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An Accurate Inverse Model for the Detection of Leaks in Sealed Landfills

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Abstract: Leaks from landfills to underlying soil layers are one of the main problems that endanger the sustainability of waste disposal in landfills. Indeed, the possible failing of in-situ equipment can give rise to serious pollution consequences or costly inspection work in the landfill body. In this paper, we develop the time dependent mathematical relationship between the concentration of water at the surface of the landfill and the flux at the bottom of the landfill. This makes it possible to detect a leak using non-expensive measurements made at the surface of the landfill. The resulting model is obtained by analytically solving Richard's equation with a piecewise linear boundary condition at the bottom. The unknown coefficients of the piecewise linear functions, which can be estimated using the measurements at the surface, provide the necessary information for detecting leaks. The algorithm has been numerically tested using simulated data of rain precipitation. The method proposed could be conveniently used to complement the usual monitoring techniques due to the limited costs of its implementation.

Keywords: monitoring; surface measurements; inverse models; Richards' equation; piecewise linear functions; boundary condition

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1. Introduction

Landfills still constitute one of the fundamental strategies of waste management. According to the latest report of IBRD/WB [1], "... some 37 percent of waste is disposed of in some form of a landfill ... [whereas] ... open dumping accounts for about 31 percent of waste" worldwide.

Indeed, some diversion of the waste presently disposed of in landfills to recycling, composting, and incineration is expected in high-income countries in the near future. However, low-income countries presently using open dumping will probably improve their waste management by extending the use of controlled landfills, which constitute locally appropriate solutions. Thus, the overall role of landfills in waste management worldwide is bound to increase over the next decades.

The overall procedure for waste disposal in landfills, as foreseen by the EU Council Directive 1999/31/EC (and amended in the Regulation N. 1882/2003) can be roughly divided into three phases [2].

A pre-treatment phase employs physical, thermal, chemical, or biological processes to enhance waste recycling and reduce its volume and its hazardous nature as well as to facilitate its handling. This is particularly important regarding some types of waste, such as Waste Electrical and Electronic Equipment (WEEE), which in many cases are a heterogeneous mix of organic, ceramic, and metallic materials that can cause serious problems for humans and the environment but at the same time represent an increasingly attractive source for recovering raw materials [3]. Well-established techniques are employed in this first phase, although less conventional approaches can sometimes be explored [4] following the guidelines of the most recent European policies, such as Circular Economy strategies [5]

(European Commission, 2017) and "best available technologies" (BAT) and "near-zero discharge" approaches, particularly when it comes to hazardous waste [6].

The second phase takes place during the operation of the landfill and can include (to various levels of technological sophistication) monitoring the integrity of the landfill containment, a landfill gas collection system, and an active strategy to change the aim of landfill from waste storage to waste treatment [7].

The third and last phase is the remediation of waste landfills (and possibly of dumpsites). A large number of processes have been developed for this purpose, although finding the most suitable approach among the numerous solutions available may not be easy as multiple and different aspects must be considered in the decision-making process [8,9], in addition to ease of implementation or efficiency. As far as the remediation of aqueous solution is concerned, adsorption techniques are often employed as a simple, easy to implement, and particularly effective approach for the removal of various contaminants, both organic [10] and inorganic, such as metals [11,12] and nitrogen compounds [13], and they possibly exploit low-cost absorbent materials [14] to reduce costs and increase process sustainability. Unfortunately, such a solution is not available for soil remediation. In such cases, the problem can first be assessed through the use of analytical approaches, particularly suitable for working in complex situations [15], and subsequently addressed through possibly innovative in-situ interventions [16], which can also exploit natural approaches [17,18]. In doing this, a Life Cycle Assessment (LCA) procedure [19] can be fundamental to assess the most acceptable option from an environmental point of view [20,21].

The scope of this paper is limited to the development of an algorithm to complement the monitoring of the integrity of landfill containment systems using non-expensive measurements at the surface of the landfill.

The monitoring of leaks in landfill liners is generally carried out using a combination of monitoring wells below the water table and sensors in the vadose zone [22]. Sensors in the vadose zone can provide an early warning for leaks before they are detected by the monitoring wells in the water table, and consequently, they can prevent costly remediation operations. However, since liner leaks generally are point sources, sensors located close to the liner might fail to detect a narrow plume [23]. On the other hand, sensors located in deeper regions of the vadose zone could fail to intercept the front of the plume if an impeding layer diverts percolating water horizontally [24]. Instrument malfunctioning can be an additional cause of data misinterpretation. Indeed, inconsistent data are frequently encountered during calibration procedures after the disposal sites are flooded with known amounts of water.

The paper is organized as follows: both the numerical algorithm that implements the inverse model and the resulting statistical hypothesis about the presence of leaks is presented in Section 2. Simulated data are used in Section 3 to evaluate significance and power of the hypothesis. Sections 4 and 5 provide an analysis of the results obtained and general conclusions.

2. Materials and Methods

The estimation of percolation rates at a fixed depth using reliable surface data can usefully complement the information provided by the vadose measurement system and possibly point to the presence of biases in it. The possibility of using inexpensive mobile equipment to carry out surface measurements is an additional advantage.

The detection of leaks at the bottom of the landfill from measurements of water content at the surface requires an inverse model that relates them to each other. In this paper, the inverse model has been obtained using the analytical solution of Richards' equation proposed by Yuan and Lu [25] and by suitably adapting it to the presence of an unknown boundary condition. Richards' equation is generally used to model the content and flow of water in soils. In this paper, it is applied to percolation problems in landfills.

While the new inverse model is based on an approximate piecewise linear approximation of the boundary condition at the bottom of the landfill, its accuracy can be arbitrarily improved by simply increasing the number of linear intervals. Thus, the model is unlike a simplified previous model [26,27] that could possibly introduce time discontinuities in the boundary condition.

The one-dimensional pressure-head form of Richards' equation is given by [25,28]

$$\frac{\partial}{\partial z}\left[K(\psi)\left(\frac{\partial\psi}{\partial z}+1\right)\right]-S(z)=C(\psi)\frac{\partial\psi}{\partial t} \tag{1}$$

where ψ is the pressure head, $K(\psi)$ is the hydraulic conductivity, $C(\psi)=\frac{d\theta}{d\psi}$ is the differential water capacity, θ is the volumetric water content, S is water uptake (which is negative when considering landfills because it gives the water generation by wastes), and z is the vertical coordinate pointing upward.

In addition to an initial condition $\psi_0(z)$, suitable boundary conditions at z = 0 (the bottom of the landfill) and at z = L (at the surface of the landfill) relating $\psi(z,t)$ to the fluxes at the boundaries are to be provided. The flux at the surface $\left[K(\psi)\left(\frac{\partial\psi}{\partial z}+1\right)\right]_{z=L}$ is equal to the amount of water released by evapotranspiration per unit surface minus the rate of rainfall infiltration amount of rain, i.e., $\left[K(\psi)\left(\frac{\partial\psi}{\partial z}+1\right)\right]_{z=L}=q_0(t)-q_1(t)$.

In cases of a sealed landfill, the condition at the bottom is given by $\left[K(\psi)\left(\frac{\partial\psi}{\partial z}+1\right)\right]_{z=0}=0$.

In the presence of leaks, this condition changes to $\left[K(\psi)\left(\frac{\partial\psi}{\partial z}+1\right)\right]_{z=0}=\beta(t)$ where the function $\beta(t)$ is unknown and has to be estimated from experimental measurements.

Following Yuan and Lu [25], exponential models to describe the dependence of the hydraulic conductivity and the water content on the pressure head are used, i.e.,

$$K(\psi)=K_s e^{\alpha\psi} \quad \text{and} \quad \theta=\theta_r+(\theta_s-\theta_r)e^{\alpha\psi} \tag{2}$$

Introducing the Kirchhoff transformation $\Phi(z,t)=K(\psi)/\alpha$ and approximating the function $\beta(t)$ with a piecewise linear function, i.e., $\beta(t)=\beta_0+\sum_{u=2}^{i}\beta_{u-1}(t_k-t_{k-1})+\beta_i(t-t_i)$, the transient water content distribution $\theta(z,t)$ and the flux potential $\Phi(z,t)$ are given (see Appendix A) by

$$\theta(z,t)=\theta_r+(\theta_s-\theta_r)\frac{\alpha}{K_s}\Phi(z,t) \tag{3}$$

$$\begin{aligned}\Phi(z,t)=&\ \Phi_s(z)+8De^{-\frac{\alpha z}{2}}\left\{\sum_{n=1}^{\infty}\frac{\left(\lambda_n^2+\frac{\alpha^2}{4}\right)sin(\lambda_n L)}{2\alpha+\alpha^2 L+4L\lambda_n^2}\cdot\right.\\ &\left[e^{\frac{\alpha L}{2}}sin(\lambda_n z)\int_0^t(q_0-q_1(\tau))e^{-D(\lambda_n^2+\frac{\alpha^2}{4})(t-\tau)}d\tau+\right.\\ &\left.\left.e^{-D(\lambda_n^2+\frac{\alpha^2}{4})t}\left(\lambda_n\cos(\lambda_n(L-z))+\frac{\alpha}{2}\sin(\lambda_n(L-z))\right)\sum_{u=1}^{i}\beta_u\xi_{un}\right]\right\}\end{aligned} \tag{4}$$

$$
\begin{aligned}
&0 \le z \le L\\
&t_i \le t \le t_{i+1}\\
&\Phi_s(z) = \beta_0 e^{-\alpha z} + \frac{q_0}{\alpha}\left(\mathrm{e}^{-\alpha z}-1\right) + \int_0^L G(z,x)S(x)dx\\
&\qquad G(z,x) = \begin{cases} \frac{\mathrm{e}^{-\alpha z}}{z}\left(1-\mathrm{e}^{-\alpha x}\right) & 0 \le x \le z \le L\\ \frac{1}{\alpha}\left(\mathrm{e}^{-\alpha z}-1\right) & 0 \le z \le x \le L \end{cases}\\
&\begin{cases} \xi_{un} = \dfrac{\left[(t_{u+1}-t_u)D\left(\lambda_n^2+\frac{\alpha^2}{4}\right)-1\right]\mathrm{e}^{-D(\lambda_n^2+\frac{\alpha^2}{4})(t-t_{u+1})}+\mathrm{e}^{-D(\lambda_n^2+\frac{\alpha^2}{4})(t-t_u)}}{\left[D\left(\lambda_n^2+\frac{\alpha^2}{4}\right)\right]^2} & u < i\\ \xi_{in} = \dfrac{\left[(t-t_i)D\left(\lambda_n^2+\frac{\alpha^2}{4}\right)-1\right]+\mathrm{e}^{-D(\lambda_n^2+\frac{\alpha^2}{4})(t-t_i)}}{\left[D\left(\lambda_n^2+\frac{\alpha^2}{4}\right)\right]^2} & \end{cases}\\
&\lambda_n \text{ n}-\text{th solution of the equation } sin(\lambda_n L) + \frac{2\lambda_n}{\alpha}cos(\lambda_n L) = 0
\end{aligned}
\tag{5}
$$

where soil parameters $\alpha, K_s, \theta_s, \theta_r, D\left(D = \frac{K_s}{\alpha(\theta_s-\theta_r)}\right)$ are the soil pore-size distribution, the hydraulic conductivity, the water content at saturation, the residual water content, and the soil moisture diffusivity, respectively. A more detailed description of these parameters can be found in [29].

The accuracy of the approximation $\beta(t) = \beta_0 + \sum\limits_{u=2}^{i} \beta_{u-1}(t_k - t_{k-1}) + \beta_i(t - t_i)$ can be improved arbitrarily by increasing the number of intervals $(t_k - t_{k-1})$, subject to the limitation that the resulting estimation problem does not become ill-posed, as shown in the next section.

This solution can be rewritten as

$$
\begin{aligned}
\theta(z,t) - \theta_r &= (\theta_s - \theta_r)\frac{\alpha}{K_s}\left[F_0(z,t) + \beta_0 F_1(z) + \sum_{u=1}^{i}\beta_u F_{2u}(z,t)\right]\\
&= R_0(z,t) + \beta_0 R_1(z) + \sum_{u=1}^{i}\beta_u R_{2u}(z,t)
\end{aligned}
\tag{6}
$$

where $F_0(z,t), F_1(z,t)$ and $F_{2u}(z,t)$ (and consequently $R_0(z,t), R_1(z,t)$ and $R_{2u}(z,t)$) can be evaluated by comparison.

In particular,

$$
\mathrm{F}_0(z,t) = \frac{q_0}{\alpha}\left(\mathrm{e}^{-\alpha z}-1\right) + \int_0^L G(z,x)S(x)dx + 8D\mathrm{e}^{-\frac{\alpha z}{2}}\sum_{n=1}^{\infty}\frac{\left(\lambda_n^2+\frac{\alpha^2}{4}\right)sin(\lambda_n L)}{2\alpha+\alpha^2 L+4L\lambda_n^2}\mathrm{e}^{\frac{\alpha L}{2}}sin(\lambda_n z)\int_0^t (q_0 - q_1(\tau))\mathrm{e}^{-D(\lambda_n^2+\frac{\alpha^2}{4})(t-\tau)}d\tau
$$

$$
\mathrm{F}_1(z) = e^{-\alpha z}
$$

$$
\mathrm{F}_{2u}(z,t) = 8D\mathrm{e}^{-\frac{\alpha z}{2}}\sum_{n=1}^{\infty}\frac{\mathrm{e}^{-D(\lambda_n^2+\frac{\alpha^2}{4})t}\left(\lambda_n^2+\frac{\alpha^2}{4}\right)sin(\lambda_n L)}{2\alpha+\alpha^2 L+4L\lambda_n^2}\left(\lambda_n\cos(\lambda_n(L-z)) + \frac{\alpha}{2}\sin(\lambda_n(L-z))\right)\xi_{un}
$$

The aim of this paper is to test the hypothesis that the flux at the bottom of the landfill at $z = 0$ given by $\frac{\partial\Phi(z,t)}{\partial z} + \alpha\Phi\Big|_{z=0}$ is equal to zero (i.e., absence of leaks) using water content measurements at the surface.

$\frac{\partial\Phi(z,t)}{\partial z}\Big|_{z=0}$ can be evaluated from $\frac{\partial\mathrm{F}_0(z,t)}{\partial z}\Big|_{z=0}$ $\frac{d\mathrm{F}_1(z)}{dz}\Big|_{z=0}$ and $\frac{\partial\mathrm{F}_{2u}(z,t)}{\partial z}\Big|_{z=0}$ (see Appendix B).

Under stationary conditions (i.e., constant values of water content), evapotranspiration equals the production of water from wastes and consequently there is no flux at the bottom even in the presence of a leak in the liner. This is confirmed by the value of the flux under stationary conditions at z = 0.

Indeed, it can easily be shown that $\frac{d\Phi_s(z)}{dz} + \alpha\Phi_s|_{z=0} = -q_0 - \int\limits_0^L S(x)dx$, which implies the absence of flux if the evapotranspiration equals the amount of water released by wastes.

Thus, the presence of fluxes at the bottom of the landfill can be detected only using the full transient model given by Equations (3)–(6).

In addition to $\theta(z,t)$ that corresponds to the values of moisture measured at the surface, it is necessary to evaluate also $\Phi(0,t)$ given by Equations (3)–(5) computed at $z = 0$ and $\left.\frac{\partial \Phi(z,t)}{\partial z}\right|_{z=0}$ which can be evaluated from $\left.\frac{\partial F_0(z,t)}{\partial z}\right|_{z=0}$ $\left.\frac{dF_1(z)}{dz}\right|_{z=0}$ and $\left.\frac{\partial F_{2u}(z,t)}{\partial z}\right|_{z=0}$ (see Appendix B).

Thus, the relation between the moisture at the surface and the flux (leak) at the bottom has been established. In other words, the measurements at the surface make it possible to estimate the coefficients β (and their variance), which are used to estimate the flux and the statistic corresponding to the hypothesis of absence of leaks.

The overall procedure is visualized in the flowchart of Figure 1.

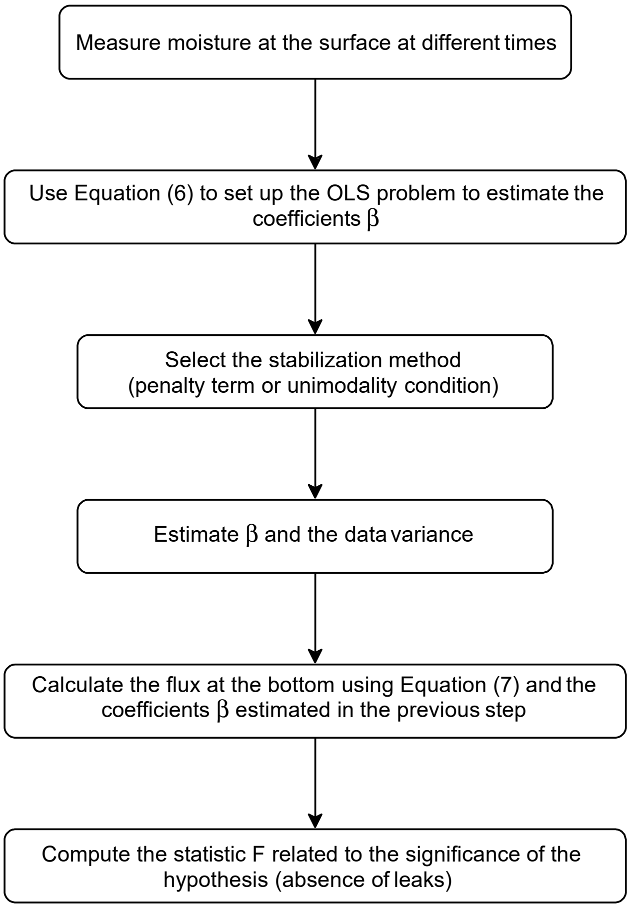

Figure 1. Flowchart of the algorithm.

The simulated data used in this paper are the theoretical values obtained from the general mathematical model to which a Gaussian random noise has been added, as shown in the next section.

The first step of the overall procedure is the estimation of the coefficients β_k from the linear relations $\theta(L,t) - \theta_r = R_0(L,t) + \beta_0 R_1(L) + \sum_{u=1}^{i} \beta_u R_{2u}(L,t)$ using standard OLS techniques. Simplifying the notation, we can write the OLS problem in the usual way.

$y(t_k) \cong \sum_{u=0}^{i} \beta_u r_{uk}$ where $\cong$ means equality in the sense of least squares, y are N measured values of $\theta(L,t_k) - \theta_r - R_0(L,t_k)$, and $r_{uk} = \{R_1, R_{2u}(t_k)\}$. The regression may possibly need regularizing. This can be done using Tikhonov's method together with Morozov's criterion to establish the penalty factor [27] or using additional information such as the unimodality of the distribution of $\beta(t)$ when it is expected that, due to a single rainfall event, it has only one minimum.

Let $\hat{\beta}_u$ be the solution of the least squares problem and $\hat{\sigma}_y$ the estimated sample variance of $y(t_k)$. The flux at the bottom can be rewritten as

$$\left.\frac{\partial\Phi(z,t_k)}{\partial z}+\alpha\Phi(z,t_k)\right|_{z=0}=c(t_k)+\sum_{u=0}^{i}\hat{\beta}_u g_{uk} \tag{7}$$

where $c(t_k)$ and g_{uk} can be computed using the values of $F_0(0,t), F_1(0,t), F_{2u}(0,t)$ $\left.\frac{\partial F_0(z,t)}{\partial z}\right|_{z=0}$, $\left.\frac{dF_1(z)}{dz}\right|_{z=0}$, $\left.\frac{\partial F_{2u}(z,t)}{\partial z}\right|_{z=0}$ as shown before.

The hypothesis $c(t_k)+\sum\limits_{u=0}^{i}\hat{\beta}_u g_{uk}=0$ (absence of leaks at times t_k) can be tested using the statistic $\frac{\left(g_k^T\hat{\beta}+c_k\right)^2}{\hat{\sigma}^2\left[g^T\left(r^Tr\right)^{-1}g\right]}$, which is known to follow an *F*-distribution with 1 and *N-i-1* degrees of freedom.

The absence of leaks at all the measured times *N* is given by the condition

$$\sum_{k=1}^{N}\left(c(t_k)+\sum_{u=0}^{i}\hat{\beta}_u g_{uk}\right)=\sum_{k=1}^{N}c(t_k)+\sum_{k=1}^{N}\left(\sum_{u=0}^{i}\hat{\beta}_u g_{uk}\right)=\sum_{k=1}^{N}c(t_k)+\sum_{u=0}^{i}\hat{\beta}_u\sum_{k=1}^{N}g_{uk}=C+\sum_{u=0}^{i}\hat{\beta}_u G_u=0$$

due to the fluxes being zero or negative and consequently by the statistic

$$\frac{\left(G^T\hat{\beta}+C\right)^2}{\hat{\sigma}^2\left[G^T\left(r^Tr\right)^{-1}G\right]} \tag{8}$$

Thus, the calculated value of the statistic can provide the required probability for the absence of leaks at the bottom of the landfill. Indeed, since the statistic (8) follows an F-distribution under the null hypothesis (absence of leaks), its value provides the probabilistic information necessary to make a decision about the possible presence of leaks.

3. Results

To test the robustness and sensitivity of the model described we have considered a numerical example using the model to simulate the data using the parameters given in Table 1. After adding Gaussian random errors, we have used them to solve the inverse problem and identify the possible presence of leaks.

Table 1. Model parameters.

Parameter	Value
L	5 m
α	1 m^{-1}
K_s	0.01 m h^{-1}
θ_s	0.45 m^3 m^{-3}
θ_r	0.20 m^3 m^{-3}

We have supposed 27 measurements of surface water content were made immediately after 1 h heavy rainfall (2 cm h^{-1}). Both evapotranspiration and water release from the waste have been neglected for the period considered due to the major contribution of the rain to the balance of water.

Furthermore, we have supposed that a leak in the liner generates a flux that can be calculated using the algorithm described in this paper, where the coefficients of the function $\beta(t)=\beta_0+\sum\limits_{u=2}^{i}\beta_{u-1}(t_k-t_{k-1})+\beta_i(t-t_i)$ are given in Table 2 and the profile of $\sum\limits_{u=2}^{i}\beta_{u-1}(t_k-t_{k-1})+\beta_i(t-t_i)$ is plotted in Figure 2. Indeed, β_0 is a steady state value and is related to the initial water content of the landfill. The water accumulated due to the rainfall is supposed to be discharged 9 h after the beginning of the rainfall. Twenty-seven measurements of surface water content are simulated using Equation (6) of the model. A Gaussian noise is added using a normal distribution with a varying standard deviation related to the accuracy of the measurements. As shown later, the precision of the measurements plays a key role in the efficiency of the algorithm developed.

Table 2. Coefficients of the piecewise linear function $\beta(t)$.

Coefficient	Value
β_0	0.70
β_1	0.00
β_2	−1.18
β_3	−1.66
β_4	−0.59
β_5	1.18
β_6	0.95
β_7	0.71
β_8	0.35
β_9	0.24

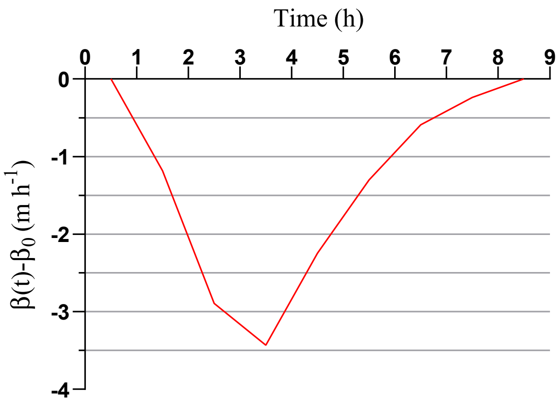

Figure 2. β profile used to simulate the data.

As anticipated, the reconstruction of the profile (i.e., the estimation of the set of β) using OLS is indeed an ill-posed problem. Of the two algorithms proposed in [27], the one based on the unimodality of the profile to be reconstructed has proved to be more reliable, especially for eliminating the necessity of introducing a penalty term whose coefficient is generally difficult to estimate. The procedure based on unimodality requires the transformation of the OLS into a quadratic programming problem with linear constraints. Both original and reconstructed profiles are plotted in Figure 3.

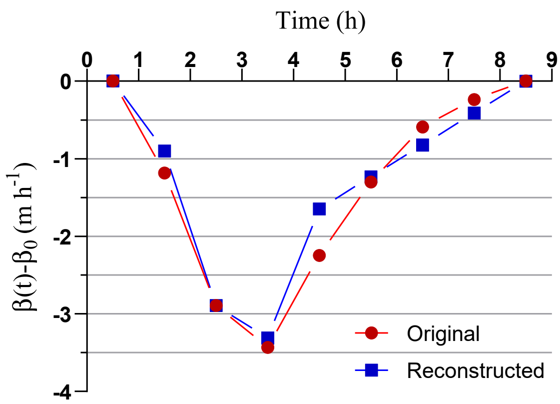

Figure 3. Original and reconstructed profiles.

The corresponding amounts of water leaked at the bottom of the landfill are shown in Figure 4.

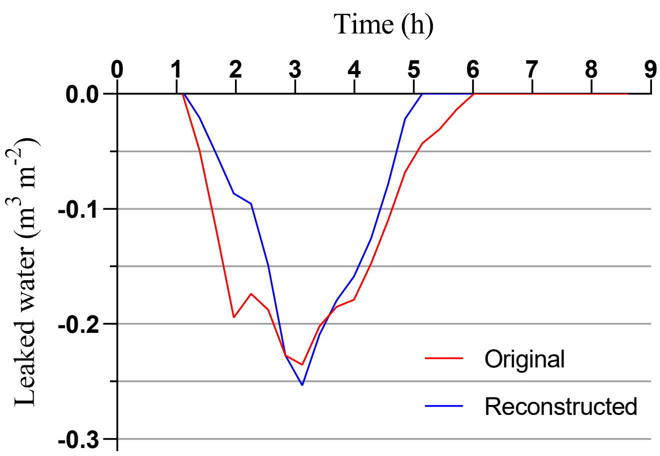

Figure 4. Original and reconstructed profiles of water leaks at the bottom of the landfill.

Using the parameters $\hat{\beta}$ obtained from the solution of the least squares problem in the analytical expression of C, G, and $\left(r^T r\right)$ (as well as the estimated variance of the data $\hat{\sigma}^2$), it is possible to compute the F-value of Equation (8).

As anticipated, the value of F depends heavily on the accuracy of the measurements. In Table 3, noise and F-values are related to the probability of the hypothesis that there is not a leak by comparing the F-values computed with the corresponding critical values of $F_{1,17}$. It shows that at the level of 2% measurement accuracy (which is the level of current technology [30]) the algorithm is capable of detecting a leak with approximately 50% of probability. As expected, a higher precision would increase the confidence in rejecting the null hypothesis that there are no leaks in the landfill.

Table 3. Dependence of F-values on measurements' accuracy.

Standard Deviation of Noise Added to the Data	F-Value	Probability of Absence of Leaks
0.001	53.77	≈0%
0.01	3.23	≈9%
0.02	0.558	≈47%
0.1	0.0004	Uninformative

Figures 2–4 as well as Table 2 provide the necessary information about the robustness of the model presented. Since the evaluation of the fluxes at the bottom of the landfill crucially depends on the profile of the reconstructed function $\beta(t)$, the reliability of the OLS analysis used to estimate it is a key issue due to the intrinsically ill-posed character of the algorithm. The results show that the unimodality regularization approach (a physically justified assumption after a strong rainfall event) provides a good reconstruction of the coefficients $\hat{\beta}$ and hence reliable estimates of the fluxes.

Table 3 summarizes the relation between the F-value of the null hypothesis (absence of leaks) with the accuracy of surface moisture measurements and constitutes the main result and goal of this contribution. Indeed, depending on the sophistication of the equipment used to measure the surface moisture, the operator is able to predict the presence of leaks with the level of confidence reported in Table 3. As outlined in the Discussion section, even off-the-shelf equipment provides a reasonable clue to the presence of leaks, but the statistical analysis can be considerably enhanced using more sophisticated equipment.

4. Discussion

The possibility of establishing a statistic for the detection of leaks obtained through a piecewise model of the boundary condition at the bottom of the landfill is the major contribution presented in this paper. Indeed, an approximate inverse model had been developed in previous articles [26,27], but the discontinuities introduced in the approximation of the boundary condition made it impossible to quantitatively estimate the probability of the presence of leaks. Rather, the inference about the presence of leaks was left to the qualitative judgment of the operator.

The example considered has shown that the algorithm possesses a high degree of robustness since it is able to identify the presence of a leak with over 50% of probability using state-of-the-art technology for measuring soil moisture. As shown by Table 3, further improvements of the measuring equipment accuracy would dramatically reduce the uncertainty in leak detection. Indeed, the 2% accuracy can be considered as an upper bound of commercial off-the-shelf equipment. Indeed, a frequently used in-situ soil moisture measuring technique is so-called Time Domain Reflectometry (TDR), which is based on the measurements of the dielectric properties of the medium over time. This method provides a theoretical accuracy of 1.2% [31], which can be further improved using empirical calibration curves obtained by fitting mathematical functions (generally third-order polynomials) of the measured dielectric constant as a function of the gravimetrically measured moisture of the particular soil considered. These curves provide an accuracy well above the default values contained in manufacturers' charts [32].

In addition to robustness, the considerable advantage provided by the method developed is an ease of implementation that considerably limits the cost of monitoring. To the best of our knowledge, there is presently no technique capable of inferring leaks without the use of sensors in the body of or below the landfill. Nor have we found in our literature search any model capable of relating leaks to the information that can be gained from surface measurements. Indeed, as reported in the Industry Code of Practice no. LGG 113 of the Landfill Guidance Group [33] " ... Mobile testing provides a one-off test after construction is complete and identifies any damage whilst a fixed system can provide information over the life span of the system which is typically a number of years". However, in addition to the reliability problems mentioned in the introduction, the cost of fixed systems limits the number of sensors, whereas the number of measurements of soil surface moisture based on mobile systems can be very extensive at a limited cost. Additionally, the procedure described could be conveniently used in many existent landfills that (especially in developing countries) do not have any sensor system installed.

5. Conclusions

The Authors are positively confident that the method described in this paper can be developed into a useful tool for landfill monitoring. Admittedly, some more theoretical analysis and an extensive experimental campaign would be necessary to fully validate our approach.

The theoretical part concerns the analysis of the sensitivity of the model with respect to the physical-chemical parameters (α, K_s, θ_s, θ_r), as well as with respect to the intensity of the rainfall (in addition to the sensitivity with respect to the accuracy of soil moisture measurements considered in this paper). Indeed, taking into account the uncertainties of the parameters for the particular landfill examined, it would be possible to evaluate the corresponding ranges of the F-values (as computed in the model) by evaluating the derivatives of the F-statistic with respect to the parameters and multiplying them by their uncertainties. The evaluation of the derivatives of the F-statistic with respect to the parameters could be carried out analytically or by automatic differentiation [34]. Though beyond the scope of the present work, this procedure would be able to estimate the role of additional sources of uncertainty and would complete the overall statistical analysis.

Furthermore, this information would help improve the design of number and frequency of measurements to be carried out for an efficient monitoring of a landfill.

These analyses constitute, in addition to field campaigns that are necessary to experimentally validate the model, the future directions of our joint research. We are fully aware of the necessity of comparing theoretical results with experimental values and of developing calibration curves with a view to further improving the accuracy of the analysis. Indeed, the present bilateral project on the design, operation, and monitoring of landfills [2] will include an extensive testing of the method outlined in this paper.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Only an outline of the procedure used to obtain the solution (3)–(5) is provided in this Appendix A. A detailed description can be found in the seminal paper by Yuan and Lu [25].

Using the transformation $K(\psi) = K_s e^{\alpha\psi}$ and $\Phi(z,t) = K(\psi)/\alpha$ provides the Richards equation

$$\frac{\partial^2\Phi}{\partial z^2} + \alpha\frac{\partial\Phi}{\partial z} - S(z) = \frac{1}{D}\frac{\partial\Phi}{\partial t}$$

Its steady-state equation is given by

$$\begin{array}{l} \frac{d^2\Phi_s}{dz^2} + \alpha\frac{d\Phi_s}{dz} - S(z) = 0 \quad \text{withtheboundaryconditions} \\ \Phi_s(0) = \frac{K_s}{\alpha}\exp(\alpha\psi_1) = \beta_0 \\ \frac{d\Phi_s}{dz} + \alpha\Phi_s\Big|_{z=L} = -q_0 \end{array}$$

where, unlike Yuan and Lu's direct problem, the boundary condition at the bottom is to be estimated in the general identification problem.

The solution of this equation is precisely the expression (5) presented in the main body, i.e.,

$$\Phi_s(z) = \beta_0 e^{-\alpha z} + \frac{q_0}{\alpha}(\mathrm{e}^{-\alpha z} - 1) + \int_0^L G(z,x)S(x)dx$$

The Laplace transformation of the transient equation gives

$$\begin{array}{l} \frac{d^2\overline{\Phi}}{dz^2} + \alpha\frac{d\overline{\Phi}}{dz} - \frac{s}{D}\overline{\Phi} + \frac{\Phi_s}{D} - \frac{S}{s} = 0 \quad \text{withtheboundaryconditions} \\ \overline{\Phi}(0) = \frac{\beta_0 + \sum\limits_{u=2}^{i}\beta_{k-1}(t_k - t_{k-1}) - \beta_i t_i}{s} + \frac{\beta_i}{s^2} \\ \frac{d\overline{\Phi}}{dz} + \alpha\overline{\Phi}\Big|_{z=L} = -\overline{q}_1 \end{array}$$

Again, the boundary condition at z = 0 is different from the one used by Yuan and Lu because its value is not fixed as in direct problems but has to be estimated from measurements at the surface.

A procedure analogous to that described in detail by Yuan and Lu in the Appendix of their article leads to the sought-after solution (3)–(6) of the main body.

Appendix B

From the definitions, the following relationships hold

$$\mathrm{F}_0(z,t) = \frac{q_0}{\alpha}(\mathrm{e}^{-\alpha z} - 1) + \int_0^L G(z,x)S(x)dx + 8D\mathrm{e}^{-\frac{\alpha z}{2}}\sum_{n=1}^{\infty}\frac{\left(\lambda_n^2 + \frac{\alpha^2}{4}\right)sin(\lambda_n L)}{2\alpha + \alpha^2 L + 4L\lambda_n^2}\mathrm{e}^{\frac{\alpha L}{2}}sin(\lambda_n z)\int_0^t (q_0 - q_1(\tau))\mathrm{e}^{-D(\lambda_n^2 + \frac{\alpha^2}{4})(t-\tau)}d\tau$$

$$\mathrm{F}_1(z) = e^{-\alpha z}$$

$$\mathrm{F}_{2u}(z,t) = 8D\mathrm{e}^{-\frac{\alpha z}{2}} \sum_{n=1}^{\infty} \frac{\mathrm{e}^{-D(\lambda_n^2 + \frac{\alpha^2}{4})t}\left(\lambda_n^2 + \frac{\alpha^2}{4}\right) sin(\lambda_n L)}{2\alpha + \alpha^2 L + 4L\lambda_n^2}\left(\lambda_n cos(\lambda_n(L-z)) + \frac{\alpha}{2} sin(\lambda_n(L-z))\right)\xi_{un}$$

Thus, for $z = L$

$$\begin{aligned}\mathrm{F}_0(L,t) =& \frac{q_0}{\alpha}\left(\mathrm{e}^{-\alpha L} - 1\right) + \frac{\mathrm{e}^{-\alpha L}}{L}\int_0^L (1-\mathrm{e}^{-\alpha x})S(x)dx + \\ & 8D\mathrm{e}^{-\frac{\alpha L}{2}} \sum_{n=1}^{\infty} \frac{\left(\lambda_n^2 + \frac{\alpha^2}{4}\right) sin(\lambda_n L)}{2\alpha + \alpha^2 L + 4L\lambda_n^2} \mathrm{e}^{\frac{\alpha L}{2}} sin(\lambda_n L) \int_0^t (q_0 - q_1(\tau))\mathrm{e}^{-D(\lambda_n^2 + \frac{\alpha^2}{4})(t-\tau)} d\tau = \\ & \frac{q_0}{\alpha}\left(\mathrm{e}^{-\alpha L} - 1\right) + \frac{\mathrm{e}^{-\alpha L}}{L}\int_0^L (1-\mathrm{e}^{-\alpha x})S(x)dx + \\ & 8D\sum_{n=1}^{\infty} \frac{\left(\lambda_n^2 + \frac{\alpha^2}{4}\right) sin^2(\lambda_n L)}{2\alpha + \alpha^2 L + 4L\lambda_n^2} \int_0^t (q_0 - q_1(\tau))\mathrm{e}^{-D(\lambda_n^2 + \frac{\alpha^2}{4})(t-\tau)} d\tau\end{aligned}$$

$$\mathrm{F}_1(L) = e^{-\alpha L}$$

$$\mathrm{F}_{2u}(L,t) = 8D\mathrm{e}^{-\frac{\alpha L}{2}} \sum_{n=1}^{\infty} \frac{\mathrm{e}^{-D(\lambda_n^2 + \frac{\alpha^2}{4})t}\left(\lambda_n^2 + \frac{\alpha^2}{4}\right) sin(\lambda_n L)}{2\alpha + \alpha^2 L + 4L\lambda_n^2}\lambda_n \xi_{un}$$

and for $z = 0$

$$\mathrm{F}_0(0,t) = 8D\sum_{n=1}^{\infty} \frac{\left(\lambda_n^2 + \frac{\alpha^2}{4}\right) sin(\lambda_n L)}{2\alpha + \alpha^2 L + 4L\lambda_n^2} \mathrm{e}^{\frac{\alpha L}{2}} sin(\lambda_n 0) \int_0^t (q_0 - q_1(\tau))\mathrm{e}^{-D(\lambda_n^2 + \frac{\alpha^2}{4})(t-\tau)} d\tau = 0$$

$$\mathrm{F}_1(0) = 1$$

$$\mathrm{F}_{2u}(0,t) = 8D\sum_{n=1}^{\infty} \frac{\mathrm{e}^{-D(\lambda_n^2 + \frac{\alpha^2}{4})t}\left(\lambda_n^2 + \frac{\alpha^2}{4}\right) sin(\lambda_n L)}{2\alpha + \alpha^2 L + 4L\lambda_n^2}\left(\lambda_n cos(\lambda_n L) + \frac{\alpha}{2} sin(\lambda_n L)\right)\xi_{un}$$

The derivatives, which are necessary for computing the flow, are given by

$$\begin{aligned}\left.\frac{\partial \mathrm{F}_0(z,t)}{\partial z}\right|_{z=0} =& -q_0 + \int_0^L \frac{\partial G(z,x)}{\partial z} S(x)dx + \\ & 8D\sum_{n=1}^{\infty} \frac{\lambda_n\left(\lambda_n^2 + \frac{\alpha^2}{4}\right) sin(\lambda_n L)}{2\alpha + \alpha^2 L + 4L\lambda_n^2} \mathrm{e}^{\frac{\alpha L}{2}} cos(\lambda_n z) \int_0^t (q_0 - q_1(\tau))\mathrm{e}^{-D(\lambda_n^2 + \frac{\alpha^2}{4})(t-\tau)} d\tau = \\ & \frac{\partial\left[\frac{1}{\alpha}(\mathrm{e}^{-\alpha z} - 1)\right]}{\partial z}\int_0^L S(x)dx \\ & +8D\sum_{n=1}^{\infty} \frac{\lambda_n\left(\lambda_n^2 + \frac{\alpha^2}{4}\right) sin(\lambda_n L)}{2\alpha + \alpha^2 L + 4L\lambda_n^2} \mathrm{e}^{\frac{\alpha L}{2}} cos(\lambda_n z) \int_0^t (q_0 - q_1(\tau))\mathrm{e}^{-D(\lambda_n^2 + \frac{\alpha^2}{4})(t-\tau)} d\tau = \\ & -\int_0^L S(x)dx + 8D\sum_{n=1}^{\infty} \frac{\lambda_n\left(\lambda_n^2 + \frac{\alpha^2}{4}\right) sin(\lambda_n L)}{2\alpha + \alpha^2 L + 4L\lambda_n^2} \mathrm{e}^{\frac{\alpha L}{2}} cos(\lambda_n z) \int_0^t (q_0 - q_1(\tau))\mathrm{e}^{-D(\lambda_n^2 + \frac{\alpha^2}{4})(t-\tau)} d\tau\end{aligned}$$

$$\left.\frac{d\mathrm{F}_1(z)}{dz}\right|_{z=0} = -\alpha$$

$$\left.\frac{\partial \mathrm{F}_{2u}(z,t)}{\partial z}\right|_{z=0} = -4\alpha D\sum_{n=1}^{\infty}\frac{\mathrm{e}^{-D(\lambda_n^2+\frac{\alpha^2}{4})t}\left(\lambda_n^2+\frac{\alpha^2}{4}\right)sin(\lambda_n L)}{2\alpha+\alpha^2 L+4L\lambda_n^2}\left(\lambda_n cos(\lambda_n L)+\frac{\alpha}{2}sin(\lambda_n L)\right)\xi_{un}+$$
$$+8D\sum_{n=1}^{\infty}\frac{\mathrm{e}^{-D(\lambda_n^2+\frac{\alpha^2}{4})t}\left(\lambda_n^2+\frac{\alpha^2}{4}\right)\lambda_n sin(\lambda_n L)}{2\alpha+\alpha^2 L+4L\lambda_n^2}\left(\lambda_n sin(\lambda_n L)-\frac{\alpha}{2}cos(\lambda_n L)\right)\xi_{un}$$

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