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What Exactly is the Nusselt Number in Convective Heat Transfer Problems and are There Alternatives?

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Abstract: The often used Nusselt number is critically questioned with respect to its physical meaning. Based on a rigorous dimensional analysis, alternative assessment numbers are found that in a systematic way separately account for the quantitative and qualitative aspect of a heat transfer process. The qualitative aspect is related to the entropy generated in the temperature field of a real, irreversible heat transfer. The irreversibility can be quantified by referring it to the so-called entropic potential of the energy involved in the transfer process.

Keywords: dimensional analysis; energy devaluation; entropic potential

1. Introduction

One of the first quantities introduced in almost every lecture about heat transfer is the convective heat transfer coefficient:

$$h \equiv \frac{\dot{q}_w}{\Delta T} \tag{1}$$

or its nondimensional version, the Nusselt number:

$$\mathrm{Nu} \equiv \frac{\dot{q}_w L}{k \Delta T} = h\,\frac{L}{k} \tag{2}$$

They combine the heat flux $\dot{q}_w$ at a heat transfer surface (index w, because often it is a wall) and a certain temperature difference ΔT due to which the heat transfer occurs. For internal flows ΔT usual is $\Delta T = T_w - T_b$, for external flows $\Delta T = \mathrm{T_w} - \mathrm{T_\infty}$ is the appropriate choice, with T_w: wall temperature, T_b: bulk temperature, and T_∞: ambient temperature. The length L is a characteristic geometrical length of the geometry under consideration and k is the thermal conductivity of the fluid.

The motivation for introducing h is often given in terms of "Newton's law of cooling", see for example [1–3], found about three centuries ago by Sir Isaac Newton: when a small hot object is cooled in a fluid flow the cooling is approximately exponential with time. With some additional assumptions (see [1], Example 1.6) this corresponds to $\dot{q}_w \sim \Delta T$ and suggests the introduction of a coefficient, which here is h.

In a systematic approach including dimensional analysis, $\dot{q}_w$ and ΔT are combined in a nondimensional group which is named the Nusselt number after Wilhelm Nusselt (1882–1957). For the formal relation between h and Nu see Equation (2). The crucial question that will be addressed here is referred to the physical interpretation of the Nusselt number Nu, or more precisely: *What kind of information is associated with a certain value of* Nu?

In standard text books this question is answered as follows:

- Incropera/DeWitt [2] (p. 314): "This parameter is equal to the dimensionless temperature gradient at the surface, and it provides a measure of the convective heat transfer occurring at the surface.

The Nusselt number is to the thermal boundary layer what the friction coefficient is to the velocity boundary layer."

- White [1] (p. 270): "The traditional dimensionless form of h is the Nusselt number Nu, which may be defined as the ratio of convection heat transfer to fluid conduction heat transfer under the same conditions."
- Özisik [3] (p. 268): "Then the Nusselt number may be interpreted as the ratio of heat transfer by convection to conduction across the fluid layer of thickness L. A larger value of the Nusselt number implies enhanced heat transfer by convection."

Taking these three statements as representative for those of the heat transfer community in general, it seems that h and Nu are routinely used when it comes to solving heat transfer problems but that their physical interpretation is qualitative in nature and not a real answer to the question "what exactly is the Nusselt number?".

In order to give a more substantial answer to the question about the information associated with a certain Nusselt number a thorough dimensional analysis of convective heat transfer problems is followed by a critical look at the nondimensional groups obtained. This analysis is performed from a thermodynamics point of view, keeping in mind that heat transfer is a fundamental process in thermodynamics; see [4] for more details about this often ignored relationship.

As it turns out the Nusselt number is a combination of two aspects: quantity and quality of the energy-transfer. When both aspects are treated separately and given by their own nondimensional groups more information is available. Thus, this study is conceptual in nature and relevant for practical applications alike.

2. Dimensional Analysis of Convective Heat Transfer

Nondimensional groups like the Nusselt number are the outcome of a dimensional analysis of a physical problem based on the Buckingham Pi-theorem. It states that from a list of n relevant system parameters $(n - m)$ independent nondimensional groups can be deduced, with m being the number of fundamental dimensions involved in the problem under consideration, see Ref. [5–7] for more details. Important steps are:

(1) Choose a target quantity and collect all system parameters which affect it (→ list of n relevant system parameters).
(2) Determine the m fundamental dimensions.
(3) Determine the $n - m$ independent nondimensional groups.

In this general procedure Step (1) is crucial, Steps (2) and (3) are purely formal. In Step (1) a certain mathematical/physical model is selected in terms of the principal parameters involved. The assessment criterion with respect to this model is not "right or wrong" but "adequate or not" in order to capture the relevant aspects of the physics with respect to the problem under consideration.

2.1. Convective Momentum Transfer

As an example in Table 1 the dimensional analysis data are collected for an incompressible fully developed flow in a smooth channel. The target quantity is the pressure gradient dp/dx. With $n = 5$ relevant system parameters and $m = 3$ fundamental dimensions two nondimensional groups appear, which here are the Reynolds number:

$$\mathrm{Re} = \frac{\rho u_m L}{\mu} \tag{3}$$

and the head loss coefficient:

$$\mathrm{K} = \frac{(-\,dp/dx)\ L}{\rho u_m^2/2} \tag{4}$$

Together they are the friction law K = K(Re) of the channel flow. Its general form is given by the dimensional analysis, its specific function has to be determined afterwards.

Table 1. Dimensional analysis data of the convective momentum transport for an incompressible fully developed flow in a smooth channel.

(1) System Parameters	(2) Fundamental Dimensions	(3) Nondimensional Groups
u_m (mean velocity)	length	$\mathrm{Re} = \frac{\rho u_m L}{\mu}$
L (length)	time	$\mathrm{K} = \frac{(-\,dp/dx)\;L}{\rho u_m^2/2}$
ρ (density)	mass	-
μ (dynamic viscosity)	-	-
dp/dx (pressure gradient)	-	-

2.2. Convective Heat Transfer

The example given in Table 1 is now resumed and heat transfer is added. Instead of the target quantity dp/dx now the heat flux $\dot{q}_w$ is selected as the system parameter of interest. Table 2 shows the standard dimensional analysis data which are (more or less explicitly) listed for this problem in standard textbooks like [1–3].

With $n = 8$ relevant system parameters and $m = 4$ fundamental dimensions now four nondimensional groups appear, which here are the Reynolds number, see Equation (3), the Prandtl number:

$$\mathrm{Pr} = \frac{\mu c_p}{k} \tag{5}$$

the Brinkman number:

$$\mathrm{Br} = \frac{\mu u_m^2}{k \Delta T} \tag{6}$$

associated with viscous heating in the flow field (which is often neglected) and instead of the nondimensional target number K now the Nusselt number Nu according to Equation (2).

As mentioned at the beginning of this section listing the n relevant system parameters is the crucial step of selecting the adequate mathematical/physical model. This listing of relevant parameters also means that certain other parameters are not included in the list. With respect to Table 2 this refers to the temperature level T_w at which the heat transfer occurs as well as to the ambient temperature T_∞. Whether both temperatures are really irrelevant in a convective heat transfer problem will be discussed next.

Table 2. Dimensional analysis data of the convective heat transfer for an incompressible fully developed flow in a smooth channel.

(1) System Parameters	(2) Primary Dimensions	(3) Nondimensional Groups
u_m (mean velocity)	length	$\mathrm{Re} = \frac{\rho u_m L}{\mu}$
L (length)	time	$\mathrm{Pr} = \frac{\mu c_p}{k}$
ρ (density)	mass	$\mathrm{Br} = \frac{\mu u_m^2}{k \Delta T}$
μ (dynamic viscosity)	temperature	$\mathrm{Nu} = \frac{\dot{q}_w L}{k \Delta T}$
$\dot{q}_w$ (heat flux)	-	-
ΔT (temperature difference)	-	-
k (thermal conductivity)	-	-
c_p (specific heat)	-	-

3. Convective Heat Transfer from a Thermodynamics Point of View

Heat transfer from a thermodynamics point of view is characterized by the entropy that is transferred as well as generated in this thermodynamic process. It is a special kind of mystery that in the heat transfer community this aspect is ignored completely, see Ref. [4] for further details.

An ideal heat transfer occurs without entropy generation transferring entropy at a rate:

$$\dot{S} = \frac{\dot{Q}}{T_w} \tag{7}$$

when there is a heat transfer rate $\dot{Q}$ with a constant temperature T_w on the heat transfer surface A. When neither the heat transfer rate $\dot{Q}$ nor the surface temperature is constant the instantaneous entropy transfer rate is:

$$\dot{S} = \int_A \frac{\dot{q}_w}{T_w}\, dA \tag{8}$$

For more details of the entropy transfer and the entropy generation described next, see for example [8,9].

The ideal heat transfer, accompanied by entropy transfer according to Equation (7) or Equation (8) occurs without operating temperature difference, *i.e.*, with $\Delta T = 0$. This corresponds to a heat transfer coefficient $h = \infty$ as well as to $\mathrm{Nu} = \infty$, *c.f.* Equations (1) and (2).

A real heat transfer will have a finite ΔT and thus finite values of h and Nu. For such a real process entropy is generated in the flow as well as in the temperature field. The instantaneous and local rate at which this occurs can be determined from an entropy balance equation as shown in Equation (8) of Ref. [4] or more extensively in Chapter 3.2 of [5]. In this equation $\dot{S}_d'''$ and $\dot{S}_c'''$ appear as source terms which (in Cartesian coordinates) read:

$$\dot{S}_d''' = \frac{\mu}{T}\left(2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)^2\right) \tag{9}$$

for entropy generation in the flow field, and:

$$\dot{S}_c''' = \frac{k}{T^2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2\right] \tag{10}$$

for that in the temperature field. Here the index d means *dissipation* while c stands for *conduction*.

The $\dot{S}_d'''$ according to Equation (9) multiplied by T is the instantaneous and local dissipation rate in the flow field. Its integration over the flow field:

$$\dot{S}_d = \int_V \dot{S}_d'''\, dV \tag{11}$$

yields $\dot{S}_d$, the instantaneous, entropy generation rate in the whole flow field. This $\dot{S}_d$ can immediately be linked to the head loss coefficient K replacing the pressure drop or drag force usually implemented in a head loss coefficient by the corresponding entropy generation as shown and discussed in [10]. The head loss coefficient then reads:

$$\mathrm{K} = \frac{2T}{\rho u_m^3 A}\, \dot{S}_d \tag{12}$$

since all losses (which are losses of exergy or available work) manifest themselves in terms of entropy generation. Here A is the channel cross section, for more details of why and how $\dot{S}_d$ can be used to determine K see [10,11]. Thus the flow field is characterized by two nondimensional groups:

- the Reynolds number, which determines the strength of the flow (its *quantity*).
- the head loss coefficient, which determines the losses of the flow (its *quality*).

Note that the quality is basically determined by the entropy generation $\dot{S}_d$ in the flow field.

From a thermodynamics point of view a very similar situation occurs in the temperature field. The heat transfer is characterized by two aspects, its *quantity* and its *quality*. Both should be determined by its own nondimensional group, with the quality being related to $\dot{S}_c'''$.

Determination of $\dot{S}_d'''$ and $\dot{S}_c'''$ is always possible when solutions are available in terms of numerical results with respect to the flow and temperature fields. Then in a post processing step local entropy generation data can be determined. Especially for the turbulent case see [10,12].

With the dimensional analysis according to Table 2, however, there is only one nondimensional group characterizing the heat transfer: the Nusselt number (keeping in mind that Pr is a mere fluid property and that Br characterizes the effect of viscous heating in a flow field, which often is neglected).

4. The Full Model of Heat Transfer

Collecting all relevant system parameters in a dimensional analysis corresponds to selecting the adequate mathematical/physical model by which the problem under consideration can be treated in order to predict all physical aspects of interest. Obviously the "aspects of interest" is the crucial step in this procedure. Analyzing the dimensional analysis data in Table 2 from a thermodynamics point of view two system parameters are missing:

(1) The temperature level of heat transfer, since according to Equation (10) it determines the strength of real heat transfer entropy generation, *i.e.*, the irreversibility of the transfer. For internal flows this temperature level is given by the caloric mean temperature in a certain cross section, also called bulk temperature T_b. It can moderately change in flow direction due to heating or cooling of the fluid and it can strongly deviate from the ambient temperature T_∞ when the heat transfer occurs in a system with a large temperature range within the whole process (like in power or refrigeration cycles).

(2) The ambient temperature T_∞, since it determines the quality of the transferred energy in terms of exergy and its losses in the real heat transfer process. This temperature is an important parameter for all processes in which exergy counts, like in power cycles.

Adding these two temperatures to the list of relevant system parameters results in two more nondimensional groups when performing the dimensional analysis, see Table 3.

Table 3. Dimensional analysis data of the convective heat transfer for an incompressible fully developed flow in a smooth channel; full model.

(1) System Parameters	(2) Fundamental Dimensions	(3) Nondimensional Groups
u_m (mean velocity)	length	$\mathrm{Re} = \rho u_m\, L/\,\mu$
L (length)	time	$\mathrm{Pr} = \mu c_p/k$
ρ (density)	mass	$\mathrm{Br} = \frac{\mu u_m^2}{k\Delta T}$
μ (dynamic viscosity)	temperature	$\mathrm{N}_q = \dot{q}_w L/\,(kT_b)$
$\dot{q}_w$ (heat flux)	-	(or: $\mathrm{Nu} = \frac{\dot{q}_w L}{k\Delta T}$)
ΔT (temperature difference)	-	$\mathrm{N}_{\Delta T} = \Delta T/T_b$
k (thermal conductivity)	-	$\mathrm{N}_T = T_\infty/T_b$
c_p (specific heat)	-	-
T_b (bulk temperature)	-	-
T_∞ (ambient temperature)	-	-

Here we keep Re, Pr and Br as before. We can also keep the Nusselt number or replace it by a nondimensional group characterizing the *quantity of heat transfer*:

$$\mathrm{N}_q = \frac{\dot{q}_w L}{k\, T_b} \tag{13}$$

For the quality of heat transfer:

$$\mathrm{N}_{\Delta T} = \frac{\Delta T}{T_b} \tag{14}$$

is introduced. Since $\mathrm{Nu} = \mathrm{N}_q/\mathrm{N}_{\Delta T}$ the two options $(\mathrm{Nu},\ \mathrm{N}_{\Delta T})$ and $(\mathrm{N}_q,\ \mathrm{N}_{\Delta T})$ are formally equivalent. In the subsequent discussion $(\mathrm{N}_q,\ \mathrm{N}_{\Delta T})$ is used, however, in order to clearly ascribe the two aspects *quantity* and *quality* of heat transfer to the two corresponding nondimensional groups.

The group:

$$\mathrm{N}_T = \frac{T_\infty}{T_b} \tag{15}$$

basically corresponds to the Carnot-factor:

$$\eta_c = 1 - \mathrm{N}_T = 1 - \frac{T_\infty}{T_b} \tag{16}$$

which (for example) determines the exergy part $\dot{Q}^E$ of a heat transfer rate $\dot{Q}$, transferred on a temperature level T_b by:

$$\dot{Q}^E = \eta_c \dot{Q} \tag{17}$$

See [8,9] for more details. Here N is the general symbol for the new nondimensional groups which are specified by their respective indices (formally different from the classical groups which all are represented by two letters, like Re, Nu, Pr, . . .).

In Equations (13)–(17) $T_b > T_\infty$ corresponds to a heating of the fluid. When $T_b < T_\infty$ the fluid is cooled and N_q, $N_{\Delta T}$, η_c and $\dot{Q}^E$ become negative. Thermodynamically the negative $\dot{Q}^E$, for example, indicates that $\dot{Q}$ and $\dot{Q}^E$ then are counter-flowing fluxes, as explained at the end of Chapter 5 in [9].

For a momentum transfer the flow *quality* is given in terms of the head loss coefficient K for a certain flow *quantity*, given by the Reynolds number Re.

For a heat transfer, in an analogous presentation the heat transfer *quality* is given in terms of $\mathrm{N}_{\Delta T}$ for a certain heat transfer quantity, given by N_q. In the next section we will discuss the quality assessment by $\mathrm{N}_{\Delta T}$, showing that it basically corresponds to the entropy generation. With the group N_T this information is turned into one about the exergy and its loss in a convective heat transfer situation.

N_q and $\mathrm{N}_{\Delta T}$ can be combined to the Nusselt number $\mathrm{Nu} = \mathrm{N}_q/\mathrm{N}_{\Delta T}$. This Nusselt number, however, can be discussed only after a discussion with respect to N_q and $\mathrm{N}_{\Delta T}$. Only after this we can answer the key question of this study: What exactly is the Nusselt number?

5. The Irreversibility of Heat Transfer

Real life heat transfer from a thermodynamics point of view is an irreversible process with the irreversibility measured by the entropy generation involved. In a convective heat transfer, irreversibilities occur in the associated flow field as well as in the temperature field involved. The first is due to the dissipation of mechanical energy and immediately determines the head loss coefficient of the flow, see Equation (12).

Entropy generation in the temperature field, however, does not immediately correspond to the number of a certain nondimensional group like $\mathrm{N}_{\Delta T}$ or the Nusselt number Nu. These nondimensional groups, however, are obviously related to the irreversibility since the limit of a reversible heat transfer is that for $\Delta T = 0$, so that $\mathrm{N}_{\Delta T} = 0$ and $\mathrm{Nu} = \infty$. In order to understand that relationship one first has to analyze the role of ΔT in a heat transfer process.

5.1. Irreversibility in the Temperature Field

With the local and instantaneous entropy generation rate $\dot{S}_c^{'''}$, *c.f.* Equation (10) for Cartesian coordinates, the entropy generation in a finite volume V is:

$$\dot{S}_c = \int_V \dot{S}_c^{'''} \, dV \tag{18}$$

As an example Figure 1 shows a fully developed laminar circular pipe flow heated with $\dot{q}_w = const$. In this example the temperature profile is known explicitly and could be used to determine $\dot{S}_c^{'''}$ and thus $\dot{S}_c$. This explicit knowledge of the temperature profile, however, is an exception so that in the following it is shown how temperature profiles can be approximated in order to determine $\dot{S}_c$.

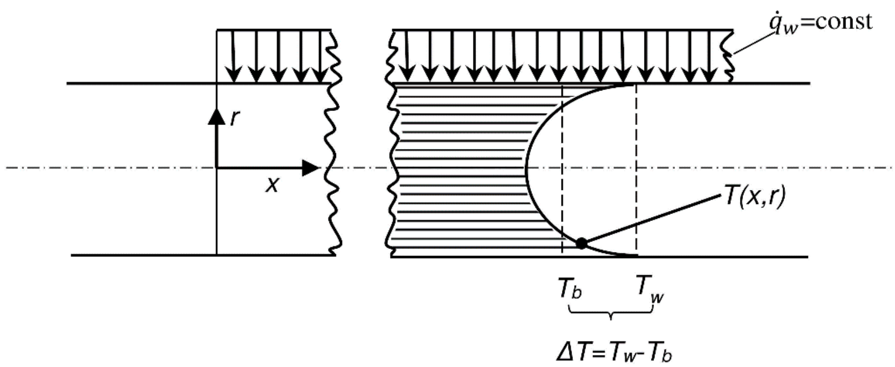


Figure 1. Temperature distribution in a fully developed pipe flow with $\dot{q}_w = const$.

5.2. Irreversibility in an Approximated Temperature Field

Since the exact temperature profile generally is not known (especially in a turbulent convective heat transfer) the entropy generation should be determined from a reasonable approximation of it. Such an approximation is shown in Figure 2, where a constant temperature gradient occurs only in a wall adjacent layer of thickness Δ. For $\Delta \to 0$ the constant temperature further away from the wall is the local bulk temperature T_b.

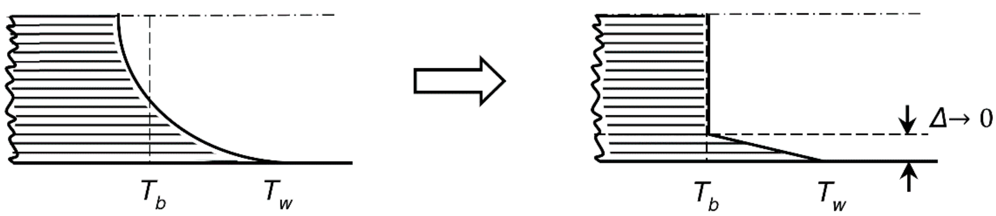


Figure 2. Approximate temperature profile.

Independent of the layer thickness Δ, integration of the entropy generation in the wall layer gives, see [11] for details:

$$\dot{S}_c = \dot{Q}_w \left(\frac{1}{T_b} - \frac{1}{T_w} \right) = \dot{Q}_w \frac{\Delta T}{T_b T_w} \tag{19}$$

This result holds for negligibly small axial conduction, *i.e.*, for $\mathrm{RePr} \to \infty$, and for a wall layer thickness $\Delta \to 0$ (because only then the second temperature is the bulk temperature T_b). Quite generally this result can be referred to a heat transfer area A so that with $\Delta T = T_w - T_b$ the entropy generation rate per wall area is:

$$\dot{s}_c = \dot{q}_w \, \frac{\Delta T}{T_b T_w} \tag{20}$$

This equation shows that the entropy generation in a heat transfer process depends on the temperature levels T_b and T_w, respectively.

In a certain heat transfer situation T_b and T_w are parameters of the process, just like a thermal boundary condition, for example. Constant values of these temperatures are either mean values in finite ranges of heating or cooling or exact values of a local analysis. Since T_b and T_w are the (absolute) thermodynamic temperatures, the ΔT will often be much smaller than the temperatures T_b and T_w. For $\Delta T \ll T_b$, *i.e.*, $T_w \approx T_b$, we then get:

$$\dot{s}_c \approx \dot{q}_w \frac{\Delta T}{T_b^2} \tag{21}$$

from which the physical meaning of ΔT becomes obvious. Within this approximation ΔT is:

$$\Delta T = \frac{\dot{S}_c}{\dot{Q}_w} \, T_b^2 = \frac{\dot{s}_c}{\dot{q}_w} \, T_b^2 \tag{22}$$

and corresponds to the relative entropy generation (entropy generation rate per heat transfer rate) for a heat transfer on the temperature level T_b. It thus is a real measure of the *quality of heat transfer*.

Since quite generally the entropy generation $\dot{S}_c$ corresponds to the loss of exergy $\dot{Q}^E$, *c.f.* Equation (17), here named $\dot{Q}_L^E$, by:

$$\dot{Q}_L^E = T_\infty \dot{S}_c \tag{23}$$

known as Gouy-Stodola theorem, we also have (index L: loss):

$$\dot{Q}_{WL}^E = T_\infty \dot{S}_c \rightarrow \dot{q}_{WL}^E = T_\infty \dot{s}_c = \frac{\dot{q}_w T_\infty}{T_b^2} \, \Delta T \tag{24}$$

with $\dot{q}_W^E$ as the exergy part of the heat flux $\dot{q}_w$.

With ΔT according to Equation (22) we can now look at the nondimensional groups introduced above. The parameter $\mathrm{N}_{\Delta T} = \Delta T / T_b$ according to Equation (14) is the nondimensional *quality of heat transfer*:

$$\mathrm{N}_{\Delta T} = \frac{\dot{S}_c}{\dot{Q}_w} \, T_b = \frac{\dot{s}_c}{\dot{q}_w} \, T_b \tag{25}$$

The Nusselt number Nu according to Equation (2) is:

$$\mathrm{Nu} = \frac{L}{k T_b^2} \left[\frac{\dot{s}_c / \dot{q}_w}{\dot{q}_w} \right]^{-1} = \frac{\mathrm{N}_q}{\mathrm{N}_{\Delta T}} \tag{26}$$

Both nondimensional groups will be discussed in the following sections.

5.3. The Nusselt Number and Its Physical Meaning

From the above considerations we can first conclude what the Nusselt number is NOT:

(1) $\mathrm{Nu} = \frac{L}{k T_b^2} \left[\frac{\dot{s}_c / \dot{q}_w}{\dot{q}_w} \right]^{-1}$ is not an immediate and general measure of the irreversibility of the heat transfer, because regardless of the (constant) factor $L / k T_b^2$ the relative entropy generation $\dot{s}_c / \dot{q}_w$ is referred to the heat flux $\dot{q}_w$ again, though in a reciprocal manner.

(2) Nu is not a measure of the exergy loss because it does not include the ambient temperature T_∞, *c.f.* Equation (24) for the exergy loss of $\dot{q}_w$.

The actual meaning of the Nusselt number can best be illustrated for the two standard thermal boundary conditions $\dot{q}_w = const$ and $T_w = const$, respectively.

The Nusselt number for the special case $\dot{q}_w = const$ in a certain problem is the information about $\Delta T = T_w - T_b$ which is a measure of the *quality of heat transfer*, *c.f.* Equation (22). However, only with the additional Equations (21) and (24) from this the entropy generation and the exergy loss rates can be determined.

The Nusselt number for the special case $T_w = const$ provides the information about $\dot{q}_w/\Delta T$ which basically is that about $\dot{q}_w^2/\dot{s}_c$. It is neither the relative entropy generation $(\dot{s}_c/\dot{q}_w \approx \dot{s}_c/(\Delta T)^2)$, *i.e.*, the *quality of heat transfer*, nor the strength of heat transfer $(\dot{q}_w)$ so that the interpretation is vague.

Often it is argued that the Nusselt number can be used twofold: Either $\dot{q}_w$ is prescribed and ΔT is the result or ΔT is given and $\dot{q}_w$ emerges as the result. For both cases increasing Nusselt numbers are interpreted as an "improvement of the convective heat transfer".

From a thermodynamics point of view this should be analysed more deeply. For a given $\dot{q}_w$ an increasing Nusselt number occurs for decreasing ΔT and thus an increase in the *quality of heat transfer* (less $\dot{s}_c$ per $\dot{q}_w$). For a given ΔT, however, the quality of heat transfer is fixed, *c.f.* Equation (22), and an increasing Nusselt number means equally increasing $\dot{q}_w$ and $\dot{s}_c$.

Thus an increasing Nusselt number means increasing *quality of heat transfer* only for $\dot{q}_w = const$ but not for $\Delta T = const$. At least the above term "improvement of the convective heat transfer" should be stated more precisely.

6. Alternatives with Respect to the Nusselt Number

When the full model in Table 3 is compared to the corresponding considerations in Table 2 it becomes obvious that the Nusselt number Nu in the full model can be replaced by the three nondimensional groups N_q, $\mathrm{N}_{\Delta T}$ and N_T. A straight forward physical interpretation can be given to the two nondimensional groups:

$$\mathrm{N}_q = \frac{\dot{q}_w L}{k T_b} \tag{27}$$

and:

$$\mathrm{N}_i \equiv \mathrm{N}_{\Delta T}\mathrm{N}_T = \frac{\Delta T\, T_\infty}{T_b^2} = \frac{\dot{s}_{c,\,i} T_\infty}{\dot{q}_w} = \frac{\dot{S}_{c,\,i}\, T_\infty}{\dot{Q}_w} \tag{28}$$

With L, k and T_b being parameters of a heat transfer problem, N_q according to Equation (27) is the nondimensional heat flux, *i.e.*, a measure of the *quantity of heat transfer*.

The *quality of heat transfer* is now given by N_i according to Equation (28). This N_i was introduced in [11] and named *energy devaluation number*. Basically it is the entropy generation in the irreversible heat transfer process referred to the so-called *entropic potential* of the energy (rate) $\dot{E}$, see Ref. [11,13] for a detailed discussion of this concept. The entropic potential $\dot{E}/T_\infty$ of a certain energy is that amount of entropy that is discharged to the ambient when the energy starts as primary energy (being pure exergy) and ends as part of the internal energy of the ambient (then being pure anergy). This energy devaluation can be looked upon as an overall *devaluation chain* of an energy starting as primary energy and ending as part of the internal energy of the ambient.

A certain heat transfer process may be one chain link i characterized by the energy devaluation number N_i. It determines how much of the entropic potential is used in this process i. In a heat transfer process the energy (rate) is $\dot{E} = \dot{Q}_w$. The entropic potential of it is (index g: generation):

$$\dot{S}_g = \frac{\dot{Q}_w}{T_\infty} \tag{29}$$

and N_i can be written as:

$$N_i = \frac{\dot{S}_{c,\,i}}{\dot{S}_g} \quad \left(= \frac{\dot{s}_{c,\,i}\, T_\infty}{\dot{q}_w} \right) \tag{30}$$

According to its definition we always have $0 \leqslant N_i \leqslant 1$ with $N_i = 0$ for a reversible process, and $\sum_i N_i = 1$ for the complete devaluation chain. Only when the energy is devaluated completely in one process i, $N_i = 1$ would occur.

The Nusselt number alternatives in Table 3 comprise three independent nondimensional groups. They now take the form N_q according to Equation (27), N_i according to Equation (30) and N_T according to Equation (15). This N_T determines the exergy part of the energy since it immediately corresponds to the Carnot factor, *c.f.* Equations (16) and (17).

Whenever solutions are available in terms of numerical results with respect to the temperature data this set of nondimensional groups can be used as an alternative to characterizing the heat transfer by the Nusselt number Nu alone as will be shown with three examples hereafter.

Applying the Nusselt Number and Its Alternatives

The following three examples may illustrate how important it is to account for entropy generation which is the crucial aspect in the *energy devaluation number* N_i according to its definition Equation (28). All three examples are about internal flows which often occur in energy transfer operations in which exergy counts and entropy generation is counterproductive. A second law analysis, however, is also applicable for external flows, as shown in [10], for example, where losses in the flow field usually specified in terms of friction coefficients are determined through the entropy generation by dissipation in the flow field.

Example 1. *Fully developed pipe flow in power cycles*

What can usually be found as the characterization of the heat transfer performance of a fully developed pipe flow is the Nusselt number Nu. Let's assume it is Nu = 100 and a heat flux $\dot{q}_w = 10^3\ \mathrm{W/m^2}$ on a length $L = 0,\ 1$ m occurs in two different power cycles:

- A steam power cycle (SPC) with water as the working fluid and an upper temperature level $T_{b,u} = 900$ K.
- A geothermal organic Rankine cycle (ORC) with ammonia (NH_3) as working fluid and an upper temperature level $T_{b,u} = 400$ K.

When in both cycles Nu, $\dot{q}_w$ and L are the same, the temperature difference ΔT in Nu according to Equation (2) is larger by a factor 2.6 for ammonia compared to water. This is due to the different values of the thermal conductivity k of water (at $T_{b,u} = 900$ K and p = 250 bar) and ammonia (at $T_{b,u} = 400$ K and p = 25 bar), assuming typical values for the temperature and pressure levels in both cycles.

For a further comparison note that the energy devaluation number N_i according to Equation (28) now is:

$$N_i = \frac{\Delta T\, T_\infty}{T_{b,\,u}^2} \tag{31}$$

Table 4 shows the *energy devaluation number* N_i for both cases according to this approximation. It shows that only 0.37% of the entropic potential is used for the heat transfer in the SPC-case, but almost 5% in the ORC-case "even though" both heat transfer situations have the same Nusselt number Nu = 100 and the same amount of energy is transferred. Note that only that part of the entropic potential that is not yet used is available for further use after the process under consideration.

Table 4. Heat transfer with Nu = 100, $\dot{q}_w = 10^3$ W/m^2, L = 0.1 m in two different power cycles.

Cycle/Fluid	$\frac{k}{\text{W/mK}}$	$\frac{T_\infty}{\text{K}}$	$\frac{T_{b,u}}{\text{K}}$	$\frac{\Delta T}{\text{K}}$	N_i Equation (28)	N_q Equation (27)	N_T Equation (15)	$\frac{q_w^E}{q_w}$
SPC/water	0.1	300	900	10	0.0037	1.11	0.33	0.67
ORC/ammonia	0.038	300	400	26	0.049	6.58	0.75	0.25

In Table 4 also N_q and N_T are listed together with $1 - \text{N}_T$ which corresponds to the relative exergy content $\dot{q}_w^E/\dot{q}_w$ of the heat flux $\dot{q}_w$.

Note that formally:

$$\text{Nu} = \frac{\text{N}_q \text{N}_T}{\text{N}_i} = \frac{\text{N}_q}{\text{N}_{\Delta T}} \tag{32}$$

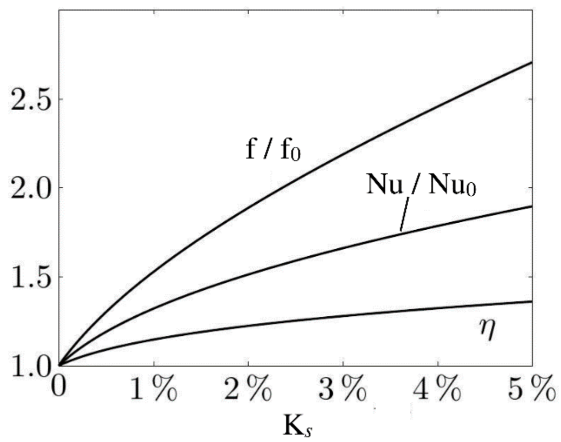
which shows that the Nusselt number is some kind of mixed information about the quantity and quality of an irreversible heat transfer process.

Example 2. *Convective heat transfer and wall roughness*

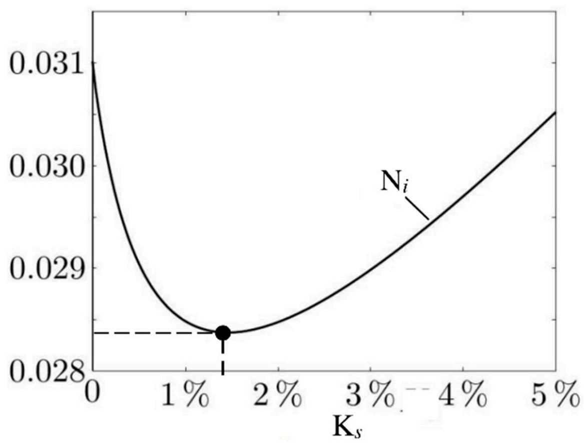
Example 1 is now extended by addressing the question whether a rough wall might perform better than a smooth wall and when this is the case, which degree of roughness performs best. The idea behind this optimization approach is the option that a better mixing of the fluid by wall roughness may decrease the exergy loss due to heat conduction such that it outweighs the increase of exergy loss due to increased frictional losses.

In [13] details of such an analysis (exemplified for the case Re = 17,000, Pr = 7, $\dot{q}_w' \approx 2$ kW/m) can be found in terms of the local entropy generation in the flow and in the temperature fields.

Figure 3 shows the friction factor f, the Nusselt number Nu and the energy devaluation number N_i for roughness parameter values K_s between 0% (smooth wall) and 5%. These results are based on friction factor and Nusselt number correlations which take into account the influence of wall roughness (Equations (11) and (12) in Ref. [13]). N_i is determined by an evaluation of $\dot{S}_{c,\,i}$ based on that Nusselt number correlation (see Equation (14) in [13]). Here K_s is a nondimensional sand roughness height. The friction factor f here is the head loss coefficient K according to Equation (4) multiplied by D/L. For $L = D$ both are equal.



(**a**)



(**b**)

Figure 3. Fully developed pipe flow with wall roughness (**a**) Friction factor and Nusselt number for rough walls referred to the smooth wall values and the thermo-hydraulic performance parameter η (**b**) Energy devaluation number, see Equation (30).

With increasing K_s there is a monotonous increase of f and Nu. Both parameters can be combined in a so-called thermo-hydraulic performance parameter proposed in [14] which here is:

$$\eta = \frac{\mathrm{Nu}/\mathrm{Nu_o}}{(\mathrm{f}/\mathrm{f_o})^{1/3}} \tag{33}$$

This parameter also increases monotonously with increasing K_s and thus does not indicate an optimum with respect to the K_s-range under consideration.

From Figure 3b which shows the energy devaluation number N_i we can conclude that about 3% of the entropic potential is used in this heat transfer process and that an optimum with respect to K_s occurs at $K_s \approx 1.5\%$.

Example 3. *Convective heat transfer in a passage within a plate heat exchanger*

Figure 4 shows a more complex geometry compared to the previous examples. It is a stripe out of a channel formed by two corrugated plates (sinusoidal fish-bone pattern, 45° corrugation angle) which by setting periodic boundary conditions is representative for the whole channel (without entrance and exit effects).

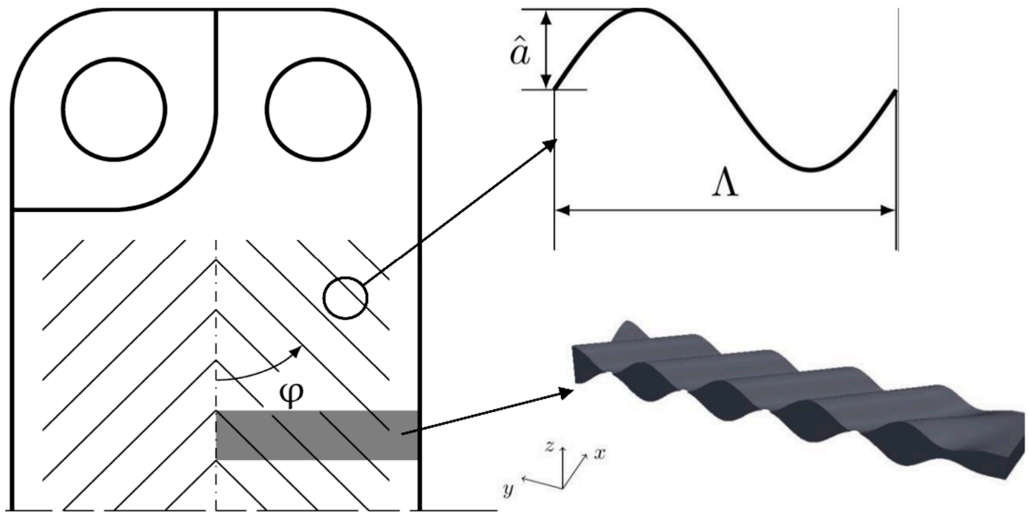


Figure 4. Geometry of the plate heat exchanger passage in [15]. Shown in grey is a cyclic element in which the flow and temperature field is determined numerically. Periodic boundary conditions are set in order to determine the entropy generation rates occurring in the flow and temperature fields.

In [15] the CFD solutions are described in detail for a range of Reynolds numbers and a fixed value of $\dot{q}_w$. Figure 5 shows how the entropy generation rates due to dissipation and conduction vary with Re and that the sum of both has a minimum at Re $\approx$ 2000.

According to the definition of the energy devaluation number this also is the Reynolds number for which N_i has the lowest value of $N_i \approx 0.0014$ indicating that in the stripe under consideration $\approx$0.14% of the entropic potential is used. This kind of information is missing completely when only the Nusselt number in such a situation is known.

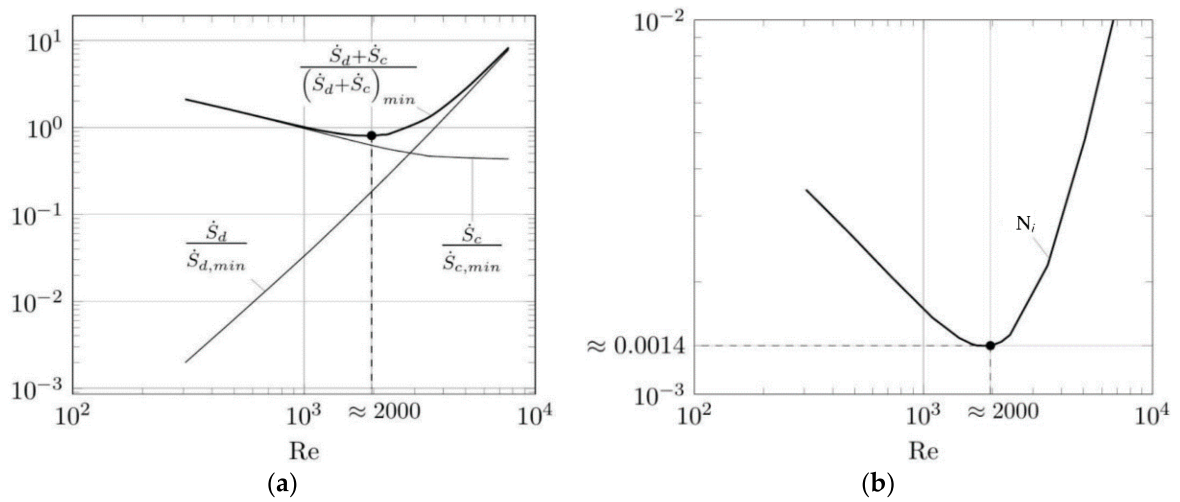


Figure 5. Entropy generation rates $\dot{S}_d$ and $\dot{S}_c$ according to Equations (11) and (18) for the cyclic element shown in Figure 4, their combination and implementation in the energy devaluation number. (**a**) Entropy generation rates by dissipation and conduction and the sum of both, referred to the minimum value at Re = 2000; (**b**) Energy devaluation number, see Equation (30).

7. Discussion and Conclusions

Convective heat transfer situations are often characterized by the nondimensional group Nu which combines the heat flux $\dot{q}_w$ and a characteristic temperature difference ΔT. Often the only reasonable interpretation of Nu is that the heat transfer is improved somehow when Nu increases. When, however, thermodynamic considerations are added to the analysis and interpretation of convective heat transfer situations, substantive statements and conclusions are available. For example it turns out, that

- the temperature level on which the heat transfer occurs is important (*c.f.* Example 1, Table 4). This information is not included in the Nusselt number.
- the quality of heat transfer can be quantified by the energy devaluation number N_i according to Equation (30) in percent-consumption of the entropic potential (*c.f.* again Example 1, Table 4, Example 2, Figure 3b or Example 3, Figure 5b). This information is not included in the Nusselt number.

The Nusselt number, however, is neither directly nor indirectly (in terms of a relative entropy generation) related to the entropy generation. Instead it is a combination of the quality, given by the entropy generation, and the quantity of heat transfer, given by the heat flux involved. The alternative approach of our study can be summarized as follows:

Nondimensional groups can be determined in an extended dimensional analysis that separately account for the *quantity* and the *quality* of an energy transfer process under consideration. The quantitative aspect of that transfer is then covered by N_q according to Equation (27) which basically is the nondimensional heat flux. The qualitative aspect is accounted for by $\mathrm{N}_{\Delta T}$ (Equation (14), *quality of heat transfer*) or by the energy devaluation number N_i according to Equation (28) or Equation (30). Based on the concept of the entropic potential of an energy it determines how much of that potential is used in the transfer process under consideration.

When a convective heat transfer situation has to be assessed as a whole, *i.e.*, including the entropy generation in the flow field, further considerations are necessary with details given in [11].

Finally for one case it should be shown that detailed information is available when the alternative approach is used:

In the second example in Section 6 it was shown that wall roughness can be beneficial for convective heat transfer, even though fluid friction may increase considerably.

From the Nusselt number perspective there seems to be no fundamental difference for the two conditions $\dot{q}_w = const$ and $\Delta T = const$ in the smooth and rough wall cases. The interpretation with N_q and $\mathrm{N}_{\Delta T}$, however, shows an important difference.

For increasing Nu number and $\dot{q}_w$ kept the same, N_q is the same and $\mathrm{N}_{\Delta T}$ according to Equation (14) decreases with ΔT getting smaller (increasing Nu). From Equation (25) it then follows that the entropy generation rate $\dot{S}_c$ also goes down. With $\dot{S}_d$ being increased in the rough wall case there will be an optimum in terms of $\dot{S}_c + \dot{S}_d$ being minimal.

For increasing Nu numbers and ΔT kept the same, however, $\dot{q}_w$ goes up (increasing Nu) and according to Equation (22) also $\dot{s}_c$ or $\dot{S}_c$ goes up. Then, however, there is no optimum of $\dot{S}_c + \dot{S}_d$ since both are increased with increasing wall roughness.

This is one more example showing that an in depth analysis of convective heat transfer problems needs more than the Nusselt number. The author is well aware of the fact, however, that heat transfer problems are often solved by practitioners without sympathy for something as abstract as entropy.

This, however, may change over times, especially when the entropy concept is included in students education as early as possible. May be this study can help on the way to such a more fundamentally oriented education.

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Abbreviations

The following abbreviations are used in this manuscript:

A	transfer area (m^2)
c_p	heat capacity (J/kgK)
D	diameter (m)
Ec	Eckert number, Equation (6)
f	friction factor
h	heat transfer coefficient ($\mathrm{W/m^2K}$)
k	thermal conductivity (W/mK)
K	head loss coefficient, Equation (4)
L	characteristic length (m)
$\mathrm{N_i}$	energy devaluation number, Equation (28)
$\mathrm{N_q}$	quantity of heat transfer, Equation (13)
$\mathrm{N_T}$	temperature ratio, Equation (15)
$\mathrm{N_{\Delta T}}$	quality of heat transfer, Equation (14)
Nu	Nusselt number, Equation (2)
p	Pressure
Pr	Prandtl number, Equation (5)
$\dot{Q}$	heat transfer rate (W)
$\dot{Q}^E$	exergy part of $\dot{Q}$ (W)
$\dot{Q}_L^E$	loss of the exergy part of $\dot{Q}$ (W)
$\dot{q}_w$	wall heat flux ($\mathrm{W/m^2}$)
$\dot{q}_W^E$	exergy part of $\dot{q}_W$ ($\mathrm{W/m^2}$)
$\dot{q}_{wL}^E$	loss of the exergy part of $\dot{q}_w$ ($\mathrm{W/m^2}$)
Re	Reynolds number, Equation (3)
$\dot{S}$	entropy transfer rate (W/K)
$\dot{S}_c$	overall entropy generation rate by conduction (W/K)
$\dot{s}_c$	overall entropy generation rate per wall area ($\mathrm{W/m^2K}$)
$\dot{S}_d$	overall entropy generation rate by dissipation (W/K)
$\dot{S}_d'''$	local entropy generation rate by dissipation ($\mathrm{W/m^3K}$)

$\dot{S}_c'''$	local entropy generation rate by conduction (W/m^{3}K)
T	temperature (K)
ΔT	temperature difference (K)
T_b	bulk temperature (K)
T_w	wall temperature (K)
u, v, w	Cartesian velocity components (m/s)
u_m	average velocity (m/s)
V	volume (m^3)
x, y, z	Cartesian coordinates (m)
η_c	Carnot factor, Equation (16)
μ	dynamic viscosity (kg/ms)
ρ	density (kg/m^3)

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