Nonextensive Statistical Mechanics*; Springer: New York, NY, USA, 2009.
20. Machado, J.T. Entropy analysis of integer and fractional dynamical systems. *Nonlinear Dyn.* **2010**, *62*, 371–378.
21. Machado, J.T. Entropy analysis of fractional derivatives and their approximation. *J. Appl. Nonlinear Dyn.* **2012**, *1*, 109–112. [CrossRef]
22. Machado, J.T. Fractional order generalized information. *Entropy* **2014**, *16*, 2350–2361. [CrossRef]
23. Ibrahim, R.W. The fractional differential polynomial neural network for approximation of functions. *Entropy* **2013**, *15*, 4188–4198. [CrossRef]
24. Ibrahim, R.W.; Jalab, H.A. Existence of entropy solutions for nonsymmetric fractional systems. *Entropy* **2014**, *16*, 4911–4922. [CrossRef]
25. Ibrahim, R.W.; Jalab, H.A. Existence of Ulam stability for iterative fractional differential equations based on fractional entropy. *Entropy* **2015**, *17*, 3172–3181. [CrossRef]
26. Laskin, N. Fractional Poisson process. *Commun. Nonlinear Sci. Numer. Simul.* **2003**, *8*, 201–213. [CrossRef]