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Article

Formulation and Analysis of an Approximate Expression for Voltage Sensitivity in Radial DC Distribution Systems

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Abstract: Voltage is an important variable that reflects system conditions in DC distribution systems and affects many characteristics of a system. In a DC distribution system, there is a close relationship between the real power and the voltage magnitude, and this is one of major differences from the characteristics of AC distribution systems. One such relationship is expressed as the voltage sensitivity, and an understanding of voltage sensitivity is very useful to describe DC distribution systems. In this paper, a formulation for a novel approximate expression for the voltage sensitivity in a radial DC distribution system is presented. The approximate expression is derived from the power flow equation with some additional assumptions. The results of approximate expression is compared with an exact calculation, and relations between the voltage sensitivity and electrical quantities are analyzed analytically using both the exact form and the approximate voltage sensitivity equation.

Keywords: DC distribution system; voltage sensitivity; formulation; analytic analysis; approximate expression

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1. Introduction

Over the past several years, there has been growing interest in DC distribution systems. There have been many reports of investigations of the advantages of DC distribution systems over AC systems. By using a DC distribution system, the transmittable power can be increased up to a factor of 10 [1]. When renewable energy resources are incorporated into a DC distribution system, energy conversion can be eliminated, which results in a saving of 2.5%–10% of the generated energy [2]. Furthermore, distributed energy resources (DERs) and electric vehicles (EVs) can be utilized efficiently in a DC distribution system because they use/produce DC power [3]. Such advantages motivate further study of the applications of DC distribution systems, including commercial facilities and shipboard power systems [4–8]. DC distribution systems can have various grid layouts, including radial, ring and meshed grids, which affect their vulnerability to faults [6]. Furthermore, in naval shipboard systems, a zonal distribution architecture can be applied to satisfy the requirements for survivability, low weight, reduced manning, and low costs [8]. In conventional distribution systems, radial distribution is most commonly used, and this structure can be use if the DC distribution system replaces a conventional AC distribution system, which is the focus of this work.

In a DC distribution system, loads are generally fed through DC/AC or DC/DC converters, which have constant-power load behavior if the converter maintains continuous load consumption under rapid current and voltage variations. In this case, voltage instability may occur in response to a negative incremental resistance [9]. For example, the load current may increase if the load voltage decreases or the load consumption increases, which leads to a drop in the voltage over the conductors. This leads to repeated decreases in the load voltage and increases in the load current, which may result in the operating point moving from the previous point to a voltage stability critical point, causing the system to become unstable. Moreover, voltage affects both current and the voltage drop over the conductors. Therefore, voltage is a primary variable that reflects system conditions, and affects power flow and voltage stability in a DC distribution system. Here, this paper focuses on the relationship between voltage magnitude and electrical quantities, which may elucidate the characteristics of a DC distribution system.

Such a relationship, expressed as the voltage sensitivity, provides a linear relation between voltage deviations and electrical quantities. The voltage–reactive power (V-Q) sensitivity is generally used for steady-state analyses of AC power systems, and V-Q sensitivity is often used in planning, operation, assessment, and control of power systems [10–20]. Moreover, voltage sensitivity can be utilized in medium- or low-voltage distribution networks with high penetrations of DERs, which underlines the importance of optimal control in a distribution network. Optimal control of a power system is often based on a linear optimal problem, where sensitivity coefficients link the control variables and the controlled quantities. Therefore, voltage sensitivity is useful for solving these types of problems [21].

Voltage sensitivity can also be use to analyze DC distribution systems. The real power and voltage magnitudes are important variables that reflect various phenomena in a DC distribution system. Therefore, this paper presents formulation and analysis of voltage sensitivity in the DC radial distribution system. An analysis of such relationships is helpful to describe and understand such systems. In addition, the proposed formulations can be use to solve practical problems in the DC network. A cooperative voltage control scheme using grid-connected converters (GCCs) and DERs in a DC distribution system has been reported [22]. The authors analyzed voltage sensitivity in response to changes in the injected power from the DERs, and a voltage control scheme was constructed based on analytic voltage sensitivity analysis to reduce required amount of power of DERs for voltage regulation. Moreover, the proposed formulation can be used for simple calculation of the voltage sensitivity in radial DC distribution systems. The voltage sensitivity can be calculated directly from measurements without power flow calculation by proposed formulation. The calculated voltage sensitivity could be utilized for voltage control. Voltage control method based on multi agent system (MAS) has been reported and voltage sensitivity is utilized [23,24]. In these researches, agents calculate voltage sensitivity factor from the inverse of the Jacobian matrix for active power control of DERs and such process could be simplified using proposed formulation to reduce calculation efforts.

In this paper, voltage sensitivity in a DC distribution system, which provides an important description of the system and is often required to provide effective system operation, is investigated. Previous studies, however, have not dealt with the details of voltage sensitivity, which is the subject of this work. This paper formulates an approximate voltage sensitivity equation for DC distribution systems, and analyze voltage sensitivity in response to changes in electrical quantities, injected power at buses, and the slack bus voltage. In Section 2, difficulties of voltage sensitivity analyses in DC distribution systems are addressed from the perspective of a general voltage equation and a voltage sensitivity equation. An approximate voltage sensitivity equation is developed in Section 3, where two types of electrical quantity that provide an analytical description of the system are considered. The resulting equation is verified in Section 4, and the results of the analytical description of the voltage sensitivity are detailed in Section 5, based on the proposed equations.

2. Power Flow and Voltage Sensitivity Analysis for a DC Distribution System

The power flow in a DC distribution system can be described using a voltage sensitivity analysis. Such an analysis is the basis of the steady-state description upon which conventional voltage sensitivity analyses are based. However, a number of difficulties arise with an analytical study based on the conventional method. This problem is addressed in this section.

2.1. Definition of the G-Bus Matrix

The Y-Bus matrix is widely used to describe various steady-state analyses in AC power systems, such as power flow calculations, sensitivity analyses, and stability assessments. In the same manner, the G-Bus matrix can be used to describe DC power systems [25]. As shown below, the conductance matrix is similar to the admittance matrix for an AC system, the only difference being the absence of reactive components. The G-Bus matrix is defined as follows:

$$G = \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ G_{n1} & G_{n2} & \cdots & G_{nn} \end{bmatrix} \tag{1}$$

where

$$G_{ii} = \sum_{j=1}^{n} g_{ij} \ and \ G_{ij}\Big|_{i \neq j} = -g_{ij}$$

2.2. Power Flow Analysis in a DC Distribution System

Figure 1 shows a diagram of a simple DC distribution system interconnected with a higher-voltage AC or DC system via a GCC, where the secondary terminal of the GCC is located at bus 1. The voltage at this bus is assumed to be constant because the secondary terminal voltage of GCC can be controlled continuously; this point is therefore considered the slack bus. Buses 2 to 6 include DC power loads that are connected to the system using DC/DC or DC/AC converters. It is assumed that each load is a constant-power load, and that these loads can be converted to current loads, where the currents are inversely proportional to the bus voltages. Based on these assumptions, the following set of nodal equations are obtained:

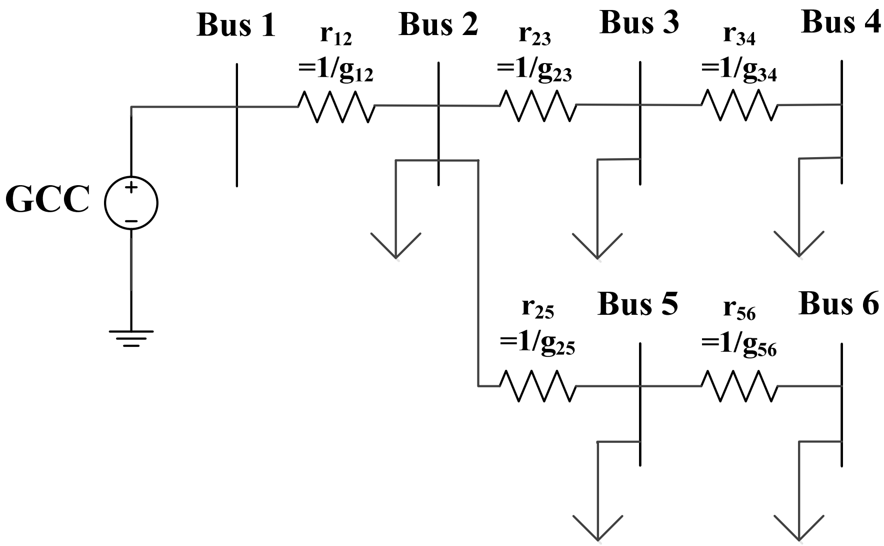


Figure 1. A simple radial DC distribution system.

$$\begin{aligned}
&g_{12}(V_2 - V_1) + g_{23}(V_2 - V_3) + g_{25}(V_2 - V_5) + P_{L,2} / V_2 = 0 \\
&g_{23}(V_3 - V_2) + g_{34}(V_3 - V_4) + P_{L,3} / V_3 = 0 \\
&g_{34}(V_4 - V_3) + P_{L,4} / V_4 = 0 \\
&g_{25}(V_5 - V_2) + g_{56}(V_5 - V_6) + P_{L,5} / V_5 = 0 \\
&g_{56}(V_6 - V_5) + P_{L,6} / V_6 = 0
\end{aligned} \tag{2}$$

to obtain general voltage equations for a DC power system, the above expressions are rearranged as follows. The voltage equation for bus 2 can be expressed as:

$$P_2 = g_{12}(V_2^2 - V_1V_2) + g_{23}(V_2^2 - V_3V_2) + g_{25}(V_2^2 - V_5V_2) \tag{3}$$

and

$$P_2 = -P_{L,2} = G_{22}V_2^2 - \sum_{k=1,k\neq 2}^{n} G_{2k}V_k \tag{4}$$

From Equations (3) and (4), the following general equation for the voltage at bus *i* is obtained:

$$P_i = G_{ii}V_i^2 + V_i \sum_{k=1,k\neq i}^{n} G_{ik}V_k \tag{5}$$

where

$$V_i = \frac{-\sum_{k=1,k\neq i}^{n} G_{ik}V_k \pm \sqrt{\left(\sum_{k=slack,k\neq i}^{n} G_{ik}V_k\right)^2 + 4G_{ii}P_i}}{2G_{ii}} \tag{6}$$

2.3. Voltage Sensitivity Calculation in a DC Distribution System

The voltage–reactive power sensitivity is widely used in steady-state analyses of AC power systems because the bus reactive power and the voltage magnitude are typically strongly correlated. In [25], a voltage–reactive power sensitivity analysis of an AC power system was reported, and the voltage sensitivity was derived using a Jacobian matrix. In the same manner, the voltage sensitivity of a DC power system can be determined. In a steady-state analysis of a DC power system, the reactive power and voltage angle can be neglected. Therefore, only the relationship between the voltage magnitude and real bus power is considered in the voltage sensitivity analysis, and a linearized form of the power flow equations can be used; *i.e.*,

$$\Delta P = [J]\Delta V \tag{7}$$

in this expression, the relationship between the deviation of the real bus power and voltage magnitude is represented directly by the Jacobian matrix. The voltage sensitivity is obtained as follows:

$$\Delta V = [J]^{-1}\Delta P] \tag{8}$$

The Jacobian matrix gives the sensitivities of the real bus power to deviations in the bus voltage magnitude. The elements of the Jacobian matrix in Equations (7) and (8) can be obtained from Equation (5), and are as follows:

$$J_{ij} = \frac{\partial P_i}{\partial V_j} = G_{ij}V_i \quad for\, i \neq j \tag{9}$$

and

$$J_{ii} = \frac{\partial P_i}{\partial V_i} = 2G_{ii}V_i + \sum_{k=1,k\neq i}^{n} G_{ik}V_k \quad for\, i = j \tag{10}$$

The expression for the voltage sensitivity of a DC power system is far simpler than the equivalent expression for an AC power system because the phase angle and reactive power can be neglected. However, as with an AC power system, the inverse matrix of the Jacobian matrix must be calculated to obtain the voltage sensitivity. Therefore, the conventional analysis has the drawback that it is difficult to describe the voltage sensitivity analytically, and hence it is difficult to find a direct relation between the voltage deviation and the deviation of the electrical quantities. In other words, it cannot provide useful information to analyze the voltage sensitivity, and the voltage sensitivity must be calculated from the inverse of the Jacobian matrix.

3. Approximate Expression for the Voltage Sensitivity in a Radial DC Distribution System

3.1. Derivative of the General Voltage Equation

The voltage sensitivity in a DC distribution system indicates the voltage deviation in response to small variations in the electrical quantities at an arbitrary bus. Therefore, the relationship between the voltage and the electrical quantities at an arbitrary bus should be investigated in the general voltage equation to derive voltage sensitivity. It follows that the voltage equation should include variables to be controlled, and the injected power at the bus and the slack bus voltage is considered controllable variables. For example, the voltage equation can be expressed as follows for the system shown in Figure 1:

$$V_i = V_i(P, V_{slack}) \tag{11}$$

Using the general voltage equation as per Equation (6), the real power at bus i is shown, whereas the real powers at other buses are not given directly. It follows that this equation cannot yield information on the direct relationship between voltage deviations and the real power deviation at an arbitrary bus. Moreover, the general voltage equation with a conventional power flow analysis is unsuitable for an analytical description of voltage sensitivity. Therefore, the set of equations as per Equation (2) should be replaced with equations that describe the voltage drop between the slack bus and the load bus to acquire a general voltage equation in the form of Equation (11); *i.e.*,

$$\begin{aligned}
V_2 &= V_1 + r_{12}\left(\frac{P_2}{V_2} + \frac{P_3}{V_3} + \cdots + \frac{P_6}{V_6}\right) \\
V_3 &= V_1 + r_{12}\left(\frac{P_2}{V_2} + \frac{P_3}{V_3} + \cdots + \frac{P_6}{V_6}\right) + r_{23}\left(\frac{P_3}{V_3} + \frac{P_4}{V_4}\right) \\
V_4 &= V_1 + r_{12}\left(\frac{P_2}{V_2} + \frac{P_3}{V_3} + \cdots + \frac{P_6}{V_6}\right) + r_{23}\left(\frac{P_3}{V_3} + \frac{P_4}{V_4}\right) + r_{34}\left(\frac{P_4}{V_4}\right) \\
V_5 &= V_1 + r_{12}\left(\frac{P_2}{V_2} + \frac{P_3}{V_3} + \cdots + \frac{P_6}{V_6}\right) + r_{25}\left(\frac{P_5}{V_5} + \frac{P_6}{V_6}\right) \\
V_6 &= V_1 + r_{12}\left(\frac{P_2}{V_2} + \frac{P_3}{V_3} + \cdots + \frac{P_6}{V_6}\right) + r_{25}\left(\frac{P_5}{V_5} + \frac{P_6}{V_6}\right) + r_{56}\left(\frac{P_6}{V_6}\right)
\end{aligned} \tag{12}$$

In Equation (12), each term except V_1 on the right-hand side of the equation describes a line voltage drop at each conductor segment. For ease of manipulation, this set of equations is arranged as follows:

$$V_i = V_{slack} + \sum_{k=1}^{n} R_{ik}\frac{P_k}{V_k} \tag{13}$$

The R-Bus matrix can be redefined as follows:

$$R = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\ R_{21} & R_{22} & \cdots & R_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ R_{n1} & R_{n2} & \cdots & R_{nn} \end{bmatrix} \tag{14}$$

and the R-Bus matrix for the system shown in Figure 1 is as follows:

$$R=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r_{12} & r_{12} & r_{12} & r_{12} & r_{12} \\ 0 & r_{12} & r_{12}+r_{23} & r_{12}+r_{23} & r_{12} & r_{12} \\ 0 & r_{12} & r_{12}+r_{23} & r_{12}+r_{23}+r_{34} & r_{12} & r_{12} \\ 0 & r_{12} & r_{12} & r_{12} & r_{12}+r_{25} & r_{12}+r_{25} \\ 0 & r_{12} & r_{12} & r_{12} & r_{12}+r_{25} & r_{12}+r_{25}+r_{56} \end{bmatrix} \tag{15}$$

each element of the R-Bus matrix R_{ij} is defined as the sum of the elements in the intersection of groups $\{R_i\}$ and $\{R_j\}$, where group $\{R_i\}$ is the set of line resistances located on the shortest path between the slack bus of the radial DC distribution system and bus i.

3.2. Formulation of an Approximate Expression for Voltage Sensitivity with Respect to Real Bus Power

The main purpose of this work is to obtain an approximate expression for voltage sensitivity. To formulate this, it is assumed that the voltage variation at bus i (the target bus) can be used to describe the variation in real power at bus j (the controlled bus).

A simple method to determine the voltage sensitivity is to use partial differentiation. However, in Equation (13), the bus voltage is expressed as a function of the bus voltage at all buses in the DC distribution system, which is too complex for a partial differentiation approach. To address this, some assumptions are made and linearization is employed.

If the real power at bus j were varied slightly, only small changes would be expected to occur in the bus voltages (except for the slack bus). Therefore, the deviation in the bus voltage and real power can be linearized because the voltage sensitivity deals with small variations in these parameters, allowing Equation (13) to be re-written as:

$$V_i+\Delta V_i=V_{slack}+R_{ij}\frac{P_j+\Delta P_j}{V_j+\Delta V_j}+\sum_{k=1,k\neq j}^{n}R_{ik}\frac{P_k}{V_k+\Delta V_k} \tag{16}$$

where i is the index of the target bus for which voltage sensitivity is investigated and j is the index of the bus for which the real bus power is varied. The voltage deviation at the slack bus is neglected because it is assumed that voltage at the slack bus is controlled continuously by GCC. To relate the deviation of the electrical quantities, the voltage deviation at bus i can be described by subtracting Equation (13) from Equation (16); *i.e.*,

$$\Delta V_i=R_{ij}\frac{V_j\Delta P_j-P_j\Delta V_j}{V_j^2+V_j\Delta V_j}-\sum_{k=1,k\neq j}^{n}R_{ik}\frac{P_k\Delta V_k}{V_k^2+V_k\Delta V_k} \tag{17}$$

For stable operation of a DC distribution system, the voltage should be maintained within an appropriate range, which is typically near to the rated voltage. IEEE Standard 1709™-2010 uses the voltage tolerance standard in IEC 60092-101, and specifies steady-state (*i.e.*, continuous) DC voltage tolerance limits of 10% [26]. Therefore, voltage deviations due to small variations in the real bus power can be expected to be smaller than the bus voltage, and Equation (17) can be approximated to Equation (19) via the assumption shown in Equation (18).

$$V_k^2+V_k\Delta V_k=V_k(V_k+\Delta V_k)\cong V_k^2 \tag{18}$$

$$\Delta V_i \cong R_{ij} \frac{V_j \Delta P_j - P_j \Delta V_j}{V_j^2} - \sum_{k=1,k\neq j}^{n} R_{ik} \frac{P_k \Delta V_k}{V_k^2} \tag{19}$$

In this case, if the voltage becomes significantly smaller than the rated voltage, voltage deviation is increased when the real bus power varies because the current flow and voltage drop over the conductors significantly increase. In this case the assumptions in Equation (18) may not hold.

Using Equation (19), the relationship between ΔV_i and ΔP_j can be expressed as follows:

$$\left(1 + \frac{R_{ii} P_i}{V_i^2}\right) \Delta V_i \cong R_{ij} \frac{\Delta P_j}{V_j} - \sum_{k=1,k\neq i}^{n} R_{ik} \frac{P_k \Delta V_k}{V_k^2} \tag{20}$$

and the voltage deviation at bus i in response to deviations in the real power at bus j (which is equivalent to the voltage sensitivity) is given by

$$\frac{\Delta V_i}{\Delta P_j} \cong \frac{V_i^2}{V_i^2 + R_{ii} P_i} \left(\frac{R_{ij}}{V_j} - \sum_{k=1,k\neq i}^{n} \frac{R_{ik} P_k}{V_k^2} \frac{\Delta V_k}{\Delta P_j} \right) \tag{21}$$

where the voltage sensitivity at bus i depends on the voltage sensitivity of the other buses. For this reason, it is not straightforward to describe the voltage sensitivity analytically. To address this problem, the following recursive substitution is carried out:

$$\begin{aligned} \frac{\Delta V_i}{\Delta P_j} \cong \frac{V_i^2}{\left(V_i^2 + R_{ii} P_i\right) V_j} (R_{ij} &- \sum_{k=1,k\neq i}^{n} \frac{R_{ik} P_k}{V_k^2 + R_{kk} P_k} R_{kj} \\ &+ \sum_{k=1,k\neq i}^{n} \frac{R_{ik} P_k}{V_k^2 + R_{kk} P_k} \sum_{k'=1,k'\neq k}^{n} \frac{R_{ik'} P_{k'}}{V_{k'}^2 + R_{k'k'} P_{k'}} R_{k'j} \\ &- \sum_{k=1,k\neq i}^{n} \frac{R_{ik} P_k}{V_k^2 + R_{kk} P_k} \sum_{k'=1,k'\neq k}^{n} \frac{R_{ik'} P_{k'}}{V_{k'}^2 + R_{k'k'} P_{k'}} \sum_{k''=1,k''\neq k'}^{n} \frac{R_{ik''} P_{k''}}{V_{k''}^2 + R_{k''k''} P_{k''}} R_{k''j} + \cdots) \end{aligned} \tag{22}$$

$$V_k^2 + R_{kk} P_k = V_k (V_k + R_{kk} \frac{P_k}{V_k}) \tag{23}$$

$$V_{Drop,k} = R_{kk} \frac{P_k}{V_k} \tag{24}$$

$$R_{kk} \geq R_{kj} \tag{25}$$

The left-hand term in Equation (23), which is the denominator of each term in Equation (22), is equivalent to the right-hand term in Equation (23). The term in Equation (24) describes the voltage drop between the slack bus and bus k, which is induced by the real power at bus k'. This voltage drop is generally smaller than the bus voltage because the bus voltage should be maintained within the voltage tolerance limits. It follows that Equation (24) is significantly smaller than the bus voltage at bus k, and therefore Equation (24) can be neglected. Furthermore, the relationship in Equation (25) is established because of the definition of the R-Bus matrix, which leads to the following expression:

$$0 < \left| \frac{R_{ik} P_k}{V_k^2 + R_{kk} P_k} \right| \leq \left| \frac{R_{kk} P_k}{V_k^2 + R_{kk} P_k} \right| << 1 \tag{26}$$

where the backward term (except the first two terms in Equation (22)) is neglected because the first two terms in the right-hand side of Equation (22) dominate. These considerations lead to the following approximate expression for the relationship of the voltage sensitivity at bus i to the real power at bus j:

$$\frac{\Delta V_i}{\Delta P_j} \cong \frac{V_i^2}{\left(V_i^2 + R_{ii}P_i\right)V_j}\left(R_{ij} - \sum_{k=1,k\neq i}^{n} \frac{R_{ik}R_{kj}P_k}{V_k^2 + R_{kk}P_k}\right) \tag{27}$$

The proposed formulation has variables that include the bus voltage at all buses, the real power at all buses except bus j, and the elements of the R-Bus matrix, which are a combination of line resistances. The expression is somewhat complex, which makes it difficult to analyze clearly how each variable affects the voltage sensitivity. The details of the analysis will be described later.

3.3. Approximate Expression for Voltage Sensitivity with Respect to the Slack Bus Voltage

In Figure 1, the slack bus is the secondary terminal of the AC/DC or DC/DC converter. The output voltage of the converter (*i.e.*, the slack bus voltage) can be controlled by the GCC depending on the control strategy of the distribution network operator (DNO) [22]. In this case, the slack bus voltage is considered as a controlled variable that reflects variations in the bus voltage in the DC distribution system, and an analysis of voltage sensitivity to slack bus voltage should be useful for establishing a voltage control strategy.

To formulate an approximate expression for voltage sensitivity to slack bus voltage, the voltage at bus i is investigated in response to changes in the slack bus voltage, V_{slack}. The bus voltage can be expressed in a linearized form, as discussed in the previous section, when there is a small deviation in the slack bus voltage. The linearized equation can be represented as follows:

$$V_i + \Delta V_i = V_{slack} + \Delta V_{slack} + \sum_{k=1}^{n} R_{ik} \frac{P_k}{V_k + \Delta V_k} \tag{28}$$

To find the relationship between deviations in V_{slack} and bus voltages, the voltage deviation at bus i can be found by subtracting Equation (13) from Equation (28); *i.e.*,

$$\Delta V_i = \Delta V_{slack} - \sum_{k=1}^{n} R_{ik} \frac{P_k \Delta V_k}{V_k^2 + V_k \Delta V_k} \tag{29}$$

This expression can be approximated as follows using the assumption in Equation (18):

$$\Delta V_i \cong \Delta V_{slack} - \sum_{k=1}^{n} R_{ik} \frac{P_k \Delta V_k}{V_k^2} \tag{30}$$

The following relationship relates V_{slack} to the voltage at other buses:

$$\left(1 + R_{ii}\frac{P_i}{V_i^2}\right)\Delta V_i \cong \Delta V_{slack} - \sum_{k=1,k\neq i}^{n} R_{ik} \frac{P_k \Delta V_k}{V_k^2} \tag{31}$$

and the voltage deviation at bus i in response to changes in V_{slack} is given by

$$\frac{\Delta V_i}{\Delta V_{slack}} \cong \frac{V_i^2}{V_i^2 + R_{ii}P_i}\left(1 - \sum_{k=1,k\neq i}^{n} R_{ik} \frac{P_k}{V_k^2}\frac{\Delta V_k}{\Delta V_{slack}}\right) \tag{32}$$

This expression is dependent on the voltage sensitivity of other buses to V_{slack}, which allows us to develop an approximate expression for voltage sensitivity as a function of the injected power at the buses. The following expression relates bus voltages to V_{slack}, and is arrived at via recursive substitution:

$$\frac{\Delta V_i}{\Delta V_{slack}} \cong \frac{V_i^2}{V_i^2 + R_{ii}P_i}\left[1 - \sum_{k=1,k\neq i}^{n} R_{ik}\frac{P_k}{V_k^2}\left\{\frac{V_k^2}{V_k^2 + R_{kk}P_k}\left(1 - \sum_{k'=1,k'\neq k}^{n} R_{ik'}\frac{P_{k'}}{V_{k'}^2}\frac{\Delta V_{k'}}{\Delta V_{slack}}\right)\right\}\right] \quad (33)$$

and

$$\frac{\Delta V_i}{\Delta V_{slack}} \cong \frac{V_i^2}{V_i^2 + R_{ii}P_i}\left[1 - \sum_{k=1,k\neq i}^{n}\frac{R_{ik}P_k}{V_k^2 + R_{kk}P_k} + \sum_{k=1,k\neq i}^{n}\frac{R_{ik}P_k}{V_k^2 + R_{kk}P_k}\sum_{k'=1,k'\neq k}^{n} R_{ik'}\frac{R_{ik'}P_{k'}}{V_{k'}^2 + R_{k'k'}P_{k'}} - \cdots\right] \quad (34)$$

The above expression has a similar form to the voltage sensitivity equation in response to changes in the injected power at buses given in Equations (22) and (26), and was established using the assumptions in Equations (23) to (25). Therefore, the backward terms on the right-hand side of Equation (34) can be neglected because the earlier terms dominate. These considerations lead to the following approximate expression for voltage sensitivity in response to variations in slack bus voltage:

$$\frac{\Delta V_i}{\Delta V_{slack}} \cong \frac{V_i^2}{V_i^2 + R_{ii}P_i}\left(1 - \sum_{k=1,k\neq i}^{n}\frac{R_{ik}P_k}{V_k^2 + R_{kk}P_k}\right) \quad (35)$$

4. Verification of the Approximate Voltage Sensitivity Equation

To verify the proposed approximate expression for voltage sensitivity, case studies are carried out to compare voltage sensitivities derived using the proposed equation with values derived using exact calculations. Figure 2 shows the 13-bus radial medium-voltage DC (MVDC) distribution test system. The base voltage was 2 kV and the base power was 1 MW; the system was constructed for case studies based on the IEEE 13-node test feeder system. Bus 1 was the slack bus, and was connected to the secondary terminal of the AC/DC converter. The converter was connected to the high-voltage power system, and its secondary terminal voltage could be controlled. Table 1 lists the constant-power loads. The total capacity was 1.38 MW. Table 2 lists the resistance and maximum currents of the branches. The line conductor parameter was determined by considering the voltage tolerance limits and line overloading when the bus real power was varied.

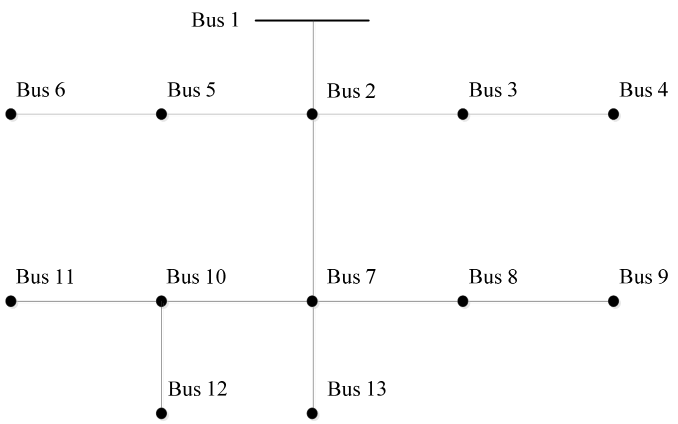


Figure 2. Diagram of a radial DC distribution system.

Table 1. Load data for the test system.

Bus No.	Bus Type	Load Capacity (kW)
1	Slack	0
2	Load	0
3	Load	200
4	Load	150
5	Load	80
6	Load	120
7	Load	200
8	Load	90
9	Load	300
10	Load	0
11	Load	80
12	Load	60
13	Load	100

Table 2. Branch data on the test system.

From Bus	To Bus	Resistance (Ω)	Maximum Current (A)
1	2	0.02940	1318
2	3	0.03460	760
2	5	0.03951	655
2	7	0.01358	1091
3	4	0.02195	655
5	6	0.06120	726
7	8	0.01730	760
7	10	0.03512	655
7	13	0.04390	655
8	9	0.01730	760
10	11	0.03073	655
10	12	0.03951	655

The voltage sensitivity was investigated as a function of the injected power at buses and the slack bus voltage. The approximate voltage sensitivity was calculated using Equations (27) and (35). The exact voltage sensitivity as a function of the injected power at the buses was obtained from the inverse of the Jacobian matrix in Equation (8), and the voltage sensitivity in response to changes in the slack bus voltage was calculated from the deviation of bus voltages and the slack bus voltage from a set of power flow solutions.

4.1. Verification of the Approximate Expression of Voltage Sensitivity with Regard to Bus Injected Power

The injected power at each bus was varied from −1 MW to 1 MW. Four cases were considered, where the slack bus voltage was 0.6, 0.8, 1.0 and 1.2 pu; however, the bus voltage tolerance limit may be exceeded in some cases. Figure 3 shows the maximum error of the approximate voltage sensitivity equation for each variation in the injected power at each bus. The controlled bus was the bus for which the injected power was varied.

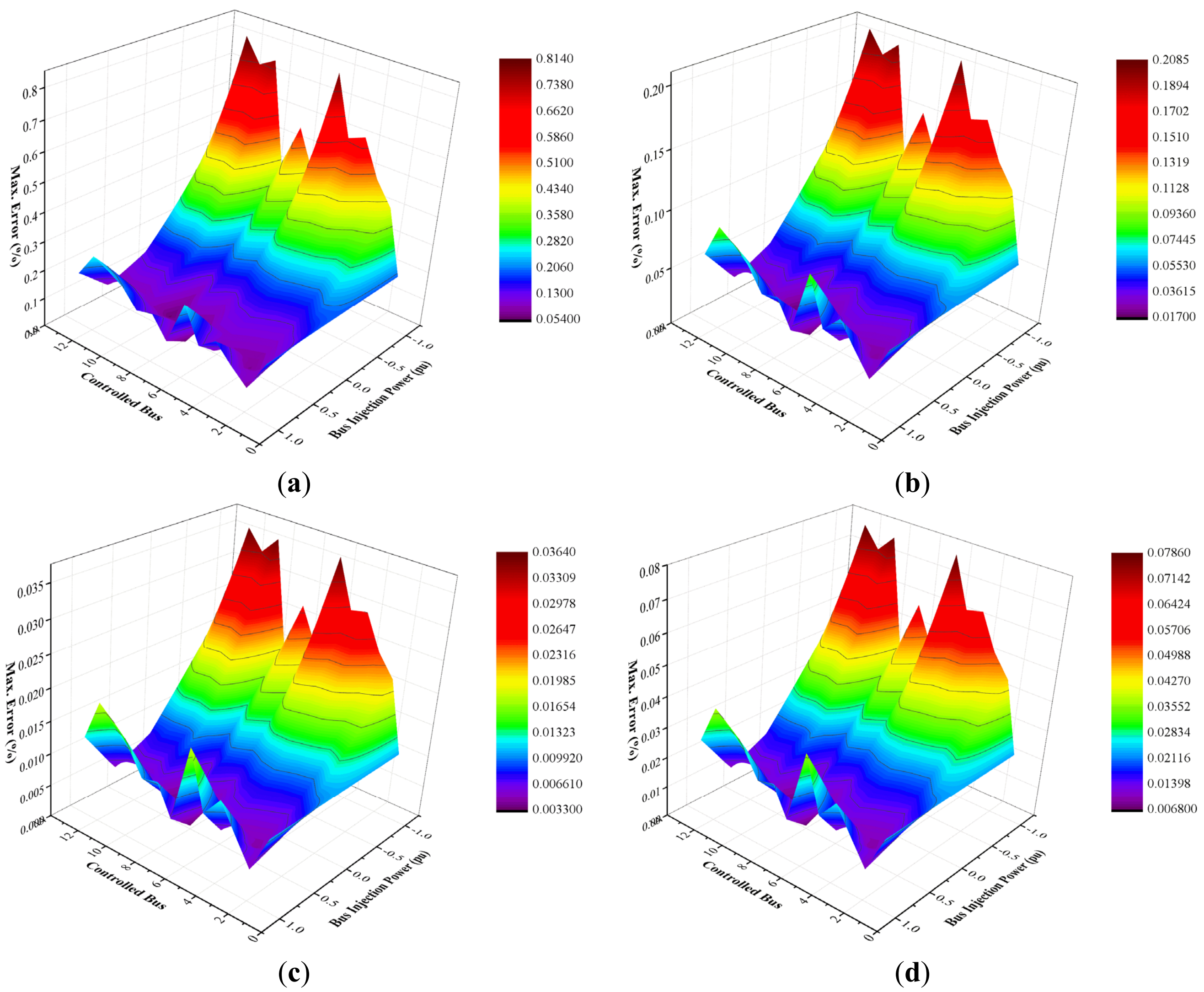


Figure 3. The maximum error of the approximate expression for voltage sensitivity as a function of the injected power at the bus. (**a**) V_{slack} = 0.6 pu; (**b**) V_{slack} = 0.8 pu; (**c**) V_{slack} = 1.0 pu; (**d**) V_{slack} = 1.2 pu.

The maximum error of the approximate expression for voltage sensitivity as a function of the injected power at the buses did not exceed 0.9% in any scenario, which indicates that the expression has good accuracy. The error increased as the negative injected power at the bus increased. This is because the voltage drop also increased due to power consumption. The later terms in Equation (22) were neglected because it was assumed that the voltage drop between line conductors was much smaller than the bus voltage. The effect of neglecting these terms becomes larger as the voltage drop increases, increasing the error. Similarly, the slack bus voltage affects the accuracy of the approximate expression. A lower slack bus voltage will lead to an increased load current, and hence an increase in the voltage drop over the conductors. Furthermore, the overall bus voltage will decrease as the slack bus voltage decreases. In addition, the ratio of the line voltage drop to the bus voltage increases, leading to an increase in the error due to the neglected terms. Consequently, the maximum error was small when slack bus voltage was maintained within the normal operating range, but increased as the slack bus voltage decreased.

4.2. Verification of the Approximate Expression of Voltage Sensitivity with Regard to the Slack Bus Voltage

The approximate expression was used to calculate the voltage sensitivity for slack bus voltages in the range 0.6–1.4 pu. The injected power was varied from −1 MW–1 MW at each bus. Figure 4 shows the maximum error in the approximate expression for voltage sensitivity as a function of V_{slack} when the injected power was varied at buses 2, 7, 9 and 12.

The maximum error did not exceed 0.7%, and was approximately 0.05% when the bus voltage was maintained within the limits. The approximate expression for voltage sensitivity as a function of the slack bus voltage therefore has good accuracy. The error increased as the slack bus voltage decreased, and the negative injected power at the bus increased because the line voltage drop became larger than the limits of normal operating conditions. Therefore, the overall bus voltage also decreased, which is consistent with the assumptions in Equation (34).

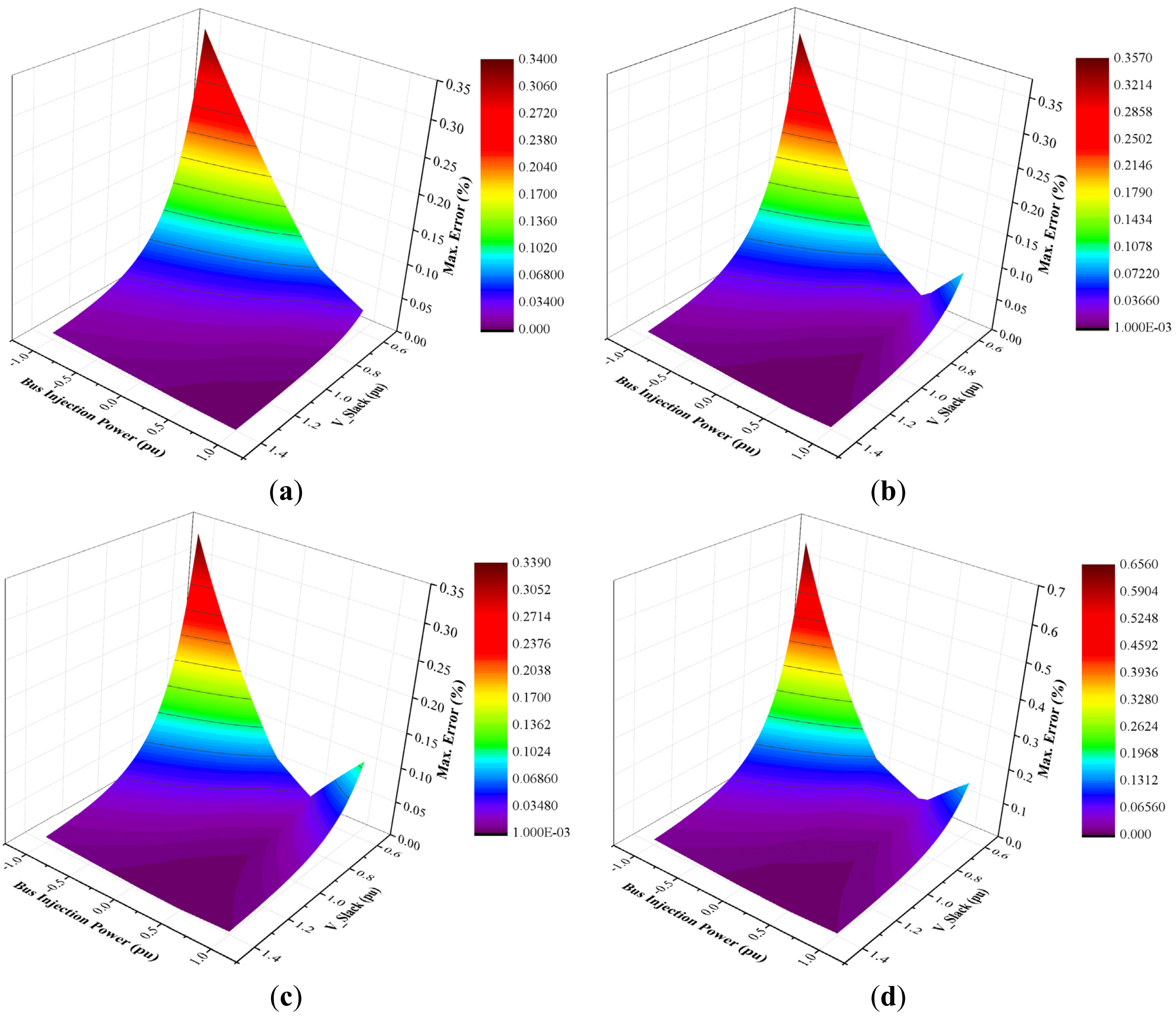


Figure 4. The maximum error in the approximate expression for voltage sensitivity as a function of V_{slack}. (**a**) Injected power varied at bus 2; (**b**) Injected power varied at bus 7; (**c**) Injected power varied at bus 9; (**d**) Injected power varied at bus 12.

5. Analysis of Voltage Sensitivity

In this section, the relationships between the voltage sensitivity and various electrical quantities are analyzed using the approximate voltage sensitivity equations. The approximate equations are separated into several terms, which is helpful for examining how each variable affects the voltage sensitivity.

5.1. Analysis of the Approximate Expression of Voltage Sensitivity with Regard to Bus Injected Power

To analyze how the variables affect the voltage sensitivity, the expression in Equation (27) is separated into the following four terms:

$$k_1 = \frac{V_i^2}{\left(V_i^2 + R_{ii} P_i\right)} \tag{36}$$

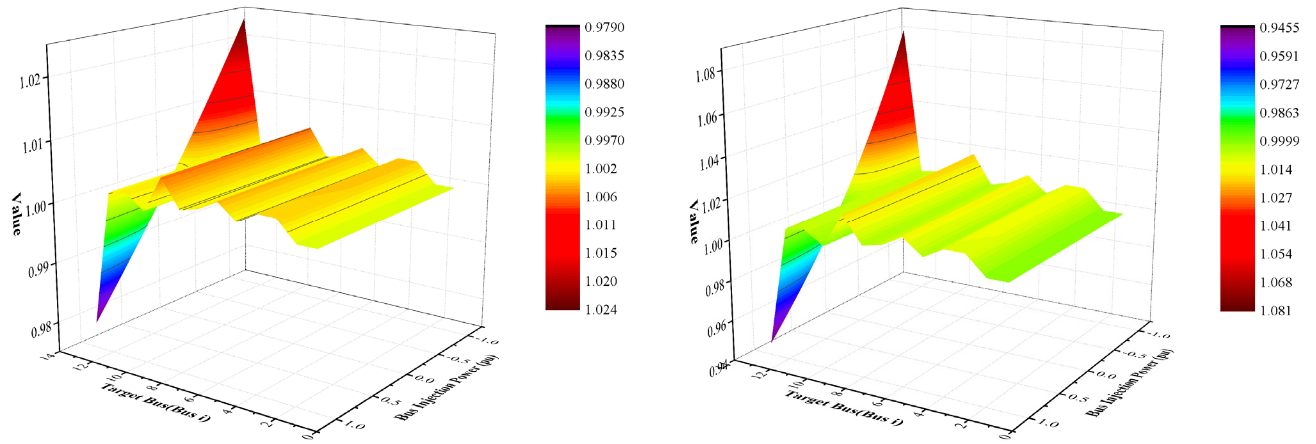
$$k_2 = \frac{1}{V_j} \tag{37}$$

$$k_3 = \sum_{k=1, k \neq j}^{n} \frac{R_{ik} R_{kj} P_k}{V_k^2 + R_{kk} P_k} \tag{38}$$

and

$$k_4 = R_{ij} \tag{39}$$

The denominator and the numerator in Equation (36) have similar values because of the assumption in Equation (24). In Equation (36), the backward term in the denominator is equivalent to the voltage difference between the slack bus and bus *i*, which is induced by the bus real power at bus *i*, which should be significantly smaller than the bus voltage. Therefore, this term should be approximately 1.0, as long as the DC voltage tolerance is small; however, this assumption may not hold for large injected powers at buses or very low bus voltages. Figure 5 shows the result of Equation (36) when the injected power was varied at bus 13. When V_{slack} = 1.0 pu, the result was in the range 1–1.0062 (except at bus 13) and then increased as the negative injected power increased because of the increase in the negative injected power at bus 13 in response to a decrease in the voltage at other buses. The result of Equation (36) at bus 13 was in the range 0.9791–1.0239, which is a larger deviation from 1.0 than at any of the other buses because of injected power variation at bus 13. Similar trends were found with V_{slack} = 0.6 pu. In this case, the result of Equation (36) was in the range 0.9457–1.0810. The larger deviation from 1.0 was investigated because the overall bus voltage decreased as the difference between line voltages increased.



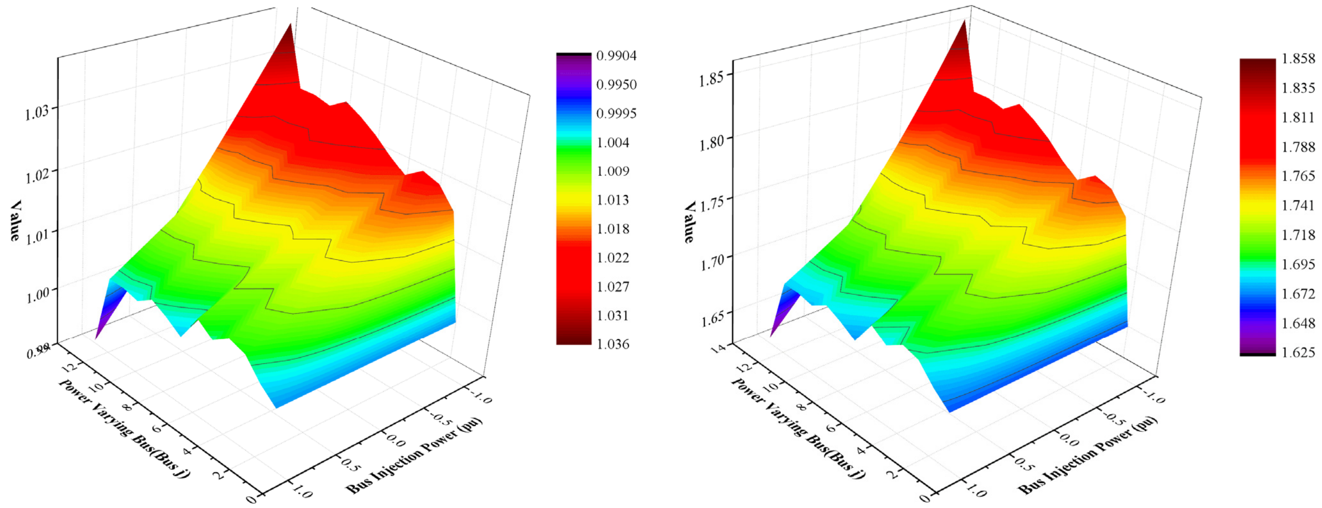
(a) **(b)**

Figure 5. The results of Equation (36) as a function of the injected power at the bus. (**a**) V_{slack} = 1.0 pu; (**b**) V_{slack} = 0.6 pu.

The expression in Equation (37) is the inverse of the voltage at bus *j* (the output bus). If the voltage is within the tolerance limits (for example, within 10%), this term should be in the range 0.9091–1.1111. In the case study, it was in the range 0.9904–1.0360 for V_{slack} = 1.0 pu; however, it increased markedly as the bus voltage decreased, and was in the range 1.625–1.858 for V_{slack} = 0.6 pu, as shown in Figure 6b. In addition, the result of Equation (37) increases with increasing negative injected power, because this induces a decrease in the bus voltage. In this case, the voltage sensitivity to changes in the injected power at bus 13 exceeded that at the other buses because the injected power was varied at bus 13. For this reason, the variation in the injected power at bus 13 was larger than that at the other buses. This will be discussed further below.

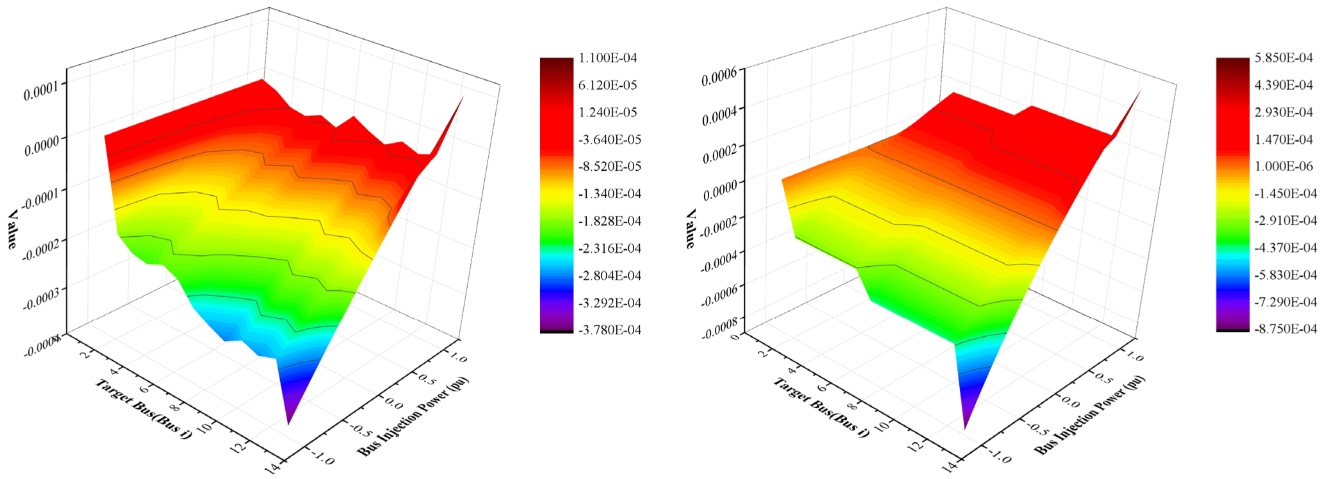
Figure 7 shows the results of Equation (38) and Figure 8 shows the result of Equation (39). The former was much smaller than the latter. This follows because the relationship in Equation (26) must be satisfied, and therefore the term in Equation (38) may be neglected if a simple equation is required; however, an error of voltage sensitivity may be increased. The term in Equation (39) is identical to the element of the R-Bus matrix between buses *i* and *j*, and thus is constant regardless of the injected power at the bus and the slack bus voltage.

Consequently, the results of Equation (38) is smaller than the result of Equation (39), and so the latter is the dominant term for the voltage sensitivity. The terms in Equations (36) and (37) are in effect coefficients of the voltage sensitivities and have values of approximately 1.0, provided that the bus voltage is maintained within the tolerance limits. An increase in the negative injected power makes these terms larger because it decreases the overall voltage. In addition, a decrease in the slack bus voltage also makes the overall bus voltage smaller, and hence increases the value of these terms.



(**a**) (**b**)

Figure 6. The results of Equation (37) as a function of the injected power at the bus. (**a**) V_{slack} = 1.0 pu; (**b**) V_{slack} = 0.6 pu.



(**a**) (**b**)

Figure 7. The results of Equation (38) as a function of the injected power at the bus. (**a**) V_{slack} = 1.0 pu; (**b**) V_{slack} = 0.6 pu.

Figure 9 shows the results for the exact calculated voltage sensitivity as a function of the injected power when the injected power was varied at bus 13. As shown in Figure 9a,b, the voltage sensitivity was largest at buses 9 and 12 because the term in Equation (39) dominates the voltage sensitivity, and this term was largest at buses 9 and 12, respectively. As shown in Figure 9c,d, the forms of the voltage sensitivity curves were similar to those in Figure 9a,b, because the element of the R-Bus matrix influences the form of the voltage sensitivity curves. Therefore, the magnitudes of the voltage sensitivities were similar to the magnitudes of the R-Bus matrix elements. The increase in the injected power at bus 13 results in a small increase in the term in Equation (36), as well as an increase in the voltage sensitivity. Overall, a low voltage results in increased voltage sensitivity, and the voltage sensitivity experiences larger changes due to variations in the injected power at the bus because a low bus voltage makes the terms in Equations (36) and (37) more significant.

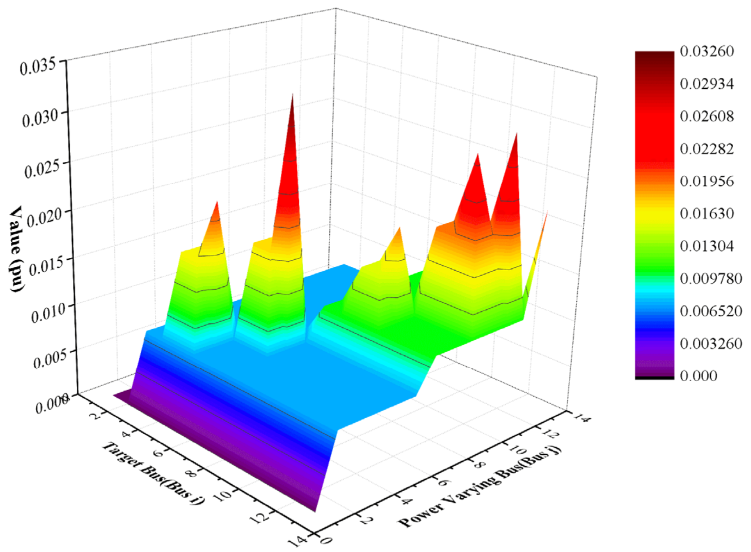


Figure 8. The results of Equation (39) as a function of the injected power at the bus.

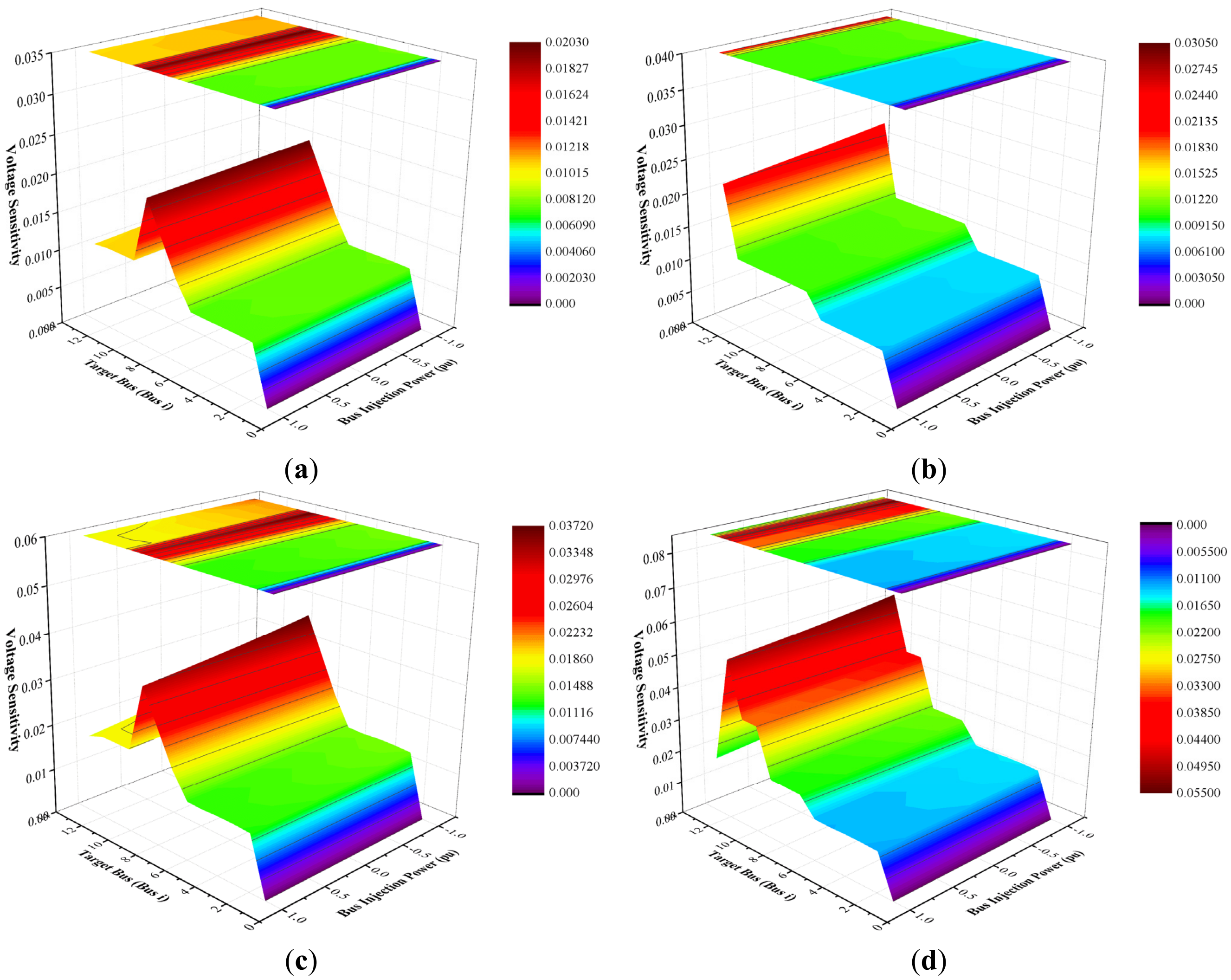


Figure 9. The results of the exact calculation of the voltage sensitivity in response to changes in the bus injected power. (**a**) j is bus 9 and V_{slack} = 1.0 pu; (**b**) j is bus 12 and V_{slack} = 1.0 pu; (**c**) j is bus 9 and V_{slack} = 0.6 pu; (**d**) j is bus 12 and V_{slack} = 0.6 pu.

5.2. Analysis of the Approximate Expression for Voltage Sensitivity to Slack Bus Voltage

The approximate expression for voltage sensitivity to changes in slack bus voltage is composed of the following three terms:

$$k_5 = \frac{V_i^2}{\left(V_i^2 + R_{ii}P_i\right)} \tag{40}$$

$$k_6 = 1 \tag{41}$$

$$k_7 = \sum_{k=1,k\neq j}^{n} \frac{R_{ik}P_k}{V_k^2 + R_{kk}P_k} \tag{42}$$

As shown in Figure 10, an increase in the negative injected power at the bus and a decrease in the slack bus voltage make the term in Equation (40) larger. The term in Equation (40) is close to 1.0 under most conditions, except when the slack bus voltage is very low and the negative injected power at the bus is very large. The magnitude of the term in Equation (42), is much smaller than 1.0, because of the inequality in Equation. Figure 11 shows the result of the term in Equation (42), which was much smaller than 1.0 at all operating points. Consequently, the approximate expression for the voltage sensitivity is equivalent to the term in Equation (40), which is equivalent to term Equation (36) if the term in Equation (42) is neglected and the term in Equation (40) dominates. The voltage sensitivity to the slack bus voltage is close to 1.0 because the term in Equation (40) is close to 1.0.

Figure 12 shows the results of the exact calculation of voltage sensitivity in response to changes in the slack bus voltage. The magnitude was close to 1.0 for all cases, as above. Both the form and the magnitude of the voltage sensitivity curve were similar to that given by the term in Equation (40); however, the form of the exact calculation was smoother than the result of the term in Equation (40) due to the effects of the term in Equation (42). Figure 12a,c,d,f show the effects of injected power at the bus. In these cases, the injected power was larger at buses 7 and 13 than at the other buses, which strongly affected the sensitivity. A large positive power injected at buses 13 and 7 with these negative powers at the other buses resulted in voltage sensitivity at these buses of less than 1.0 (see Figure 12c,f).

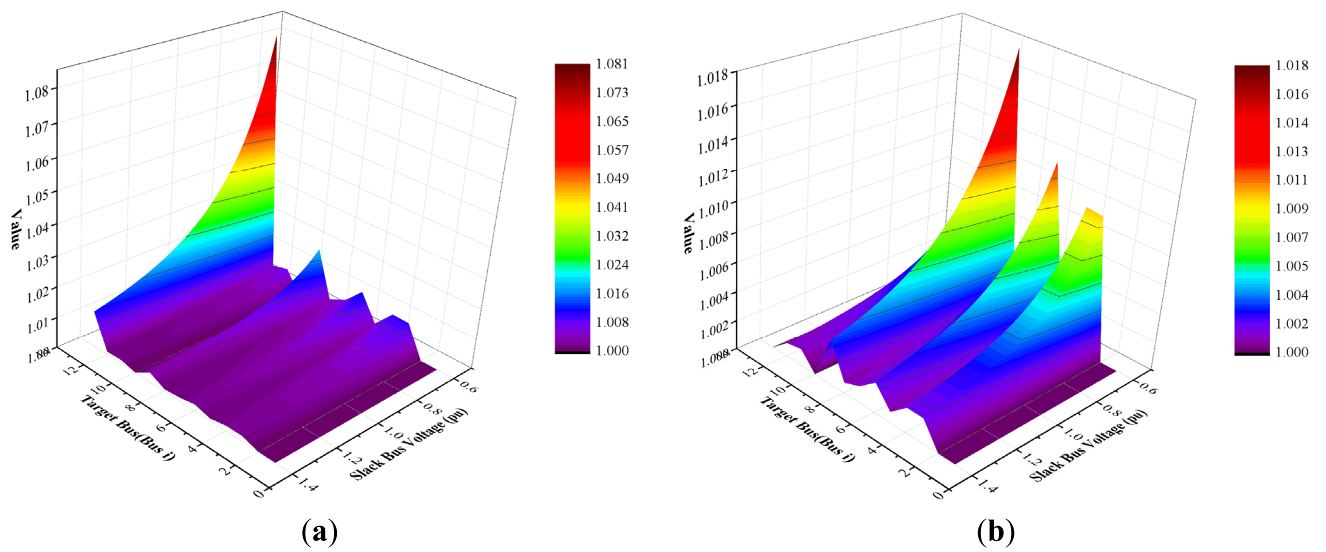
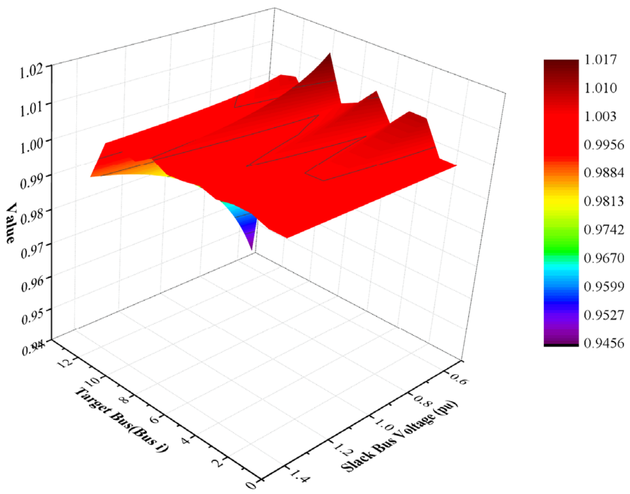
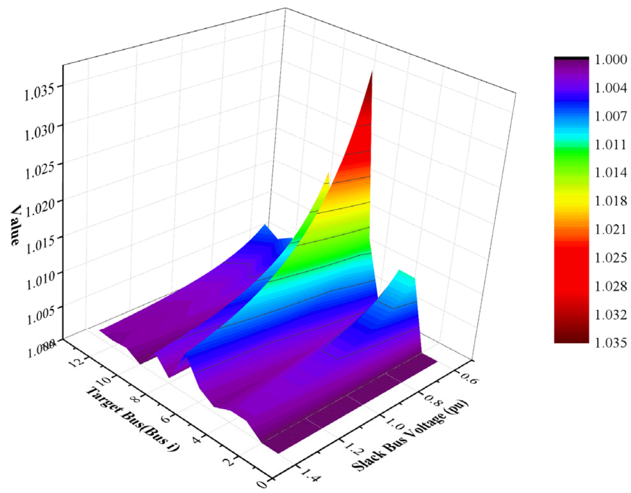


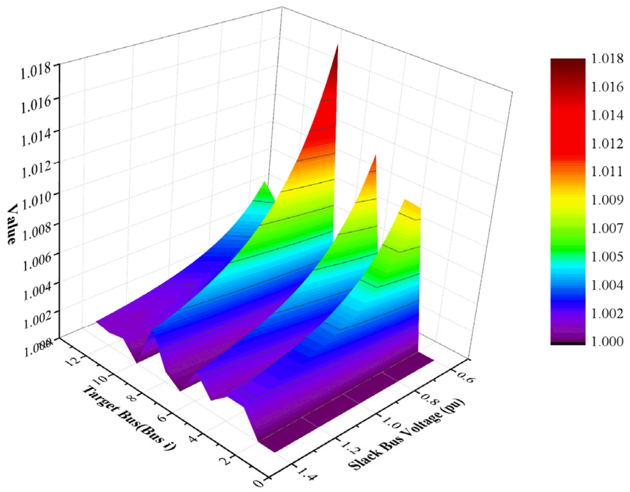
Figure 10. *Cont.*



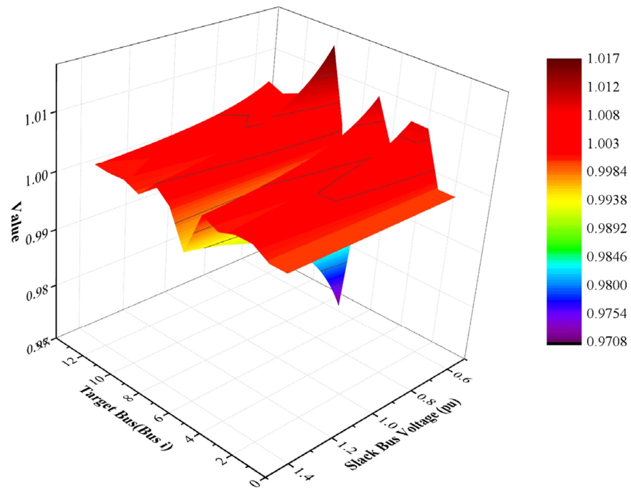
(c)



(d)

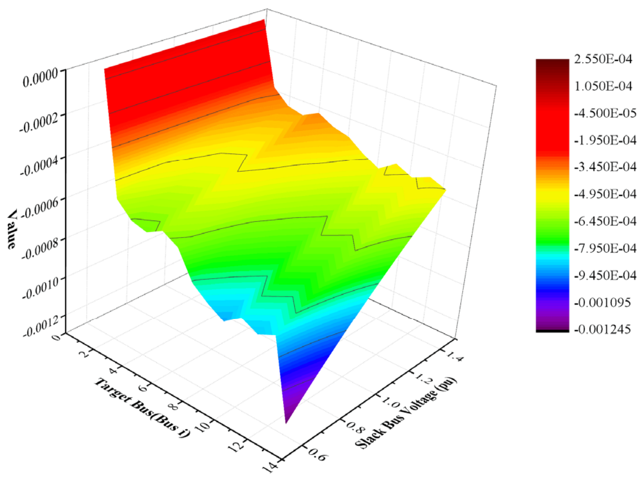


(e)

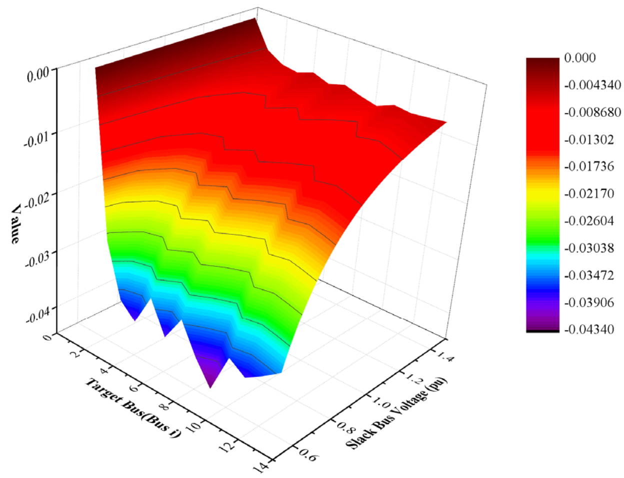


(f)

Figure 10. The results of Equation (40). (**a**) The injected power at bus 13 is −1 MW; (**b**) The injected power at bus 13 is 0 MW; (**c**) The injected power at bus 13 is 1 MW; (**d**) The injected power at bus 7 is −1 MW; (**e**) The injected power at bus 7 is 0 MW; (**f**) The injected power at bus 7 is 1 MW.



(a)



(b)

Figure 11. *Cont.*

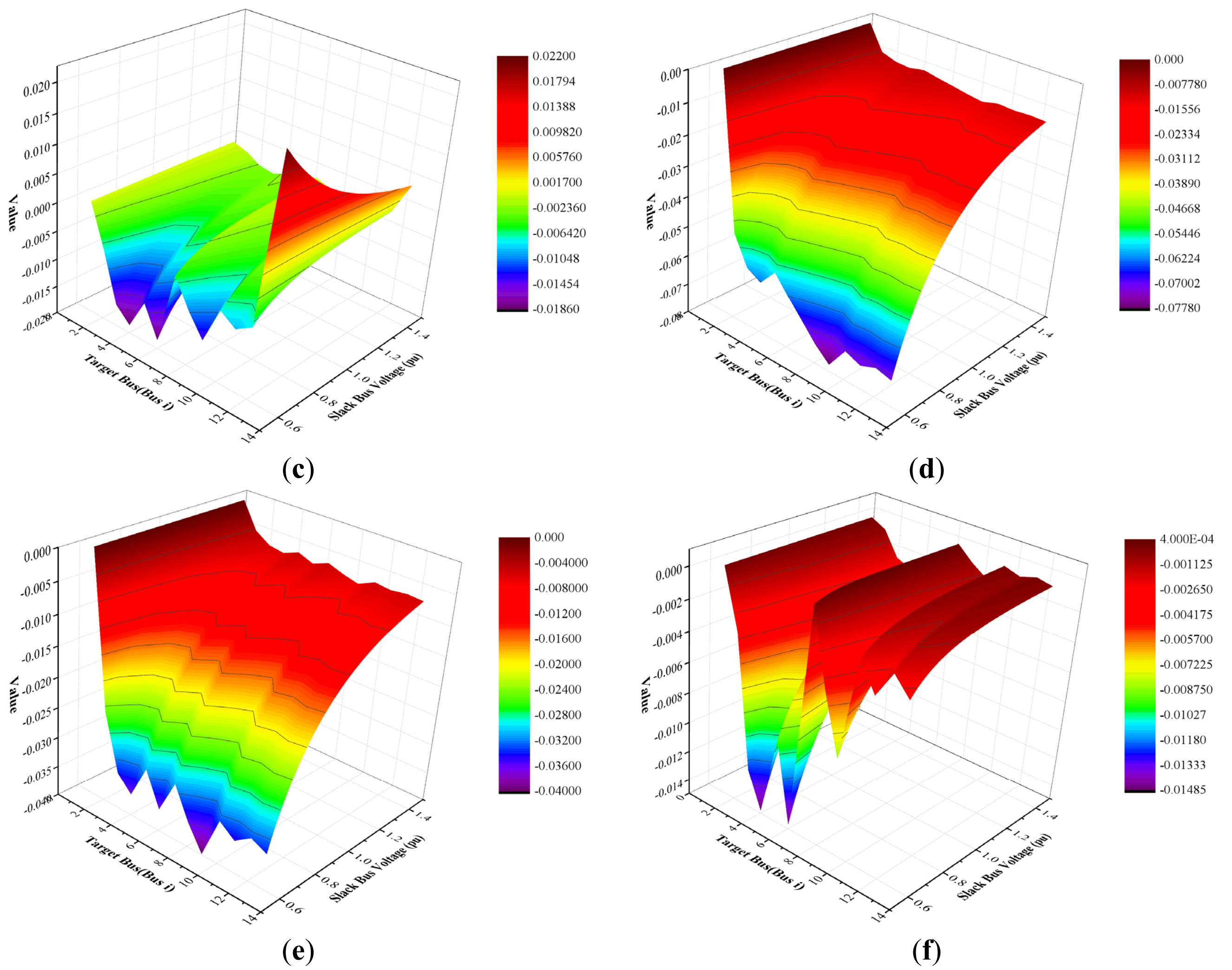


Figure 11. The results of Equation (42). (**a**) The injected power at bus 13 is −1 MW; (**b**) The injected power at bus 13 is 0 MW; (**c**) The injected power at bus 13 is 1 MW; (**d**) The injected power at bus 7 is −1 MW; (**e**) The injected power at bus 7 is 0 MW; (**f**) The injected power at bus 7 is 1 MW.

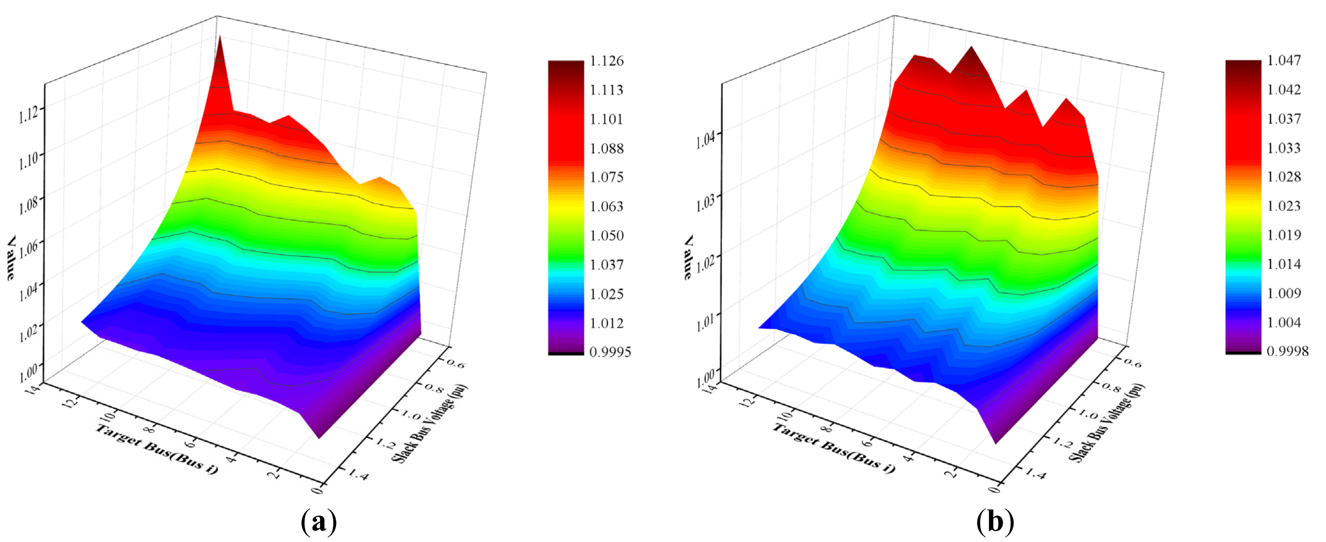


Figure 12. *Cont.*

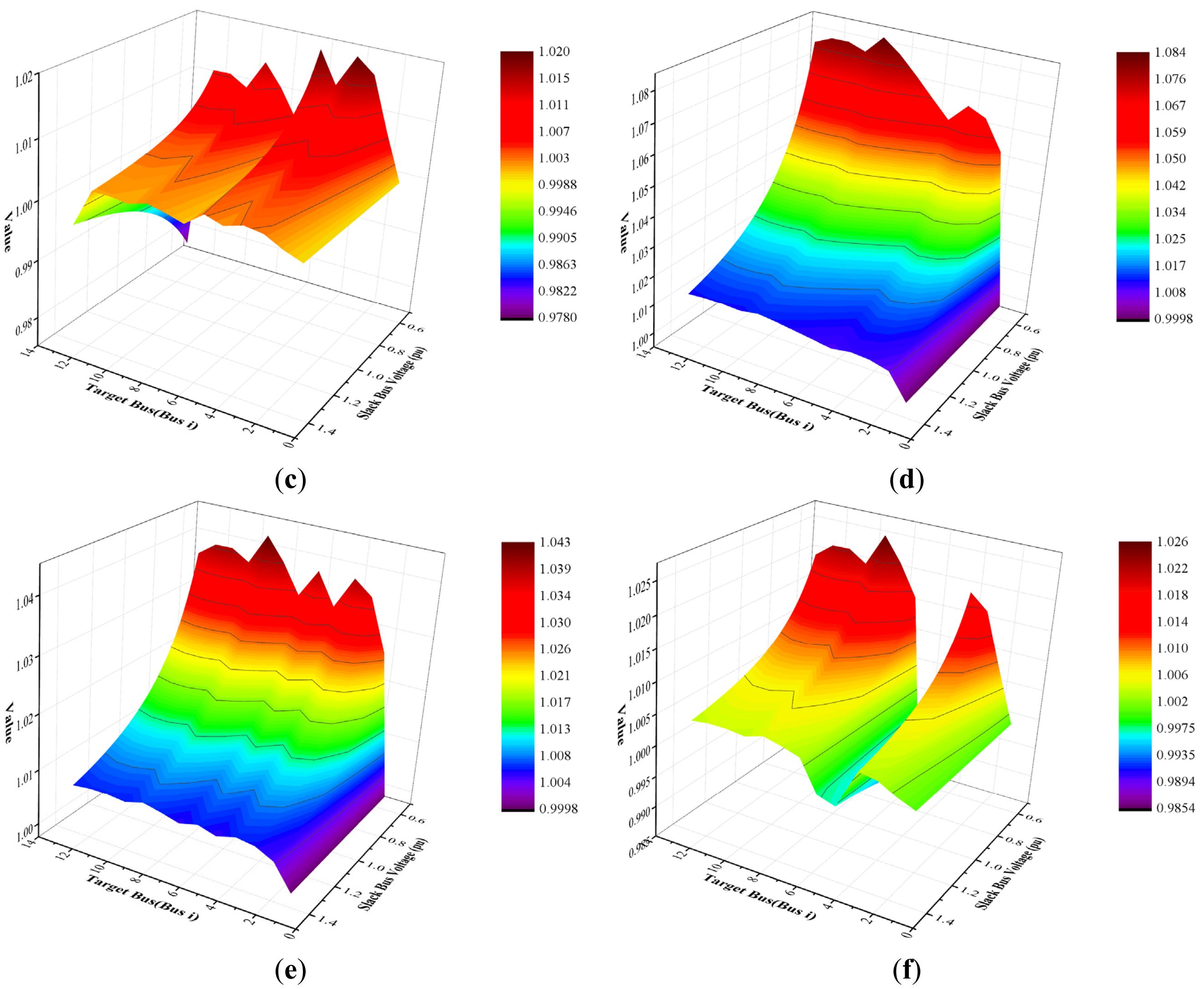


Figure 12. The results of the exact calculation of the voltage sensitivity in response to changes in the slack bus voltage. (**a**) The injected power at bus 13 is −1 MW; (**b**) The injected power at bus 13 is 0 MW; (**c**) The injected power at bus 13 is 1 MW; (**d**) The injected power at bus 7 is −1 MW; (**e**) The injected power at bus 7 is 0 MW; (**f**) The injected power at bus 7 is 1 MW.

6. Conclusions

This paper described the formulation and analysis of an approximate expression for the voltage sensitivity of a radial DC distribution system. Conventional methods to calculate voltage sensitivity were discussed, and an approximate expression for voltage sensitivity in response to changes in the injected power at the bus and the slack bus voltage was formulated. The results of this expression were investigated using a 13-bus radial MVDC system, and were compared with the results of an exact calculation. It is found that the approximate expression was in good overall agreement with the results of the exact calculation. This paper analyzed the way in which each variable affected the voltage sensitivity. The proposed formulation can be used for simple calculation of voltage sensitivity without power flow calculation in radial DC distribution systems. Moreover, these results are helpful in elucidating radial DC distribution systems, and can be used for controller design or to establish operating strategies.

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Author Contributions

Ho-Yong Jeong proposed method to formulate equations and analyzed proposed formulations. Jong-Chan Choi provided technical support to formulate equations and contributed to establish idea. Dong-Jun Won, Seon-Ju Ahn, and Seung-il Moon validated and double-checked the proposed formulation, the results, and the whole manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

Abbreviation

g_{ij}	Conductance of the line between bus *i* and bus *j*
g_{ii}	Conductance of a constant resistive load at bus *i*
n	Number of buses in the DC distribution system
$P_{L,i}$	The load power of constant power load at bus *i*
$[J]$	Jacobian matrix for the DC power system
P_i	Net bus real power (*i.e.*, generation minus the load consumption)
R_{ik}	Element of an *R*-Bus matrix at *i*-th row and *k*-th column
$\{R_i\}$	Set of line resistances located on the shortest path between the slack bus and bus *i*
R_{ij}	The sum of elements at the intersection of $\{R_i\}$ and $\{R_j\}$

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