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Cost-Sharing Contracts for Energy Saving and Emissions Reduction of a Supply Chain under the Conditions of Government Subsidies and a Carbon Tax

Yuyin Yi * and Jinxi Li

School of Management, Jinan University, Guangzhou 510632, China; leejinxi@stu2015.jnu.edu.cn
* Correspondence: yiyuyin@jnu.edu.cn

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Abstract: To study the cooperation of upstream and downstream enterprises of a supply chain in energy saving and emissions reduction, we establish a Stackelberg game model. The retailer moves first to decide a cost-sharing contract, then the manufacturer determines the energy-saving level, carbon-emission level, and wholesale price successively. In the end, the retailer determines the retail price. As a regulation, the government provides subsidies for energy-saving products, while imposing a carbon tax on the carbon emitted. The results show that (1) both the energy-saving cost-sharing (ECS) and the carbon emissions reduction cost-sharing (CCS) contracts are not the dominant strategy of the two parties by which they can facilitate energy savings and emissions reductions; (2) compared with single cost-sharing contracts, the bivariate cost-sharing (BCS) contract for energy saving and emissions reduction is superior, although it still cannot realise prefect coordination of the supply chain; (3) government subsidy and carbon tax policies can promote the cooperation of both the upstream and downstream enterprises of the supply chain—a subsidy policy can always drive energy saving and emissions reductions, while a carbon tax policy does not always exert positive effects, as it depends on the initial level of pollution and the level of carbon tax; and (4) the subsidy policy reduces the coordination efficiency of the supply chain, while the influences of carbon tax policy upon the coordination efficiency relies on the initial carbon-emission level.

Keywords: supply chain management; cost-sharing contract; energy-saving level; carbon-emission level

1. Introduction

In recent years, problems such as energy shortages and environmental pollution have become increasingly prominent. In such a context, energy saving and emissions reduction, as important measures for sustainable development, are paid increasing attention by various countries [1,2]. In reality, governments of various countries have formulated energy-saving subsidy policies to encourage manufacturers to produce products that save more energy. For example, the Chinese government has supported the use of energy-saving products by ordinary households with subsidies since 2012 [3]. In the programme, the government provides subsidies for purchasing high-efficiency energy-saving products, including variable-frequency air conditioners, LCD TV sets, and energy-saving automobiles [4]. Northern European countries, such as Norway and Sweden, have implemented all kinds of tax preferences relating to energy-saving products [5]. On the other hand, manufacturers always emit carbon in the production process [6]. Therefore, to control carbon contamination in the production process, the governments of Sweden, Norway, Denmark, et al. levy a carbon tax on enterprises. Canada also actively implements a carbon tax policy [7]. Apart from the close attention from the government, consumers' environmental protection consciousness has also grown. People tend to purchase more energy-efficient, environment-friendly products, despite their high price [8,9]. Therefore, with close attention from government and the public, manufacturers have to enhance their energy-saving product research and development (R&D) and reduce carbon contamination in the production process.

As is well-known, energy saving and emissions reduction costs a lot, which apparently reduces manufacturers' motivation to act. As upstream and downstream enterprises in supply chains have closer relationships and interact more frequently, retailers, as the downstream partners, see benefits that are closely related to energy saving and emissions reduction among manufacturers. Therefore, retailers have the motivation to cooperate with manufacturers in order to save energy and reduce emissions. For instances, Wal-Mart launched its green supply chain plan (in 2008) to jointly abate pollution with upstream manufacturers. At the same time, Wal-Mart also increased its input to energy-saving products, to increase the energy efficiency of the majority of energy-intensive products by 25%, by cooperating with manufacturers [10,11]. However, the question is: what is the proper approach for a retailer to cooperate with manufacturers to improve the energy saving and emissions reduction? Cost-sharing contracts are a commonly used cooperative approach, and have been widely used in practical operations. For example, Alpha Labs (a small public U.S. biotech company) and Mega Pharmaceuticals (a large Fortune 500 pharmaceutical company) agreed to equally share the cost of drug development investment [12]. Besides, numerous studies have revealed that cost-sharing contracts can remit the cost pressure on manufacturers and improve profits along the supply chain [13,14]. However, these studies generally separate the energy-saving level of products and carbon emission reduction in production, and simply study the sharing of emissions reduction costs. In practice, these two problems are often intertwined. For example, when manufacturers produce household electrical appliances, including refrigerators and air conditioners, they generate carbon emissions and pollute the environment. Meanwhile, these products consume energy while being used [15]. In addition, European countries, such as Norway and The Netherlands, have provided subsidy policies for energy-saving products [5], and carbon taxes on carbon emissions arising from production [7]. Therefore, similar to the green supply chain plan of Wal-Mart, manufacturers and retailers not only have to consider how to cooperate in order to reduce carbon emissions during production, but also need to consider how to cooperate to save energy while their products are in service. This gives rise to a series of interesting academic questions:

(1) If the retailer cooperates with the manufacturer to save energy and reduce emissions using cost-sharing contracts, which contract should that retailer select: sharing the energy-saving R&D cost, carbon emissions reduction cost during production, or both? Which represents the optimal cost-sharing strategy?
(2) How do the above three cost-sharing strategies influence the energy-saving and emissions reduction strategies of the manufacturer? In particular, how does energy-saving cost-sharing on the part of the retailer impact the carbon emissions reduction strategy of the manufacturer? Can cost-sharing for carbon emissions reduction from the retailer improve the level of energy saving among manufacturers?
(3) What are the influences of government subsidies and carbon tax policies on cooperation, energy saving and emissions reduction, and the profits of node enterprises in the supply chain?

To answer the above questions, a two-echelon supply chain consisting of one manufacturer and one retailer is constructed. In the supply chain, the manufacturer produces certain energy-saving products (e.g., energy-saving air conditioners or refrigerators) during which it emits carbon. The government provides subsidies for energy-saving products, while imposing a carbon tax on the carbon emitted. The retailer decides whether it shares the manufacturer's energy-saving R&D cost and emission reduction cost or not, and sells the final products to consumers. The research results demonstrate that both the energy-saving cost-sharing (ECS) and the carbon emission-reduction cost-sharing (CCS) contracts are not the dominant strategy of the two parties, whereas they can facilitate energy savings and emissions reductions. Compared with single cost-sharing contracts, the bivariate cost-sharing (BCS) contract for energy-saving and emissions reduction is superior. Government subsidies and carbon tax policies improve the cost-sharing proportion. A subsidy policy can always drive energy saving and emissions reductions, while a carbon tax policy does not always exert positive effects, but depends on the initial level of pollution from the enterprises and the level of carbon tax. For clean manufacturers with less initial pollution, the increase in carbon tax can promote carbon emissions reduction and improve the energy-saving level of products. In comparison, a government that increases the carbon tax not only fails to reduce carbon emissions, but also inhibits energy-saving R&D into products made by polluting manufacturers with heavy initial pollution loads.

The remainder of the paper is organised as follows. Section 2 is the literature review. Section 3 describes the model and basic hypotheses. Section 4 summarises the centralised decision process and applies it as a benchmark. Section 5 investigates decentralised decisions and the influences of cost-sharing contracts on the manufacturer's energy-saving and emissions reduction decisions and the profit of the supply chain. Section 6 discusses the influences of government subsidy and carbon tax policies on the energy saving and emission reduction of the supply chain and on the profit of the enterprises therein. Section 7 verifies the major conclusions of the research through numerical analysis. Finally, Section 8 draws conclusions, and points out the main deficiencies of the research and future research directions.

2. Literature Review

Firstly, the energy-saving decisions in atwo-echelon supply chain under a government subsidy policy is the primary topic of this research.

Xie [16] built a two-echelon supply chain comprised of one manufacturer and one retailer, in which the former decided the energy-saving level of the products, while the retailer determined the product price. On this basis, Xie investigated the optimal strategies of the manufacturer and retailer under centralised and decentralised decision-making paradigms, and further explored the coordination effects of the wholesale price contract, profit-sharing contract, and a one-time transfer contract on the supply chain. Afterwards, Xie [17] considered competitive conditions and established a two-echelon supply chain consisting of two competitive suppliers and a manufacturer. At first, the two suppliers decided the energy-saving level, and then one supplier decided whether or not to cooperate with the manufacturer. Finally the other supplier determines whether or not to cooperate with the manufacturer, after learning the decisions of the aforementioned supplier and manufacturer. By discussing different scenarios, Xie found that a cooperative strategy is able to improve the sustainability of the supply chain. Based on previous research [16,17], Hafezalkotob [18] constructed two competitive supply chains, each of which was formed by one manufacturer and one retailer. Then, he studied the energy-saving and wholesale price decisions of the manufacturers and the retail price decisions of retailers under four supply-chain configurations (D-D competitive, D-D cooperative, C-C competitive, and C-C cooperative). Zhang et al. [19] analysed the influences of different subsidy policies (tax preference, price subsidy, sales subsidy, and investment subsidy) on the production output of the manufacturer, the repurchase price of the recycler, and the profits of both. Zhou and Huang [4] supposed that government provides a quota subsidy contract or a price discount subsidy contract, to encourage consumers to purchase energy-saving products. Then, they studied the energy-saving level and price decisions of enterprises and the form of contract selection by government. By doing so, it was found that the quota subsidy contract is better when government budgets are low—otherwise, the government is likely to choose a price discount subsidy contract. Both contracts are beneficial to the environment.

Another important topic of this research is emissions reduction decisions in a two-echelon supply chain under a carbon tax policy. He et al. [20] built an emission-dependent dyadic supply chain, made up of a supplier and a manufacturer, where the manufacturer can pay certain fees to supplier or third party for emissions reduction. It was found that, whether or not a third party provides services for emissions reduction, manufacturer's payments to the supplier are beneficial to carbon emissions reduction and profits in the supply chain. Du [21] constructed a two-echelon supply chain, composed of one manufacturer and one retailer. To begin with, the manufacturer determined its emissions-reduction effort level and the wholesale price, and then the retailer decided its emission-reduction effort level and the retail price. On this basis, the optimal decisions of the manufacturer and the retailer were discussed under centralised and decentralised decision-making frameworks, and a price discount contract was designed to coordinate the supply chain. Aside from research on single supply chains, some scholars also discussed conditions under competition. Choi [22] assumed the existence of a domestic manufacturer and an overseas manufacturer. Carbon tax would be levied when the retailer purchased products from the overseas manufacturer, but not when it bought goods from the domestic manufacturer. It was found that, when the carbon tax is high, whether or not the manufacturers provide the wholesale price contract or the price discount contract, the retailer will choose to purchase products from the domestic manufacturer. Liu et al. [23] discussed two supply-chain configurations: monopolistic and competitive. Each supply chain is composed of one manufacturer and one retailer, in which the manufacturer produces ordinary or low-carbon products. They found that whether the manufacturer produces low-carbon products or not depends on the substitutability of the products and primary market demand. In addition, when the products have high substitutability, the members of the supply chains are likely to get trapped in the prisoner's dilemma, where acting in their own interests results in negative effects for each party. Apart from the supply chain structure, some literature considers the power of enterprises in the supply chain. Xiao et al. [24] supposed that the government only levies carbon tax on the supplier, and the retailer provides the shared-tax contract for the supplier. The emissions-reduction and price decisions of the supplier and the retailer were investigated under retailer- and supplier-led conditions, as well as under the condition that the two parties have similar market status. Yang et al. [25] establish a supply chain system consisting of a manufacturer and a retailer, in order to explore the effect of revenue sharing and first-mover advantage in manufacturer's emission abatement and the firms' profitability. The authors found that a revenue-sharing contract does not necessarily dull the manufacturer's emission reduction effort; it depends on whether that manufacturer is dominant in the market and whether consumers have environmental awareness. Considering that there is the spillover effect when upstream and downstream enterprises reduce their emissions, Xu et al. [14] studied the influences of the low-carbon preferences of consumers, and the vertical spillover effect on the emission-reduction decision and profits of enterprises. In addition, they also coordinated the supply chain using a bargaining-coordination contract. Yenipazarli [26] explored the influences of the carbon tax on the price and production decisions of the manufacturer. Yenipazarli also investigated how a government should formulate its carbon tax, so as to realise an all-win outcome for the economy, the environment, and society as a whole.

The aforementioned studies show that existing research basically separates the energy-saving and emission-reduction decisions of enterprises; however, in reality, the two problems are intertwined, and manufacturers commonly need to solve the two problems at the same time. Taking energy-efficient air conditioners as an example, due to presence of carbon emissions in the production process, enterprises like Midea and Gree not only actively raise the energy-saving level of their products, but also constantly improve the manufacturing process to control carbon emissions (e.g., green production). Therefore, the largest difference in the current study from those mentioned previously is that the current one considers the energy-saving and emission-reduction decisions of enterprises simultaneously, and explores the influences of government subsidies and carbon tax policies on the energy-saving and emission-reduction decisions of enterprises.

The third important topic for this research is supply chain coordination. There is much research on the coordination of upstream and downstream enterprises using contracts, such as quantity-discount contracts [27], return contracts [28], two-part tariff contracts [29], and revenue sharing contracts [30]. What is directly related to the current study are cost-sharing contracts. Ghosh and Shah [13] constructed a green supply chain consisting of one manufacturer and one retailer, in which the retailer offered a green cost-sharing contract to the manufacturer. They found that the cost-sharing contract can not only coordinate the supply chain, but also can bring about a cleaner environment. Suppose that consumers are all environmentalists. Wang et al. [31] found that the CCS contract, under retailer-led conditions, causes Pareto improvement; however, when consumers merely have high low-carbon consciousness, only the wholesale price contract can improve the profit of the supply chain. When the retailer and the manufacturer are of equal status, both the wholesale price contract and the CCS cause Pareto improvement. Not only considering consumers with low-carbon preference, but also adding advertising factors to promote the need for low-carbon products, Zhou et al. [32] studied the influences of advertising cooperation, as well as advertising and CCS contracts, on a low-carbon supply chain. Moreover, the decisions relating emission reduction and advertising input, as well as the coordination of the supply chain, were investigated when the retailer has a fairness preference. Some scholars have considered different contracts and compared their coordination effects; for example, Xu et al. [14] explore emission-reduction decisions of manufacturers, and the size of orders placed by retailers under both wholesale price contracts and CCS contracts. By doing so, they find that the optimal emissions-reduction level rises with increasing carbon trading prices at first, and then tends to stabilize; in addition, both the wholesale price contract and the CCS contract are able to coordinate the supply chain. Yu and Han [33] investigate the influence of carbon tax on emissions reduction and retail price in a two-level supply chain composed of a manufacturer and a retailer. By designing the modified wholesale price and the modified cost-sharing contract, supply chain coordination is achieved, but neither of the two contracts benefits the manufacturer. Yang and Chen [34] supposed that the retailer provides a CCS contract, a revenue-sharing contract, or both at the same time for the manufacturer. In the supply chain, the manufacturer decides the emissions reduction level and the wholesale price, while the retailer determines the size of the order placed. It was found that the CCS contract and the revenue-sharing contract can both improve the supply chain when compared under a decentralised decision-making regime; however, the revenue-sharing contract endows the supply chain system with higher efficiency, and the condition that the retailer simultaneously provides the CCS and revenue-sharing contracts has the same effect as when the retailer only offers a revenue-sharing contract.

It can be seen that the aforementioned research mainly investigates the CCS contract. However, retailers and manufacturers cooperate in many ways, including energy saving and emissions reduction (the green supply chain plan of Wal-Mart, for instance). Therefore, differing from previous research, the current study investigates conditions in which the retailer shares two costs of the manufacturer—namely, energy-saving and emissions-reduction costs. On this basis, the influences of different cost-sharing contracts on the energy saving, emissions reduction and the profits of enterprises are explored.

3. Model Description and Hypotheses

The focus of the research is to explore the cooperative energy-saving and emissions-reduction decisions of the supply chain under government subsidies and carbon tax policies. We consider a two-echelon supply chain comprised of one manufacturer and one retailer, in which the manufacturer produces certain energy-saving products (e.g., energy-efficient air conditioners or refrigerators). Carbon emissions are generated during production. The schematic diagram of the two-echelon supply chain operation is depicted by Figure 1:

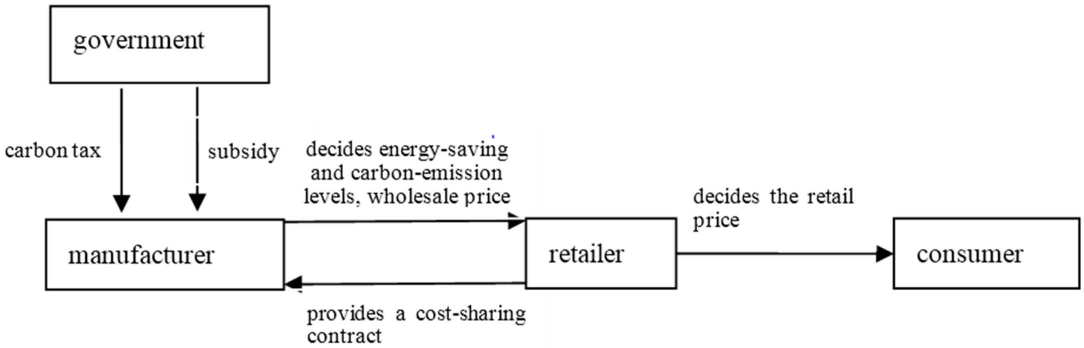

Figure 1. Schematic diagram of the supply chain operation.

The following hypotheses and symbol definitions are proposed:

(1) The manufacturer has a marginal production cost of c and it sells products to the retailer at wholesale price w. Then, the retailer sells the products to consumers at retail price p.

(2) The energy-saving level of products is g $(g > 0)$. The higher the energy-saving level, the lower the energy consumption of the products per unit service life. After the products are designed, all finished products have the same energy-saving level. In reference to [16], the energy-saving R&D cost of the products is $\frac{1}{2}bg^2$, where b $(b > 0)$ is the energy-saving cost coefficient.

(3) The manufacturer has carbon emissions during the production process, and the initial carbon-emissions level for producing per unit of product is e_0. To reduce the tax burden or for the sake of social responsibility, the manufacturer is likely to reduce their carbon emissions. The carbon-emissions level while producing the unit product is e $(0 \le e \le e_0)$ after emissions reduction. The emission-reduction cost is $\frac{1}{2}z(e_0 - e)^2$, where z $(z > 0)$ is the cost coefficient of carbon emissions reduction [23].

(4) On the one hand, currently more than ten countries and regions have implemented a carbon labelling system. By using the system, the amount of greenhouse gas discharged during the production of goods is indicated using quantitative indices on product labels, so as to inform consumers of the carbon information of products. Therefore, as consumers' consciousness of environmental protection gets stronger, market demand is influenced by the carbon-emissions level of enterprises [23,34]. On the other hand, many countries have implemented an energy efficiency labelling system for energy-saving products, to inform consumers of the energy consumption grade of products. Therefore, market demand is also affected by the energy-saving level of products [16–18]. By combining these studies, it is assumed that the market demand is jointly influenced by the retail price, carbon-emissions level, and energy-saving level of products. The demand function is:

$$q = a - p - \beta e + kg,$$

where a $(a > c)$ is the potential market capacity, p is the retail price of products, $\beta(\beta > 0)$ is the sensitivity coefficient of consumers to the carbon-emission level of enterprises, and $k(k > 0)$ is the sensitivity coefficient of consumers to the energy-saving level of products.

(5) To encourage enterprises to produce more energy-saving products, the government entrusts a third party to carry out free energy-saving certification for enterprises, and then provides subsidies for the enterprises according to the certification results. Zhang et al. [19] set the government subsidy to be a fixed amount that is unrelated to product/process energy-saving levels; however, in reality, the subsidy intensity is classified according to the energy efficiency index of products. For example, the Chinese government provides subsidies of 240 to 400 yuan for each variable-frequency air conditioner, and 100 to 400 yuan for each LCD TV set [3]. Therefore, the research supposes that the government provides subsidies for manufacturers based on the energy-saving level of products. The energy-saving subsidy per unit of product is sg—that

is, the higher the energy-saving level, the larger the subsidy, in which s denotes the subsidy coefficient $(s > 0)$.

(6) To protect the environment, the government levies a carbon tax on enterprises, to force them to reduce their carbon emissions. It imposes carbon tax t $(t > 0)$ per unit carbon emission [34].

(7) Owing to the downstream retailer selling the final products to consumers, that retailer shares a common interest with the manufacturer. For this reason, the retailer has the motivation to encourage the manufacturer to save energy and reduce emissions, so as to improve market demand and profits. For example, Wal-Mart abates pollution jointly with upstream manufacturers, and also cooperates with them to improve the energy efficiency of most energy-intensive products by 25%. Suppose that the retailer provides a cost-sharing contract (μ, θ) for the manufacturer, where μ and θ represent the shared proportion of the energy-saving and emission-reduction costs, respectively—in that case, (i) when $\theta = 0,\ \mu = 0$, the retailer does not provide any contract and does not cooperate with the manufacturer, or NCS; (ii) if $0 < \mu \leq 1,\ \theta = 0$, then the retailer only offers an ECS contract; (iii) if $0 < \theta \leq 1,\ \mu = 0$, then the retailer only provides a CCS contract; (iv) when $0 < \theta \leq 1,\ 0 < \mu \leq 1$, then it represents the BCS for energy saving and emissions reduction.

(8) According to the above descriptions, the profit function of the retailer is:

$$\pi_r = (p - w)(a - p - \beta e + kg) - \frac{1}{2}\mu b g^2 - \frac{1}{2}\theta z (e_0 - e)^2 \tag{1}$$

The profit function of the manufacturer is:

$$\pi_m = (w - c)q + sgq - \frac{1}{2}(1 - \mu) b g^2 - teq - \frac{1}{2}(1 - \theta) z (e_0 - e)^2 \tag{2}$$

(9) To make our research realistic and avoid trivial results, we assume $2bz > (k + s)^2 z + b(t + \beta)^2$, and $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \leq e_0 \leq \frac{(a-c)}{t+\beta}$ (see the proof of Theorem 1 for more details).

According to the aforementioned hypotheses, the game timeline of the manufacturer and the retailer is shown in Figure 2.

Stage 1 (contract incentive): the retailer decides the cost-sharing contract (θ, μ).
Stage 2 (product R&D): the manufacturer decides the energy-saving level (g) of the products.
Stage 3 (product production): the manufacturer determines the carbon emissions level (e).
Stage 4 (product wholesale): the manufacturer sets the wholesale price (w) of the products.
Stage 5 (product retail): the retailer determines the retail price (p) of the products.

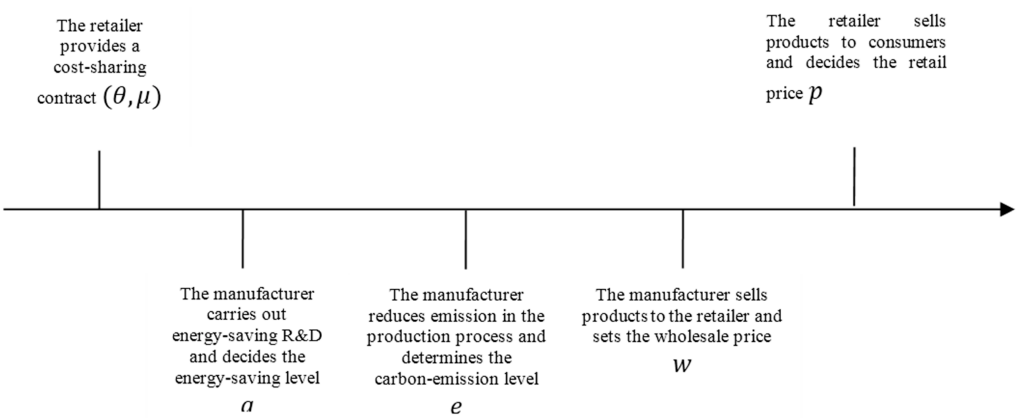

Figure 2. Game timeline.

The following sections discuss the conditions under a centralised decision-making regime for NCS, ECS, CCS, and BCS contracts. The proofs of each of the theorems and propositions are provided in the Appendix A.

4. Centralised Decision-Making

At first, centralised decision-making is considered, and applied as the benchmark. The total profit of the supply chain is:

$$\pi = \pi_r + \pi_m = (p - c)q + sgq - \frac{1}{2}bg^2 - teq - \frac{1}{2}z(e_0 - e)^2 \tag{3}$$

where the first to fifth items represent the product profit, amount of energy-saving subsidy provided by government, energy-saving cost, amount of carbon tax levied by government, and the emission-reduction cost, respectively.

Under a centralised decision-making regime, the manufacturer and the retailer are just like an enterprise, and they jointly decide the price, carbon-emission level, and energy-saving level, in order to maximise the profit of the whole supply chain. In this way, Theorem 1 is obtained.

Theorem 1. *Under centralised decision-making, when* $2bz > (k+s)^2 z + b(t+\beta)^2$ *and* $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \le e_0 \le \frac{a-c}{t+\beta}$ *, then*

$$p^{C*} = \frac{a\left[s(k+s)z + b\left(t^2 - z + t\beta\right)\right] + z(k+s)(ck + kte_0 - s\beta e_0) + b\left[c\beta(t+\beta) - cz + z(\beta - t)e_0\right)}{(k+s)^2 z + b(t^2 - 2z + 2t\beta + \beta^2)},$$

$$e^{C*} = e_0 - \frac{b(t+\beta)(c + te_0 + \beta e_0 - a)}{(k+s)^2 z + b(t^2 - 2z + 2t\beta + \beta^2)},\ g^{C*} = \frac{z(k+s)(c + te_0 + \beta e_0 - a)}{(k+s)^2 z + b(t^2 - 2z + 2t\beta + \beta^2)},$$

$$q^{C*} = \frac{bz(c + te_0 + \beta e_0 - a)}{(k+s)^2 z + b(t^2 - 2z + 2t\beta + \beta^2)},\quad \pi^{C*} = \frac{bz(c + te_0 + \beta e_0 - a)^2}{2\left[2bz - (k+s)^2 z - b(t+\beta)^2\right]}.$$

5. Decentralised Decision-Making

5.1. No Cost-Sharing (NCS) Contract

When the retailer does not provide cost-sharing contracts—in other words, when an NCS contract is used:

$$\pi_r = (p - w)(a - p - \beta e + kg) \tag{4}$$

$$\pi_m = (w - c)q + sgq - \frac{1}{2}bg^2 - teq - \frac{1}{2}z(e_0 - e)^2 \tag{5}$$

Under that condition, the manufacturer determines the energy-saving level, carbon-emission level, and wholesale price to maximise its profit. Then, the retailer sets the retail price to maximise its profit. In this way, Theorem 2 is attained.

Theorem 2. *Under a decentralised decision-making framework, when* $2bz > (k+s)^2 z + b(t+\beta)^2$ *and* $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \le e_0 \le \frac{a-c}{t+\beta}$ *, then*

$$g^{NCS*} = \frac{(k+s)z(c + te_0 + \beta e_0 - a)}{(k+s)^2 z + b(t^2 - 4z + 2t\beta + \beta^2)},\ e^{NCS*} = e_0 - \frac{b(t+\beta)(c + te_0 + \beta e_0 - a)}{(k+s)^2 z + b(t^2 - 4z + 2t\beta + \beta^2)},$$

$$w^{NCS*} = \frac{a\left[b\left(t^2 - 2z + t\beta\right) + s(k+s)z\right] + (k+s)z(ck + kte_0 - s\beta e_0) + b\left[c\left(\beta^2 - 2z + t\beta\right) + 2z(\beta - t)e_0\right]}{(k+s)^2 z + b(t^2 - 4z + 2t\beta + \beta^2)},$$

$$p^{NCS*} = w^{NCS*} + q^{NCS*},\ q^{NCS*} = \frac{bz(c + te_0 + \beta e_0 - a)}{(k+s)^2 z + b(t^2 - 4z + 2t\beta + \beta^2)},$$

$$\pi_r^{NCS*} = \frac{b^2z^2(c+te_0+\beta e_0-a)^2}{[(k+s)^2z+b(t^2-4z+2t\beta+\beta^2)]^2},\ \pi_m^{NCS*} = \frac{bz(c+te_0+\beta e_0-a)^2}{-2[(k+s)^2z+b(t^2-4z+2t\beta+\beta^2)]}.$$

As this research mainly investigates the cooperation between the retailer and the manufacturer, the following sections pay attention to the conditions whereby the retailer provides cost-sharing contracts.

5.2. Energy-Saving Cost-Sharing Contract

Here, we consider the condition in which the retailer offers a single cost-sharing contract, or an ECS contract:

$$\pi_r^{ECS} = (p-w)(a-p-\beta e+kg) - \frac{1}{2}\mu bg^2 \tag{6}$$

$$\pi_m^{ECS} = (w-c)q + sgq - \frac{1}{2}(1-\mu)bg^2 - teq - \frac{1}{2}z(e_0-e)^2 \tag{7}$$

Under this condition, the retailer decides the sharing proportion for energy-saving cost first, then the manufacturer decides the energy-saving level, carbon-emission level, and wholesale price, and finally the retailer determines the retail price.

Theorem 3. *When the retailer provides an ECS, if* $2bz > (k+s)^2z + b(t+\beta)^2$ *and* $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \le e_0 \le \frac{(a-c)}{t+\beta}$*, then*

$$\mu^{ECS*} = \frac{(k+s)^2z+b(t+\beta)^2}{[8z-(t+\beta)^2]b},\ g^{ECS*} = \frac{z(k+s)[8z-(t+\beta)^2](a-c-te_0-\beta e_0)}{2[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2]},$$

$$e^{ECS*} = e_0 - \frac{z(t+\beta)(k+s)^2(a-c+te_0+\beta e_0)+2b[4z-(t+\beta)^2][4ze_0-(a-c)(t+\beta)]-12(k+s)^2z^2e_0}{2[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2]},$$

$$w^{ECS*} = \frac{A}{2[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2]},\ p^{ECS*} = w^{ECS*} + q^{ECS*},$$

$$q^{ECS*} = \frac{z[8bz-2b(t+\beta)^2-z(k+s)^2](a-c-te_0-\beta e_0)}{2[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2]},$$

$$\pi_r^{ECS*} = \frac{[8b+(k+s)^2]z^2(a-c-te_0-\beta e_0)^2}{8[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2]},$$

$$\pi_m^{ECS*} = \frac{z[8bz-2b(t+\beta)^2-z(k+s)^2](a-c-te_0-\beta e_0)^2}{4[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2]}.$$

where $A = a((k+s)z(k(t^2-2z+t\beta)+s(2t^2-10z+3t\beta+\beta^2))+2b(t^4+3t^3\beta+2z(4z-\beta^2)+t^2(-6z+3\beta^2)+t(-8z\beta+\beta^3)))+2b(t^2-4z+2t\beta+\beta^2)(c(-2z+\beta(t+\beta))+2z(-t+\beta)e_0)+(k+s)z(c(s(-2z+\beta(t+\beta))+k(t^2+3t\beta+2(-5z+\beta^2)))+(k(t^3+2t^2\beta+2z\beta+t(-10z+\beta^2))-s(t^2\beta-10z\beta+\beta^3+2t(z+\beta^2)))e_0)$.

By comparing the conditions of ECS and NCS, Proposition 1 is obtained.

Proposition 1. *(i)* $g^{ECS*} > g^{NCS*}$, $e^{ECS*} < e^{NCS*}$; *(ii)* $\pi_r^{ECS*} > \pi_r^{NCS*}$, $\pi_m^{ECS*} > \pi_m^{NCS*}$.

Proposition 1 indicates that (i) the ECS contract provided by the retailer can improve the energy-saving level and reduce carbon emissions. When the retailer offers an ECS contract, the manufacturer has some motivation to improve their energy saving level, and more capital to invest in emissions reduction owing to the retailer bearing some of the cost, thus decreasing the final level of carbon emissions.

It also indicates that (ii) provided with an ECS contract, the retailer not only improves its profit but also increases the profit of the manufacturer, thereby realising a win-win situation. This is because the ECS contract promotes energy saving and emissions reduction by the manufacturer and therefore increases market demand. In that context, the increase of product revenue can sufficiently offset the shared energy-saving cost, thus increasing the profit of the retailer. For the manufacturer, the increases in product revenue and amount of subsidy, and the decrease in the energy-saving cost, all augment the profit of the manufacturer.

5.3. Carbon Emissions Reduction Cost-Sharing Contract

Under the condition that the retailer provides a CCS contract to the manufacturer:

$$\pi_r^{CCS} = (p - w)(a - p - \beta e + kg) - \frac{1}{2}\theta z(e_0 - e)^2 \quad (8)$$

$$\pi_m^{CCS} = (w - c)q + sgq - \frac{1}{2}bg^2 - teq - \frac{1}{2}(1 - \theta)z(e_0 - e)^2 \quad (9)$$

Similar to the analysis in Section 5.2, Theorem 4 is obtained.

Theorem 4. *Under the condition that the retailer offers a CCS contract, when* $2bz > (k+s)^2 z + b(t+\beta)^2$ *and* $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \leq e_0 \leq \frac{(a-c)}{t+\beta}$ *, then*

$$\theta^{CCS*} = \frac{(k+s)^2 z + b(t+\beta)^2}{[8b-(k+s)^2]z},\ g^{CCS*} = \frac{(k+s)[8bz-2(k+s)^2 z - b(t+\beta)^2](a-c-te_0-\beta e_0)}{2[(k+s)^4 z + b(k+s)^2(t^2-8z+2t\beta+\beta^2) - 2b^2(3t^2-8z+6t\beta+3\beta^2)]},$$

$$e^{CCS*} = e_0 - \frac{b(t+\beta)[8b-(k+s)^2](a-c-te_0-\beta e_0)}{2[(k+s)^4 z + b(k+s)^2(t^2-8z+2t\beta+\beta^2) - 2b^2(3t^2-8z+6t\beta+3\beta^2)]},$$

$$w^{CCS*} = \frac{B}{2[(k+s)^4 z + b(k+s)^2(t^2-8z+2t\beta+\beta^2) - 2b^2(3t^2-8z+6t\beta+3\beta^2)]},\ p^{CCS*} = w^{ccs*} + q^{CCS*},$$

$$q^{CCS*} = \frac{b[8bz-2(k+s)^2 z - b(t+\beta)^2](a-c-te_0-\beta e_0)}{2[(k+s)^4 z + b(k+s)^2(t^2-8z+2t\beta+\beta^2) - 2b^2(3t^2-8z+6t\beta+3\beta^2)]},$$

$$\pi_r^{CCS*} = \frac{b^2[8z+(t+\beta)^2](a-c-te_0-\beta e_0)^2}{8[(k+s)^4 z + b(k+s)^2(t^2-8z+2t\beta+\beta^2) - 2b^2(3t^2-8z+6t\beta+3\beta^2)]},$$

$$\pi_m^{CCS*} = \frac{b[8bz-2(k+s)^2 z - b(t+\beta)^2](a-c-te_0-\beta e_0)^2}{4[(k+s)^4 z + b(k+s)^2(t^2-8z+2t\beta+\beta^2) - 2b^2(3t^2-8z+6t\beta+3\beta^2)]}.$$

where $B = a(2s(k+s)^3 z - 2b^2(5t^2 - 8z + 6t\beta + \beta^2) + b(k+s)(k(t^2 - 4z + t\beta) + s(2t^2 - 12z + 3t\beta + \beta^2))) + 2(k+s)^3 z(ck + kte_0 - s\beta e_0) - 2b^2(c(t^2 - 8z + 6t\beta + 5\beta^2) + (t-\beta)(t^2 - 8z + 2t\beta + \beta^2)e_0) + b(k+s)(c(s(-4z + \beta(t+\beta)) + k(t^2 + 3t\beta + 2(-6z + \beta^2))) + (k(t^3 - 12tz + 2t^2\beta + 4z\beta + t\beta^2) - s(4tz + t^2\beta - 12z\beta + 2t\beta^2 + \beta^3))e_0)$.

Proposition 2 can be obtained by comparing the CCS contract and the ECS contract.

Proposition 2. *(i)* $g^{CCS*} < g^{ECS*}$, $e^{CCS*} < e^{ECS*}$; *(ii)* If $t + \beta > (k+s)\sqrt{\frac{z}{b}}$, *then* $\pi_r^{CCS*} > \pi_r^{ECS*}$, $\pi_m^{CCS*} > \pi_m^{ECS*}$, *and* $\theta^{CCS*} < \mu^{ECS*}$; *If* $k + s > (t+\beta)\sqrt{\frac{b}{z}}$, *then* $\pi_r^{CCS*} < \pi_r^{ECS*}$, $\pi_m^{CCS*} < \pi_m^{ECS*}$, *and* $\theta^{CCS*} > \mu^{ECS*}$.

Proposition 2 suggests that (i) compared with the ECS contract, the manufacturer has a lower energy-saving level and carbon emissions level under the CCS contract. Apparently, if the retailer chooses to cooperate with the manufacturer in emissions reduction, and shares a certain part of the emissions reduction cost with the manufacturer, then the manufacturer has more capital and motivation to reduce their carbon emissions. If the retailer offers an ECS contract, the manufacturer will place more attention to energy-saving R&D in order to improve energy-savings; however, whether under an ECS or CCS contract, it can always promote energy savings and emissions reduction on the part of the manufacturer.

Proposition 2 also indicates (ii) that under conditions that the carbon tax of the government and the carbon-emissions consciousness of consumers is high enough, the manufacturer and the retailer will pay more attention to emissions reduction. In such a context, a CCS can better enhance the profits of the two parties compared with the case under an ECS contract and the retailer shares a smaller proportion of the cost. When government subsidies, and consumers awareness of energy conservation, are high enough, the manufacturer and retailer will attach more importance to energy saving. Under this condition, compared with the CCS contract, the ECS contract can lift the profits of the two parties and reduce the cost-share proportions of each.

It can be seen from Proposition 2 that single cost-sharing contracts (ECS or CCS) have their own merits and disadvantages, but both have no absolute advantages. Then the question becomes whether or nota BCS contract can more effectively prompt the cooperation between the manufacturer and the retailer with regards to energy saving and emissions reduction. This question is discussed below.

5.4. Bivariate Cost-Sharing Contract

Here, we study the condition in which the retailer provides a BCS contract for the manufacturer, in which case:

$$\pi_r^{BCS} = (p-w)(a-p-\beta e+kg) - \frac{1}{2}\mu b g^2 - \frac{1}{2}\theta z(e_0-e)^2 \tag{10}$$

$$\pi_m^{BCS} = (w-c)q + sgq - \frac{1}{2}(1-\mu)bg^2 - teq - \frac{1}{2}(1-\theta)z(e_0-e)^2 \tag{11}$$

Similar to the solution procedure described previously, Theorem 5 is obtained:

Theorem 5. *Under the condition that the retailer offers a BCS contract, if* $2bz > (k+s)^2 z + b(t+\beta)^2$ *and* $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \le e_0 \le \frac{(a-c)}{t+\beta}$ *, then*

$$\mu^{BCS*} = \theta^{BCS*} = \frac{(k+s)^2 z + b(t+\beta)^2}{8bz},\ g^{BCS*} = \frac{2z(k+s)(a-c-te_0-\beta e_0)}{8bz-3z(k+s)^2-3b(t+\beta)^2},$$

$$e^{BCS*} = e_0 - \frac{2b(t+\beta)(a-c-te_0-\beta e_0)}{8bz-3z(k+s)^2-3b(t+\beta)^2},\ w^{BCS*} = \frac{E}{2[8bz-3z(k+s)^2-3b(t+\beta)^2]},$$

$$p^{BCS*} = w^{ccs*} + q^{BCS*},\ q^{BCS*} = \frac{[8bz-z(k+s)^2-b(t+\beta)^2](a-c-te_0-\beta e_0)}{4[8bz-3z(k+s)^2-3b(t+\beta)^2]},$$

$$\pi_r^{BCS*} = \frac{[8bz+z(k+s)^2+b(t+\beta)^2](a-c-te_0-\beta e_0)^2}{16[8bz-3z(k+s)^2-3b(t+\beta)^2]},$$

$$\pi_m^{BCS*} = \frac{[8bz-z(k+s)^2-b(t+\beta)^2](a-c-te_0-\beta e_0)^2}{8[8bz-3z(k+s)^2-3b(t+\beta)^2]}.$$

where $E = (a((k^2+6ks+5s^2)z + b(5t^2-8z+6t\beta+\beta^2)) + (k+s)z(c(5k+s) + (5kt+st-k\beta-5s\beta)e_0) + b(c(t^2-8z+6t\beta+5\beta^2) + (t-\beta)(t^2-8z+2t\beta+\beta^2)e_0))$.

By comparing the BCS contract with single cost-sharing contracts, Propositions 3 and 4 are posited.

Proposition 3. *(i)* $\mu^{BCS*} < \mu^{ECS*}$, $\theta^{BCS*} < \theta^{CCS*}$; *(ii)* $g^{BCS*} > g^{ECS*}$, $e^{BCS*} < e^{CCS*}$.

Proposition 3 suggests that (i) compared with a single cost-sharing contract, the shared proportions for energy-saving costs and emissions-reduction costs are smaller when the retailer provides a BCS contract. When the retailer and manufacturer cooperate, both towards energy saving and emissions reduction, the retailer is likely to reduce their cost-share to decrease the cost, so as to avoid too large a cost-share affecting its own profit.

Proposition 3 also suggests that (ii) in comparison with the single cost-sharing contracts, the energy-saving level is higher and the carbon-emission level is lower when a BCS contract is used. This is because, although the cost-sharing proportion is reduced when the retailer offers a BCS contract, the total shared cost is higher ($\frac{1}{2}b\mu^{BCS*}(g^{BCS*})^2 + \frac{1}{2}z\theta^{BCS*}(e_0-e^{BCS*})^2 > max\{\frac{1}{2}b\mu^{ECS*}(g^{ECS*})^2,\ \frac{1}{2}z\theta^{CCS*}(e_0-e^{CCS*})^2\}$). Therefore, the manufacturer has more capital with which to promote energy saving and emissions reduction. This also suggests that overall cooperation between the retailer and the manufacturer can more greatly facilitate energy saving and emissions reduction than some level of partial cooperation.

Proposition 4. *(i)* $\pi_r^{BCS*} > \pi_r^{ECS*}$, $\pi_r^{BCS*} > \pi_r^{CCS*}$; *(ii)* $\pi_m^{BCS*} > \pi_m^{ECS*}$, $\pi_m^{BCS*} > \pi_m^{CCS*}$.

Proposition 4 indicates that (i) the retailer earns a higher profit under a BCS contract thanunder single cost-sharing contracts. This is because the improvement of the energy-saving and emission-reduction levels vastly enlarges market demand. The increase in the sales revenue offsets the shared energy-saving and emission-reduction costs, so the total profit still increases.

Proposition 4 (ii) also demonstrates that a BCS contract also improves the profit of the manufacturer. On the one hand, the increasing demand brings more orders (and sales volume) and revenue for the manufacturer. On the other hand, the rising energy saving level also generates greater government subsidies, and the growing emissions reduction level also reduces the carbon tax cost. The retailer also shares the energy-saving and emission-reduction costs of the manufacturer. Therefore, the manufacturer benefits more from a BCS contract.

It can be seen from Propositions 3 and 4 that overall cooperation between the retailer and the manufacturer is superior to partial cooperation. Overall cooperation can not only increase profits among node enterprises in the supply chain, but can also improve the environment, thus realising both energy savings and emissions reductions at a higher level.

In comparison with the single cost-sharing contracts, the BCS contract is absolutely dominant. Then, whether or nota BCS contract can achieve ideal conditions under a centralised decision-making framework becomes the question. The following section compares the BCS contract with the benchmark model (centralised decision-making), and Proposition 5 is obtained.

Proposition 5. $e^{C*} < e^{BCS*}$, $g^{C*} > g^{BCS*}$, *and* $\pi^{C*} > \pi^{BCS*}$.

Proposition 5 suggests that the carbon-emission level under a centralised decision-making regime is lower, while the energy-saving level and total profits are higher, than those under a BCS contract. This indicates that, although the BCS contract realises a Pareto improvement in the supply chain, it cannot achieve prefect coordination.

6. Influence of Government Policies on Energy Saving and Emissions Reduction of the Supply Chain

The section explores the influences of government subsidies and carbon tax policies on the energy-saving and emission-reduction decisions as well as the profits of the supply chain. In addition, the impacts of government policies on the cost-sharing proportion, energy-saving level, carbon-emissions level, and the profits of nodal enterprises in the supply chain are highlighted. As mentioned above, the NCS, single cost-sharing, and BCS models lead to similar analytical conclusions, and the retailer's BCS contract is the dominant strategy. For this reason, this section merely analyses the influences of government policies on the energy-saving and emission-reduction decisions and the profits of the supply chain under a BCS contract, and compares them with those under the centralised decision-making framework.

Proposition 6. *(i)* $\frac{\partial \mu^{BCS*}}{\partial s} = \frac{\partial \theta^{BCS*}}{\partial s} > 0$ *and (ii)* $\frac{\partial \mu^{BCS*}}{\partial t} = \frac{\partial \theta^{BCS*}}{\partial t} > 0$.

Proposition 6 implies that (i) the increase in the subsidy coefficient increases the cost-sharing proportion of the retailer. The conclusion seems to be counterintuitive. In fact, the increase in the energy-saving subsidy intensity of the government relieves the cost pressure on energy saving and emissions reduction for the manufacturer, elevates the energy-saving level of the manufacturer, and increases market demand. Due to the reduction of the energy-saving and emission-reduction costs, as well as the increasing demand, the retailer is motivated to improve its cost-sharing proportion, so as to further enlarge market demand.

The increase in the carbon tax (ii) also enlarges the cost-sharing proportion of the retailer. Apparently, when the government improves the carbon tax regime, it can increase the cost to the manufacturer. To afford the manufacturer more capital with which to save energy, reduce emissions, and improve market demand, the retailer will raise their share of the energy-saving and emission-reduction costs.

Proposition 6 indicates that both the energy-saving subsidy and the carbon tax policy can enhance cooperation between the upstream and the downstream enterprises and strengthen the partnership.

Proposition 7. *(i)* $\frac{\partial g^{BCS*}}{\partial s} > 0$, $\frac{\partial e^{BCS*}}{\partial s} < 0$; *(ii) When* $t < \sqrt{\frac{z[8b-3(k+s)^2][2b-(k+s)^2]}{b[10b-3(k+s)^2]}} - \beta$, *then* ① *If* $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \leq e_0 < \frac{(a-c)[8bz-3(k+s)^2z+3b(t+\beta)^2]}{2z(t+\beta)[8b-3(k+s)^2]}$, *then* $\frac{\partial e^{BCS*}}{\partial t} < 0$; ② *If* $\frac{(a-c)[8bz-3(k+s)^2z+3b(t+\beta)^2]}{2z(t+\beta)[8b-3(k+s)^2]} < e_0 \leq \frac{(a-c)}{t+\beta}$, *then* $\frac{\partial e^{BCS*}}{\partial t} > 0$; *When* $t > \sqrt{\frac{z[8b-3(k+s)^2][2b-(k+s)^2]}{b[10b-3(k+s)^2]}} - \beta$, *then* $\frac{\partial e^{BCS*}}{\partial t} > 0$. *(iii) When* $t < \sqrt{\frac{z[4b-3(k+s)^2]}{3b}} - \beta$, *then* ① *If* $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \leq e_0 \leq \frac{6b(a-c)(t+\beta)}{8bz-3(k+s)^2z+3b(t+\beta)^2}$, *then* $\frac{\partial g^{BCS*}}{\partial t} > 0$; ② *If* $\frac{6b(a-c)(t+\beta)}{8bz-3(k+s)^2z+3b(t+\beta)^2} \leq e_0 \leq \frac{(a-c)}{t+\beta}$, *then* $\frac{\partial g^{BCS*}}{\partial t} < 0$; *When* $t > \sqrt{\frac{z[4b-3(k+s)^2]}{3b}} - \beta$, *then* $\frac{\partial g^{BCS*}}{\partial t} < 0$.

Proposition 7 suggests that (i) the larger the energy-saving subsidy coefficient of the government, the higher the energy-saving and emissions-reduction levels of the enterprises. Under the condition that the government enhances its subsidy intensity, the manufacturer is motivated to increase its energy-saving level, so as to obtain a larger subsidy. Meanwhile, the manufacturer has more capital with which to reduce emissions, so as to remit the carbon tax pressure and promote market demand. Therefore, government subsidy policy plays a positive, guiding role in energy saving and emissions reductions behaviours of enterprises.

When the government imposes a lower carbon tax, (ii) the carbon emission amount per unit product declines with increasing carbon tax if the initial carbon-emissions level is low. On the contrary, if the initial carbon-emission level is high, the carbon emissions per unit product increases with an increasing carbon tax.

This indicates that for a manufacturer with heavy initial pollution, an improved carbon tax regime exerts the opposite effect. This is mainly because the emission reduction demand calls for higher costs for this kind of manufacturer, and the increase in the carbon tax costs the manufacturer more profit, and therefore generates a lack of funds for emissions reduction. Consequently, the manufacturer reduces their emissions passively. For this kind of manufacturer, the government should implement a conciliation policy and improve the energy-saving subsidy at the same time, instead of implementing too strict a carbon tax policy. In this way, the manufacturer can have enough funds with which to innovate, develop new technologies, and thereby reducetheir carbon emissions, thus forming a favourable green corporate image. On the other hand, due to the low carbon-emissions cost, a manufacturer with low initial pollution levels is motivated to reduce these emissions when the government increases the carbon tax. Therefore, the government is suggested to provide subsidies and levy a carbon tax simultaneously, so as to more effectively guide the manufacturer to reduce emissions.

When the government imposes too high a carbon tax, the carbon emitted per unit product increases regardless of the initial carbon-emissions level of the manufacturer. This implies that it is not wise for the government to implement a high carbon tax blindly from the perspective of promoting carbon emissions reductions. For example, in 2012, Australia implemented a nationwide carbon tax and set the highest rates thereof of any nation, but it did not achieve the desired effect. Given that, we suggest that it is not a good idea for any government to raise its carbon taxes blindly.

Finally, Proposition 7 (iii) indicates that under the condition that the government imposes a low carbon tax, the energy-saving level of the products increases with an increasing carbon tax if the initial carbon-emissions level is low, and vice versa.

For the manufacturer causing heavy initial pollution, the pressure of a carbon tax cost forces the manufacturer to cut production as the tax increases ($\frac{\partial q^*}{\partial t} < 0$), thereby leading to a reduction in profit. As a result, the R&D investment available for raising the energy-saving level of products is influenced, so the energy-saving level declines. Under these conditions, to obtain highly energy-efficient products, governments should not levy too heavy a carbon tax burden, but instead devote more attention to energy-saving subsidies.

For the manufacturer with low initial pollution levels, the increasing carbon tax has slight negative effects on the profit of the enterprises, while a larger demand can earn the manufacturer more profit (under the condition, $\frac{\partial q^*}{\partial t} > 0$. Therefore, enterprises are motivated to raise their product energy-saving levels so as to generate more demand. Under that condition, the government can guide manufacturers to produce highly energy-efficient products by simultaneously using energy-saving subsidy and carbon tax policies.

When the government imposes too heavy a carbon tax, the energy-saving level of most products decreases regardless of the initial carbon emissions of the manufacturer; therefore, it is unwise for a government to blindly levy a high carbon tax.

In summary, the above conclusions suggest that the subsidy policy is able to facilitate energy saving and emission reduction, while the influences of the carbon tax policy are more complex. When the government imposes a low carbon tax, the carbon tax policy is able to prompt energy savings and emissions reductions among enterprises with low initial pollution levels, while the policy shows inhibitory effects on enterprises with high initial pollution levels. In the case that the government imposes too high a carbon tax, the policy always presents adverse effects on the energy saving and emissions reduction of all types of enterprises.

Proposition 8. *(i)* $\frac{\partial \pi_r^{BCS*}}{\partial s} > 0$, $\frac{\partial \pi_m^{BCS*}}{\partial s} > 0$; *(ii)* $\frac{\partial \pi_r^{BCS*}}{\partial t} < 0$, $\frac{\partial \pi_m^{BCS*}}{\partial t} < 0$.

Proposition 8 indicates that (i) the increase in the subsidy coefficient increases the profits of the retailer and manufacturer. On the one hand, the improvements in government subsidies reduce the cost pressure on enterprises for energy saving and emission reduction; on the other hand, Proposition 7 implies that an increase in the government subsidy coefficient can improve the energy-saving level of products and decrease the associated carbon-emissions level, thus greatly increasing market demand. Therefore, the manufacturer and the retailer both benefit.

Proposition 8 also indicates that (ii) an increase in the carbon tax reduces the profits of the manufacturer and the retailer. Considering this, the manufacturer should actively reduce its pollution output, so that the government will not be overly concerned about such an environmental problem, and thereby increase the carbon tax again. The retailer also needs to actively respond to the energy-saving and emission-reductions policy set by the government and adopt a strategy of cooperative emissions reduction, in order to help the manufacturer reduce carbon emissions and build a desirable industry operating environment that is recognised as such by the market.

Proposition 9. *(i)* $\frac{\partial\left(e^{C*}-e^{BCS*}\right)}{\partial s} < 0$; $\frac{\partial\left(g^{C*}-g^{BCS*}\right)}{\partial s} > 0$; *(ii) If* $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \le e_0 \le \bar{e}$, *then* $\frac{\partial\left(e^{C*}-e^{BCS*}\right)}{\partial t} < 0$; *if* $\bar{e} \le e_0 \le \frac{(a-c)}{t+\beta}$, *then* $\frac{\partial\left(e^{C*}-e^{BCS*}\right)}{\partial t} > 0$, *where* $\bar{e} = \frac{(a-c)\{3(k+s)^6z^3-b(k+s)^4z^2[26z-3(t+\beta)^2]-b^2(k+s)^2z[3(t+\beta)^4+4z(t+\beta)^2-72z^2]-b^3[3(t+\beta)^6-2z\left(11(t+\beta)^4-4z(t+\beta)^2-32z^2\right)]\}}{2z(t+\beta)\{3(k+s)^6z^2+2bz(k+s)^4[3(t+\beta)^2-13z]-2b^3[(t+\beta)^4+32z^2-16z(t+\beta)^2]+b^2(k+s)^2[3(t+\beta)^4+72z^2-28z(t+\beta)^2]\}}$.
(iii) If $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \le e_0 \le \widetilde{e}$, *then* $\frac{\partial\left(g^{C*}-g^{BCS*}\right)}{\partial t} > 0$; *if* $\widetilde{e} \le e_0 \le \frac{(a-c)}{t+\beta}$, *then* $\frac{\partial\left(g^{C*}-g^{BCS*}\right)}{\partial t} < 0$, *in which* $\widetilde{e} = \frac{2b(a-c)(t+\beta)\{40b^2z^2-24bz[(k+s)^2z+b(t+\beta)^2]+3[(k+s)^2z+b(t+\beta)^2]^2\}}{[8bz-3(k+s)^2z-3b(t+\beta)^2][4bz-(k+s)^2z-b(t+\beta)^2][2bz-(k+s)^2z-b(t+\beta)^2]+4zb^2(t+\beta)^2[8bz-3(k+s)^2z-3b(t+\beta)^2]}$.

It can be seen from Proposition 9 that (i) with the increase in the subsidy coefficient, the centralised decision-making regime and the BCS contract show an increasing difference in carbon-emission and energy-saving levels. This indicates that the subsidy policy can better facilitate energy saving and emissions reductions under a centralised decision-making framework, or that the subsidy policy drives a greater loss of efficiency with regards to energy-saving and emissions-reduction decisions under each of the two decision models.

In addition, (ii) at a low initial carbon-emission level, the carbon-emissions and energy-saving levels under the centralised decision-making framework show increasingly greater differences (efficiency loss in decision-making) with production under a BCS contract and an increase in the carbon tax. When the initial carbon-emissions level is high, the above difference (efficiency loss in decisions) decreases continuously as the carbon tax increases. In line with Proposition 7, the carbon tax prompts energy saving and emissions reduction measures among manufacturers with low initial pollution levels. However, the energy saving and emissions reductions present greater efficiency under a centralised decision-making framework, so the carbon tax policy increases the loss of efficiency under the two decision models. For a manufacturer with a heavy initial pollution output, the carbon tax policy inhibits emissions reduction, so the carbon tax policy decreases the efficiency loss under the two decision models.

Proposition 10. *(i)* $\frac{\partial\left(\pi^{C*}-\pi^{BCS*}\right)}{\partial s} > 0$; *(ii) If* $\frac{b(a-c)(t+\beta)}{\left[2b-(k+s)^2\right]z} \le e_0 \le \underline{e}$, *then* $\frac{\partial\left(\pi^{C*}-\pi^{BCS*}\right)}{\partial t} > 0$; *if* $\underline{e} \le e_0 \le \frac{(a-c)}{t+\beta}$, *then* $\frac{\partial\left(\pi^{C*}-\pi^{BCS*}\right)}{\partial t} < 0$.

Proposition 10 reveals that (i) the difference between the total profits of the supply chain under centralised decision-making and a BCS contract increases with an increase in the subsidy coefficient. This suggests that, although a BCS contract realises the Pareto improvement of the supply chain, the subsidy policy reduces the coordination efficiency $\mu(\mu = \frac{\pi^{BCS*}}{\pi^{C*}})$ of the contract for the supply chain.

Proposition 10 also shows that (ii) the difference between the total profits of the supply chain under centralised decision-making and a BCS contract increases with increasing carbon tax when the initial carbon-emissions level is low. With a high initial carbon-emissions level, the difference between the profits decreases with an increase in the carbon tax. That is to say, the influences of the carbon tax policy upon the coordination efficiency of the supply chain depend on the initial carbon-emissions level.

7. Numerical Analysis

To verify the correctness of the Propositions 1 to 10, without loss of generality, we suppose that $a = 20,\ c = 10,\ \beta = 0.5,\ t = 2(0.3), e_0 = 3,\ k = 1,\ z = 8,\ b = 4$, and $s = 1$, on the premise of meeting the assumptions ($2bz > (k+s)^2 z + b(t+\beta)^2$ and $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \le e_0 \le \frac{(a-c)}{t+\beta}$) of the model. In this way, Figures 3–17 can be plotted.

As shown in Figure 3, compared with conditions under an NCS contract, the cost-sharing contracts offered by the retailer can improve the energy-saving level. Among the cost-sharing contracts, a BCS contract can obtain the optimal effect, followed by the ECS contract; the CCS contract shows the least effect. Regardless of conditions, the energy-saving level always increases with increasing subsidy coefficient.

Figure 4 indicates that cost-sharing contracts provided by the retailer are able to decrease the carbon-emissions level more than NCS contracts. The BCS contract exhibits the optimal effect, which is followed by the CCS and ECS contracts, successively; the carbon-emissions level always decreases with an increasing subsidy coefficient, under any conditions.

The following two conclusions can be drawn from Figure 5: first, the retailer can increase its profit by providing cost-sharing contracts to a greater extent than is possible under an NCS contract. Retailer profits are highest under a BCS contract. When the subsidy coefficient is small, the CCS contract presents more prominent promotional effects, while the ECS contract has the better effect when the subsidy coefficient is large. Second, the profits of the retailer always grow with an increasing subsidy coefficient.

Figure 6 shows, firstly, that compared with the NCS contract, cost-sharing contracts provided by the retailer are able to improve the profits of the manufacturer; of the contracts, the BCS contract endows the manufacturer with the highest profit. The CCS contract has the more apparent facilitation effect when the subsidy coefficient is small; when the subsidy coefficient is large, the ECS contract exhibits the better effect. Figure 6 also shows that regardless of conditions, the profit of the manufacturer continues to grow with an increasing subsidy coefficient.

As shown in Figure 7, the proportional shares are larger when the retailer provides single cost-sharing contracts, compared with those under a BCS contract. The share of the energy-saving costs is greater when the subsidy coefficient is small; while when the subsidy coefficient is large, the sharing proportion for emission-reduction cost is greater. In addition, the share always increases with increases in the subsidy coefficient, under any conditions.

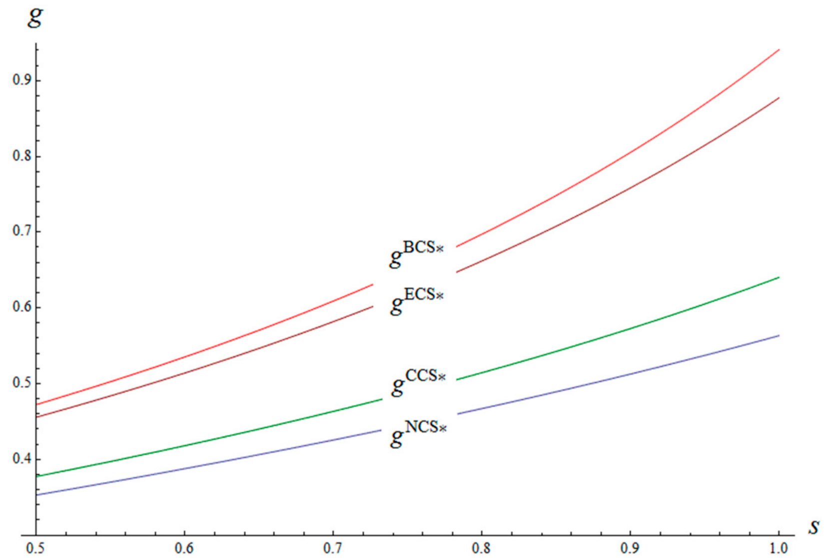

Figure 3. Comparison of energy-saving levels under different contracts.

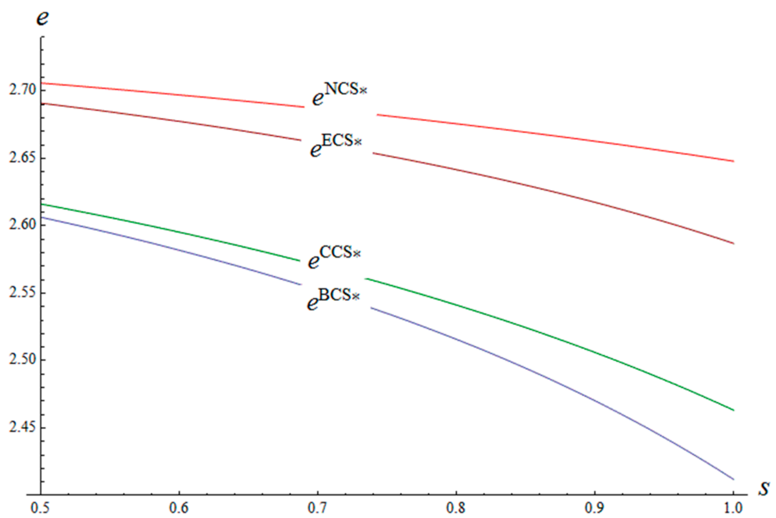

Figure 4. Comparison of carbon-emission levels under different contracts.

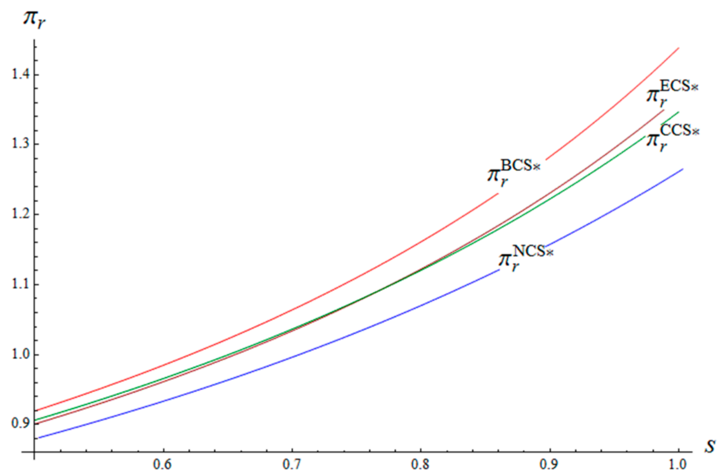

Figure 5. Comparison of the profits of the retailer under different contracts.

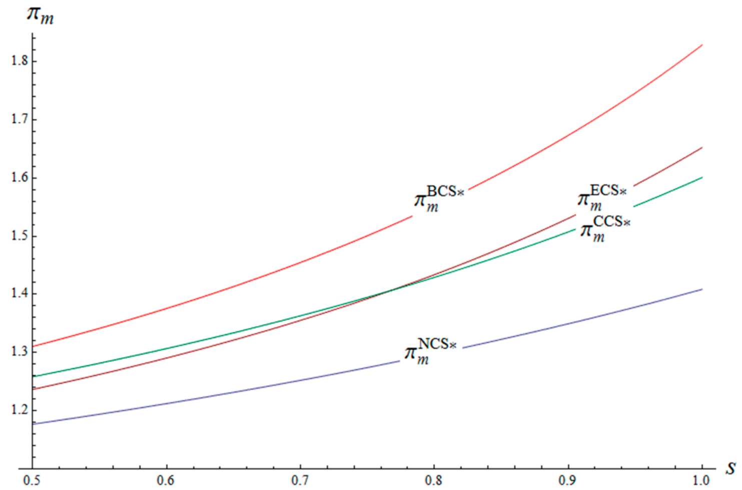

Figure 6. Comparison of the profits of the manufacturer under different contracts.

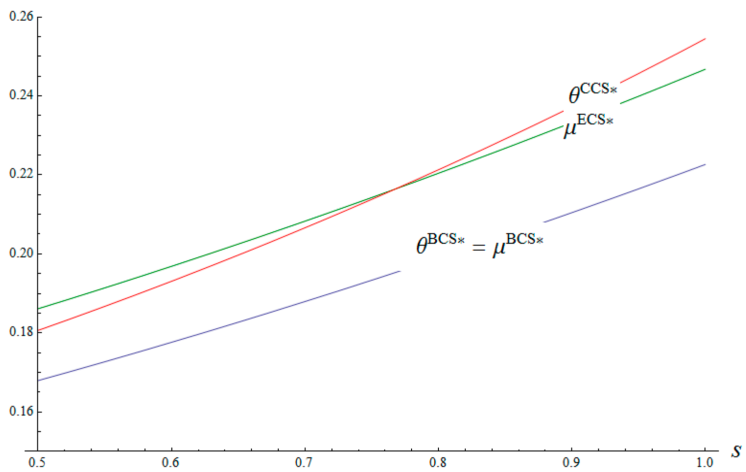

Figure 7. Comparison of cost-sharing proportions under different contracts.

It can be observed from Figure 8 that the energy-saving level continues to grow with increases in the subsidy coefficient, under either centralised decision-making conditions or a BCS contract;

however, the energy-saving level under a centralised decision-making framework is higher than that under the BCS contract, and their difference increases with an increasing subsidy coefficient.

Figure 9 shows that the carbon-emissions level under centralised decision-making regimes is lower than that under a BCS contract, and the gap between them increases with an increasing subsidy coefficient. The carbon-emissions levels under both conditions increase with the growing subsidy coefficient.

It can be seen from Figure 10 that the total profit of the supply chain under a centralised decision-making regime is greater than that under a BCS contract, and the difference between them also increases with an increasing subsidy coefficient. Under both conditions, the subsidy coefficient always promotes a greater total profit in the supply chain.

As shown in Figures 11 and 12, with an increasing carbon tax under a BCS contract, the cost-share increases as the profits of the manufacturer and retailer decrease.

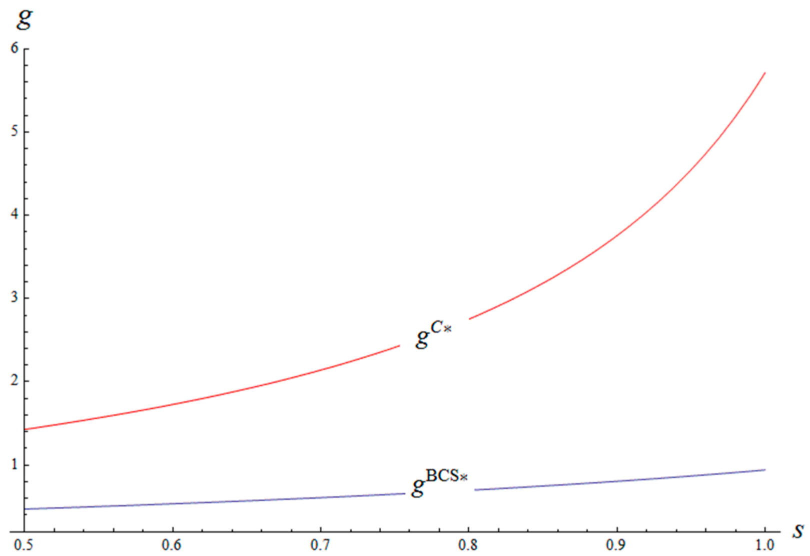

Figure 8. Comparison of energy-saving levels under a bivariate cost-sharing (BCS) model and centralised model.

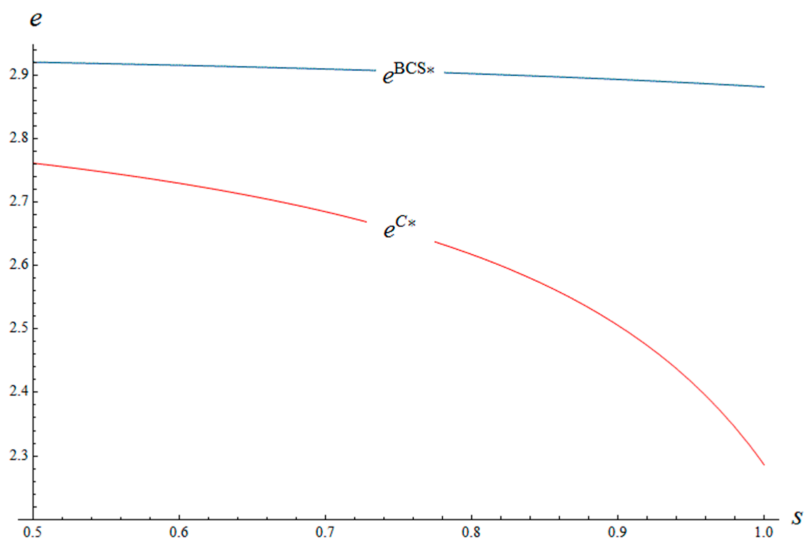

Figure 9. Comparison of the carbon-emissions levels under a BCS model and centralised model.

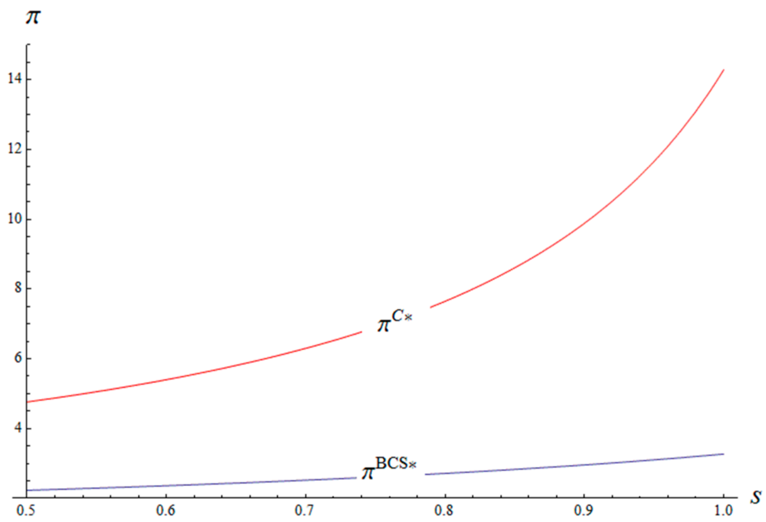

Figure 10. Comparison of the profits of the supply chain under a BCS model and centralised model.

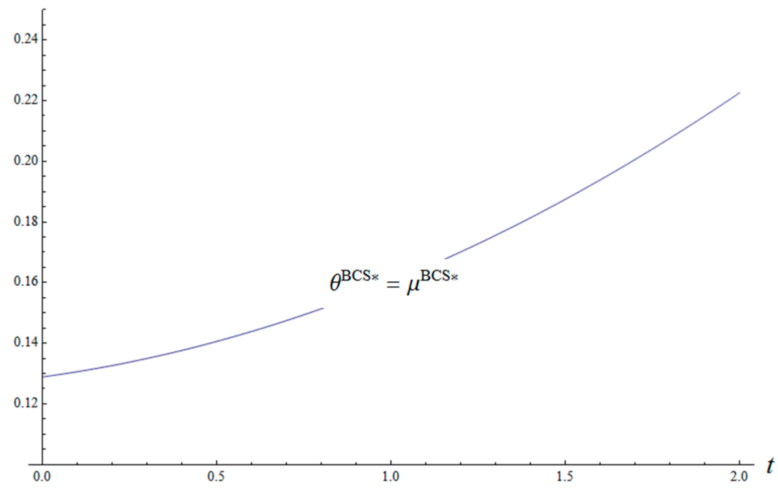

Figure 11. Influence of carbon tax on the proportional cost-share (BCS contract).

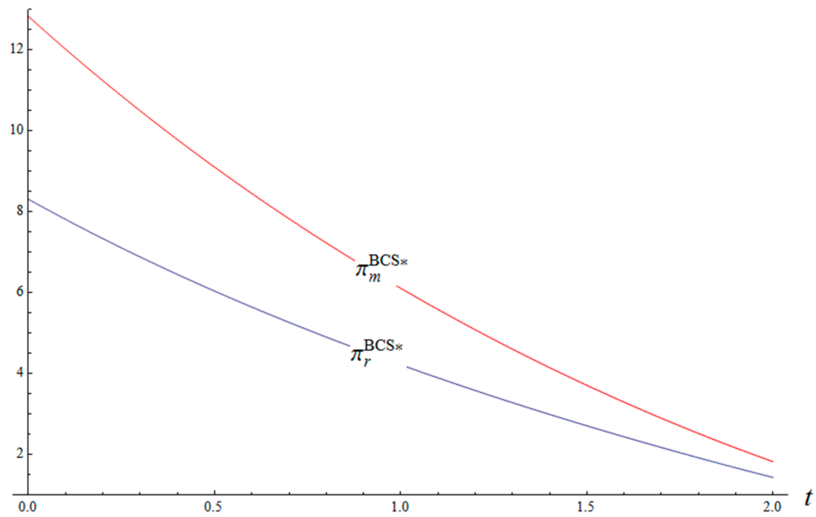

Figure 12. Influence of carbon tax on profit (BCS contract).

Figure 13 shows that, when the carbon tax is small, the carbon-emissions level decreasesas the carbon tax increases, when the initial carbon-emissions level is low, and vice versa.

Figure 14 shows that under a heavy carbon tax, the carbon-emissions level always increases with increasing carbon tax, regardless of the initial carbon-emissions level.

Combining Figures 13 and 14, it can be seen that, regardless of the level of carbon tax, the gap between the carbon-emissions levels under centralised decision-making regimes and a BCS contract increases with increases in the carbon tax, if the initial carbon-emissions level is low; when the initial carbon-emissions level is high, the gap between them narrows with the increasing carbon tax.

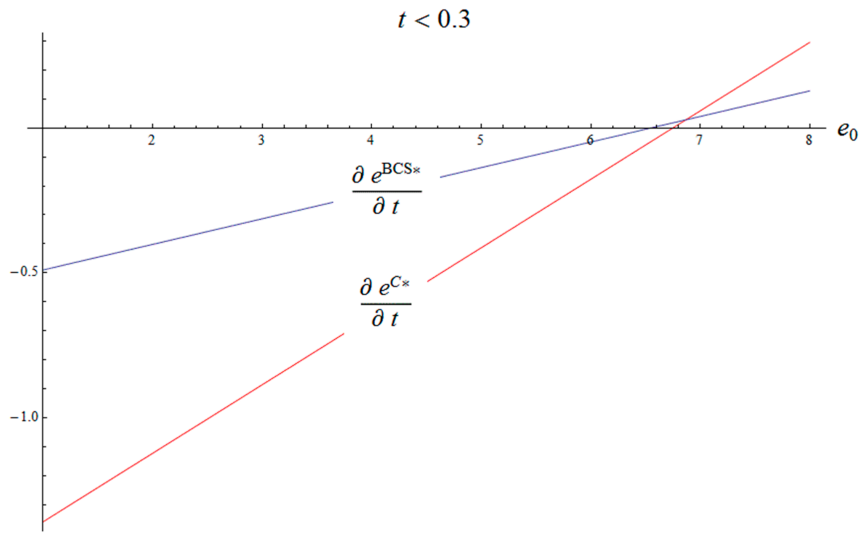

Figure 13. Influence of carbon tax on carbon-emissions level ($t < 0.3$).

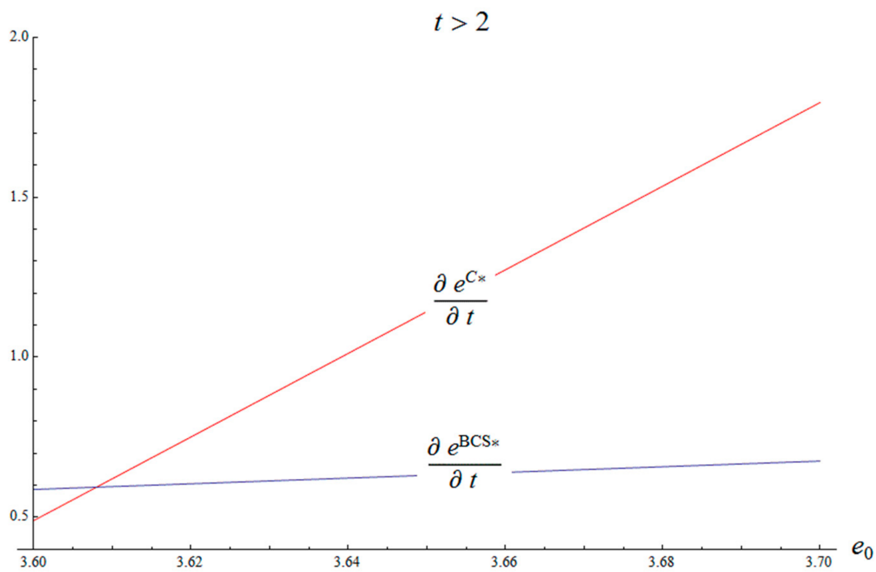

Figure 14. Influence of carbon tax on carbon-emissions level ($t > 2$).

When the carbon tax is small, the energy-saving grows with the increasing carbon tax, if the initial carbon-emissions level is low, and vice versa (Figure 15).

Figure 16 shows that, under a high carbon tax, the energy-saving level declines with an increasing carbon tax, regardless of initial carbon-emissions level.

Figures 15 and 16 show that the difference between the energy-saving levels under a centralised decision-making regime and a BCS contract increases with an increasing carbon tax if the initial carbon-emissions level is low; however, when the initial carbon-emissions level is high, the difference becomes smaller as the carbon tax increases.

As shown in Figure 17, the total profit of the supply chain decreases with increases in the carbon tax. When the initial carbon-emissions level is low, the difference between the total profits of the supply chain under a centralised decision-making regime and a BCS contract increases; if the initial carbon-emissions level is high, the difference gradually decreases.

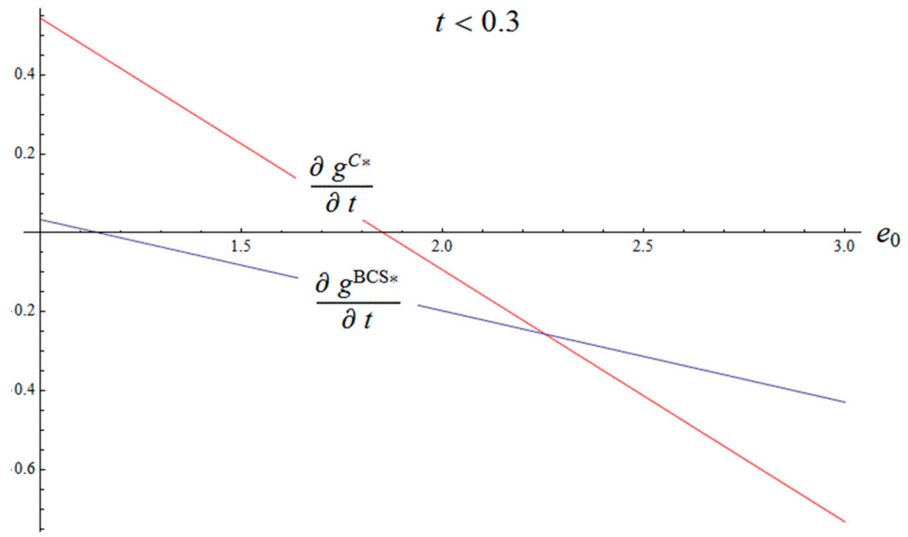

Figure 15. Influence of carbon tax on energy-saving levels ($t < 0.3$).

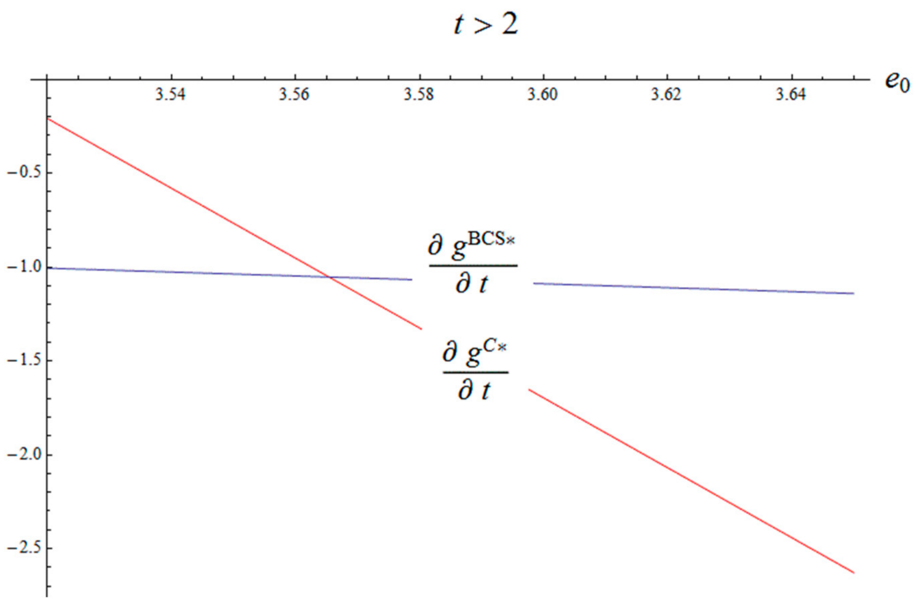

Figure 16. Influence of carbon tax on energy-saving levels ($t > 2$).

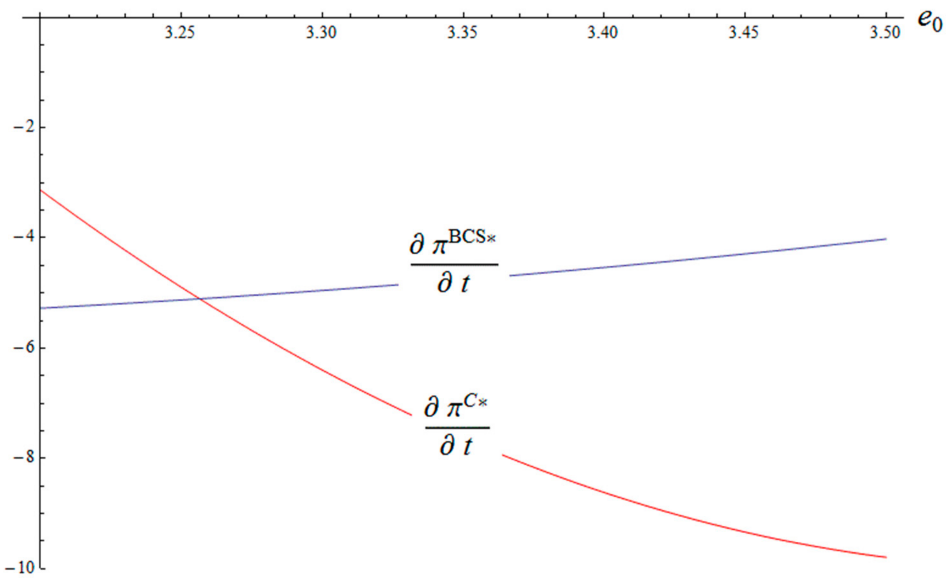

Figure 17. Influence of carbon tax on the total profit of the supply chain.

8. Conclusions

This research focuses on the influences of cost-sharing contracts and government policies on the energy-saving and emissions-reduction decisions of the manufacturer and the profits of the upstream and downstream enterprises in the supply chain. For this purpose, a two-echelon supply chain consisting of one manufacturer and one retailer was built. In the supply chain, the retailer initially provides an incentive contract (NCS, ECS, CCS, or BCS contract) to the manufacturer. Then, the manufacturer decided the energy-saving level, emissions-reduction level, and wholesale price. Finally, the retailer determined the retail price. Afterwards, the cost-sharing proportions, energy-saving levels, carbon-emissions levels, prices, and profits of the enterprises under different contracts were compared under the following conditions: the government provides subsidies for energy-saving products and levies a carbon tax on carbon emissions, and consumers prefer energy-saving, environmentally-friendly products. We also analysed the influences of subsidies and carbon tax policies on decision-making and profits in the supply chain. The following conclusions may be drawn:

(1) Compared with an NCS contract, cost-sharing contracts can better promote energy saving and emissions reduction among manufacturers, and are also beneficial to the profits of the enterprises in the supply chain.

(2) If the retailer only offers a single cost-sharing contract, then the following conclusions apply: the manufacturer pays more attention to the input with regards to emissions reduction (the carbon-emission level thus decreases) when the retailer provides a CCS contract, while the manufacturer focuses on energy-saving R&D (energy-saving levels become higher) when the retailer offers an ECS contract. The manufacturer and the retailer pay more attention to emissions reduction when the carbon tax is imposed by the government and the carbon emissions consciousness of consumers are high enough. Under these conditions, a CCS contract is better able to improve the profits of the two parties, compared with an ECS contract. When the energy-saving subsidies from the government and consumer awareness of energy conservation are high enough, the manufacturer and retailer tend to pay closer attention to energy saving. In that context, the ECS contract earns both parties more profit than under a CCS contract.

(3) The BCS contract (from the retailer) is superior to single cost-sharing contracts, and brings about smaller cost-shares than those under a single cost-sharing contract. Although the BCS contract improves the supply chain, the energy-saving level and carbon-emissions level of the manufacturer,

as well as the total profit of the supply chain, are lower than those under a centralised decision-making framework. In other words, the BCS contract fails to coordinate the supply chain perfectly.

(4) An increase in the subsidy coefficient not only improves cost-sharing proportions, but also facilitates energy saving and emissions reduction along the supply chain. Aside from these, it also boosts the profits of the retailer and the manufacturer. Therefore, the government should actively implement a subsidy policy and enhance the intensity thereof.

(5) The improvement of the carbon tax regime increases the cost-sharing proportion and reduces the profits of the retailer and the manufacturer. Under conditions in which the government levies a lower carbon tax, a carbon tax policy is able to promote energy savings and emissions reduction among enterprises with low initial pollution levels; however, for enterprises generating high initial pollution levels, such a policy curbs energy savings and emissions reductions. If the government imposes too high a carbon tax, the policy always exerts adverse effects, on any type of enterprise. Therefore, government probably cannot obtain the expected result, but instead gets just the opposite, if it blindly levies too high a carbon tax. The government should impose a carbon tax in a discriminative fashion for different types of enterprises: for enterprises with heavy initial pollution loads, the government should not enact too strict a carbon tax policy, but is advised to use a conciliatory policy and increase the energy-saving subsidy thereto. In this way, the manufacturer can have enough funds to carry out technology innovation to reduce carbon emissions and build a benign environmental corporate image. While due to the low emission reduction cost, the manufacturer with a lower initial pollution load is motivated to reduce emissions under the pressure imposed by government increases in carbon tax. Under these conditions, the government is suggested to impose a carbon tax and provide energy-saving subsidies at the same time, to more effectively guide the manufacturer to reducing carbon emissions.

(6) The carbon-emissions level, energy-saving level, and total profit of the supply chain under a centralised decision-making framework have increasingly greater differences from those under a BCS contract with an increasing subsidy coefficient. A subsidy policy decreases the coordination efficiency along the supply chain; therefore, while increasing subsidies, government needs to advocate for more coordination of the supply chain, so that upstream and downstream enterprises therein can systematically carry out energy saving and emission reductions.

(7) For enterprises with low initial pollution levels, the carbon-emissions level, energy-saving level, and total profit of the supply chain under a centralised decision-making framework show growing differences with those under the BCS contract with an increasing carbon tax. This indicates that the increasing carbon tax decreases coordination efficiency in the supply chain. However, for enterprises with a high initial pollution load, these differences, under a centralised decision-making framework and a BCS contract decrease with an increasing carbon tax. This implies that an increasing carbon tax increases the coordination efficiency of the supply chain; therefore, for those manufacturers initially generating less pollution, government needs to support close cooperation between upstream and downstream enterprises and encourage production when it increases its carbon tax. As for manufacturers generating heavy initial pollution loads, the government is advised to reduce their total carbon emissions by limiting the productivity of the supply chain, rather than encouraging joint decisions between upstream and downstream enterprises. This is because, even given cooperation between upstream and downstream enterprises in the supply chain, this fails to obtain satisfactory emissions reduction effects.

Some aspects of the research warrant further work: (1) the research assumes that the manufacturer decides the wholesale price. In fact, considering that the retailer provides a cost-sharing contract to help the manufacturer, the manufacturer, in return, will probably offer a wholesale price discount contract or bargain with the retailer to decide the wholesale price jointly. Next, (2) we studied a two-echelon supply chain under monopolistic conditions. Therefore, the influences of the contract coordination of upstream and downstream enterprises on energy savings and emissions reductions, and the profit in the supply chain under competitive conditions (two competitive manufacturers or retailers), remain to be discussed. Finally, (3) the study is based on complete information—however, in reality, the manufacturer knows its R&D cost, which is not clearly learnt by the retailer, and the retailer has more information about market conditions. Therefore, how the decisions of the retailer and the manufacturer will change under conditions of incomplete information remains to be seen.

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Author Contributions: Yi Yuyin designed the model, improved the model, wrote and revised the paper. Li Jinxi analyzed the model and performed the numerical simulation. All authors contributed to discussion and revision of the manuscript.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix

Appendix A.1 Proof of Theorem 1

Under centralized decision-making, the total profit of the supply chain is derived from p, g, e:

$$\frac{\partial\pi}{\partial p} = a + c + gk - 2p - gs + et - e\beta,$$

$$\frac{\partial\pi}{\partial g} = -bg - ck + kp + as + 2gks - ps - ekt - es\beta,$$

$$\frac{\partial\pi}{\partial e} = -at + pt - ez + c\beta - p\beta + 2et\beta - g(kt + s\beta) + ze_0.$$

The corresponding Hessian Matrix is $H(p,g,e) = \begin{bmatrix} -2 & k-s & t-\beta \\ k-s & 2ks-b & -kt-s\beta \\ t-\beta & -kt-s\beta & 2t\beta-z \end{bmatrix}$. As $2bz > (k+s)^2z + (t+\beta)^2b$, then $(b-2ks)z + (z-2t\beta)b > \left(k^2+s^2\right)z + \left(t^2+\beta^2\right)b$. We have the first-order master $-2 < 0$, $2ks - b < 0$, $2t\beta - z < 0$, the second-order master $H_1 = (-2)(2ks-b) - (k-s)^2 = 2b - (k+s)^2 > 0$, $H_2 = (-2)(2t\beta - z) - (t-\beta)^2 = 2z - (t+\beta)^2 > 0$, $H_3 = (2ks-b)(2t\beta - z) - (-kt-s\beta)^2 > \left(k^2+s^2\right)\left(t^2+\beta^2\right) - (kt+s\beta)^2 = (st-k\beta)^2 > 0$, and the third-order master $(k+s)^2z + (t+\beta)^2b - 2bz < 0$. Therefore, the total profit of the supply chain is a concave function of p, g, e. Let $\frac{\partial\pi}{\partial p} = 0$, $\frac{\partial\pi}{\partial g} = 0$, $\frac{\partial\pi}{\partial e} = 0$, and solving the equations can obtain equilibrium retail price, energy-saving level, and carbon emissions level. In order to make the equilibrium solution e^{C*} satisfy the condition of $0 \le e \le e_0$, we have $\frac{b(a-c)(t+\beta)}{\left[2b-(k+s)^2\right]z} \le e_0 \le \frac{(a-c)}{t+\beta}$. Finally, the equilibrium solutions are used to replace the profit and demand function in the supply chain. Theorem 1 is available.

Appendix A.2 Proof of Theorem 2

Goingbackwards, the retailer decides the retail price first,

$$\frac{\partial\pi_r^{NCS}}{\partial p} = a + gk - 2p + w - e\beta,$$

Make it equal to 0, and we get $p^* = \frac{1}{2}(a + gk + w - e\beta)$. And $\frac{\partial^2\pi_r^{NCS}}{\partial p^2} = -2 < 0$, i.e., π_r^{NCS} is a concave function of p.

Next, the manufacturer decides the energy-saving level, carbon-emissions level and wholesale price,

$$\frac{\partial\pi_m^{NCS}}{\partial w} = a + c + gk - 2p - gs + et - e\beta,$$

$$\frac{\partial \pi_m^{NCS}}{\partial g} = \frac{1}{2}(-2bg - ck + as + 2gks - ekt + kw - sw - es\beta),$$

$$\frac{\partial \pi_m^{NCS}}{\partial e} = \frac{1}{2}[-at + tw - 2ez + c\beta + 2et\beta - w\beta - g(kt + s\beta) + 2ze_0].$$

The corresponding Hessian Matrix is $F(w, g, e) = \begin{bmatrix} -1 & \frac{k-s}{2} & \frac{t-\beta}{2} \\ \frac{k-s}{2} & ks - b & \frac{1}{2}(-kt - s\beta) \\ \frac{t-\beta}{2} & \frac{1}{2}(-kt - s\beta) & t\beta - z \end{bmatrix}$. By the condition of $2bz > (k+s)^2 z + (t+\beta)^2 b$ and the proof of Theorem 1, we have that F is a negative definite matrix, i.e., π_m^{NCS} is a concave function of w, g, e. Let $\frac{\partial \pi_m^{NCS}}{\partial w} = 0$, $\frac{\partial \pi_m^{NCS}}{\partial g} = 0$, $\frac{\partial \pi_m^{NCS}}{\partial e} = 0$, and solving the equations can obtain the equilibrium wholesale price, energy-saving level, and carbon-emissions level. Also, when $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \leq e_0 \leq \frac{(a-c)}{t+\beta}$, then $e^{NCS*} \in [0, e_0]$. Finally, the equilibrium solutions are used to replace the retail price, profit, and demand function in the supply chain. Theorem 2 is available.

Appendix A.3 Proof of Theorem 5

Since the proofs of Theorems 3 and 4 are similar to the proof of Theorem 5, we mainly present the proof of Theorem 5, and omitted the proofs of Theorems 3 and 4 here.

The decision-making of the retail price is similar to Theorem 2. Solve for the manufacturer's decision-making:

$$\frac{\partial \pi_m^{BCS}}{\partial w} = \frac{1}{2}(a + c + gk - gs + et - 2w - e\beta),$$

$$\frac{\partial \pi_m^{BCS}}{\partial g} = \frac{1}{2}[as - ck + 2gks - ekt + kw - sw - es\beta - 2bg(1-\mu)],$$

$$\frac{\partial \pi_m^{BCS}}{\partial e} = \frac{1}{2}[tw - at - 2ez + c\beta + 2et\beta - w\beta - g(kt + s\beta) + 2ez\theta + 2ze_0 - 2z\theta e_0].$$

Let $\frac{\partial \pi_m^{BCS}}{\partial w} = 0$, $\frac{\partial \pi_m^{BCS}}{\partial g} = 0$, $\frac{\partial \pi_m^{BCS}}{\partial e} = 0$, and solving the equations can obtain $w^* = \frac{1}{2}(a + c + g^*k - g^*s + e^*t - e^*\beta)$, $e^* = \frac{ab(t+\beta)(1-\mu) + (k+s)^2 z(1-\theta)e_0 - b(1-\mu)[c(t+\beta) + 4z(1-\theta)e_0]}{(k+s)^2 z(1-\theta) + b(1-\mu)[(t+\beta)^2 - 4z(1-\theta)]}$, $g^* = \frac{(k+s)z(1-\theta)(c + te_0 + \beta e_0 - a)}{(k+s)^2 z(1-\theta) + b(1-\mu)[(t+\beta)^2 - 4z(1-\theta)]}$.

Substitute w^*, e^*, g^* for the retailer's profit, and take the derivative of the energy saving cost-sharing ratio and emissions reduction cost-sharing ratio,

$$\frac{\partial \pi_r^{BCS}}{\partial \theta} = (bz(((k+s)^2 z(-1+\theta) + b(t^2 + 2t\beta + \beta^2 + 4z(-1+\theta))(-1+\mu))(b(-(t+\beta)^2 + 4z(-1+\theta))(-1+\mu)^2 - 2(k+s)^2 z(-1+\theta)\mu) - 2z((k+s)^2 + 4b(-1+\mu))(b(2z(-1+\theta)^2 - (t+\beta)^2\theta)(-1+\mu)^2 - (k+s)^2 z(-1+\theta)^2\mu))(-a + c + (t+\beta)e_0)^2) / (2((k+s)^2 z(-1+\theta) + b(t^2 + 2t\beta + \beta^2 + 4z(-1+\theta))(-1+\mu))^3),$$

$$\frac{\partial \pi_r^{BCS}}{\partial \mu} = (bz(((k+s)^2 z(-1+\theta) + b(t^2 + 2t\beta + \beta^2 + 4z(-1+\theta))(-1+\mu))(-(k+s)^2 z(-1+\theta)^2 + 2b(2z(-1+\theta)^2 - (t+\beta)^2\theta)(-1+\mu)) - 2b(t^2 + 2t\beta + \beta^2 + 4z(-1+\theta))(b(2z(-1+\theta)^2 - (t+\beta)^2\theta)(-1+\mu)^2 - (k+s)^2 z(-1+\theta)^2\mu))(-a + c + (t+\beta)e_0)^2) / (2((k+s)^2 z(-1+\theta) + b(t^2 + 2t\beta + \beta^2 + 4z(-1+\theta))(-1+\mu))^3).$$

Let $\frac{\partial \pi_r^{BCS}}{\partial \theta} = 0$, $\frac{\partial \pi_r^{BCS}}{\partial \mu} = 0$, and we have three sets of solutions: $\mu_1^* = \theta_1^* = \frac{(k+s)^2 z + b(t+\beta)^2}{8zb}$; $\mu_2^* = 1, \theta_2^* = \frac{8bz - (k+s)^2 z - 2b(t+\beta)^2}{[8b-(k+s)^2]z}$; and $\mu_3^* = \frac{8bz - 2z(k+s)^2 - b(t+\beta)^2}{[8z-(k+s)^2]b}$, $\theta_3^* = 1$.

Substitute $\mu_2^* = 1,\ \theta_2^* = \frac{8bz-(k+s)^2z-2b(t+\beta)^2}{[8b-(k+s)^2]z}$ for the reaction function of the energy-saving level, and we can obtain $g^* = \frac{c+te_0+\beta e_0-a}{k+s}$. Since $e_0 \le \frac{(a-c)}{t+\beta}$, $g^* \le 0$ does not fit the hypothesis of $g > 0$, delete the solution.

Similarly, substitute $\mu_3^* = \frac{8bz-2z(k+s)^2-b(t+\beta)^2}{[8z-(k+s)^2]b}$, $\theta_3^* = 1$ for the reaction function of the energy-saving level, and we obtain $g^* = 0$, it doesn't fit the hypothesis of $g > 0$, so it is deleted.

Finally, we need to verify that (μ_1^*, θ_1^*) is the optimal solution for retailer. When $\mu_1^* = \theta_1^* = \frac{(k+s)^2z+b(t+\beta)^2}{8zb}$, we have $\frac{\partial^2\pi_r^{BCS}}{\partial\mu^2} = \frac{32b^2(k+s)^2z^4[(k+s)^4z+b(k+s)^2(t^2-16z+2t\beta+\beta^2)-8b^2(3t^2-8z+6t\beta+3\beta^2)](c+te_0+\beta e_0-a)^2}{[(k+s)^2z+b(t^2-8z+2t\beta+\beta^2)]^2[3(k+s)^2z+b(3t^2-8z+6t\beta+3\beta^2)]^3}$. By $2bz > (k+s)^2z + (t+\beta)^2b$, then $(k+s)^4z + b(k+s)^2(t^2-16z+2t\beta+\beta^2) - 8b^2(3t^2-8z+6t\beta+3\beta^2) > 0$, $3(k+s)^2z + b(3t^2-8z+6t\beta+3\beta^2) < 0$, $\frac{\partial^2\pi_r^{BCS}}{\partial\mu^2} < 0$. Similarly, we can obtain $\frac{\partial^2\pi_r^{BCS}}{\partial\theta^2} = \frac{32b^4z^2(t+\beta)^2[(k+s)^2z(t^2-24z+2t\beta+\beta^2)+b(t^2-8z+2t\beta+\beta^2)^2](c+te_0+\beta e_0-a)^2}{[(k+s)^2z+b(t^2-8z+2t\beta+\beta^2)]^2[3(k+s)^2z+b(3t^2-8z+6t\beta+3\beta^2)]^3} < 0$ and $\frac{\partial^2\pi_r^{BCS}}{\partial\mu^2} * \frac{\partial^2\pi_r^{BCS}}{\partial\theta^2} - \frac{\partial^2\pi_r^{BCS}}{\partial\mu\partial\theta} * \frac{\partial^2\pi_r^{BCS}}{\partial\theta\partial\mu} = \frac{8192b^6(k+s)^2z^6(t+\beta)^2(c+te_0+\beta e_0-a)^4}{[(k+s)^2z+b(t^2-8z+2t\beta+\beta^2)]^2[8bz-3(k+s)^2z-3b(t+\beta)^2]^5} > 0$ when $\mu_1^* = \theta_1^* = \frac{(k+s)^2z+b(t+\beta)^2}{8zb}$. Therefore, the Hessian Matrix corresponding to (μ, θ) is negative, and $\mu_1^* = \theta_1^* = \frac{(k+s)^2z+b(t+\beta)^2}{8zb}$ is the optimal solution.

Substitute $\mu_1^*,\ \theta_1^*$ for w^*, e^*, g^*, and we have

$$g^* = \frac{2z(k+s)(a-c-te_0-\beta e_0)}{8bz-3z(k+s)^2-3b(t+\beta)^2},\ e^* = e_0 - \frac{2b(t+\beta)(a-c-te_0-\beta e_0)}{8bz-3z(k+s)^2-3b(t+\beta)^2},$$
$$w^* = \frac{E}{2[8bz-3z(k+s)^2-3b(t+\beta)^2]}.$$

where $E = (a((k^2+6ks+5s^2)z + b(5t^2-8z+6t\beta+\beta^2)) + (k+s)z(c(5k+s) + (5kt+st-k\beta-5s\beta)e_0) + b(c(t^2-8z+6t\beta+5\beta^2) + (t-\beta)(t^2-8z+2t\beta+\beta^2)e_0))$.

The manufacturer's Hessian Matrix corresponding to (w, g, e) is $N = \begin{bmatrix} -1 & \frac{k-s}{2} & \frac{t-\beta}{2} \\ \frac{k-s}{2} & ks - b(1-\frac{(k+s)^2z+b(t+\beta)^2}{8zb}) & \frac{1}{2}(-kt-s\beta) \\ \frac{t-\beta}{2} & \frac{1}{2}(-kt-s\beta) & t\beta - z(1-\frac{(k+s)^2z+b(t+\beta)^2}{8zb}) \end{bmatrix}$. By $2bz > (k+s)^2z + (t+\beta)^2b$ and the proof of Theorem 1 and 2, we have that N is a negative definite matrix. Therefore, g^*, e^*, and w^* maximize the manufacturer's profit. In addition, when $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \le e_0 \le \frac{(a-c)}{t+\beta}$, $e^* \in [0, e_0]$. Finally, the equilibrium solutions are used to replace the retail price, profit, and demand function in the supply chain. Theorem 5 is available.

Appendix A.4 Proof of Proposition 1

(i) $g^{ECS*} - g^{NCS*} = \frac{z(k+s)[8z-(t+\beta)^2](a-c-te_0-\beta e_0)}{2[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2]} - \frac{(k+s)z(c+te_0+\beta e_0-a)}{(k+s)^2z+b(t^2-4z+2t\beta+\beta^2)} = \frac{z(k+s)[4z-(t+\beta)^2][(k+s)^2z+b(t+\beta)^2](a-c-te_0-\beta e_0)}{2[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2][4bz-(k+s)^2z-(t+\beta)^2b]}$.. Since $\frac{a-c}{t+\beta} \ge e_0$ and $2bz > (k+s)^2z + (t+\beta)^2b$, we can obtain $g^{\ ECS*} - g^{NCS*} > 0$, i.e., $g^{ECS*} > g^{NCS*}$.

$$e^{ECS*} - e^{NCS*} = -\frac{z(t+\beta)(k+s)^2[(k+s)^2z+b(t+\beta)^2](a-c-te_0-\beta e_0)}{2[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2][4bz-(k+s)^2z-(t+\beta)^2b]} < 0.$$

(ii) $\pi_r^{ECS*} - \pi_r^{NCS*} = \frac{[8b+(k+s)^2]z^2(a-c-te_0-\beta e_0)^2}{8[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2]} - \frac{b^2z^2(a-c-te_0-\beta e_0)^2}{[(k+s)^2z+b(t^2-4z+2t\beta+\beta^2)]^2} = \frac{(k+s)^2z^2[(k+s)^2z+b(t+\beta)^2]^2(a-c-te_0-\beta e_0)^2}{8[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2][(k+s)^2z+b(t^2-4z+2t\beta+\beta^2)]^2} > 0$, i.e., $\pi_r^{ECS*} > \pi_r^{NCS*}$.

$$\pi_m^{ECS*} - \pi_m^{NCS*} = \frac{(k+s)^2z^2[(k+s)^2z+b(t+\beta)^2](a-c-te_0-\beta e_0)^2}{4[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2][4bz-(k+s)^2z-(t+\beta)^2b]} > 0.$$

Appendix A.5 Proof of Proposition 2

(i) $g^{ECS*} - g^{CCS*} = \frac{z(k+s)[8z-(t+\beta)^2](a-c-te_0-\beta e_0)}{2[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2]} - \frac{(k+s)[8bz-2(k+s)^2z-b(t+\beta)^2](a-c-te_0-\beta e_0)}{2[(k+s)^4z+b(k+s)^2(t^2-8z+2t\beta+\beta^2)-2b^2(3t^2-8z+6t\beta+3\beta^2)]} = \frac{(k+s)(a-c-te_0-\beta e_0)[(k+s)^2z+b(t+\beta)^2]C}{2[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2][(k+s)^4z+b(k+s)^2(t^2-8z+2t\beta+\beta^2)-2b^2(3t^2-8z+6t\beta+3\beta^2)]}$, where $C = (k+s)^2z(t^2-4z+2t\beta+\beta^2) + b[t^4+16z^2+4t^3\beta-10z\beta^2+\beta^4+4t\beta(\beta^2-5z)+t^2(6\beta^2-10z)]$. By the conditions of $2bz > (k+s)^2z + b(t+\beta)^2$ and $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \le e_0 \le \frac{(a-c)}{t+\beta}$, we know whether $g^{ECS*} - g^{CCS*}$ is positive or not asdetermined by C.

Since $\frac{\partial C}{\partial z} = 32bz - 10b(t+\beta)^2 - 8z(k+s)^2 + (k+s)^2(t+\beta)^2$, and by $2bz > (k+s)^2z + b(t+\beta)^2$, we can get $\frac{\partial C}{\partial z} > 0$, i.e., C is monotonically increasing with z. As $C_{min} = C\left(\frac{b(t+\beta)^2}{2b-(k+s)^2}\right) = \frac{4b^2(k+s)^2(t+\beta)^4}{[2b-(k+s)^2]^2} > 0$, we have $C > 0$, i.e., $g^{ECS*} > g^{CCS*}$.

$e^{ECS*} - e^{CCS*} = \frac{(t+\beta)(a-c-te_0-\beta e_0)[(k+s)^2z+b(t+\beta)^2]D}{2[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2][(k+s)^4z+b(k+s)^2(t^2-8z+2t\beta+\beta^2)-2b^2(3t^2-8z+6t\beta+3\beta^2)]}$, where $D = (k+s)^4z + b(k+s)^2(t^2-10z+2t\beta+\beta^2) - 4b^2(t^2-4z+2t\beta+\beta^2)$. Owing to $2bz > (k+s)^2z + b(t+\beta)^2$ and $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \le e_0 \le \frac{(a-c)}{t+\beta}$, we know whether $e^{ECS*} - e^{CCS*}$ is positive or not, as determined by D.

Since $\frac{\partial D}{\partial z} = [2b-(k+s)^2][8b-(k+s)^2]$, and by $2bz > (k+s)^2z + b(t+\beta)^2$, we can obtain $\frac{\partial D}{\partial z} > 0$, i.e., D is monotonically increasing with z. As $D_{min} = D\left(\frac{b(t+\beta)^2}{2b-(k+s)^2}\right) = 4b^2(t+\beta)^2 > 0$, we have $D > 0$, i.e., $e^{ECS*} > e^{CCS*}$.

(ii) $\pi_r^{ECS*} - \pi_r^{CCS*} = \frac{[(k+s)^2z-b(t+\beta)^2][(k+s)^2z+b(t+\beta)^2]^2(a-c-te_0-\beta e_0)^2}{8[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2][(k+s)^4z+b(k+s)^2(t^2-8z+2t\beta+\beta^2)-2b^2(3t^2-8z+6t\beta+3\beta^2)]}$, $\pi_m^{ECS*} - \pi_m^{CCS*} = \frac{[(k+s)^2z-b(t+\beta)^2][(k+s)^2z+b(t+\beta)^2][4bz-(k+s)^2z-b(t+\beta)^2](a-c-te_0-\beta e_0)^2}{4[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2][(k+s)^4z+b(k+s)^2(t^2-8z+2t\beta+\beta^2)-2b^2(3t^2-8z+6t\beta+3\beta^2)]}$, $\mu^{ECS*} - \theta^{CCS*} = \frac{(k+s)^2z+b(t+\beta)^2}{[8z-(t+\beta)^2]b} - \frac{(k+s)^2z+b(t+\beta)^2}{[8b-(k+s)^2]z} = \frac{[(k+s)^2z+b(t+\beta)^2][b(t+\beta)^2-(k+s)^2z]}{[8z-(t+\beta)^2][8b-(k+s)^2]bz}$. Therefore, $\pi_r^{CCS*} > \pi_r^{ECS*}$, $\pi_m^{CCS*} > \pi_m^{ECS*}$, $\theta^{CCS*} < \mu^{ECS*}$ if $t+\beta > \sqrt{\frac{z}{b}}(k+s)$; and $\pi_r^{CCS*} < \pi_r^{ECS*}$, $\pi_m^{CCS*} < \pi_m^{ECS*}$, $\theta^{CCS*} > \mu^{ECS*}$ if $t+\beta < \sqrt{\frac{z}{b}}(k+s)$.

Appendix A.6 Proof of Proposition 3

(i) $\theta^{BCS*} - \theta^{CCS*} = \frac{(k+s)^2z+b(t+\beta)^2}{8bz} - \frac{(k+s)^2z+b(t+\beta)^2}{[8b-(k+s)^2]z} = -\frac{(k+s)^2[(k+s)^2z+b(t+\beta)^2]}{[8b-(k+s)^2]8bz} < 0$, $\mu^{BCS*} - \mu^{ECS*} = \frac{(k+s)^2z+b(t+\beta)^2}{8bz} - \frac{(k+s)^2z+b(t+\beta)^2}{[8z-(k+s)^2]b} = -\frac{(k+s)^2[(k+s)^2z+b(t+\beta)^2]}{[8z-(k+s)^2]8bz} < 0$.

(ii) $g^{BCS*} - g^{ECS*} = \frac{2z(k+s)(a-c-te_0-\beta e_0)}{8bz-3z(k+s)^2-3b(t+\beta)^2} - \frac{z(k+s)[8z-(t+\beta)^2](a-c-te_0-\beta e_0)}{2[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2]} = \frac{z(k+s)(t+\beta)^2(a-c-te_0-\beta e_0)[(k+s)^2z+b(t+\beta)^2]}{2[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2][8bz-3z(k+s)^2-3b(t+\beta)^2]} > 0$, $e^{BCS*} - e^{CCS*} = e_0 - \frac{2b(t+\beta)(a-c-te_0-\beta e_0)}{8bz-3z(k+s)^2-3b(t+\beta)^2} - \left(e_0 - \frac{b(t+\beta)[8b-(k+s)^2](a-c-te_0-\beta e_0)}{2[(k+s)^4z+b(k+s)^2(t^2-8z+2t\beta+\beta^2)-2b^2(3t^2-8z+6t\beta+3\beta^2)]}\right) = \frac{b(t+\beta)(k+s)^2(a-c-te_0-\beta e_0)[(k+s)^2z+b(t+\beta)^2]}{2[(k+s)^4z+b(k+s)^2(t^2-8z+2t\beta+\beta^2)-2b^2(3t^2-8z+6t\beta+3\beta^2)][8bz-3z(k+s)^2-3b(t+\beta)^2]} < 0$.

Appendix A.7 Proof of Proposition 4

(i) $\pi_r^{BCS*} - \pi_r^{ECS*} = \frac{[8bz+z(k+s)^2+b(t+\beta)^2](a-c-te_0-\beta e_0)^2}{16[8bz-3z(k+s)^2-3b(t+\beta)^2]} - \frac{z^2[8b+(k+s)^2](a-c-te_0-\beta e_0)^2}{8[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2]} = \frac{(t+\beta)^2(a-c-te_0-\beta e_0)^2[(k+s)^2z+b(t+\beta)^2]^2}{16[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2][8bz-3z(k+s)^2-3b(t+\beta)^2]} > 0$, $\pi_r^{BCS*} - \pi_r^{CCS*} = \frac{[8bz+z(k+s)^2+b(t+\beta)^2](a-c-te_0-\beta e_0)^2}{16[8bz-3z(k+s)^2-3b(t+\beta)^2]} - \frac{b^2[8z+(t+\beta)^2](a-c-te_0-\beta e_0)^2}{8[(k+s)^4z+b(k+s)^2(t^2-8z+2t\beta+\beta^2)-2b^2(3t^2-8z+6t\beta+3\beta^2)]} = \frac{(k+s)^2(a-c-te_0-\beta e_0)^2[(k+s)^2z+b(t+\beta)^2]^2}{16[(k+s)^4z+b(k+s)^2(t^2-8z+2t\beta+\beta^2)-2b^2(3t^2-8z+6t\beta+3\beta^2)][8bz-3z(k+s)^2-3b(t+\beta)^2]} > 0.$

(ii) $\pi_m^{BCS*} - \pi_m^{ECS*} = \frac{[8bz-z(k+s)^2-b(t+\beta)^2](a-c-te_0-\beta e_0)^2}{8[8bz-3z(k+s)^2-3b(t+\beta)^2]} - \frac{z[8bz-2b(t+\beta)^2-z(k+s)^2](a-c-te_0-\beta e_0)^2}{4[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2]} = \frac{(t+\beta)^2(a-c-te_0-\beta e_0)^2[(k+s)^2z+b(t+\beta)^2][4bz-z(k+s)^2-b(t+\beta)^2]}{8[(k+s)^2z(t^2-6z+2t\beta+\beta^2)+b(t^2-4z+2t\beta+\beta^2)^2][8bz-3z(k+s)^2-3b(t+\beta)^2]} > 0$, $\pi_m^{BCS*} - \pi_m^{CCS*} = \frac{[8bz-z(k+s)^2-b(t+\beta)^2](a-c-te_0-\beta e_0)^2}{8[8bz-3z(k+s)^2-3b(t+\beta)^2]} - \frac{b[8bz-2(k+s)^2z-b(t+\beta)^2](a-c-te_0-\beta e_0)^2}{4[(k+s)^4z+b(k+s)^2(t^2-8z+2t\beta+\beta^2)-2b^2(3t^2-8z+6t\beta+3\beta^2)]} = \frac{(k+s)^2(a-c-te_0-\beta e_0)^2[(k+s)^2z+b(t+\beta)^2][4bz-z(k+s)^2-b(t+\beta)^2]}{8[(k+s)^4z+b(k+s)^2(t^2-8z+2t\beta+\beta^2)-2b^2(3t^2-8z+6t\beta+3\beta^2)][8bz-3z(k+s)^2-3b(t+\beta)^2]} > 0.$

Appendix A.8 Proof of Proposition 5

$e^{C*} - e^{BCS*} = \left[e_0 - \frac{b(t+\beta)(c+te_0+\beta e_0-a)}{(k+s)^2z+b(t^2-2z+2t\beta+\beta^2)}\right] - \left[e_0 - \frac{2b(t+\beta)(c+te_0+\beta e_0-a)}{3(k+s)^2z+b(3t^2-8z+6t\beta+3\beta^2)}\right] = -\frac{b(t+\beta)[(k+s)^2z+b(t+\beta)^2-4bz](c+te_0+\beta e_0-a)}{[(k+s)^2z+b(t^2-2z+2t\beta+\beta^2)][3(k+s)^2z+b(3t^2-8z+6t\beta+3\beta^2)]}$. Since $2bz > (k+s)^2z + b(t+\beta)^2$, we can obtain $e^{C*} - e^{BCS*} < 0$, i.e., $e^{C*} < e^{BCS*}$.

$g^{C*} - g^{BCS*} = \frac{(k+s)z(c+te_0+\beta e_0-a)}{(k+s)^2z+b(t^2-2z+2t\beta+\beta^2)} - \frac{2(k+s)z(c+te_0+\beta e_0-a)}{3(k+s)^2z+b(3t^2-8z+6t\beta+3\beta^2)} = \frac{(k+s)z[(k+s)^2z+b(t+\beta)^2-4bz](c+te_0+\beta e_0-a)}{[(k+s)^2z+b(t^2-2z+2t\beta+\beta^2)][3(k+s)^2z+b(3t^2-8z+6t\beta+3\beta^2)]}$. By $2bz > (k+s)^2z + b(t+\beta)^2$, we get $g^{C*} - g^{BCS*} > 0$, i.e., $g^{C*} > g^{BCS*}$.

Note $M = \frac{3bz(c+te_0+\beta e_0-a)^2}{8[2bz-(k+s)^2z-b(t+\beta)^2]}$, we can get $\pi^{C*} - M = \frac{bz(c+te_0+\beta e_0-a)^2}{8[2bz-(k+s)^2z-b(t+\beta)^2]} > 0$, $M - \pi^{BCS*} = \frac{[8bz-(k+s)^2z-b(t+\beta)^2][(k+s)^2z+b(t+\beta)^2](c+te_0+\beta e_0-a)^2}{16[(k+s)^2z+b(t^2-2z+2t\beta+\beta^2)][3(k+s)^2z+b(3t^2-8z+6t\beta+3\beta^2)]} > 0$, so $\pi^{C*} > \pi^{BCS*}$.

Appendix A.9 Proof of Proposition 6

(i) As $\mu^{BCS*} = \theta^{BCS*} = \frac{(k+s)^2z+b(t+\beta)^2}{8bz}$ and $k, s, b > 0$, we get $\frac{\partial\, \mu^{BCS*}}{\partial s} = \frac{\partial\, \theta^{BCS*}}{\partial s} = \frac{k+s}{4b} > 0$.

(ii) Since $t, z, \beta > 0$, we have $\frac{\partial\, \mu^{BCS*}}{\partial t} = \frac{\partial\, \theta^{BCS*}}{\partial t} = \frac{t+\beta}{4z} > 0$.

Appendix A.10 Proof of Proposition 7

(i) $g^{BCS*} = \frac{2z(k+s)(a-c-te_0-\beta e_0)}{8bz-3z(k+s)^2-3b(t+\beta)^2}$, $e^{BCS*} = e_0 - \frac{2b(t+\beta)(a-c-te_0-\beta e_0)}{8bz-3z(k+s)^2-3b(t+\beta)^2}$. Since $2bz > (k+s)^2z + b(t+\beta)^2, e_0 \leq \frac{(a-c)}{t+\beta}$, we have $\frac{\partial g^{BCS*}}{\partial s} = \frac{2z[8bz-3b(t+\beta)^2+3(k+s)^2z](a-c-te_0-\beta e_0)}{[8bz-3(k+s)^2z-3b(t+\beta)^2]^2} > 0$, $\frac{\partial e^{BCS*}}{\partial s} = -\frac{12bz(k+s)(t+\beta)(a-c-te_0-\beta e_0)}{[8bz-3(k+s)^2z-3b(t+\beta)^2]^2} < 0$.

(ii) $\frac{\partial e^{BCS*}}{\partial t} = \frac{-2b\{(a-c)[8bz-3(k+s)^2z+3b(t+\beta)^2]-2[8b-3(k+s)^2](t+\beta)ze_0\}}{[8bz-3(k+s)^2z-3b(t+\beta)^2]^2}$. Let $\frac{\partial e^{BCS*}}{\partial t} = h(e_0)$, we note $h(e_0)$ is a function of e_0. As $2bz > (k+s)^2z + b(t+\beta)^2$, we can obtain $\frac{\partial h(e_0)}{\partial e_0} = \frac{4bz(t+\beta)[8b-3(k+s)^2]}{[8bz-3(k+s)^2z-3b(t+\beta)^2]^2} > 0$, i.e., $h(e_0)$ is monotonically increasing with e_0. Since $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \leq e_0 \leq \frac{(a-c)}{t+\beta}$, taking the upper bound value $e_0 = \frac{(a-c)}{t+\beta}$, we have $h(e_0) = \frac{2b(a-c)}{8bz-3(k+s)^2z-3b(t+\beta)^2} > 0$. And taking the lower bound value $e_0 = \frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}$, we have $h(e_0) = \frac{-2b(a-c)\{[8bz-3(k+s)^2z+3b(t+\beta)^2][2b-(k+s)^2]+2b(t+\beta)^2[3(k+s)^2-8b]\}}{[2b-(k+s)^2][8bz-3(k+s)^2z-3b(t+\beta)^2]^2}$. Obviously, when $[8bz - 3(k+s)^2z + 3b(t+\beta)^2][2b - (k+s)^2] + 2b(t+\beta)^2[3(k+s)^2 - 8b] < 0$, i.e.,

$z < \frac{b[10b-3(k+s)^2](t+\beta)^2}{[8b-3(k+s)^2][2b-(k+s)^2]}$, we have $h\left(\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}\right) > 0$, that is to say, $h(e_0) > 0$ in the interval of $[\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}, \frac{a-c}{t+\beta}]$. Therefore, $\frac{\partial e^{BCS*}}{\partial t} > 0$ when $z < \frac{b[10b-3(k+s)^2](t+\beta)^2}{[8b-3(k+s)^2][2b-(k+s)^2]}$. However, when $[8bz-3(k+s)^2z+3b(t+\beta)^2][2b-(k+s)^2]+2b(t+\beta)^2[3(k+s)^2-8b] > 0$, i.e., $z > \frac{b[10b-3(k+s)^2](t+\beta)^2}{[8b-3(k+s)^2][2b-(k+s)^2]}$, $h\left(\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}\right) < 0$. According to the mean value theorem, there is a value $\hat{e} \in [\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}, \frac{a-c}{t+\beta}]$ to make $h(\hat{e}) = 0$, where $\hat{e} = \frac{(a-c)[8bz-3(k+s)^2z+3b(t+\beta)^2]}{2z(t+\beta)[8b-3(k+s)^2]}$. Hence, when $z > \frac{b[10b-3(k+s)^2](t+\beta)^2}{[8b-3(k+s)^2][2b-(k+s)^2]}$, we have $h(e_0) < 0$ i.e., $\frac{\partial e^{BCS*}}{\partial t} < 0$ if $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \le e_0 \le \frac{(a-c)[8bz-3(k+s)^2z+3b(t+\beta)^2]}{2z(t+\beta)[8b-3(k+s)^2]}$ and $h(e_0) > 0$ i.e., $\frac{\partial e^{BCS*}}{\partial t} > 0$ if $\frac{(a-c)[8bz-3(k+s)^2z+3b(t+\beta)^2]}{2z(t+\beta)[8b-3(k+s)^2]} \le e_0 \le \frac{(a-c)}{t+\beta}$.

(iii) $\frac{\partial g^{BCS*}}{\partial t} = \frac{2(k+s)z[6b(a-c)(t+\beta)+3(k+s)^2ze_0-be_0(3t^2+8z+6t\beta+3\beta^2)]}{[8bz-3(k+s)^2z-3b(t+\beta)^2]^2}$. Let $\frac{\partial g^{BCS*}}{\partial t} = f(e_0)$, we note $f(e_0)$ is a function of e_0. As $2bz > (k+s)^2z+b(t+\beta)^2$, we can obtain $\frac{\partial f(e_0)}{\partial e_0} = \frac{2(k+s)z[3(k+s)^2z-8bz-3b(t+\beta)^2]}{[8bz-3(k+s)^2z-3b(t+\beta)^2]^2} < 0$, i.e., $f(e_0)$ is monotonically decreasing with e_0. Since $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \le e_0 \le \frac{(a-c)}{t+\beta}$, taking the upper bound value $e_0 = \frac{(a-c)}{t+\beta}$, we have $f(e_0) = -\frac{2(a-c)(k+s)z}{(t+\beta)[8bz-3(k+s)^2z-3b(t+\beta)^2]} < 0$. And taking the lower bound value $e_0 = \frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}$, we have $f(e_0) = \frac{2b(a-c)(k+s)(t+\beta)[4bz-3(k+s)^2z-3b(t+\beta)^2]}{[2b-(k+s)^2][8bz-3(k+s)^2z-3b(t+\beta)^2]^2}$. Obviously, when $4bz-3(k+s)^2z-3b(t+\beta)^2 < 0$, i.e., $b < \frac{3(k+s)^2z}{4z-3(t+\beta)^2}$, we have $f\left(\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}\right) < 0$, that is to say, $f(e_0) < 0$ in the interval of $[\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}, \frac{a-c}{t+\beta}]$. Therefore, $\frac{\partial g^{BCS*}}{\partial t} < 0$ when $b < \frac{3(k+s)^2z}{4z-3(t+\beta)^2}$. However, when $4bz-3(k+s)^2z-3b(t+\beta)^2 > 0$, i.e., $b > \frac{3(k+s)^2z}{4z-3(t+\beta)^2}$, $f\left(\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}\right) > 0$. According to the mean value theorem, there is a value $\check{e} \in [\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}, \frac{a-c}{t+\beta}]$ to make $f(\check{e}) = 0$, where $\check{e} = \frac{6b(a-c)(t+\beta)}{8bz-3(k+s)^2z+3b(t+\beta)^2}$. Hence, when $b > \frac{3(k+s)^2z}{4z-3(t+\beta)^2}$, we have $f(e_0) > 0$ i.e., $\frac{\partial g^{BCS*}}{\partial t} > 0$ if $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \le e_0 \le \frac{6b(a-c)(t+\beta)}{8bz-3(k+s)^2z+3b(t+\beta)^2}$ and $f(e_0) < 0$ i.e., $\frac{\partial g^{BCS*}}{\partial t} < 0$ if $\frac{6b(a-c)(t+\beta)}{8bz-3(k+s)^2z+3b(t+\beta)^2} \le e_0 \le \frac{(a-c)}{t+\beta}$.

Appendix A.11 Proof of Proposition 8

(i) $\pi_r^{BCS*} = \frac{[8bz+z(k+s)^2+b(t+\beta)^2](a-c-te_0-\beta e_0)^2}{16[8bz-3z(k+s)^2-3b(t+\beta)^2]}$, $\pi_m^{BCS*} = \frac{[8bz-z(k+s)^2-b(t+\beta)^2](a-c-te_0-\beta e_0)^2}{8[8bz-3z(k+s)^2-3b(t+\beta)^2]}$. We have $\frac{\partial \pi_r^{BCS*}}{\partial s} = \frac{4b(k+s)z^2(a-c-te_0-\beta e_0)^2}{[8bz-3z(k+s)^2-3b(t+\beta)^2]^2} > 0$, $\frac{\partial \pi_m^{BCS*}}{\partial s} = \frac{4b(k+s)z^2(a-c-te_0-\beta e_0)^2}{[8bz-3z(k+s)^2-3b(t+\beta)^2]^2} > 0$.

(ii) $\frac{\partial\ \pi_r^{BCS*}}{\partial t} = \frac{X}{8[8bz-3z(k+s)^2} - 3b(t+\beta)^2]^2$, where $X = \{64b^2z^2+16b^2z(t+\beta)^2-16bz^2(k+s)^2-3[(k+s)^2z+b(t+\beta)^2]^2\}(t+\beta)e_0{}^2- (-3a(k+s)^4z^2+3c(k+s)^4z^2+32ab^2z(t+\beta)^2-32b^2cz(t+\beta)^2-2ab(k+s)^2z(3t^2+8z+6t\beta+3\beta^2)+2bc(k+s)^2z(3t^2+8z+6t\beta+3\beta^2)-ab^2(3t^4-64z^2+12t^3\beta-16z\beta^2+3\beta^4-2t^2(8z-9\beta^2)-4t(8z\beta-3\beta^3))+b^2c(3t^4-64z^2+12t^3\beta-16z\beta^2+3\beta^4-2t^2(8z-9\beta^2)-4t(8z\beta-3\beta^3)))e_0+32(a-c)^2b^2z(t+\beta)$. We note that X is a quadratic function of e_0. Since $2bz > (k+s)^2z+b(t+\beta)^2$, we can obtain the quadratic coefficient $\{64b^2z^2+16b^2z(t+\beta)^2-16bz^2(k+s)^2-3[(k+s)^2z+b(t+\beta)^2]^2\}(t+\beta) > 0$, i.e., X opens up. As $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \le e_0 \le \frac{(a-c)}{t+\beta}$, taking the upper bound value $e_0 = \frac{(a-c)}{t+\beta}$ to X, we have $X = 0$. And taking the lower bound value $e_0 = \frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}$, we have $X = -\frac{b(a-c)^2(t+\beta)[2bz-(k+s)^2z-b(t+\beta)^2][16bz-(k+s)^2z-b(t+\beta)^2][(k+s)^2z+b(t+\beta)^2]}{z^2[2b-(k+s)^2]^2} < 0$. Therefore, according to the property of the quadratic function, we have $X < 0$ in the interval of $[\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}, \frac{a-c}{t+\beta}]$, i.e., $\frac{\partial\ \pi_r^{BCS*}}{\partial t} < 0$.

$\frac{\partial \pi_m^{BCS*}}{\partial t} = \frac{Y}{8[8bz-3z(k+s)^2-3b(t+\beta)^2]^2}$, where $Y = \{64b^2z^2 - 16b^2z(t+\beta)^2 - 32bz^2(k+s)^2 + 3[(k+s)^2z + b(t+\beta)^2]^2\}(t+\beta)e_0{}^2 - (-3a(k+s)^4z^2 + 3c(k+s)^4z^2 - 16ab^2z(t+\beta)^2 + 16b^2cz(t+\beta)^2 - 2ab(k+s)^2z(3t^2 - 16z + 6t\beta + 3\beta^2) + 2bc(k+s)^2z(3t^2 - 16z + 6t\beta + 3\beta^2) - ab^2(3t^4 + 64z^2 + 12t^3\beta - 16z\beta^2 + 3\beta^4 - 2t^2(8z - 9\beta^2) - 4t(8z\beta - 3\beta^3)) + b^2c(3t^4 + 64z^2 + 12t^3\beta - 16z\beta^2 + 3\beta^4 - 2t^2(8z - 9\beta^2) - 4t(8z\beta - 3\beta^3)))e_0 + 16(a-c)^2b^2z(t+\beta)$. We note that Y is a quadratic function of e_0. Since $2bz > (k+s)^2z + b(t+\beta)^2$, we can obtain the quadratic coefficient $\{64b^2z^2 - 16b^2z(t+\beta)^2 - 32bz^2(k+s)^2 + 3[(k+s)^2z + b(t+\beta)^2]^2\}(t+\beta) > 0$, i.e., Y opens up. As $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \le e_0 \le \frac{(a-c)}{t+\beta}$, taking the upper bound value $e_0 = \frac{(a-c)}{t+\beta}$ to Y, we have $Y = 0$. And taking the lower bound value $e_0 = \frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}$, we have $Y = -\frac{b(a-c)^2(t+\beta)[2bz-(k+s)^2z-b(t+\beta)^2]\{16bz[2bz-(k+s)^2z-b(t+\beta)^2]+3[(k+s)^2z+b(t+\beta)^2]^2\}}{z^2[2b-(k+s)^2]^2} < 0$. Therefore, according to the property of the quadratic function, we have $Y < 0$ in the interval of $[\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}, \frac{a-c}{t+\beta}]$, i.e., $\frac{\partial \pi_m^{BCS*}}{\partial t} < 0$.

According to the mean value theorem, there is a value $\bar{e} = \frac{(a-c)\{3(k+s)^6z^3 - b(k+s)^4z^2[26z-3(t+\beta)^2] - b^2(k+s)^2z[3(t+\beta)^4+4z(t+\beta)^2-72z^2] - b^3[3(t+\beta)^6-2z(11(t+\beta)^4-4z(t+\beta)^2-32z^2)]\}}{2z(t+\beta)\{3(k+s)^6z^2+2bz(k+s)^4[3(t+\beta)^2-13z]-2b^3[(t+\beta)^4+32z^2-16z(t+\beta)^2]+b^2(k+s)^2[3(t+\beta)^4+72z^2-28z(t+\beta)^2]\}}$. Therefore, $V < 0$ i.e., $\frac{\partial (e^{C*}-e^{BCS*})}{\partial t} < 0$ if $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \le e_0 \le \bar{e}$ and $V > 0$ i.e., $\frac{\partial (e^{C*}-e^{BCS*})}{\partial t} > 0$.

Appendix A.12 Proof of Proposition 9

(i) $\frac{\partial (e^{C*}-e^{BCS*})}{\partial s} = -\frac{(a-c-te_0-\beta e_0)2bz(k+s)(t+\beta)U}{[8bz-3z(k+s)^2-3b(t+\beta)^2]^2[2bz-z(k+s)^2-b(t+\beta)^2]^2}$, where $U = 40b^2z^2 - 24bz[z(k+s)^2 + b(t+\beta)^2] + 3[z(k+s)^2 + b(t+\beta)^2]^2$. We know that $\frac{\partial (e^{C*}-e^{BCS*})}{\partial s}$ is negative with U. Since $2bz > (k+s)^2z + b(t+\beta)^2$, we have $\frac{\partial^2 U}{\partial z^2} = 80b^2 - 48b(k+s)^2 + 6(k+s)^4 > 0$, i.e., $\frac{\partial U}{\partial z}$ is monotonically decreasing with z. Since $z > \frac{b(t+\beta)^2}{2b-(k+s)^2}$ and $\frac{\partial U}{\partial z}|_{z=\frac{b(t+\beta)^2}{2b-(k+s)^2}} = \frac{4b^2(8b-3(k+s)^2)(t+\beta)^2}{2b-(k+s)^2} > 0$, U is monotonically decreasing with z. Hence, $U_{min} = U|_{z=\frac{b(t+\beta)^2}{2b-(k+s)^2}} = \frac{4b^4(t+\beta)^4}{[2b-(k+s)^2]^2} > 0$, i.e., $\frac{\partial (e^{C*}-e^{BCS*})}{\partial s} < 0$.

$\frac{\partial (g^{C*}-g^{BCS*})}{\partial s} = -\frac{(a-c-te_0-\beta e_0)zN}{[8bz-3z(k+s)^2-3b(t+\beta)^2]^2[2bz-z(k+s)^2-b(t+\beta)^2]^2}$, where $= -3(k+s)^6z^3 + b(k+s)^4z^2[22z - 3(t+\beta)^2] - b^3[4z - (t+\beta)^2][2z - (t+\beta)^2][8z - 3(t+\beta)^2] + b^2(k+s)^2z[3(t+\beta)^4 - 4z(t+\beta)^2 - 8z^2]$. We know $\frac{\partial (g^{C*}-g^{BCS*})}{\partial s}$ is negative with N. Since $2bz > (k+s)^2z + b(t+\beta)^2$, we have $\frac{\partial^3 N}{\partial z^3} = 6[-64b^3 - 8b^2(k+s)^2 + 22b(k+s)^4 - 3(k+s)^6] < 0$, i.e., $\frac{\partial^2 N}{\partial z^2}$ is monotonically decreasing with z. As $z > \frac{b(t+\beta)^2}{2b-(k+s)^2}$ and $\frac{\partial^2 N}{\partial z^2}|_{z=\frac{b(t+\beta)^2}{2b-(k+s)^2}} = -\frac{4b[24b^3+52b^2(k+s)^2-32b(k+s)^4+3(k+s)^6](t+\beta)^2}{2b-(k+s)^2} < 0$, then $\frac{\partial N}{\partial z}$ is monotonically decreasing with z. As $\frac{\partial N}{\partial z}|_{z=\frac{b(t+\beta)^2}{2b-(k+s)^2}} = -\frac{4b^3[2b^2+17b(k+s)^2-6(k+s)^4](t+\beta)^4}{[2b-(k+s)^2]^2} < 0$, then N is monotonically decreasing with z. Hence, $N_{max} = N|_{z=\frac{b(t+\beta)^2}{2b-(k+s)^2}} = -\frac{8b^5(k+s)^2(t+\beta)^6}{[2b-(k+s)^2]^3} < 0$, i.e., $\frac{\partial (g^{C*}-g^{BCS*})}{\partial s} > 0$.

(ii) $\frac{\partial (e^{C*}-e^{BCS*})}{\partial t} = \frac{bV}{[8bz-3z(k+s)^2-3b(t+\beta)^2]^2[2bz-z(k+s)^2-b(t+\beta)^2]^2}$, where $V = (t+\beta)[4bz - (k+s)^2z - b(t+\beta)^2][2bz - (k+s)^2z - b(t+\beta)^2][8bz - 3(k+s)^2z - 3b(t+\beta)^2]e_0 + 6b(t+\beta)^2[4bz - (k+s)^2z - b(t+\beta)^2][2bz - (k+s)^2z - b(t+\beta)^2](te_0 + \beta e_0 + c - a) + 2b(t+\beta)^2[4bz - (k+s)^2z - b(t+\beta)^2][8bz - 3(k+s)^2z - 3b(t+\beta)^2](te_0 + \beta e_0 + c - a) - 2b(t+\beta)^2[2bz - (k+s)^2z - b(t+\beta)^2][8bz - 3(k+s)^2z - 3b(t+\beta)^2](te_0 + \beta e_0 + c - a) + [4bz - (k+s)^2z - b(t+\beta)^2][2bz - (k+s)^2z - b(t+\beta)^2]$

$[8bz-3(k+s)^2z-3b(t+\beta)^2](te_0+\beta e_0+c-a)$. We note that V is a linear function with e_0 and $\frac{\partial\left(e^{C*}-e^{BCS*}\right)}{\partial t}$ is positive with V. As $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \le e_0 \le \frac{(a-c)}{t+\beta}$, taking the lower bound value $e_0 = \frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}$ to V, we have $V = \frac{(a-c)[2bz-(k+s)^2z-b(t+\beta)^2]}{2b-(k+s)^2}v$, where $v = 3(k+s)^6z^2 + 2b(k+s)^4z(3t^2-13z+6t\beta+3\beta^2) + 3b^2(k+s)^2(t^4+24z^2+4t^3\beta-8z\beta^2+\beta^4+4t\beta(-4z+\beta^2)+t^2(-8z+6\beta^2)) + 2b^3(t^4-32z^2+4t^3\beta+12z\beta^2+\beta^4+6t^2(2z+\beta^2)+4t(6z\beta+\beta^3))$. Regarding v as a quadratic function of z, by $2b > (k+s)^2$, $\frac{\partial^2 v}{\partial z^2} = -128b^3+144b^2(k+s)^2-52b(k+s)^4+6(k+s)^6 < 0$, i.e., $\frac{\partial v}{\partial z}$ is monotonically decreasing with z. Since $z > \frac{b(t+\beta)^2}{2b-(k+s)^2}$ and $\frac{\partial v}{\partial z}\Big|_{z=\frac{b(t+\beta)^2}{2b-(k+s)^2}} = -8b^2[5b-2(k+s)^2](t+\beta)^2 < 0$, v is monotonically decreasing with z. Hence $v_{max} = v|_{z=\frac{b(t+\beta)^2}{2b-(k+s)^2}} = -\frac{4b^4(t+\beta)^4}{2b-(k+s)^2} < 0$ and $V\left(\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}\right) < 0$. Taking the upper bound value $e_0 = \frac{a-c}{t+\beta}$ to V, we have $V\left(\frac{a-c}{t+\beta}\right) = (a-c)[4bz-(k+s)^2z-b(t+\beta)^2][2bz-(k+s)^2z-b(t+\beta)^2][8bz-3(k+s)^2z-3b(t+\beta)^2] > 0$. According to the mean value theorem, there is a value $\bar{e} \in [\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}, \frac{a-c}{t+\beta}]$ to make $V(\bar{e}) = 0$, where $\bar{e} = \frac{(a-c)\{3(k+s)^6z^3-b(k+s)^4z^2[26z-3(t+\beta)^2]-b^2(k+s)^2z[3(t+\beta)^4+4z(t+\beta)^2-72z^2]-b^3[3(t+\beta)^6-2z(11(t+\beta)^4-4z(t+\beta)^2-32z^2)]\}}{2z(t+\beta)\{3(k+s)^6z^2+2bz(k+s)^4[3(t+\beta)^2-13z]-2b^3[(t+\beta)^4+32z^2-16z(t+\beta)^2]+b^2(k+s)^2[3(t+\beta)^4+72z^2-28z(t+\beta)^2]\}}$.

Therefore, $V < 0$ i.e., $\frac{\partial\left(e^{C*}-e^{BCS*}\right)}{\partial t} < 0$ if $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \le e_0 \le \bar{e}$ and $V > 0$ i.e., $\frac{\partial\left(e^{C*}-e^{BCS*}\right)}{\partial t} > 0$ if $\bar{e} \le e_0 \le \frac{(a-c)}{t+\beta}$.

(iii) $\frac{\partial\left(g^{C*}-g^{BCS*}\right)}{\partial t} = \frac{(k+s)zR}{[8bz-3z(k+s)^2-3b(t+\beta)^2]^2[2bz-z(k+s)^2-b(t+\beta)^2]^2}$, where $R = -[4bz-(k+s)^2z-b(t+\beta)^2][2bz-(k+s)^2z-b(t+\beta)^2][8bz-3(k+s)^2z-3b(t+\beta)^2]e_0 - 6b(t+\beta)[4bz-(k+s)^2z-b(t+\beta)^2][2bz-(k+s)^2z-b(t+\beta)^2](te_0+\beta e_0+c-a) - 2b(t+\beta)[4bz-(k+s)^2z-b(t+\beta)^2][8bz-3(k+s)^2z-3b(t+\beta)^2](te_0+\beta e_0+c-a) + 2b(t+\beta)[2bz-(k+s)^2z-b(t+\beta)^2][8bz-3(k+s)^2z-3b(t+\beta)^2](te_0+\beta e_0+c-a)$. We note R is a linear function with e_0 and $\frac{\partial\left(g^{C*}-g^{BCS*}\right)}{\partial t}$ is positive with R. As $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \le e_0 \le \frac{(a-c)}{t+\beta}$ and $2bz > (k+s)^2z+b(t+\beta)^2$, taking the lower bound value $e_0 = \frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}$ to R, we have $R = \frac{b(a-c)(t+\beta)[2bz-(k+s)^2z-b(t+\beta)^2]\{48b^2z^2-28bz[(k+s)^2z+b(t+\beta)^2]+3[(k+s)^2z+b(t+\beta)^2]^2\}}{[2b-(k+s)^2]z} > 0$, i.e., $R\left(\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}\right) > 0$. Taking the lower bound value $e_0 = \frac{a-c}{t+\beta}$ to R, we have $R\left(\frac{a-c}{t+\beta}\right) = -\frac{(a-c)[4bz-(k+s)^2z-b(t+\beta)^2][2bz-(k+s)^2z-b(t+\beta)^2][8bz-3(k+s)^2z-3b(t+\beta)^2]}{t+\beta} < 0$. According to the mean value theorem, there is a value $\widetilde{e} \in [\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}, \frac{a-c}{t+\beta}]$ to $R(\widetilde{e}) = 0$, where $\widetilde{e} = \frac{2b(a-c)(t+\beta)\{40b^2z^2-24bz[(k+s)^2z+b(t+\beta)^2]+3[(k+s)^2z+b(t+\beta)^2]^2\}}{[8bz-3(k+s)^2z-3b(t+\beta)^2][4bz-(k+s)^2z-b(t+\beta)^2][2bz-(k+s)^2z-b(t+\beta)^2]+4zb^2(t+\beta)^2[8bz-3(k+s)^2z-3b(t+\beta)^2]}$.

Therefore, $R < 0$ i.e., $\frac{\partial\left(g^{C*}-g^{BCS*}\right)}{\partial t} > 0$ if $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z} \le e_0 \le \widetilde{e}$ and $R > 0$ i.e., $\frac{\partial\left(g^{C*}-g^{BCS*}\right)}{\partial t} < 0$ if $\widetilde{e} \le e_0 \le \frac{(a-c)}{t+\beta}$.

Appendix A.13 Proof of Proposition 10

(i) $\frac{\partial\left(\pi^{C*}-\pi^{BCS*}\right)}{\partial s} = \frac{b(k+s)z^2(a-c-te_0-\beta e_0)^2\{32b^2z^2-16bz[(k+s)^2z+b(t+\beta)^2]+[(k+s)^2z+b(t+\beta)^2]^2\}}{[8bz-3z(k+s)^2-3b(t+\beta)^2]^2[2bz-z(k+s)^2-b(t+\beta)^2]^2}$. As $2bz > (k+s)^2z+b(t+\beta)^2$, we have $\frac{\partial\left(\pi^{C*}-\pi^{BCS*}\right)}{\partial s} > 0$.

(ii) $\frac{\partial\left(\pi^{C*}-\pi^{BCS*}\right)}{\partial t}=-\frac{(a-c-te_0-\beta e_0)T}{[8bz-3z(k+s)^2-3b(t+\beta)^2]^2[2bz-z(k+s)^2-b(t+\beta)^2]^2}$, where $T=2[2bz-(k+s)^2z-b(t+\beta)^2][8bz-3(k+s)^2z-3b(t+\beta)^2]\{16b^2z^2+2bz[(k+s)^2z+b(t+\beta)^2]-[(k+s)^2z+b(t+\beta)^2]^2\}e_0+4b(t+\beta)[2bz-(k+s)^2z-b(t+\beta)^2][bz-(k+s)^2z-b(t+\beta)^2][8bz-3(k+s)^2z-3b(t+\beta)^2](te_0+\beta e_0+c-a)+4b(t+\beta)[7bz-2(k+s)^2z-2b(t+\beta)^2]\{16b^2z^2+2bz[(k+s)^2z+b(t+\beta)^2]-[(k+s)^2z+b(t+\beta)^2]^2\}(te_0+\beta e_0+c-a)$. We note that T is a linear function with e_0 and $\frac{\partial\left(\pi^{C*}-\pi^{BCS*}\right)}{\partial t}$ is negative with T. Since $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}\leq e_0\leq\frac{(a-c)}{t+\beta}$ and $2bz>(k+s)^2z+b(t+\beta)^2$, taking the lower bound value $e_0=\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}$ to T, we have $T=-\frac{2b(a-c)(t+\beta)[2bz-(k+s)^2z-b(t+\beta)^2]^2\{64b^2z^2-16bz[(k+s)^2z+b(t+\beta)^2]+3[(k+s)^2z+b(t+\beta)^2]^2\}}{[2b-(k+s)^2]z}<0$, i.e., $T\left(\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}\right)<0$. Taking the upper bound value $e_0=\frac{a-c}{t+\beta}$ to T, we have $T\left(\frac{a-c}{t+\beta}\right)=\frac{2(a-c)[8bz-3(k+s)^2z-3b(t+\beta)^2][2bz-(k+s)^2z-b(t+\beta)^2]\{16b^2z^2+2bz[(k+s)^2z+b(t+\beta)^2]-[(k+s)^2z+b(t+\beta)^2]^2\}}{t+\beta}>0$.
According to the mean value theorem, there is a value $\underline{e}\in[\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z},\frac{a-c}{t+\beta}]$ to make $T(\underline{e})=0$, where $\underline{e}=\frac{16b^2(a-c)z(t+\beta)\{32b^2z^2-16bz[(k+s)^2z+b(t+\beta)^2]+[(k+s)^2z+b(t+\beta)^2]^2\}}{4b(t+\beta)^2K+2[8bz-3(k+s)^2z-3b(t+\beta)^2][2bz-(k+s)^2z-b(t+\beta)^2]\{16b^2z^2+2bz[(k+s)^2z+b(t+\beta)^2]-[(k+s)^2z+b(t+\beta)^2]^2\}}$, and $K=[8bz-3(k+s)^2z-3b(t+\beta)^2][bz-(k+s)^2z-b(t+\beta)^2][2bz-(k+s)^2z-b(t+\beta)^2]+[7bz-2(k+s)^2z-2b(t+\beta)^2]\{16b^2z^2+2bz[(k+s)^2z+b(t+\beta)^2]-[(k+s)^2z+b(t+\beta)^2]^2\}$.
Therefore, $T<0$ i.e., $\frac{\partial\left(\pi^{C*}-\pi^{BCS*}\right)}{\partial t}>0$ if $\frac{b(a-c)(t+\beta)}{[2b-(k+s)^2]z}\leq e_0\leq\underline{e}$ and $T>0$ i.e., $\frac{\partial\left(\pi^{C*}-\pi^{BCS*}\right)}{\partial t}<0$ if $\underline{e}\leq e_0\leq\frac{(a-c)}{t+\beta}$.

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