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Transient Thermal Field Analysis in ACCC Power Lines by the Green's Function Method

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Abstract: The paper investigates the dynamics of the thermal field of the ACCC (aluminum conductor composite core) line. The system was heated by solar radiation and current flow. Conductor cooling was modeled using the total heat transfer coefficient as the sum of convective and radiative components. The temperature increase generated by the current is described by a system of parabolic differential equations with an appropriate set of boundary, initial and continuity condition. The mentioned boundary-initial problem was solved by a modified Green's method, adapted to the layered structure of the system. For this purpose, Green's functions, as the kernels of integral operators inverse to differential ones, were determined. Aluminum resistivity and heat transfer coefficient change significantly with temperature. For this reason, the solution to the problem is presented in the form of a lower and upper estimation of the heating curve and local time constant. A steady-state current rating was also determined. The results are presented graphically and verified by other methods (power balance and finite element). The physical interpretation of the presented solution is also given.

Keywords: ACCC lines; Green's functions; heating curves; local time constants; steady-state current ratings

1. Introduction

Power lines produced in the ACCC (aluminum conductor composite core) technology have a number of advantages over the classic ACSR (aluminum conductor steel reinforced) conductors. First of all, composite cores are 70% lighter and twice as strong [1] as steel ones. A core braid is made of fully annealed 99.7% aluminum (type 1350-O) [2]. Of all alloys of this metal, the resistivity of 1350-O aluminum is the lowest. In order to increase the effective cross section of aluminum (with the same diameter of the conductor) profiled trapezoidal strands are used. The above mentioned solutions lead to a significant reduction in the resistance of ACCC conductors and thus to a reduction in power loss (by 25–30%) compared to ACSR systems. Moreover, composite cores can operate at much higher temperatures than steel ones (180 °C and 85 °C, respectively) [2]. This results in a double increase in transmitted power. There is no need to change the diameter of the conductor or to strengthen/raise towers. In addition, composite cores have a very low coefficient of thermal expansion (1.6×10^{-6} K⁻¹). Combined with the low core weight, this results in low sag of ACCC conductors. In this way, a safe clearance between the line and objects below is maintained. In addition, covering the core and surface of the ACCC conductor with additional coatings significantly reduces thermal aging and retains flexural strength [3–5].

The disadvantages of ACCC conductors are the high price and the requirement for accurate installation (especially tensioning must be in accordance with the manufacturer's instructions). The full annealing of aluminum makes the surface soft and susceptible to damage. However, the advantages of ACCC conductors described earlier make them an object of interest of companies dealing with

electrical power transmission and distribution. Evidence of this is the many thousands of kilometers of ACCC conductors installed worldwide after 2005. For this reason, research on ACCC lines meet the needs of modern power engineering and electrical engineering.

Each technology has its own limit parameters. If the composite core sustainedly heats up above the maximum operating temperature, its accelerated aging process will occur. It may be manifested by a decrease in the flexibility of the core [3,4]. On the other hand, too large thermal field gradients lead to internal mechanical stresses and thus to smaller or larger displacement of the strands. In extreme cases of very high temperature (generated, e.g., by short-circuit current), the strands delaminate to form a kind of "birdcage" on the section of the conductor. Therefore, thermal field analysis is an important part of checking the correctness of the line design and its optimization.

The stationary surface temperature of the ACCC conductor can be determined [6] by the CIGRE [7] or IEEE [8] methods. They rely on an iterative solution to heat balance equations with respect to temperature. An interesting variation [8] is the conductor replica method [9]. The finite element method (FEM) [10] is much stronger than [7–9]. Transient thermal field in the cross-section of the ACCC conductor was analyzed in [3,11] using commercial FEM software. In [3], however, the skin effect and solar radiation were omitted. In [11] additional carbon nanostructure coatings were considered. In the thermal field analysis of ACCC systems, methods previously used to study classic ACSR lines will also be effective. The following methods should be mentioned here: finite difference (FDM) [12,13], finite volume (FVM) [14,15] and equivalent thermal circuit (ETC) [16–18].

The experimental methods [6] are also worth mentioning. The method of measuring variable temperature in a cylindrical current-heated system is briefly described in [19] (chapter experimental verification). The calculations given in this paper can be experimentally verified in the same way.

It follows from the above that the thermal field of the ACCC conductor can be calculated numerically or analytically. The first method allows for taking into account material non-linearities and conveniently maps the heterogeneity of the ACCC cross-section. The mesh of the numerical method imposes, however, a finite number of degrees of freedom. In turn, the main advantage of analytical methods is the result in the form of formulas. It facilitates physical interpretation of phenomena and discussion of the impact of individual parameters. In addition, the analytical solution enables quick field estimation at any selected points in the area and moments of time. Due to greater accuracy, analytical results are benchmark values for the numerical solutions tested. For the above-mentioned reasons, the thermal field of the ACCC line is investigated in this paper using the Green's function [20–22] (i.e., by the analytical method). The Green's function remains an important field theory tool used, among others, in mechanics, electrical engineering and physics. This is confirmed by three extensive monographs [20–22] devoted to this subject and published in the current decade. To the best of the authors' knowledge, the Green's method has not been used in published works dedicated to ACCC systems.

From the application point of view, it is important to estimate the following parameters and operational characteristics of the ACCC line: steady-state current rating, heating curve and time constant. The first value ensures the core thermal security. In turn, the last two values are the basic parameters of thermal field dynamics (e.g., when switching the line on and off, changing the load current, short circuit, discontinuous running, etc.). As previously mentioned, the problems transient states in ACCC conductors are relatively rare topics of other publications. For this reason, this paper contributes to filling this gap.

2. Physical Model of the ACCC Line

The cross section of the investigated system is shown in Figure 1. The bearing element of the ACCC line is the central core with R_1 radius. The core is made of carbon fiber epoxy composite (CFC). The next layer with R_1 , R_2 radiuses, protects aluminum against corrosion from carbon fibers. The layer 2 material is a glass fiber epoxy composite (GFC). The additional task of GFC is to increase the strength and flexibility of the core. The last area (index 3) is the current conductor. The conductor is twisted

from soft and annealed aluminum trapezoidal strands. This cross-section ensures large contact surfaces between individual strands and their tight arrangement in the annulus $R_2 \le r \le R_3$ (Figure 1). For this reason, a monolithic conductor structure was assumed. Helical stranding of strands was also omitted. This last simplification was compensated by introducing the stranding factor k_1 [23] into the formula for volumetric power density. Additionally, it was assumed that the aluminum surface ($r = R_3$) is moderately oxidized (i.e., emissivity $\varepsilon = 0.5$). The outer diameter of the conductor (2 R_3) is much smaller than its length and dimensions of surrounding objects. The latter are at ambient temperature $T_a = const$.



Figure 1. Cross-section of the aluminum conductor composite core (ACCC) conductor.

The thermal field is generated from the moment of switching on the current with the power frequency f and the root mean square current |I|. Low frequency allows one to neglected dielectric losses in composites relative to Joule heat produced in aluminum. The influence of the air boundary layer around the line was also omitted. It means uniformly heat transfer by the surface $r = R_3$. The last assumption together with the axisymmetric geometry of the system allows one to approximate the thermal field with the radial coordinate *r* and time *t* functions.

Solar radiation can be taken into account by the surface density of its power (W/m²) [7,8] or the temperature of the sun sensor [9,24]. The second method was chosen in this paper. The simplest sensor is a replica of the conductor [9,24] located next to the real system. It was assumed that T_s means the temperature of the entire volume of the replica, heated by solar radiation. Therefore, T_s is also the conductor temperature before turning on the power (i.e., for t < 0). The replica is never charged with electricity—there is only solar input. For this reason, a constant value of the replica's temperature is assumed T_s (even after turning on the power supply of the real line). The $T_s - T_a$ increase is a measure of the intensity of solar radiation.

The system cooling was modeled using the total heat transfer coefficient α . It is the sum of [25] convection γ_n and radiation α_r coefficients. The first component includes, among others, wind speed V [19]. The cross flow direction relative to the line was assumed. The second component depends, among others, on emissivity ε [25] (p. 10). For the lower and upper estimation of the heating curve, temperature-time profiles will be determined for the following parameters.

$$\alpha_L = \alpha(V, \varepsilon, T_p = T_s, T_f = 0.5(T_s + T_a)), \ \rho_L = \rho(T_s), \tag{1a}$$

$$\alpha_{M} = \alpha(V, \varepsilon, T_{p} = 0.5(T_{s} + T_{\max}), T_{f} = 0.25(T_{s} + T_{\max} + 2T_{a})), \ \rho_{M} = \rho(0.5(T_{s} + T_{\max})),$$
(1b)

$$\alpha_H = \alpha(V, \varepsilon, T_p = T_{\max}, T_f = 0.5(T_{\max} + T_a)), \ \rho_H = \rho(T_{\max}), \tag{1c}$$

where the following nomenclature was used:

 T_p —conductor surface temperature (or average),

 $T_f = 0.5(T_p + T_a)$ —average boundary layer temperature, T_{max} —maximum operating temperature (or sustained maximum temperature),

$$\rho(T) = \rho(20\,^{\circ}\text{C})[1 + \beta(T - 20\,^{\circ}\text{C})] \text{--resistivity of annealed aluminum,}$$
(2)

at temperature $T < 200 \,^{\circ}\text{C}$ [23],

 β —temperature coefficient of electric resistivity.

3. Mathematical Model of the Thermal Field in the ACCC Line

The temperature increases $v_i(r,t)$ and $T_s - T_a$ are calculated relative to T_a and generated, respectively, by current flow and solar radiation (i = 1 for CFC, i = 2 for GFC, i = 3 for Al). According to the Electric Power Research Institute—USA [24], these increases can be analyzed separately. After averaging the thermal and material parameters, the system is linear. From the superposition of increases it results that

$$T_i(r,t) = T_s + v_i(r,t), \tag{3}$$

where: $T_i(r, t)$ is the spatial-temporal distribution of temperature in the *i*-th zone. The temperature increase forced by current was defined by the following initial-boundary problem for the heat equation [22,25].

$$\frac{\partial^2 v_i(r,t)}{\partial r^2} + \frac{1}{r} \frac{\partial v_i(r,t)}{\partial r} - \frac{1}{\chi_i} \frac{\partial v_i(r,t)}{\partial t} = -\frac{g_i(r)}{\lambda_i} \text{ for } R_{i-1} \le r \le R_i, \ t \ge 0, \ i = 1, \ 2, \ 3, \tag{4}$$

where: $R_0 = 0$ and $\chi_i = \lambda_i / (c_i \mu_i)$ is the diffusivity of the *i*-th zone, λ_i is the thermal conductivity of the *i*-th zone, c_i is the specific heat of the *i*-th zone, μ_i is the density of the *i*-th zone and R_1, R_2, R_3 are the consecutive radiuses of the conductor zones (Figure 1). The efficiency of the spatial heat source $g_3(r)$ in the aluminum annulus was determined based on [26].

$$g_3(r) = k_1 \cdot \rho \cdot \left| \frac{m|I|}{2\pi R_3} \frac{K_1(mR_2)I_0(mr) + I_1(mR_2)K_0(mr)}{I_1(mR_3)K_1(mR_2) - I_1(mR_2)K_1(mR_3)} \right|^2,$$
(5)

where: $m = \sqrt{j2\pi f \mu_o / \rho}$, ρ is the resistivity of the conductor, μ_o is the magnetic permeability of a vacuum, k_1 —stranding factor, f—the frequency and $j = \sqrt{-1}$ is an imaginary unit, $I_p(...), K_p(...)$ are modified Bessel functions of the first and second kind of the order of p (p = 0 or p = 1), respectively.

There are no heat sources in the core, therefore $g_1(r) = g_2(r) = 0$.

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In the considered model it was assumed that the current flow heats the system from t = 0. The temperature increase generated by the current is calculated relative to T_a . This results in zero initial conditions in all zones of the conductor

$$v_i(r, t = 0) = 0$$
 for $R_{i-1} \le r \le R_i$ for $i = 1, 2, 3, R_0 = 0.$ (6)

The outer surface of the conductor ($r = R_3$) gives up heat by convection and radiation. The mentioned transfer is described by Hankel's boundary condition [25]

$$\frac{\partial \nu_3(r,t)}{\partial r} \bigg|_{r=R_3} = -\frac{\alpha}{\lambda_3} \nu_3(r=R_3,t) \text{ for } t \ge 0,$$
(7)

where α is the total heat transfer coefficient.

The particular layers of the conductor are closely adherent to each other. Therefore, the conditions for continuity of temperature increase and heat flux are met at the boundaries of regions

$$v_i(R_i, t) = v_{i+1}(R_i, t) \text{ for } i = 1, 2 \text{ and } t \ge 0,$$
 (8a)

$$\lambda_{i} \frac{\partial v_{i}(r,t)}{\partial r} \bigg|_{r = R_{i}} = \lambda_{i+1} \frac{\partial v_{i+1}(r,t)}{\partial r} \bigg|_{r = R_{i}} \text{ for } i = 1, 2 \text{ and } t \ge 0.$$
(8b)

Equations (4)–(8) form the boundary-initial problem of the transient increase of the temperature field.

4. Green's Functions of the Mathematical Model of the Thermal Field in the ACCC Line

The boundary-initial problem (4)–(8) was solved by the Green's method [20–22]. In this method, it is unnecessary to determine the particular solution [27] of the heterogeneous Equation (4) for i = 3. This is the advantage of the Green's function method, which does not depend on the forcing function $g_3(r)$ [22]. In addition, this method does not require a separate determination of the stationary and transient temperature field components for later superposition. The above reasons prompted the authors to choose the Green method. Details on the definition, physical interpretation and description of the boundary-initial problem of Green's function are given in [22,27,28]. In the considered model, the current-related increase in temperature in the *i*-th zone can be represented by the following integral relationship [28].

$$\nu_i(r,t) = \frac{\chi_3}{\lambda_3} \int_0^t \int_{R_2}^{R_3} g_3(\xi) G_i(r,t,\xi,\eta) \xi d\xi d\eta \text{ for } R_{i-1} \le r \le R_i, \ t \ge \eta, \ i = 1, \ 2, \ 3, \ R_0 = 0,$$
(9)

where $g_3(\xi)$ is given by the relation (5) (after conversion $r \to \xi$), $G_i = G_i(r, t, \xi, \eta)$ is the Green's function in the i-th zone, (ξ, η) are parameters related to the definition of the Green's function. The condition for using (9) is knowledge of the Green's functions G_i . One way to determine them is to solve the auxiliary (homogeneous) problem using two methods: separation of variables [25,27] and Green's function [22,27].

The homogenous auxiliary problem in relation to the function $\Psi_i(r, t)$ is defined as below:

$$\frac{\partial^2 \Psi_i(r,t)}{\partial r^2} + \frac{1}{r} \frac{\partial \Psi_i(r,t)}{\partial r} - \frac{1}{\chi_i} \frac{\partial \Psi_i(r,t)}{\partial t} = 0 \text{ for } R_{i-1} \le r \le R_i, \ t \ge 0, \ i = 1, \ 2, \ 3, \ R_0 = 0,$$
(10)

$$\frac{\partial \Psi_3(r,t)}{\partial r} \bigg|_{r=R_3} = -\frac{\alpha}{\lambda_3} \Psi_3(r=R_3,t) \text{ for } t \ge 0,$$
(11)

$$\Psi_i(R_i, t) = \Psi_{i+1}(R_i, t) \text{ for } i = 1, 2 \text{ and } t \ge 0,$$
(12a)

$$\lambda_{i} \frac{\partial \Psi_{i}(r,t)}{\partial r} \bigg|_{r = R_{i}} = \lambda_{i+1} \frac{\partial \Psi_{i+1}(r,t)}{\partial r} \bigg|_{r = R_{i}} \text{ for } i = 1, 2 \text{ and } t \ge 0,$$
(12b)

$$\Psi_3(r,0) = F_3(r) \text{ for } R_2 \le r \le R_3 \text{ and } \Psi_i(r,0) = 0 \text{ for } R_{i-1} \le r \le R_i \text{ at } i = 1, 2,$$
(13)

where $F_3(r)$ is any distribution of the initial condition in the source zone (i.e., the third).

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Energies 2020, 13, 280

The boundary-initial problem (10)–(13) was solved first by the separation of variables method [25,27]. According to the method, after separating time and location variables and rejecting the singular component for r = 0 the following was obtained.

$$\Psi_1(r,t) = \sum_{n=1}^{\infty} C_n J_0(\gamma_n r/R_1) \cdot e^{-\gamma_n^2 \frac{\chi_1}{R_1^2} t} \text{ for } 0 \le r \le R_1, \ t \ge 0,$$
(14)

$$\Psi_{2}(r,t) = \sum_{n=1}^{\infty} \left(D_{n} J_{0}(\gamma_{n} s_{1} r/R_{1}) + E_{n} Y_{0}(\gamma_{n} s_{1} r/R_{1}) \right) \cdot e^{-\gamma_{n}^{2} \frac{\chi_{1}}{R_{1}^{2}} t} \text{ for } R_{1} \le r \le R_{2}, \ t \ge 0,$$
(15)

$$\Psi_{3}(r,t) = \sum_{n=1}^{\infty} \left(F_{n} J_{0}(\gamma_{n} s_{1} s_{2} r/R_{1}) + H_{n} Y_{0}(\gamma_{n} s_{1} s_{2} r/R_{1}) \right) \cdot e^{-\gamma_{n}^{2} \frac{\chi_{1}}{R_{1}^{2}} t} \text{ for } R_{2} \le r \le R_{3}, \ t \ge 0,$$
(16)

where: $s_1 = \sqrt{\chi_1/\chi_2}$, $s_2 = \sqrt{\chi_2/\chi_3}$ and γ_n are dimensionless eigenvalues (rescaled separation constants) of the boundary-initial problem (10)–(13), C_n, D_n, E_n, F_n and H_n are the coefficients of eigenfunctions and $J_k(...), Y_k(...)$ are Bessel functions of the first and second kind of order k. Then using the continuity (12a,b) and boundary (11) conditions, equation of the eigenvalues of the problem (10)–(13) was determined and the number of coefficients was reduced. For this purpose, the relations (14)–(16) were substituted to (12a,b) and (11). This leads to a homogeneous system of five equations relative to unknown coefficients C_n, D_n, E_n, F_n and H_n .

$$C_n J_0(\gamma_n) - D_n J_0(\gamma_n s_1) - E_n Y_0(\gamma_n s_1) = 0,$$
(17)

$$D_n J_0(\gamma_n p_1 s_1) + E_n Y_0(\gamma_n p_1 s_1) - F_n J_0(\gamma_n p_1 s_1 s_2) - H_n Y_0(\gamma_n p_1 s_1 s_2) = 0,$$
(18)

$$K_1 C_n J_1(\gamma_n) - D_n J_1(\gamma_n s_1) - E_n Y_1(\gamma_n s_1) = 0,$$
⁽¹⁹⁾

$$K_2 D_n J_1(\gamma_n p_1 s_1) + K_2 E_n Y_1(\gamma_n p_1 s_1) - F_n J_1(\gamma_n p_1 s_1 s_2) - H_n Y_1(\gamma_n p_1 s_1 s_2) = 0,$$
(20)

$$F_{n}(\vartheta \cdot J_{0}(p_{1}p_{2}s_{1}s_{2}\gamma_{n})/(\gamma_{n}s_{1}s_{2}) - J_{1}(p_{1}p_{2}s_{1}s_{2}\gamma_{n})) + H_{n}(\vartheta \cdot Y_{0}(p_{1}p_{2}s_{1}s_{2}\gamma_{n})/(\gamma_{n}s_{1}s_{2}) - Y_{1}(p_{1}p_{2}s_{1}s_{2}\gamma_{n})) = 0,$$
(21)

where: $K_1 = \lambda_1 / (\lambda_2 s_1), K_2 = \lambda_2 / (\lambda_3 s_2), \vartheta = \alpha R_1 / \lambda_3$ and $p_1 = R_2 / R_1, p_2 = R_3 / R_2$.

The system of Equations (17)–(21) has a non-trivial solution if its main determinant is zero

$$\Delta(\gamma_n) = a_1[b_1b_2 + b_3b_4 + b_5b_6 + b_7b_8] - a_2[b_9b_2 + b_{10}b_4 + b_{11}b_6 + b_{12}b_8] = 0,$$
(22)

where:

 b_9

$$\begin{aligned} a_1 &= \vartheta \cdot Y_0(p_1 p_2 s_1 s_2 \gamma_n) / (\gamma_n s_1 s_2) - Y_1(p_1 p_2 s_1 s_2 \gamma_n), \\ a_2 &= \vartheta \cdot J_0(p_1 p_2 s_1 s_2 \gamma_n) / (\gamma_n s_1 s_2) - J_1(p_1 p_2 s_1 s_2 \gamma_n), \\ b_1 &= K_1 J_1(\gamma_n) J_1(\gamma_n p_1 s_1 s_2), \ b_2 &= J_0(\gamma_n s_1) Y_0(\gamma_n p_1 s_1) - Y_0(\gamma_n s_1) J_0(\gamma_n p_1 s_1), \\ b_3 &= K_2 J_0(\gamma_n) J_0(\gamma_n p_1 s_1 s_2), \ b_4 &= J_1(\gamma_n s_1) Y_1(\gamma_n p_1 s_1) - Y_1(\gamma_n s_1) J_1(\gamma_n p_1 s_1), \\ b_5 &= K_1 K_2 J_1(\gamma_n) J_0(\gamma_n p_1 s_1 s_2), \ b_6 &= Y_0(\gamma_n s_1) J_1(\gamma_n p_1 s_1) - J_0(\gamma_n s_1) Y_1(\gamma_n p_1 s_1), \\ b_7 &= J_0(\gamma_n) J_1(\gamma_n p_1 s_1 s_2), \ b_8 &= Y_1(\gamma_n s_1) J_0(\gamma_n p_1 s_1) - J_1(\gamma_n s_1) Y_0(\gamma_n p_1 s_1), \\ &= K_1 J_1(\gamma_n) Y_1(\gamma_n p_1 s_1 s_2), \ b_{10} &= K_2 J_0(\gamma_n) Y_0(\gamma_n p_1 s_1 s_2), \ b_{11} &= K_1 K_2 J_1(\gamma_n) Y_0(\gamma_n p_1 s_1 s_2), \\ &= J_0(\gamma_n) Y_1(\gamma_n p_1 s_1 s_2). \end{aligned}$$

The condition (22) is an equation of eigenvalues.

Using of Equations (17)–(21), one can also reduce the number of unknown coefficients. For this purpose, the first four Equations (17)–(20) were used. As a result D_n , E_n , F_n and H_n coefficients were

made dependent on one C_n . The coefficients determined in this way were substituted to (15) and (16) and, after appropriate reduction, the following was obtained.

$$\Psi_{2}(r,t) = \sum_{n=1}^{\infty} C_{n} Z_{0}(\gamma_{n} s_{1} r/R_{1}) \cdot e^{-\gamma_{n}^{2} \frac{\lambda_{1}}{R_{1}^{2}}t} \text{ for } R_{1} \le r \le R_{2}, \ t \ge 0,$$
(23)

$$\Psi_{3}(r,t) = \sum_{n=1}^{\infty} C_{n} \Lambda_{0}(\gamma_{n} s_{1} s_{2} r/R_{1}) \cdot e^{-\gamma_{n}^{2} \frac{\chi_{1}}{R_{1}^{2}} t} \text{ for } R_{2} \le r \le R_{3}, \ t \ge 0,$$
(24)

where:

$$Z_k(\gamma_n s_1 r/R_1) = w_1 J_k(\gamma_n s_1 r/R_1) + w_2 Y_k(\gamma_n s_1 r/R_1),$$
(25)

$$\Lambda_k(\gamma_n s_1 s_2 r/R_1) = w_3 J_k(\gamma_n s_1 s_2 r/R_1) + w_4 Y_k(\gamma_n s_1 s_2 r/R_1),$$
(26)

$$w_{1} = (K_{1}J_{1}(\gamma_{n})Y_{0}(\gamma_{n}s_{1}) - J_{0}(\gamma_{n})Y_{1}(\gamma_{n}s_{1}))/(J_{1}(\gamma_{n}s_{1})Y_{0}(\gamma_{n}s_{1}) - J_{0}(\gamma_{n}s_{1})Y_{1}(\gamma_{n}s_{1})),$$

$$w_{2} = (J_{0}(\gamma_{n})J_{1}(\gamma_{n}s_{1}) - K_{1}J_{1}(\gamma_{n})J_{0}(\gamma_{n}s_{1}))/(J_{1}(\gamma_{n}s_{1})Y_{0}(\gamma_{n}s_{1}) - J_{0}(\gamma_{n}s_{1})Y_{1}(\gamma_{n}s_{1})),$$

$$w_{3} = (b_{6}b_{11} + b_{2}b_{9} + b_{4}b_{10} + b_{8}b_{12})/b_{13}, w_{4} = -(b_{5}b_{6} + b_{1}b_{2} + b_{3}b_{4} + b_{7}b_{8})/b_{13},$$

$$b_{13} = (J_1(\gamma_n s_1)Y_0(\gamma_n s_1) - J_0(\gamma_n s_1)Y_1(\gamma_n s_1)) \cdot (J_1(\gamma_n p_1 s_1 s_2)Y_0(\gamma_n p_1 s_1 s_2) - J_0(\gamma_n p_1 s_1 s_2)Y_1(\gamma_n p_1 s_1 s_2)).$$

The form of Equation (14) remained unchanged.

It is also necessary to determine the unknown coefficient C_n visible in the formulas (14), (23), (24). For this purpose initial conditions (13) were used. After substituting distributions (14), (23), (24) into (24) into (13) the following was obtained

$$\sum_{n=1}^{\infty} C_n J_0(\gamma_n r/R_1) = 0 \text{ for } 0 \le r \le R_1,$$
(27)

$$\sum_{n=1}^{\infty} C_n Z_0(\gamma_n s_1 r / R_1) = 0 \text{ for } R_1 \le r \le R_2,$$
(28)

$$\sum_{n=1}^{\infty} C_n \Lambda_0(\gamma_n s_1 s_2 r / R_1) = F_3(r) \text{ for } R_2 \le r \le R_3.$$
(29)

Subsequently, the relation (27) was multiplied by $(\lambda_1/\chi_1) \cdot J_0(\gamma_m r/R_1)$ and integrated in the range $\langle 0, R_1 \rangle$. Analogous operations were performed on expressions (28) and (29) (respectively multiplied by $(\lambda_2/\chi_2) \cdot Z_0(\gamma_m s_1 r/R_1)$ and $(\lambda_3/\chi_3) \cdot \Lambda_0(\gamma_m s_1 s_2 r/R_1)$ and integrated in ranges $\langle R_1, R_2 \rangle$ and $\langle R_2, R_3 \rangle$). The relations obtained in this way were added to each other. Subsequently, the orthogonality condition of the radial coordinate eigenfunctions in cylindrical multi-zone systems (proven in [27]) was used

$$\frac{\lambda_{1}}{\lambda_{1}} \int_{0}^{R_{1}} r J_{0}(\gamma_{n}r/R_{1}) J_{0}(\gamma_{m}r/R_{1}) dr + \frac{\lambda_{2}}{\lambda_{2}} \int_{R_{1}}^{R_{2}} r Z_{0}(\gamma_{n}s_{1}r/R_{1}) Z_{0}(\gamma_{m}s_{1}r/R_{1}) dr + + \frac{\lambda_{3}}{\lambda_{3}} \int_{R_{2}}^{R_{3}} r \Lambda_{0}(\gamma_{n}s_{1}s_{2}r/R_{1}) \Lambda_{0}(\gamma_{m}s_{1}s_{2}r/R_{1}) dr = \begin{cases} 0 & \text{for } \gamma_{n} \neq \gamma_{m} \\ \|N(m)\|^{2} & \text{for } \gamma_{n} = \gamma_{m} \end{cases}$$

$$(30)$$

where $||N(m)||^2$ —square of the norm (32).

After calculating the suitable integrals in (30), ordering and replacing $m \rightarrow n$, the C_n coefficient was obtained

$$C_{n} = \frac{(\lambda_{3}/\chi_{3}) \cdot \int\limits_{R_{2}}^{R_{3}} rF_{3}(r)\Lambda_{0}(\gamma_{n}s_{1}s_{2}r/R_{1})dr}{\|N(n)\|^{2}},$$
(31)

where:

$$\|N(n)\|^{2} = \frac{\lambda_{1}}{2\chi_{1}}R_{1}^{2} \Big(J_{0}^{2}(\gamma_{n}) + J_{1}^{2}(\gamma_{n})\Big) + \frac{\lambda_{2}}{2\chi_{2}}R_{2}^{2} \Big(Z_{0}^{2}(p_{1}s_{1}\gamma_{n}) + Z_{1}^{2}(p_{1}s_{1}\gamma_{n})\Big) - \frac{\lambda_{2}}{2\chi_{2}}R_{1}^{2} \Big(Z_{0}^{2}(s_{1}\gamma_{n}) + Z_{1}^{2}(s_{1}\gamma_{n})\Big) + \frac{\lambda_{3}}{2\chi_{3}}R_{3}^{2} \Big(\Lambda_{0}^{2}(p_{1}p_{2}s_{1}s_{2}\gamma_{n}) + \Lambda_{1}^{2}(p_{1}p_{2}s_{1}s_{2}\gamma_{n})\Big) - \frac{\lambda_{3}}{2\chi_{3}}R_{2}^{2} \Big(\Lambda_{0}^{2}(p_{1}s_{1}s_{2}\gamma_{n}) + \Lambda_{1}^{2}(p_{1}s_{1}s_{2}\gamma_{n})\Big).$$

$$(32)$$

Then the integration variable was changed in (31) ($r \rightarrow \xi$). The expression thus modified was substituted to (14), (23) and (24).

The auxiliary problem (10)–(13) was solved again by another method (i.e., using the Green's function [22])

$$\Psi_i(r,t) = \int_{R_2}^{R_3} G_i(r,t,\xi,\eta=0) F_3(\xi) \xi d\xi \text{ for } R_{i-1} \le r \le R_i, \ i=1,\ 2,\ 3,\ R_0=0,\ t\ge 0.$$
(33)

The coefficient C_n is the integral (31) and occurs in (14), (23) and (24) with the replaced variable $r \rightarrow \xi$. This makes it possible to compare the relations (14), (23)and (24) with (33) for i = 1, 2, 3 (including conversion $t \rightarrow t - \eta$), respectively. As a result, the following was obtained.

$$G_{1}(r,t,\xi,\eta) = \frac{\lambda_{3}}{\chi_{3}} \sum_{n=1}^{\infty} \frac{\Lambda_{0}(\gamma_{n}s_{1}s_{2}\xi/R_{1})J_{0}(\gamma_{n}r/R_{1})}{\|N(n)\|^{2}} \cdot e^{-\gamma_{n}^{2}\frac{\chi_{1}}{R_{1}^{2}}(t-\eta)},$$
(34)

$$G_{2}(r,t,\xi,\eta) = \frac{\lambda_{3}}{\chi_{3}} \sum_{n=1}^{\infty} \frac{\Lambda_{0}(\gamma_{n}s_{1}s_{2}\xi/R_{1})Z_{0}(\gamma_{n}s_{1}r/R_{1})}{\|N(n)\|^{2}} \cdot e^{-\gamma_{n}^{2}\frac{\chi_{1}}{R_{1}^{2}}(t-\eta)},$$
(35)

$$G_{3}(r,t,\xi,\eta) = \frac{\lambda_{3}}{\chi_{3}} \sum_{n=1}^{\infty} \frac{\Lambda_{0}(\gamma_{n}s_{1}s_{2}\xi/R_{1})\Lambda_{0}(\gamma_{n}s_{1}s_{2}r/R_{1})}{\|N(n)\|^{2}} \cdot e^{-\gamma_{n}^{2}\frac{\chi_{1}}{R_{1}^{2}}(t-\eta)},$$
(36)

where $||N(n)||^2$ is the square of the norm (32). Green's functions (34)–(36) are kernels of integral operators (9) inverse to differential ones (4).

5. The Time-and-Space-Variable Heating Curves, Steady State Current Ratings and the Time Constants

The time-and-space-variable heating curves were obtained after substitution of (34)–(36)–(9), integration and use of relationship (3).

$$T_1(r,t) = T_s + \sum_{n=1}^{\infty} Q_n L_n J_0(\gamma_n r/R_1) \phi_n(t) \text{ for } 0 \le r \le R_1, \ t \ge 0,$$
(37)

$$T_2(r,t) = T_s + \sum_{n=1}^{\infty} Q_n L_n Z_0(\gamma_n s_1 r / R_1) \phi_n(t) \text{ for } R_1 \le r \le R_2, \ t \ge 0,$$
(38)

$$T_3(r,t) = T_s + \sum_{n=1}^{\infty} Q_n L_n \Lambda_0(\gamma_n s_1 s_2 r / R_1) \phi_n(t) \text{ dla } R_2 \le r \le R_3, \ t \ge 0,$$
(39)

where:

$$Q_n = \frac{R_1^2}{\chi_1 \gamma_n^2 ||N(n)||^2},$$
(40a)

$$\phi_n(t) = \left(1 - e^{-\gamma_n^2 \frac{\lambda_1}{R_1^2} t}\right),$$
(40b)

$$L_n = \int_{R_2}^{R_3} g_3(\xi) \Lambda_0(\gamma_n s_1 s_2 \xi / R_1) \xi d\xi,$$
 (40c)

and $g_3(\xi)$ is given by relation (5) (after conversion $r \to \xi$). The integral (40c) is calculated numerically.

The temperature distributions (37)–(39) and other parameters of the ACCC conductor were calculated using a software developed in the Mathematica 10.4 environment [29]. The following data [3,19,23,25] were adopted:

$$R_1 = 0.0034$$
 m, $R_2 = 0.004765$ m, $R_3 = 0.016425$ m, $\lambda_1 = 0.17$ W/(mK), $\lambda_2 = 0.2$ W/(mK),

$$\lambda_{3} = 238 \text{ W/(mK)}, c_{1} = 800 \text{ J/(kgK)}, c_{2} = 1000 \text{ J/(kgK)}, c_{3} = 900 \text{ J/(kgK)}, \mu_{1} = 1043 \text{ kg/m}^{3}, \\ \mu_{2} = 1168 \text{ kg/m}^{3}, \mu_{3} = 2700 \text{ kg/m}^{3}, k_{1} = 1.02, \rho(20 \,^{\circ}\text{C}) = 2.79 \cdot 10^{-8} \Omega m, \beta = 4.07 \cdot 10^{-3} \text{ 1/ °C}, \\ V = 0.6 \text{ m/s}, \varepsilon = 0.5, T_{a} = 40 \,^{\circ}\text{C}, T_{s} = 55 \,^{\circ}\text{C}, \alpha_{L} = 17.45 \text{ W/(m}^{2}\text{K}), \alpha_{M} = 18.88 \text{ W/(m}^{2}\text{K}), \\ \alpha_{H} = 20.17 \text{ W/(m}^{2}\text{K}), \rho_{L} = 3.18744 \cdot 10^{-8} \Omega m, \rho_{M} = 3.89714 \cdot 10^{-8} \Omega m, \rho_{H} = 4.60685 \cdot 10^{-8} \Omega m, \\ f = 50 \text{ Hz}, \mu_{o} = 4\pi \cdot 10^{-7} \text{ H/m}.$$
(41)

Resistivity ρ and heat transfer coefficients α were defined according to (1a–c) and calculated using [19,25] Equation (2).

One of the most important parameters of the current path is steady-state current rating I_{cr} . After loading the system with such current, the steady-state aluminum temperature ($R_2 \le r \le R_3$) should be equal to T_{max} (maximum operating temperature). According to the manufacturer's guidelines [1,6] $T_{max} = 180 \,^{\circ}\text{C}$ was adopted. This value ensures core security under the highest thermal load conditions (i.e., for parameters α_{H}, ρ_{H}). Therefore, in order to determine the steady-state current rating, one should solve the following equation with respect to I_{cr} .

$$T_{3H}(r = R_2, t \to \infty, |I| = I_{cr}) = T_{\max}$$

$$\tag{42}$$

Equation (42) was iteratively solved using (39) and (41) and while loops in Mathematica 10.4 [29]. The result was $I_{cr} = 2057$ A. The calculated current value was verified by the steady-state power balance. For this purpose, the following theorem was used [9]: if in the convection and radiation terms, the ambient temperature T_a is replaced by the solar temperature T_s , then the solar radiation power can be zeroed in the balance.

$$I_{cr}^2 R(T_{\max}) = \alpha_k S(T_{\max} - T_s) + \varepsilon \delta S[(T_{\max} + 273.15)^4 - (T_s + 273.15)^4],$$
(43a)

where the following nomenclature was used:

$$R(T_{\max}) = k_1 k_2 \rho_H l / [\pi (R_3^2 - R_2^2)] - aluminum resistance at temperature T_{\max},$$
(43b)

 k_1 —stranding factor [23] (p. 116), k_2 —skin factor [23] (Figure 7.2), l—length of conductor's section,

 $\alpha_k = \alpha_k(V, T_p = T_{\text{max}}, T_f = 0.5(T_{\text{max}} + T_a)) = 13.58 \text{ W}/(\text{m}^2 K)$ —convective component of the total heat transfer coefficient (67.33%) [19],

 $S = 2\pi R_3 l$ —conductor surface,

 $\delta = 5.67 \cdot 10^{-8} \text{ W}/(\text{m}^2 K^4)$ —Stefan-Boltzmann constant.

From the relationship (43a) was obtained $I_{cr} = 2070$ A. Therefore, the relative difference between solution (42) and (43a) is approx. 0.63%, which was considered good enough accordance. The discussed

difference results from the approximation of the AC resistance with the expression $R_{AC} \cong k_1 k_2 R_{DC}$ (43b).

On the basis (39), the heating curves $T_{3L}(r = R_2, t)$, $T_{3M}(r = R_2, t)$ and $T_{3H}(r = R_2, t)$ were plotted with heavy lines, which were respectively marked with letters L, M and H (Figure 2). They correspond to the parameters (α_L, ρ_L) , (α_M, ρ_M) and (α_H, ρ_H) . The load current is constant and is $||I| = I_{cr} = 2057 \text{ A}$). For $0 \le t \le t_1 = 800 \text{ s}$ real time-profile $T_3(r = R_2, t)$ is initially close to L and then gradually approaches to M (window in Figure 2). For $t \ge t_1 = 800$ s the sought temperature-time history it increases analogously from the curve M to H. Therefore, the gray area visible in Figure 2 shows the range of changes of the real heating curve $T_3(r = R_2, t)$.



Figure 2. Lower and upper estimation of the heating curve $T_3(r = R_2, t)$ with the current load $I_{cr} = 2057$ A.

At the moment t = 100 s the system parameters are very close to (α_L, ρ_L) , and in t = 800 s and $t \to \infty$ respectively to (α_M, ρ_M) and (α_H, ρ_H) . Figure 3a–c show radial temperature distributions $T_{iL}(r, t = 100 \text{ s})$, $T_{iM}(r, t = 800 \text{ s})$ and $T_{iH}(r, t \to \infty)$, where i = 1, 2 and i = 3 Gradual equalization of the core temperature as a result of its heating is visible $(T_{2L}(r = R_2, t = 100 \text{ s}) - T_{1L}(r = 0, t = 100 \text{ s}) \approx 2.35 \text{ °C}$, $T_{2M}(r = R_2, t = 800 \text{ s}) - T_{1M}(r = 0, t = 800 \text{ s}) \approx 1.42 \text{ °C}$ and $T_{2H}(r = R_2, t \to \infty) - T_{1H}(r = 0, t \to \infty) \approx 0 \text{ °C}$. Due to inequality $\lambda_1 \approx \lambda_2 << \lambda_3$, this process is relatively slow. On the other hand, a high value λ_3 means that in the annulus $\{R_2 \le r \le R_3\}$ the Biot number (Bi) is very low (<< 0.1). This Bi value justifies the almost uniform temperature distribution in aluminum at any given time [25], e.g., from Figure 3c it follows: $T_{3H}(r = R_2, t \to \infty) - T_{3H}(r = R_3, t \to \infty) < 0.07 \text{ °C}$. Since the outer layer is a source of heat, its temperature increases rapidly over time (for t = 100 s approx. 64 °C, for t = 800 s approx. 117.35 °C, and in steady state it reaches 180 °C).

64.0





Figure 3. (a) Radial temperature distribution at the instant t = 100 s with the current load $I_{cr} = 2057$ A, (b) radial temperature distribution at the instant t = 800 s with the current load $I_{cr} = 2057$ A and (c) radial temperature distribution in the steady state ($t \rightarrow \infty$) with the load current $I_{cr} = 2057$ A.

In order to verify the correctness of formulas (37)–(39), the boundary-initial problem (4)–(8) was solved again. This was done using the finite element method (FEM) [10]. The Green and FEM methods are completely different. In the second case, the commercial NISA software was used [30]. The discrepancy of the results was evaluated using the following relation

$$\delta_{Ti} = 100\% \frac{\nu_i(r,t) - \nu_i^{(FE)}(r,t)}{\nu_i(r,t)} \text{ for } i = 1, 2, 3,$$
(44)

where $v_i(r, t)$ is the temperature increase (above T_a) caused by the current flow calculated by the Green method, and $v_i^{(FE)}(r, t)$ is the same increase but determined by the finite element method. Figure 4 shows the relation (44) for parameters (α_M , ρ_M). As one can see $\delta_{Ti}^{(M)} < 0.11\%$, which was considered good enough accordance. For parameters (α_L , ρ_L) and (α_H , ρ_H) the relative differences (44) are of the same order as in Figure 4.



Figure 4. Relative difference (44) of temperature increases determined by means of the Green's and finite element methods for (α_M , ρ_M) parameters.

The local time constant method $\tau_i(r)$ consists in approximating the dynamics of each point of the field with the first order lag element. The step response of such an object is well known and equals

$$T_i(r,t) = T_i(r,t \to \infty) [1 - \exp(-t/\tau_i(r))] + T_i(r,t=0) \exp(-t/\tau_i(r)),$$
(45)

where *i* = 1, 2 and 3. From (45) it results [31–33]

$$\tau_i(r) = \int_0^\infty \frac{T_i(r,t) - T_i(r,t \to \infty)}{T_i(r,t=0) - T_i(r,t \to \infty)} dt.$$
(46)

Heating curves (37)–(39) were substituted to (46) and, after integration, the following was finally obtained

$$\tau_{1}(r) = \frac{R_{1}^{2}}{\chi_{1}} \frac{\sum_{n=1}^{\infty} A_{n}(r)/\gamma_{n}^{2}}{\sum_{n=1}^{\infty} A_{n}(r)}, \text{ where } A_{n}(r) = \frac{J_{0}(\gamma_{n}r/R_{1})}{\gamma_{n}^{2} \|N(n)\|^{2}} L_{n} \text{ for } 0 \le r \le R_{1},$$
(47)

$$\tau_{2}(r) = \frac{R_{1}^{2}}{\chi_{1}} \frac{\sum_{n=1}^{\infty} B_{n}(r)/\gamma_{n}^{2}}{\sum_{n=1}^{\infty} B_{n}(r)}, \text{ where } B_{n}(r) = \frac{Z_{0}(\gamma_{n}s_{1}r/R_{1})}{\gamma_{n}^{2}||N(n)||^{2}} L_{n} \text{ for } R_{1} \le r \le R_{2},$$
(48)

$$\tau_{3}(r) = \frac{R_{1}^{2}}{\chi_{1}} \frac{\sum_{n=1}^{\infty} U_{n}(r) / \gamma_{n}^{2}}{\sum_{n=1}^{\infty} U_{n}(r)}, \text{ where } U_{n}(r) = \frac{\Lambda_{0}(\gamma_{n}s_{1}s_{2}r/R_{1})}{\gamma_{n}^{2}||N(n)||^{2}} L_{n} \text{ for } R_{2} \le r \le R_{3}.$$
(49)

Graphs of functions (47)–(49) are shown in Figure 5. The inequality $\tau_{iL}(r) > \tau_{iM}(r) > \tau_{iH}(r)$ shows that at any point *r* the heating curve with parameters (α_H , ρ_H) increases the fastest, and with parameters (α_L , ρ_L) the slowest. For $r = R_2$ this is confirmed by Figure 2. In turn, the inequality $\tau_{jK}(r) > \tau_{3K}(r)$ (for j = 1, 2, K = L, M, H) makes the temperature in the composite core (CFC + GFC) increase more slowly than in aluminum. The high speed of heat propagation in the metal means that in its whole volume the time constant does not change. The same conclusions were drawn in the analysis of Figure 3 based on inequality $\lambda_1 \approx \lambda_2 << \lambda_3$. The steady state arose as a result of a gradual transition from M to H curve in Figure 2. For this reason, the duration of the transient state was estimated between $5\tau_{1H}(r = 0) = 4840$ s and $5\tau_{1M}(r = 0) = 5155$ s. This result is also confirmed by Figure 2.



Figure 5. Distributions of local time constants with parameters (α_H, ρ_H) , (α_M, ρ_M) and (α_L, ρ_L) .

6. Final Remarks

The conclusions below follow from the presented analysis:

- The lower and upper estimates of the heating curve determined for the point $r = R_2$ (Figure 2) was almost the same in the whole volume of aluminum. This is due to the high thermal conductivity of this metal and is confirmed by the graphs of Figures 3a–c and 5 in the range $R_2 \le r \le R_3$.
- The heating curves in the composite core increased more slowly than in aluminum. It justifies the inequality λ₁ ≈ λ₂ << λ₃ and was confirmed by the graphs of Figures 3a–c and 5 in the range 0 ≤ *r* ≤ *R*₂. At the beginning of the transient state (*t* < 25 s), even apparent dead time occurred inside the core (*r* = 0).
- The presented solution also included the worst case analysis (i.e., the highest thermal load). It will occur for parameters α_H and ρ_H defined by formulas (1c). Thus, the critical aluminum heating curve is presented in the diagram H in Figure 2.
- The solution in the form of estimation of the heating curve and ACCC line time constant results from the strong influence of temperature on aluminum resistivity (change by 44.5%) and on the

13 of 15

total heat transfer coefficient (change by 15.6%). Sensitivity of other parameters to temperature was much lower.

- Since the current rating was calculated in a steady state, the value of this parameter was precisely determined (i.e., without estimation).
- Solution verification carried out by other methods than Green's function gave a positive result (power balance (43), finite elements—Figure 4).
- The presented solution had a very good physical interpretation (comments to Figures 2, 3a–c and 5 in the previous chapter).
- The analysis included 25 terms of the series (37)–(39) and (47)–(49). The following characteristics and parameters were determined: heating curves (37)–(39) for three different values of heat transfer coefficient and resistivity, time constants (47)–(49) and spatial temperature distributions presented in Figure 3a–c. The calculations were made in the Mathematica 10.4 program within 50 min.

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