

Article

Restrictions of Historical Tissues on Urban Growth, Self-Sustaining Agglomeration in Walled Cities of Chinese Origin

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Abstract: This article uses a fractal observation to help delineate the constraints placed by multiple city walls on the growth of historical East Asian cities. By applying advanced technologies from economic geography and fractal indices, a staged scaling process within urban dimension coherence can be applied to both indices. In this study, a discovery is proposed based on the urban organism concept that is capable of indicating a proportional intra-urban structure from a fundamental wall-bounded urban element (local specificity) to other greater walled spatial properties (global variables). This local specificity potentially performs approximate scaling regularities, and spatially denotes an average historical threshold of urban growth for its overall size, with similar scaling law constraints. This finding involves territorial, urban planning, and ancient architectural perspectives, providing a historical and local response to the expansion of contemporary cities. By employing growing fractal estimation, data processing enables the logarithmic city size to be obtained by measuring each wall's specific features using the Ordinary Least Squares (OLS) method. On the basis of two-dimensional allometric scaling patches, a spatial unfolding mechanism is utilized to reproduce these dynamic changes with city walls as a result of the human trajectories in time geography.

Keywords: historical agglomeration; urban equilibrium; allometric scaling; walled city; organizational hierarchy

1. Introduction

As a morphological phenomenon, cities have maintained relatively stable structural dynamics and texture complexity in a long historical period to maintain an urban equilibrium, which has recently transitioned towards a fast expansion due to urban sprawling, thus placing an increasing anthropogenic effect on the natural environment. More importantly, the uncontrolled population growth and the increased burden on environmental resources will eventually lead to a major dependence on the intensive use of pesticides and fertilizers [1], ultimately pushing the integrity of many biomes' biodiversity to beyond the limits of safety [2]. Therefore, research on the upper limit of the urban growth scale in a historical period will provide a comparison for the current excessive population growth and, at the same time, will remind us to describe spatiotemporal changes in urban growth boundaries from the perspective of historical growth by using some simple and clear growth mechanisms.

The current trend is to use a clustering algorithm to sample cities as environmental and socioeconomic scenarios, which represent urban expansion through statistical cluster aggregation and

a discrete agglomeration boundary definition. In line with the history of urbanization, truly evolved urban areas also invoke a spontaneous result from their embryonic state, given that it shows a connection to the “traditional individual’s development trajectory of the overall organization” when attracting and disseminating capital and information flows [3,4]. Those features that are considered to be localized in global chain command controls are generalized as institutional settings for correlated urban patterns in reasonable endogenous processes and represent the structural inhomogeneity of the material (urban) fabrics being created. According to the long line of literature on urban geography and economics, the simulated agglomeration model for urban growth, however, still focuses on quantifying the environmental and socioeconomic outcomes of cities with respect to their sizes (i.e., the city size for population outcomes), mainly confronting a polarized city size, defined as the whole city (system) size with urban scaling law domains [5–8]; internal city structures have networks and form pattern controls [9–13], yet they comparatively contribute to a universal definition by which cluster integration can be used to help classify site ontologies from urban areas across various macro variables of different cities with differing sizes. This approach, in line with the concept of underlying self-organization [14], uses a biological analog to express the way in which urbanized land use varies and correlates with the city size distribution, laying emphasis on the scaling laws and organizational tensions across the organism’s metabolism and location of the relative temporal and spatial dynamics. The related literature has been principally empirical and focused on the following: the urban size distribution of a city’s population, particularly its power-law distribution [15]; the city’s relational driving force to aggregate [16,17]; and the structural endogeneity of the growth framework in various domains [13,18–20].

However, despite the fact that all the aforementioned delineations agree that urban agglomeration is not only a geographically continuous entity but also a closely integrated spatial existence of networks (people, cargo, capital and information) and nodes (central and peripheral cities) [21], the fuzzy physical boundary and multiple perspectives from multiple ideologies still make quantitative criteria a huge and complex responsive system, remaining in universal or fluid dimensions and having no consensus on unfolding such dimensional sedimentations through any relational trajectories of locational potentials for growth. In other words, research in this field still lacks the concept of a ‘material substratum’ that affords the physical and socioeconomic benefits of cities’ allometry from their initial emergence to each appearing growth radius of single or multiple production center aggregates. Nordbeck [22], Martin [23], and Batty and Longley [24] have characterized such observational sprawling within the allometric equation. We herein continue using this biological scaling law to delve into fundamental elements of the city organism.

On the other hand, alongside the rapid changes and activities from non-fixed growth driving forces, developed cities’ pattern formation endures increasingly complex dynamics from the original structural interactions to the subsequent urban renewal and population growth. This occurs in contrast to the cities’ historical textures (fabric), such as street networks, old quarters, ramparts, and heritage building arrangements, as well as the rises and falls of various sequences of technology cycles. Thus, when looking at the evolution process of a city’s geometric pattern and material fabric from the perspective of non-linear development, the city in its embryonic developmental stage has provided a relatively stable state that involves the overall organizational structure in consistent development from a self-sustainable urban equilibrium (order) to a dynamic non-equilibrium (chaos) [14,25], evolutionarily anchoring the agglomeration object into each random land consumption starting point. Regardless of the spatiotemporal unfolding practice, little attention has been paid to this urban agglomeration delineation with territory-associated features [26–29]. In other words, these studies are believed to provide insights that illuminate the unpacking of agglomeration bodies, aiming to link the cities’ activities resulting from an endogenous intra-urban process with the fundamental land-use properties as locales.

For this case study, 21 multiple-wall-enclosed historical cities in East Asia are utilized to establish a database to assess the universal macroscopic pattern of the pre-industrial city model. Hence,

the well-kept wall records allow for a population investigation by using the physical size proxy of unavailable historical population data [30]. Depending on the pre-industrial wall-enclosed feature, the material built-up environment of these cities, including buildings, wards, city blocks, and communities with walled boundary classifications, have also been considered to preserve an introverted spatial living mode for different land-use properties, where most of these amenities have demonstrated their territory isolation in accordance with the compound construction for respective environmental and socioeconomic outcomes.

Under the organismic concept, the physical form of these cities has maintained their overall embryonic state through continuous wall mosaics, granting a leap-frog proportion to sample coherence land consumption laws. In the context of the endogeneity of cities' systems, multiple walls have physically limited the expansion of city sizes and created an organic force from the center to the periphery [31,32]. These walls also demonstrate a spontaneous motivation for capital allocation and coalescence of material tissues [12,33], which can assist in implementing urban clustering supported by different systems of behavior, along with the correspondence of its physical environment and socioeconomic outcome as appropriate urban scaling scales. Hence, in the case of urban embryos surrounded by these walls, their historical development status can be summarized as a kind of organizational elasticity in which the tissues are all glued together by a common growth process.

On the basis of these grounds, the purpose of this article is to find the scaling regulation principle formed by the historical growth state of Sino-influenced wall formation. The variable of their envelope tissue is hopefully expressed through a simple set of scaling mechanisms. At the same time, the two polarized aspects of urban modeling discussed in the aforementioned literature (globally integrated cities' systems and locally heterogeneous internal structures) are herein represented by the aim of empirical analysis for the organic force fueling urban growth in historical walled boundary constraints, namely, reflected by the urban profile contingent (the homothetic scaling level from the same location) and locational growth radius (an endogenous force from allometric consistency). The contingent of urban profiles is capable of being analyzed through diffusion and aggregation thinking, for which the area-perimeter method is used to pursue their hypothesized scaling center from the Ordinary Least Squares (OLS) estimate, whereas the locational growth radius is restored by allometric scaling that uses the estimated city's logarithmic size to trace the hierarchical endogeneity of the entire urban dimension within different environmental and socioeconomic benefits. As a successive boundary of urban growth restrictions, we assume that all studied cities in wall enclosures are completed artifacts as organisms. Consequently, we expect that the urban individual element will be locally proportional to the whole city's growth, including both allometric and homothetic growth. This involves examining a political or social network in this research that forms both the urban systems and internal structures located in the same actual places and that embodies the emerging urban agglomeration.

This article is organized as follows: the associated literature and theoretical background, as well as data interpretations of Sino-influenced wall-enclosed city models, are introduced in Section 2. Section 3 introduces the database that is accessed with a methodological innovation and generation of the dynamic model, involving the spatial organization evaluation for the endogenous growth level and structural stability of each walled city. In Section 4, we summarize and discuss the estimated results of our findings. Finally, the complementary theory and possible contributions are discussed in Section 5.

2. Sino-Influenced Walled City Formation

2.1. Social and Historical Context in Chinese Walled Cities

Wall-enclosed cities formed in ancient East Asia were probably most highly influenced by 'Sinocentric' cultural traditions for their similar special planning morphology and socio-spatial delimiting [34], which artificially categorizes their socioeconomic and geographic evolution patterns as an organization delineation of urban cluster integration. Herein, the wall enclosure, as the fundamental urban built-up indicator, materially emphasizes the localized spatial living patterns emanating with

respect to their land-use property of relational organization hierarchy. Its globally attributed walled compound structures of clustering significance are attributed to all introverted urban profiles within the same generic metric of locational potential quantification. Each individual living mode that constitutes a localized building environment (urban fabric) is demarcated by the multiple-wall enclosure complex that separates private and quiet residential spaces from the public and chaotic urban environment [32]. This special urban classification is generally supported by the ‘centralized planning economy’ for each social community and political network in a highly concentric manner. Moreover, the hierarchical wall clustering process has facilitated a proportional land consumption rate that makes the whole city structure a self-organized system for its increase in total size and unchanging population densities, especially for some historical capital cities in their embryonic periods. (The ancient Chinese developed a unified urban theory involving spatial form, artistic planning and aesthetics values, all of which are now the unique wealth inherited and developed by East Asian countries whose societies were once a part of Chinese civilization and whose scripts are still widely acknowledged as derivatives of Classical Chinese script, which is broadly alive in today’s Japan, Korea, Mongolia and Vietnam [35]). Under the mutual influence from cultural dissemination, the doctrine for such urban form development is commonly inherited by the historic states in the mainland and peripheral regions of East Asia.

Upon locating the historical agglomeration state within significant wall enclosures, each wall forming an urban profile with its environmental and socioeconomic development has received the most consideration possible in associated literature [36], since the wall and the tower should be the most distinguishable symbols marking a distinct urban space from the surrounding countryside [37]. The walled discrimination (a combined road network with the street plan from a walled-ward) [38] is commonly accepted as the spatial significance factor that indicates the consumption of land size properties and is employed as the manual proxy to clarify the population distribution [39]. In other words, the urban structure of Sino-influence walled cities is more inclined to “use the enclosure of the wall to replace the changes in a large number of land-use areas on the edge of the city, thereby describing the expansion of the city” [40], and the origin of the city, starting from proportional planning right from the very beginning [41], can prevent the tendency of the large discontinuity of closely settled areas haphazardly intermingled [42].

In view of endogenized urban land consumption, one of the ubiquitous locational factors used to link the agglomeration subject to a localized wall enclosure feature is the feudal monarchy domain land-use framework, which is a politically dependent territorial emphasis from which introverted spatial living patterns emanate. This essentially positions the origin of the entire urban system to be a certain local physical size, which can be specific to the wall-enclosing form of a certain public building scale. In this situation, the global socioeconomic outcomes that imprint urban agglomeration within the corresponded activities hierarchically invoke urban internal structures into the same allometric progress. These structures include the city edge, building envelope, planning layout, social aspects, and political administration. The global socioeconomic outcomes also adapt to the historical urban fabric, such as single buildings, patios, communities, districts, small towns, and other compound structures, in a homothetic scaling relationship according to their political representative generic allometry (Figure 1). Formally, the inner wall area encompasses a higher range of wall enclosures (up to the city boundary), and this higher-level area has a more complex socioeconomic activity environment. Such a strict division makes each wall enclosure adhere to respective social rank hierarchies and quantified in pieces of laps with at least four or five levels, while the dividing characteristics can commonly start from the wall-enclosed range from the hall, palace, imperial city, administrative city, town city, and outer city (Table 1). Each profile with respective urban dimensions has contributed differently to yield the environmental and socioeconomic benefits as homothetic scaling proportions.

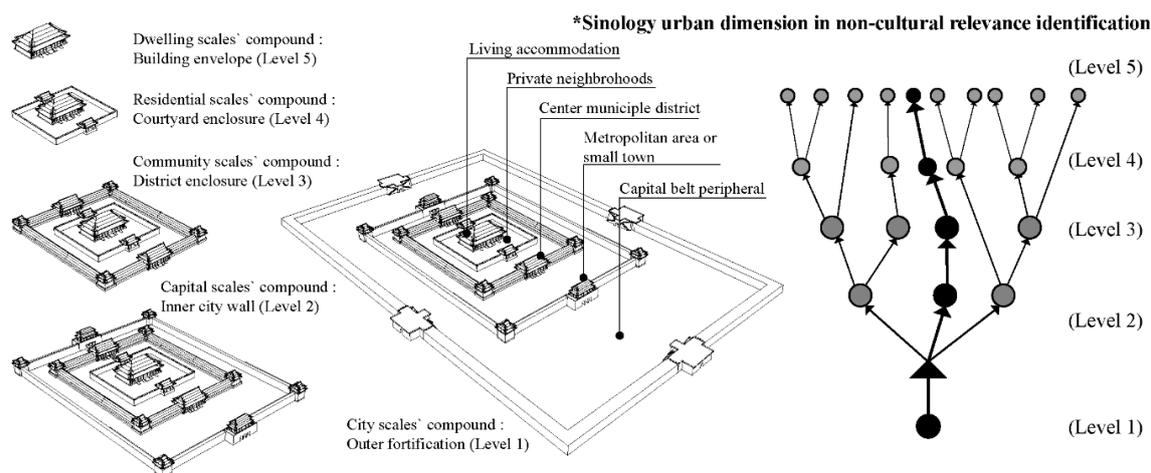


Figure 1. Walled spaces that are significant for urban cities with homothetic territorial features demarcated from their relational global force imprints in macro-levels.

Table 1. Dimension and organization hierarchy of wall enclosure urban profiles in a macrostructural to a microstructural process of Sinocentric land development using a scaling framework.

Urban Spatial Ranks	Space Centralized Dissemination			Social Organization Hierarchy	
	Urban Enclosures	Living Pattern Emanation	Compound Typology	Social Class (Political Case)	Interacting Type
Rank 1	City area	Socioeconomic activity	Agglomeration	Citizen	Society
Rank 2	District area	Municipal	Capital area	Seignior	Bloc
Rank 3	Residential quarters	Residential environment	Community	Cabal	Clan
Rank 4	Courtyard	Neighborhood	Courtyard	Royalties	Family
Rank 5	Interior space	Individual space	Housing	Emperor	Individual

2.2. Data Acquisition and Processing

The empirical analysis of this study was based on the notion that all Sino-influenced cities impart morphological statistics on the surrounding walls with respect to their geographic spatial characteristics, and all these cities have existed as capitals of the country at certain historical times. A large amount of historical investigation work collected all earlier experience and theoretical literature to help reconstruct the characteristics of these cities (for details, see Table A1 in Appendix A). At the same time, by using the data gathered manually as a reference, the existing preserved wall sites were corrected using remote sensing images. The materials adopted the principle notion of “China’s core and peripheral regions” from Shuji Funo [43] and experience wide distribution, currently including mainland China, Mongolia, Korea, Japan, Vietnam, and some peripheral regions of East Asia. Most of the data came from the long-term research project of historical literature, which is mainly led by the Chinese Academy of Social Sciences (CASS) [44], Nara National Research Institution for Culture Properties (Japan) [45], and Culture Heritage Administration of Korea (CHA) [46], which are known for their historical approach to urban recovery, heritage conservation, and urban planning study. A large group of public databases on urban form, walled city size, stratified walled areas, and walled city envelopes from exhaustive surveying was recorded, enabling us to access the original allometry status of those walls surrounding cities from various geometric measures. All the data, including of 21 historic walled cities, could support the empirical analysis. Most of the databases have well-maintained detailed information on each wall’s structure and can help us restore its pre-modern city type. Their distribution locations are marked in Figure 2.

Combining the remote sensing images from Google Maps with the complementary data from correlated historical literature and drawings, the major source of the formed area and perimeter of each wall-enclosed urban dimension can verify their data accuracy through manual error correction.

All related remote sensing images were downloaded from the LocaSpace Viewer center [47] of the commercial software. We first converted the images of these cities into the GCS_WGS_1984 geographic coordinate system and used Asia_Lambert_Conformal_Conic as the project coordinate system. Subsequently, we used CAD software to refer to historical documents and drawings in creating vector graphics for cities in the form of city walls. By applying the spatial calibration tool in Esri ArcGIS, and by setting the geometric feature points to the appropriate spectral features and key landmarks, the shape and perimeter recorded by the vectored wall surrounding the city and the historical wall relics actually saved or excavated the performed alignment transformation. The hierarchical wall-embedded city forms and boundaries could then train their cross-sectional levels within the geometrical feature clipping and extraction, in which their spatial extrusion showed an exponential relationship from the center to frontiers. This geographic information with spatial hierarchy classification and manual corrections are illustrated in the array in Table 2 to reflect the actual city size.

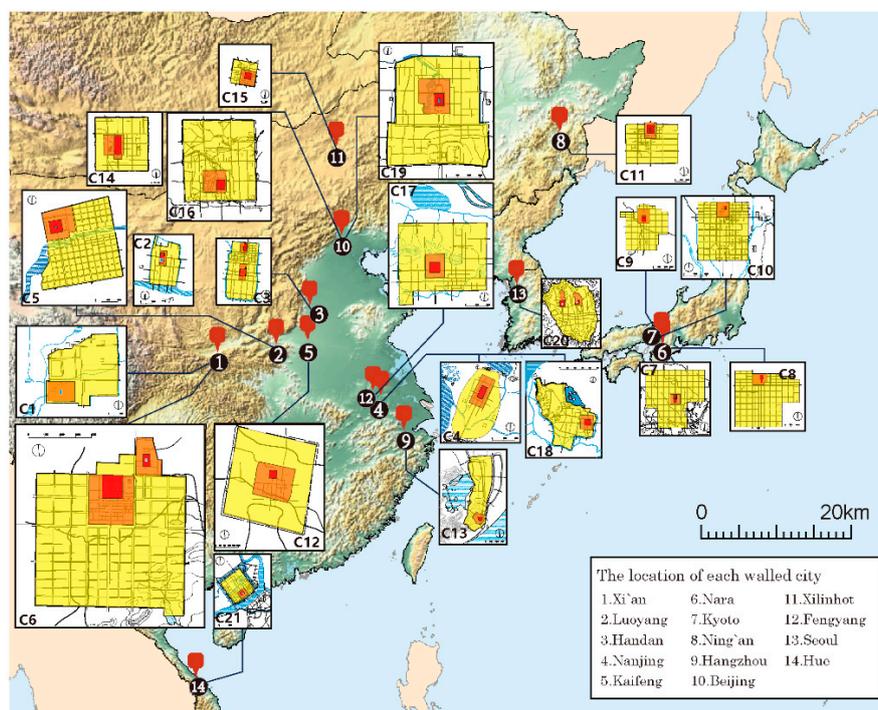


Figure 2. Wall concentric distribution pattern in different historical places with the current location or city names, East Asia.

The scaling laws of the Sino-influence city in wall enclosures can be obtained by modeling and depicting the two-dimensional geometric patterns of its introverted living space and increased land occupation. Then, each individual mode of private landscape extension can characterize its potential (scaling) trajectory contingent on relational global and local activities [48] with formed diffusion or aggregation. It resembles the intrinsic urban tissue variables within single command controls. Meanwhile, as central dissemination proceeds with wall enclosure constraints, structural tension can be found from the walled spatial property extensions and can be relatively scaled in accordance with the core place size for its encapsulated spatial unit [49].

Generally, when applying wall enclosures as a ubiquitous feature of urban cluster statistics, the extensive part of the city in subsequent fractions is assumed to be able to proportionally regress to its embryonic form and function as a structurally self-sustaining or sometimes self-regulating organization. Thus, if the current city size greater or smaller than the former (S) is $S + a$ or $S - a$, respectively, then the city in an organism can vary under a power-law relationship. As the most common symbol of a walled city's clustering, the introverted spatial enclosure feature allowed us to

correlate the size of the fundamental element in urban growth with the overall structure scale through the control of chain instructions while, at the same time, linking the existing enclosure. The evolution law between forms serves as a prediction of the space-enclosing pattern in a larger area [50]. In order to describe this process, the impact of all urban growth models, including the observed evolution of urban geographic space, as well as various artificial infrastructures and political and cultural activities, were conceptualized as a simple rule-controlled operating mechanism [9] that made a meaningful attempt to simulate walled cities.

Table 2. Wall-enclosed city areas and perimeters with respect to different clustering representatives of a specific growth trajectory (political domains) in 21 Sino-influenced historical cities, East Asia.

Abbr.	Capital Name	Outer Wall		Inner Wall		Palace Wall		Courtyard Wall		Hall Building	
		Perimeter P (Km)	Area A (Km ²)	Perimeter P (Km)	Area A (Km ²)	Perimeter P (Km)	Area A (Km ²)	Perimeter P (m)	Area A (m ²)	Perimeter P (m)	Area A (m ²)
C1	Chang'an (Han)	25.700	35.749	8.800	4.767	N/A	N/A	1221	80,534	N/A	N/A
C2	Luo yang (Han)	14.184	9.156	4.010	0.838	2.815	0.396	1360	103,891	338	6779
C3	Ye	15.800	14.467	4.410	1.008	3.508	0.606	N/A	N/A	N/A	N/A
C4	Jian kang	18.054	21.743	8.647	4.095	3.464	0.638	N/A	N/A	N/A	N/A
C5	Luo yang (Tang)	27.900	45.389	9.444	5.502	4.174	1.088	985	60,057	306	5248
C6	Chang'an (Tang)	42.511	86.755	7.647	3.256	3.277	0.465	N/A	N/A	N/A	N/A
C7	Fujiwara Kyo	20.250	18.873	3.694	0.853	2.003	0.191	577	20,379	N/A	N/A
C8	Heijo Kyo	22.263	25.715	4.588	1.228	1.857	0.151	420	10,643	137	1049
C9	Nagaoka Kyo	16.819	13.090	N/A	N/A	N/A	N/A	468	13,564	125	903
C10	Heian Kyo	19.455	23.530	4.878	1.474	1.368	0.087	676	20,103	149	1132
C11	Balhae	16.400	16.086	4.827	1.437	2.708	0.455	767	36,708	163	1447
C12	Bianjing	28.518	52.942	11.550	8.480	2.521	0.394	N/A	N/A	N/A	N/A
C13	Lin'an	17.994	11.381	2.725	0.495	1.096	0.061	N/A	N/A	N/A	N/A
C14	Jin Zhongdu	18.690	22.055	9.797	5.338	5.05	1.451	N/A	N/A	N/A	N/A
C15	Yuan Xanadu	8.805	4.852	5.620	1.972	1.755	0.328	N/A	N/A	N/A	N/A
C16	Dadu	28.600	50.896	9.289	5.103	3.45	0.732	1281	96,406	230	3156
C17	Ming Zhongdu	30.365	48.904	7.670	3.617	3.702	0.855	N/A	N/A	N/A	N/A
C18	Nanjing	35.267	43.240	9.950	4.643	3.572	0.803	N/A	N/A	N/A	N/A
C19	Beijing	34.368	65.055	11.104	6.773	3.44	0.736	1353	96,253	205	2428
C20	Seoul	21.443	23.088	3.900	0.692	2.404	0.329	507	15,852	121	896
C21	Hué	10.746	5.016	2.646	0.387	1.269	0.1	387	8627	159	1551

3. The Mathematics of Walled Cities: Allometric Growth

Despite non-cultural relevant conditions, the gear shifts between each walled urban enclosure are still considered to have the traits of a city system's relational dynamics of the tissue variable, where the walled-bounded agglomeration body and the localized urban enclosure feature are both relationally responsive to the individual spatial living modes. This locally situated process uses an initial emergence to constraints on cities from their priority growth force to impose a localized urban unit size for mass quantities. Thus, in this study, the wall-delimited land occupation of the Royal Assembly Hall, a building type, could be used as a representative of the basic unit of the city. While the range of wall enclosure spatial characteristics encompasses the scaling relationship of the spatial life model, it represents and provides a self-similarity based on actual location factors for the entire multi-layered wall enclosure system. For such geospatial observation of spatial organization diffusion and boundary evolution (in multiple overlapping cities' perimeters), a fractal scaling behavior can be appropriately adaptive to these locational emanated growth boundaries and describe their structural intensity within different wide ranges and diffusion scales.

Prevalent methods of fractal estimation are mainly implemented by the area-radius scaling method [51–54], the area-perimeter scaling method [55–57], and the box-counting method [58–60]. The area-perimeter scaling method that we used in this paper aims to reveal the evolution mode of city morphology by switching between different homogeneous growth boundaries. This method has been recently employed to characterize the urban size evolution in a fractal description [61]. In practice, the walled area-perimeter features on a geometrical basis are equivalent in construction by the system of scaling equations to the numerical illustration of the dimension relation [62] and to equipotential evolution of scaling hierarchies [18,63–65]. This approach applies the scaling relationship to reveal the urban system's spatiotemporal evolution dynamics.

3.1. Fractal Measurement of Urban Dimensions Evolution

Under the description of the fractal dimension, the change in the geometric measure between the non-Euclidean dimension indices ensures the consistency of the evolution of the spatial form during the extrusion change and can be established in the traditional shape lineage from linear space to the correspondence of the columnar space. The concept here is that such a clarified dimension relationship can only be found in situations where the dimension value of geometric measure X is equal to another measure Y with geometrical proportionality; hence, we can say that they have a dimensional consistency [66,67]. The dimension consistency has a proportional relationship, such as $Y \propto X$ or $Y = kX$, where k is a proportionality constant for the coefficient. Between different measures, the consistency of the dimension means that, genealogically, any spatial quantity M (their volume amount 'Mass'), such as length L , area A , and volume V , can have their relationship expressed as follows:

$$L^{1/1} \propto A^{1/2} \propto V^{1/3} \propto M^{1/d}, \quad (1)$$

where the index df refers to a general dimension [60]. If the value emerges as an integer, it denotes the traditional Euclidean object. For instance, $df = 0$ defines a point, $df = 1$ defines a line, $df = 2$ defines a plane with its surface, and $df = 3$ defines a cube.

In the case of wall enclosures of a city in practice, the wall-surrounded city form's territorial features can be regarded as having general 2D dimensions $df = 2$ for the Euclidean plane and boundary perimeters as $df = 1$ for a line. In the fractal theory of dimension consistency, the geometric pattern that varies from area to boundary involves more than a Euclidian dimensional process in which their relational spatial measurements of the accordant shape index are all attributed to a quantitative spatial mass, and the dimension evolution is estimated through the regression analysis of the Ordinary Least Squares (OLS) method [60,68]. Their dimensional relationship in a fractal follows the power-law expression:

$$\left(\frac{P}{k}\right)^{1/D_l} = A^{1/2}, \quad (2)$$

where P refers to the urban boundary perimeter, which demarcates the irregular form shape with a successive enclosed walled curve, while A refers to the bounded distribution area. For the boundary dimension in the fractal relationship, D_l indicates a cascade proportion that derives the urban size (geographical) from the conventional dimension to a fractal abundance, and k is the proportional constant reflecting the complexity of boundary allometry.

As a geometric measure, D_l is used to express the exponential relationship in the spatial dimensional relationship, and it is expressed by the concept of "initial size" because it reveals the ratio required to distinguish between different conventional dimensional indexes. By using the data from the real-world system with geometric measures, it is easy to evaluate the value of k and P/k through a regression analysis, where P/k can be regarded as the instant quantity of spatiotemporal growth that is available in both geometric and mathematical descriptions. The data measured from the perimeter (line) and area (plane) are used to describe the spatial allometry of urban agglomerations, and their dimensional consistency can satisfy the following relationship:

$$D_l = \frac{2 \ln(P/k)}{\ln(A)}. \quad (3)$$

In order to avoid the overestimation of insignificant sizes of values in the OLS method, the morphological measurement related to the two-dimensional area has been improved for identifying the quasi-dimension D_l estimate [51,62,69] as follows:

$$D_b = \frac{(1 + D_l)}{2}, \quad (4)$$

where D_b denotes the revised boundary dimension and is designed to establish compatibility in the dimension consistency of fractal abundance. By using the regression analysis with $b = D_l/d = D_l/2$, the value of D_b can be derived from D_l , as their linear correlations are characterized by the log-linear regression and reflect an initially fixed entire dimension quantity.

3.2. Spontaneous Growth and Spatial Allometric Scales

In the general fractal growth model, the underlying mechanism of self-similarity [70] is often considered to be able to respond appropriately to the growth trajectory of correlations that exist in natural and social systems. If a city can use the spatial expansion resulting from its endogenous growth as a physical diffusion to its neighbors and can subdivide the possibility of such scaling due to diffusion in a cascading land distribution method, then the continuous emergence of this growth phenomenon, that is, its evolution law, can be summarized as a structural elasticity controlled by the central government (Figure 3). Their spatial scaling has been thus generalized as allometric growth [24,67]. Moreover, in many cases, the scale index of allometric growth plays a more important role than the fractal dimension in the spatial analysis of urban systems [71]. This relationship actually illustrates a hyperbolic relationship during urban form aggregation, and the correlated forms emerge as territory features and adapt through the fractal dimension description.

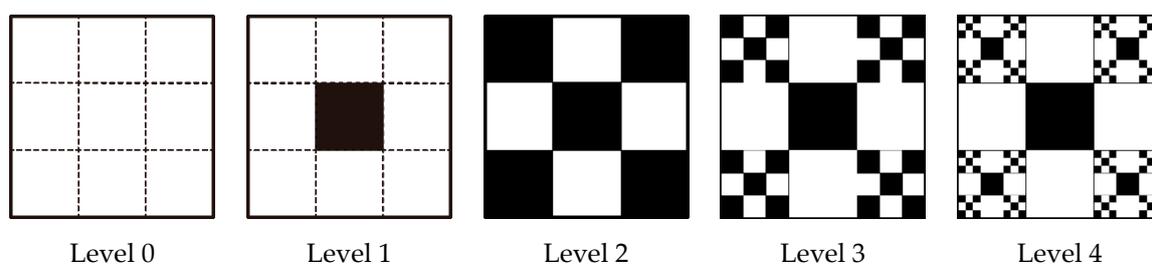


Figure 3. Regular spatial emergence steps in growth property subdivisions with its spontaneous driving force diffusion.

Given that the scale of the city enclosed by the wall is based on the evolution of the environment and the social economy, the amount of space enclosed by the wall of the city can be regarded as the volume of the entire space aggregate, which is in (2), and the area-perimeter model for allometric growth listed within can be written as:

$$(kP)^{1/D_b} = A^{1/D_f}. \quad (5)$$

On the basis of the proportion regulation from the structure constant of coefficient k , the scaling relationship on both the linear and square dimensions of fractal parameters in (5) provides a *quasi-allometric scale* as the initial fractal dimension, which we suggest in terms of *non-dimensional* features to estimate the urban form instantaneously [72]. For the urban area of growth elasticity, we have:

$$A_{f1} = (kP)^{1/D_f}. \quad (6)$$

For D_f and D_b in the initial dimension value D_l correlation, we have a hyperbolic relationship, as:

$$\frac{1}{D_f} = 1 - \frac{1}{2D_b}, \quad (7)$$

where the form enclosure $1/D_b$ of more boundary complexity adaptivity is used to derive the more fractal dimension $1/D_f$. D_f here represents the form emerging dimension, which departs from the initial place as the formula $D_f = 1 + 1/D_l$. Depending on the corresponding *non-dimension*, we can derive

the value from kP as the entire quantity of the fractal property that enables access to any dimension segment with fixed geometry.

Through regression analysis of (2), (5), and (7), the scaling relationship in an index can be found in the allometry of spatial organization, and the dimensional relationship corresponding to their scaling scale can be defined as:

$$\sigma = \frac{D_f}{D_b} = \frac{A(\varepsilon)}{A_{f1}} \quad , \quad (8)$$

where σ refers to the scaling exponent variation for the revised area-perimeter allometry, and 'proportionally' implies a ratio between two cross-sectional measurements of self-similarities, while ε means the changing of areas and length or other geometric measures, D_f stands for the fractal entity, and D_b is the spatial boundary variable. By applying this scaling exponent relationship with Equation (6), we can generalize the structural self-sustaining model of urban form evolution into power-law degenerate growth as:

$$A(\varepsilon) = A_{f1}^\sigma = (kP)^{-\sigma} \quad , \quad (9)$$

where $A(\varepsilon)$ represents the entire quantity from 2D planar allometry, and A_{f1} denotes the OLS estimate of the initial emerging size (mean logarithm size). This method enables the approximation of the core place of an organization in proportional growth. Conversely, by applying such a hypothesized organization kernel, material dependence form features can be utilized as a hub for the localized growth potential from relational trajectories of global responsiveness, approaching the entire organization quality by tracing the initial scales of allometry.

3.3. Allometric Regularity of Spatial Extension

Under the structure interaction condition, the growth potential of any locale (such as the building-occupied site in this study) is assumed to be responsive to the universal range of its private land use probability with allometric scaling. In this process, it is important to confirm the regularity and stability of this locational scaling during the valid city form evolution, that is, how these special locations gradually allocate their growth potential to different evolutionary subdivisions in a cascade formation. Herein, the entire emergence of perimeter length (edges) and allometric scales (mass) are coupled with double logarithmic relations (D_b and D_f), using fractal abundance to describe the boundary complexity (D_l) within continuous scaling. In other words, the consistency of the urban outcome in locational potentials yields an inversed power-law relationship that reflects outside force perturbances on urban size distributions (total mass in 1) with its flattening pattern evolutions.

For any portion of the global force domain organization, a suitable structure strength has always existed for its cross-sectional tissues (even single-mode) with a respective aggregate state to adapt, while in a fractal, the *non-dimension* quantity M is preferred to depict those aggregation entities as $(kP) = M$, where \hat{M} denotes the estimated value and \underline{M} is the potentiality locale. By applying the basic fractal algorithm, we attempt to depict such a spatial evolution by yielding the allometric scaling, which is given as:

$$A(\varepsilon) = \hat{M}^{(1/D_{f1})^\sigma} \quad . \quad (10)$$

If S is taken as the actual urban size (in physical space) related to location potential in walled cities, then the change in its overall organizational size can be characterized by the proportional function $f(s)$, which is written as:

$$f(S) = 1 - \frac{A_1}{A(\varepsilon)} = 1 - \frac{M^{1/D_{f1}}}{M^{(1/D_{f1})^{\Delta\lambda}}} \quad \forall M \geq \underline{M} \quad . \quad (11)$$

If $f(s)$ in (11) is assumed to be the proportion of the socioeconomic level R related to the location growth radius when the non-dimensional quantity is quantified, then $\mathcal{R} = [1 - f(s)]$. In order to

observe the interference caused by the consistency of the structure or other external factors on the existing scaling law, we take the natural logarithmic rate as:

$$\ln R = -(1/D_{f1})^{\Delta\lambda} . \quad (12)$$

Thus, on the basis of the scaling regularity from the allometric relationship, the locational scaling trajectory at different endogenous states can cross-sectionally obtain their structural tension variation $\Delta\lambda$ from the corresponding proportion regulation. In this research, the consistent radius from cities of their homothetic wall enclosures provides such regularity for describing their organic force and socioeconomic activities.

4. Empirical Analysis and Results

According to the processed data and the previous formula, in a historic walled city, the sequence of urban contour scaling (as a hierarchy of homothetic scaling) and the locational growth radius (tissue tightness due to allometric laws) can be divided into two steps for analysis. The analytic framework used to process the historical agglomeration state, illustrated in Figure 4, was as follows.

For walled urban profiles with the whole city area scales (trajectory), we first used the regression method to evaluate the proportionality coefficient k of the form fractality from each walled urban profile (Equations (1) and (2)). Then, we assessed the mean logarithmic size's dimension D_l and its boundary dimension D_b from each local estimated wall enclosure for preparing the subsequent calculations (Equations (3) and (4)). Furthermore, each of these staged allometric profiles with respect to the cross-sectional scaling σ could be calculated from the quasi-least-squares A_f for its represented core place size of different growth states to regress (Equations (5)–(8)). Additionally, by observing the trend line fit of the area-perimeter regression from each local estimated urban profile, we could then globally build an experimental model to check if this contingent urban scaling from an individual walled locale to the whole city's boundary was divided into appropriate cross-sections or not, that is, the comparatively homothetic scaling (unfolding) process for the walled urban clusters in general algorithm controls. The multiple-wall enclosure patterns from 21 Sino-influence cities could provide this part's estimation as an acquired database.

When the radius from one potential location was taken as the internal structure of the whole city, the interval regularity between the local-global radius could then be considered to be the organic force of the organizational structure performance. This radius attributed an overall urban system's linkage that is proportionally associated with the exponent of some locational growth possibilities, and therefore, a local response between each initial increase in land area and the whole aggregate mass M could be obtained by estimating the minimum logarithm size A_{f1} from the walled urban elements' locations (Equations (8)–(10)). Afterwards, the environmental and socioeconomic outcomes differentiated from this initial growth site, even to the divergent edge of each layer with the wall as the boundary, could be developed by the accumulation of the allometric scaling index σ . This process was described dynamically. The fractal spectrum $\Delta\lambda$ and correlated parameters herein can help characterize the consistent allometry of 'scaling regularities' that 'measure' the structure endogeneity with organic force fluctuations (Equations (10)–(12)). In this part, nine cities had successfully preserved their political domain urban profiles (including their imperial audience hall for a socially organized center to represent it) that are available for the act of locational growth potential and that materially characterize both the physical and socioeconomic evolutions within the same particular locales.

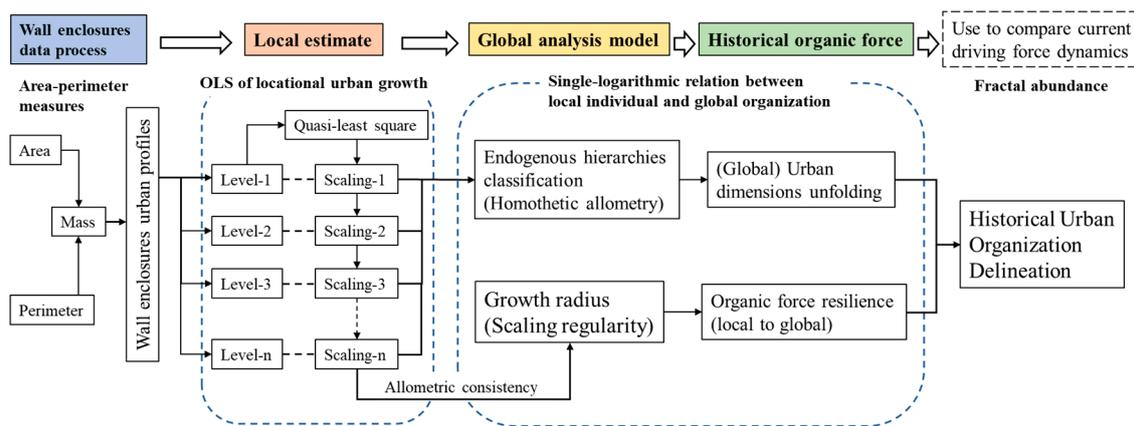


Figure 4. The workflow of historical urban organization delineation.

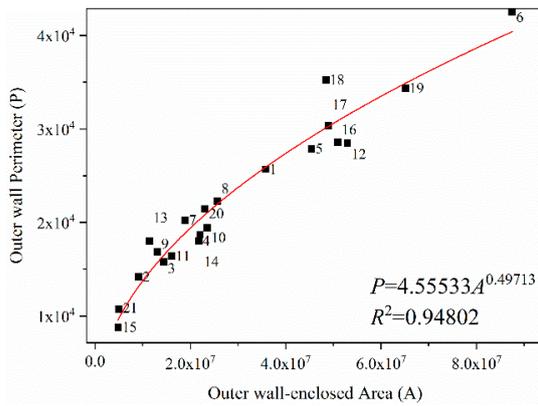
4.1. Growth Stage of Wall Enclosure Urban Profiles

On the basis of the Ordinary Least Square (OLS) estimate, each wall-bounded urban profile could be evaluated from the highly integrated agglomeration boundary (urban form boundary) to the individual evolution potential (local territory features) for responsive relational trajectories. By implementing the area-perimeter regression, the curve of the trend line in each local growth estimation (Figure 5) was employed to demonstrate the fitting of how much the wall enclosure feature deviated from the same scaling sequence or same cross-sectional hierarchy that can represent their allometric stages as the generic proportion of an urban empirical model, where the goodness-of-fit R^2 indicates the local estimation deviation, and k implies a boundary fractality for the contents of relational growth potentialities. Depending on the successive boundary of each wall-enclosed urban profile, the area-perimeter scaling exponent b can be directly associated with allometry consistency. By taking natural logarithms of both sides of Equation (2), the area-perimeter scaling approach to analyzing enclosed city forms could yield a linear relation as depicted below:

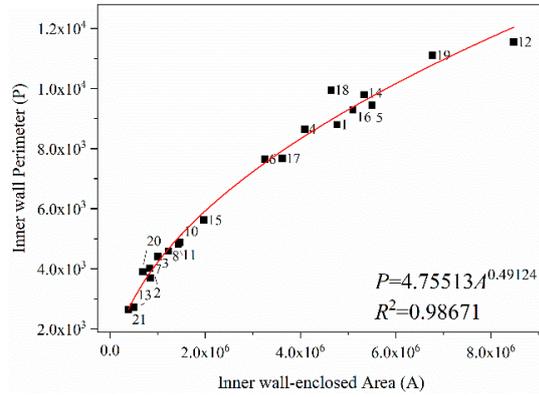
$$\ln\left(\frac{P}{k}\right) = \ln k + \frac{D_l}{2} \cdot \ln A = \ln k + b \cdot \ln A \quad , \quad (13)$$

where $\ln(P/k)$ estimates the logarithmic size of growth emergence, and b is the scaling exponent for the form dimension $D_l/2$. The database from Table 2 allowed us to calculate the least squares of each wall-enclosed area hierarchically. By applying Equations (4)–(7), we could calculate D_l , D_b for the fractal parameter and D_f for further estimation, where D_l stands for the initial boundary dimension of the wall-enclosed area, D_b is revised from D_l as a quasi-dimension value and D_f is the fractal form dimension.

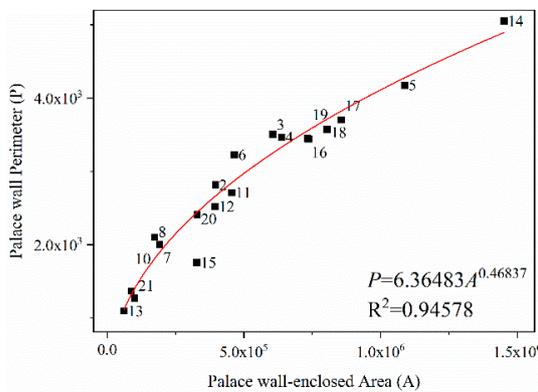
For the global fractal basis analyzed with regression results from different urban profiles of the same cross-section level, series values were given and tabulated in Table 3, including the general scaling exponent of wall-enclosed area b , the goodness-of-fit R^2 , and the boundary complexities k (or the coefficient of evolution consistency). The k value here also represents the proportional coefficient for the least square of some cross-sectional urban profile's emergence, which in this research was the minimum proportion of locational growth to the urban endogeneity, attributed to local fractal parameters. The correlated value involves the non-dimensional value D_l from Equation (3), the boundary dimension D_b , and the form dimension D_f from (4) and (5) accordingly, which are all tabulated in Table 4 after the empirical calculation.



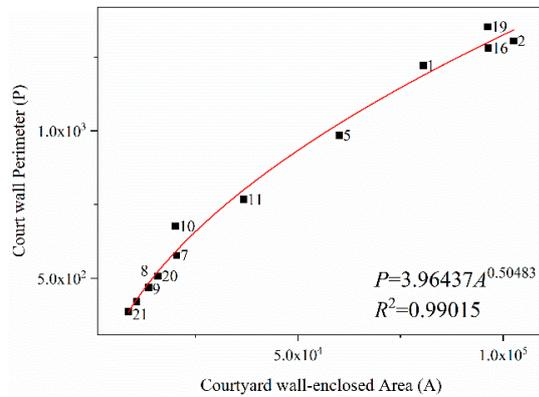
(a) Outer fortification for urban dimension feature



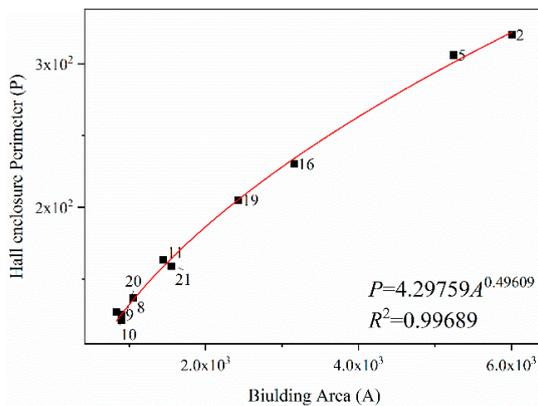
(b) Inner city wall for urban dimension feature



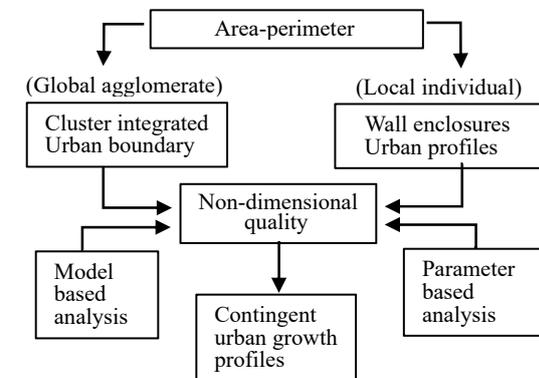
(c) Palace enclosure for urban dimension feature



(d) Courtyard enclosure for urban dimension feature



(e) Building envelope for urban dimension feature



(f) Analysis process separated in global and local estimate

Figure 5. The area-perimeter scaling relationships between wall-enclosed areas and boundary perimeters of each wall enclosure features with different compound clusters. (a) fit result of mass and shapes of outer fortification bounded cities' enclosure; (b) fit result of mass and shapes of inner-city wall bounded cities' enclosure; (c) fit result of mass and shapes of palace wall bounded cities' enclosure; (d) fit result of mass and shapes of courtyard wall bounded cities' enclosure; (e) fit result of mass and shapes of building envelope bounded cities' enclosure; (f) schematic diagram for both the local cities and their universal and empirical model based on the area-perimeter scaling.

Table 3. The global fractal parameters of the walled city unfolding model in different scaling levels of potential trajectories with homothetic cross-sections.

Wall-Enclosed Urban Stratify	Original Result (Model Basis)			Revised Result		
	k	b	R ²	D ₁ (2b)	D _b = (1 + D ₁)/2	D _f = 1 + 1/D ₁
Outer wall	4.5553	0.49713	0.94802	0.99426	0.99713	2.00577
Inner wall	4.75513	0.49124	0.98671	0.98248	0.99124	2.01783
Palace wall	6.36483	0.46837	0.94578	0.93674	0.96837	2.06753
Courtyard	3.96437	0.50483	0.99015	1.00966	1.00483	1.99043
Hall envelope	4.29759	0.49609	0.99689	0.99218	0.99609	2.00788
Average	4.7874	0.49153	0.97351	0.983064	0.991532	2.017888

The notion here is that when allometric growth transitions from a linear dimension or a shape dimension to another non-Euclidean measurement, the structural evolution process with cross-sectional growth layers emerges and follows the self-affine fractal for its growth radius variation. Combining (Table 3) the universal fractal model obtained by the area-perimeter calculation and (Table 4) the estimation of the local fractal parameters of every single instance, we could determine whether it is the boundary dimension D_1 or the revised boundary dimension D_b and the form dimension D_f , and they both showed a similar fractal property in their respective dimensions' measures. This indicates applicability for the global model's patch in which all estimated urban dimensions can follow a universal (political domains) growth framework to unfold. This suggests homothetic cross-sections that characterize all Sino-influence walled cities into the same hierarchy classification of endogeneity. As a result of the historical agglomeration state situation, each cross-sectional walled urban profile harboring different socioeconomic activities can express their locational growth state, contingently associated with the hyperbolic relationship between D_b and D_f for their scaling exponent $-\sigma$ from Equations (7) and (8). While their least-squares value A_f in the non-dimensional fractal calculated from (6) and (9) can be categorized as the hypothesized aggregation center for the growth potential locales (Table 5), the correlated local average values of the global analysis model to be adopted are tabulated in Table 6.

After calculating nine well-preserved city walls (Table 5), the least-squares area A_{f1} – A_{f5} in each of their cross-sectional growth stage and their central-peripheral-associated scaling exponent $-\sigma$ associated with them were calculated locally and subsequently averaged. The values are displayed separately and show a relatively fixed scaling relationship between each walled territory boundary and its assumed core place. In addition, for the quasi-least-squares area A_f of the urban aggregates in each homothetic cross-section, their scaling ability shows an empirical constant from the macro-global to the micro-individuals in Table 1, which is the general scaling capacity of each endogenic hierarchy. Accordingly, the enclosure area of the city's outer fortification was $\sigma_{f1} = 2.01289$, which is the upper scale of the city size, and the inner city wall was $\sigma_{f2} = 2.03461$ as the capital scale; the palace wall range was $\sigma_{f3} = 2.13545$ as the scale of the community; the scale of the courtyard fence, $\sigma_{f4} = 1.98044$, is the scale of the living environment; the hall building enclosure range, $\sigma_{f5} = 2.01493$, is the scale of the dwelling. As the global average, the mean scaling capacity of the cities' wall enclosing was close to 2.035, suggesting a relatively constant proportionality for any locational potential to reach its maximum scaling range of urban boundaries' allometry.

Table 4. Empirical and theoretical results of fractal parameters in different walled allometric stages.

Abbr.	Outer Wall			Inner Wall			Palace Wall			Yard Wall			Hall Wall		
	D _l	D _b	D _f	D _l	D _b	D _f	D _l	D _b	D _f	D _l	D _b	D _l	D _b	D _f	
C1	0.9933	0.9967	2.0067	0.9785	0.9892	2.0219	N/A	N/A	N/A	1.0145	1.0073	N/A	N/A	N/A	
C2 *	1.0036	1.0018	1.9964	0.9879	0.9939	2.0123	0.9452	0.9726	2.0579	1.0047	1.0024	0.9908	0.9954	2.0093	
C3	0.9888	0.9944	2.0113	0.9885	0.9942	2.0117	0.9481	0.9741	2.0547	N/A	N/A	N/A	N/A	N/A	
C4	0.9808	0.9904	2.0196	0.9859	0.9929	2.0143	0.9426	0.9713	2.0609	N/A	N/A	N/A	N/A	N/A	
C5 *	0.9892	0.9946	2.0109	0.9786	0.9893	2.0219	0.9332	0.9666	2.0716	1.0025	1.0013	0.9959	0.9979	2.0041	
C6	0.9998	0.9999	2.0002	0.9846	0.9923	2.0156	0.9545	0.9772	2.0477	N/A	N/A	N/A	N/A	N/A	
C7	1.0027	1.0014	1.9973	0.9747	0.9873	2.026	0.9457	0.9729	2.0574	1.0039	1.0019	N/A	N/A	N/A	
C8 *	0.9957	0.9978	2.0044	0.9802	0.9901	2.0202	0.9616	0.9808	2.0399	1.0057	1.0029	0.9954	0.9977	2.0046	
C9	1.0025	1.0012	1.9975	N/A	N/A	N/A	N/A	N/A	N/A	1.0028	1.0014	0.9904	0.9952	2.0097	
C10 *	0.9849	0.9925	2.0152	0.9763	0.9881	2.0243	0.9434	0.9717	2.0599	1.0373	1.0186	1.0070	1.0035	1.9930	
C11 *	0.9869	0.9935	2.0132	0.9765	0.9883	2.0240	0.9292	0.9646	2.0762	1.0019	1.0009	1.0000	1.0000	1.9999	
C12	0.983	0.9916	2.0172	0.9773	0.9886	2.0233	0.9285	0.9642	2.0771	N/A	N/A	N/A	N/A	N/A	
C13	1.0194	1.0097	1.9809	0.9686	0.9843	2.0324	0.9338	0.9668	2.0709	N/A	N/A	N/A	N/A	N/A	
C14	0.9841	0.9921	2.0162	0.9852	0.9926	2.0150	0.9411	0.9706	2.0626	N/A	N/A	N/A	N/A	N/A	
C15	0.9830	0.9915	2.0173	0.9762	0.9881	2.0244	0.8849	0.9424	2.1301	N/A	N/A	N/A	N/A	N/A	
C16 *	0.9856	0.9928	2.0146	0.9812	0.9906	2.0192	0.9323	0.9662	2.0726	1.0069	1.0035	0.9879	0.9939	2.0122	
C17	0.9946	0.9973	2.0055	0.9782	0.9891	2.0223	0.9321	0.9660	2.0729	N/A	N/A	N/A	N/A	N/A	
C18	1.0121	1.0060	1.9881	0.9962	0.9981	2.0039	0.9311	0.9655	2.0740	N/A	N/A	N/A	N/A	N/A	
C19 *	0.9926	0.9963	2.0075	0.9862	0.9931	2.0139	0.9316	0.9658	2.0735	1.0166	1.0083	0.9917	0.9958	2.0084	
C20 *	0.9976	0.9988	2.0024	0.9978	0.9989	2.0022	0.9344	0.9672	2.0703	1.0033	1.0016	0.9819	0.9909	2.0184	
C21 *	1.0067	1.0034	1.9933	0.9826	0.9913	2.0177	0.9196	0.9598	2.0875	1.0109	1.0055	0.9829	0.9915	2.0173	
AVG	0.9946	0.9973	2.0055	0.9821	0.9910	2.0183	0.9354	0.9677	2.0694	1.0092	1.0046	0.9924	0.9962	2.0076	

Note: * Denotes the full data acquirable in the wall surrounding the city of a social ordered hierarchy.

Table 5. Scaling capacity (potentiality) from the core to the edge with respect to each wall-embedded urban enclosure for structure diffusion cross-sections.

Abbr.	Capital Name	Outer Wall		Inner Wall		Palace Wall		Yard Wall		Hall Wall		Realistic Built Scale
		A_{f1}	$-\sigma_{f1}$	A_{f2}	$-\sigma_{f2}$	A_{f3}	$-\sigma_{f3}$	A_{f4}	$-\sigma_{f4}$	A_{f5}	$-\sigma_{f5}$	A_5 (m ²)
C2	Luo yang	256.70963	1.99289	134.00571	2.0245	116.60713	2.11595	72.65835	1.99056	36.46797	2.01863	6779
C5	Luo yang	345.29542	2.02186	199.96298	2.04382	136.68084	2.14321	62.81304	1.99501	36.00166	2.0081	5248
C8	Heijo Kyo	314.50259	2.00869	140.51896	2.04034	105.44578	2.07985	41.23969	1.9886	24.08827	2.00917	1049
C10	Heian Kyo	285.15146	2.03047	143.38227	2.04861	81.77081	2.11992	55.64149	1.92818	23.62127	1.98605	1132
C11	Balhae	263.46053	2.02638	142.73511	2.04806	109.76223	2.15245	55.3479	1.99629	26.50927	1.99993	1447
C16	Dadu	345.85405	2.02923	199.75665	2.03836	124.38089	2.14522	72.32346	1.98619	30.78978	2.02437	3156
C19	Beijing	386.95322	2.01496	221.27391	2.02796	123.95823	2.14694	75.87624	1.9673	29.26393	2.0168	2428
C20	Seoul	310.36536	2.00486	135.44732	2.0044	105.10422	2.1405	45.1084	1.99355	22.16468	2.03669	896
C21	Hué	225.28692	1.98665	107.61288	2.03546	74.43002	2.17497	39.96131	1.97828	25.41877	2.03461	1551
Average		303.73102	2.01289	158.29953	2.03461	108.68223	2.13545	57.88554	1.98044	28.2584	2.01493	22,631.778

Table 6. The global average value of the allometric scaling in each wall-limited growth stage.

Wall Enclosure Hierarchy	Local Fractal Parameter (in 9 cities)			Least-Squares Average	Scaling Capacity
	D_l	D_b	D_f	A_f (m ²)	$-\sigma$
Outer wall	0.99364	0.99683	2.00643	303.73102	2.01289
Inner wall	0.98303	0.99151	2.0173	158.29953	2.03461
Palace wall	0.93672	0.96836	2.06771	108.68223	2.13545
Courtyard	1.00997	1.00483	1.99023	57.88554	1.98044
Hall envelope	0.99261	0.99628	2.00746	28.2584	2.01493
Average	0.983194	0.991562	2.017826	131.37134	2.03566

4.2. Spatial Allometry and Structure Compactness

For an empirical urban growth analysis, the city's phased allometry under the same scaling trajectory provides geometric self-similarity to help characterize its endogeneity, covering the range from the initial growth potential of that locale to the actual evolved urban boundary of the agglomeration carrier. The proportional form of the cross-section of the urban scaling trajectory due to the continuous wall enclosing also allows the walled cities to predict their future growth trend from a central-peripheral scaling law of the universal model. This implies a cascading distribution of the urban structure force with distance-dependent degeneration. In the mathematical model, the ordinary least-squares area A_{f1} of the outline of the city enclosed by any wall (in this study, we took the least-squares area A_{f5} from the wall enclosure range of the building as the initially presented city scale and to serve as the anchoring of urban agglomeration) in Equations (9) and (10) can be used for the initial location of urban allometry and shares self-similar growth (Table 5). The application of an allometric scaling (fractional abundance) can also be used to approach the proportionality of urban organizations in different endogeneity hierarchies. Furthermore, if we can ensure that the wall cross-sections' city scaling layer stays in constant proportion when it scales with the entire mass M (the proportional rate of endogeneity hierarchy) and assume the least-squares area A_{f5} as an actualized location for overall dynamics of the city growth, then a city's agglomeration entity based on the single logarithmic relationship can be described by the power-law-related distribution mechanism. The coherence of fractal abundance across whole city areas due to allometric scales can be used to predict the structural tensions or tightness of a walled urban system with its scaling changes from A_{f5} .

By using Equation (11) to simulate the allometry of a city linearly scaled from a fixed location, the continuous set of walls surrounding the city outline could be obtained by the least-squares value A_{f5} (quasi-allometric scaling) in Table 5. This estimated value is the initial emergence of city logarithmic size, which comes from the fundamental walled urban element's (building) feature. The proportional rate resulting from the hierarchical endogeneity of each city can be roughly expressed through this locational least-squares scaling $-\sigma$ and is displayed in Figure 6a. The results show high similarity in the allometric scaling of each city's growth state with the same locational growth radius (as the scaling trajectory) classification, revealing the whole system's mass and rates from the basic urban elements (buildings) to the highly aggregated land consumption borders (outer fortification). For the urban growth force with these radius constraints, the average scaling exponent from the least-squares scaling is around $\sigma_1 \approx 4.18$, $\sigma_2 \approx 5.51$, $\sigma_3 \approx 6.89$, $\sigma_4 \approx 7.80$, and $\sigma_5 \approx 9.15$. In the percentage calculation, their cross-sectional proportion is shown in Figure 6b. Since the land consumption rates in the locational growth radius are associated with the size of urban agglomeration, the compactness of these walled urban cross-sections is then proportional to the organic force accumulated by the locational scaling index, with its tensions degenerating, as shown in Figure 6c. Regardless of whether it is a phased allometric scaling or linearly related locational growth dynamics, the similarity of the scaling exponent at each homothetic walled urban profile is summarized as a common universal model. It is applicable for global access to every local Sino-influenced city and its hierarchical endogeneity. The results show that these cities have historically maintained relatively constant proportional regulations due to different historical conditions and growth force dynamics, suggesting a conceptualized organism model. For each proportional walled city scale's unfolding, the average value is in the respective local zoom index, namely, the dwelling scale with 12.5%, the residential scale with 16.6%, the community scale with 20.3%, the small-town or capital scale with 23.3%, and the city size with 27.3%.

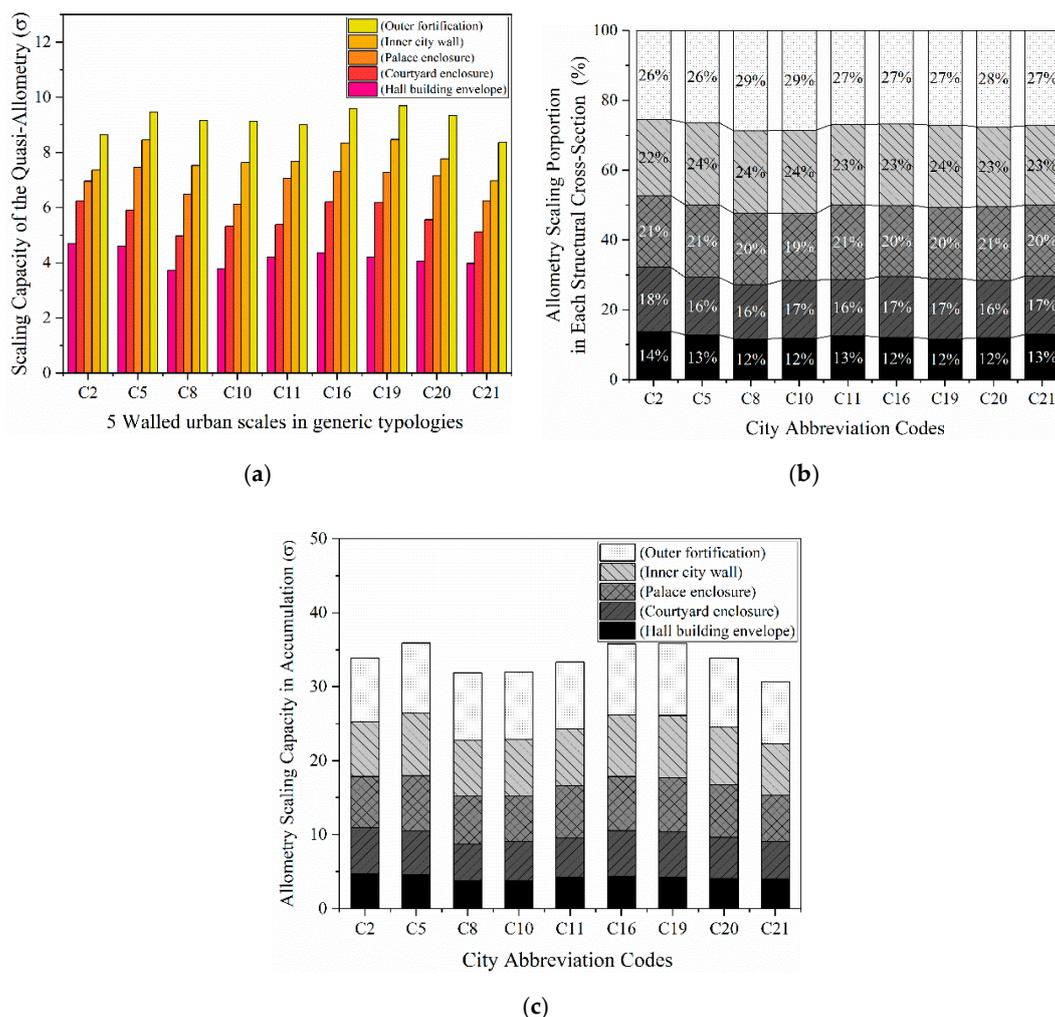


Figure 6. Scaling rate (proportionality) of the quasi-allometry property in different wall-embedded urban enclosures of its staged emergence: (a) Quasi-allometric property for each wall-enclosed urban range; (b) proportion of fitting of percentage; (c) accumulation of scaling capacity for initiator allometric.

Combining the regularity of urban allometry in Equations (11) and (12), a city of a distribution size with coherent walled urban profile constraints could be manifested as a dimensional boundary evolution process through power-law dependence scaling regularities, and the fluctuations observed around the generic trend are shown in Figure 7. The estimation results from nine cities all show a better fitting to the overall distribution trend line and reveal an approximate urban evolution from top to bottom. With the exception of C8 and C10 showing a relatively large fluctuation accident during the third evolution stage, all other cities have maintained a stable evolutionary state due to the quasi-least-squares allometry, which means that in addition to the endogenized land consumption rate, each of these cities' internal structure is less disturbed by other factors. The broken line represents the possible growth trajectory under a fixed scaling range, which reflects the local response of the quasi-least-square when it is embodied by the walled urban elements of the city distribution from the rate 1.011 to 1.648. Owing to the implied fractal cascade relationship, the parameter values of this kind of allometry process can provide an optimized urban development model for the evolution of real cities. The fractal spectrum in this model can help describe a city's persistent dynamics $\lambda(r)$ in (12). This dynamic represents the average growth radius from the regional aggregation center to the edge of the aggregation pattern. The model was universally adapted to the city cluster integration algorithm. The clusters were aggregated upwards into spatial agglomeration and implemented downwards

to the locational growth potential of specific artificial shapes. The result reflects a historical urban growth tension and its coherent growth, and it is closely related to the urban form and social and economic vitality.

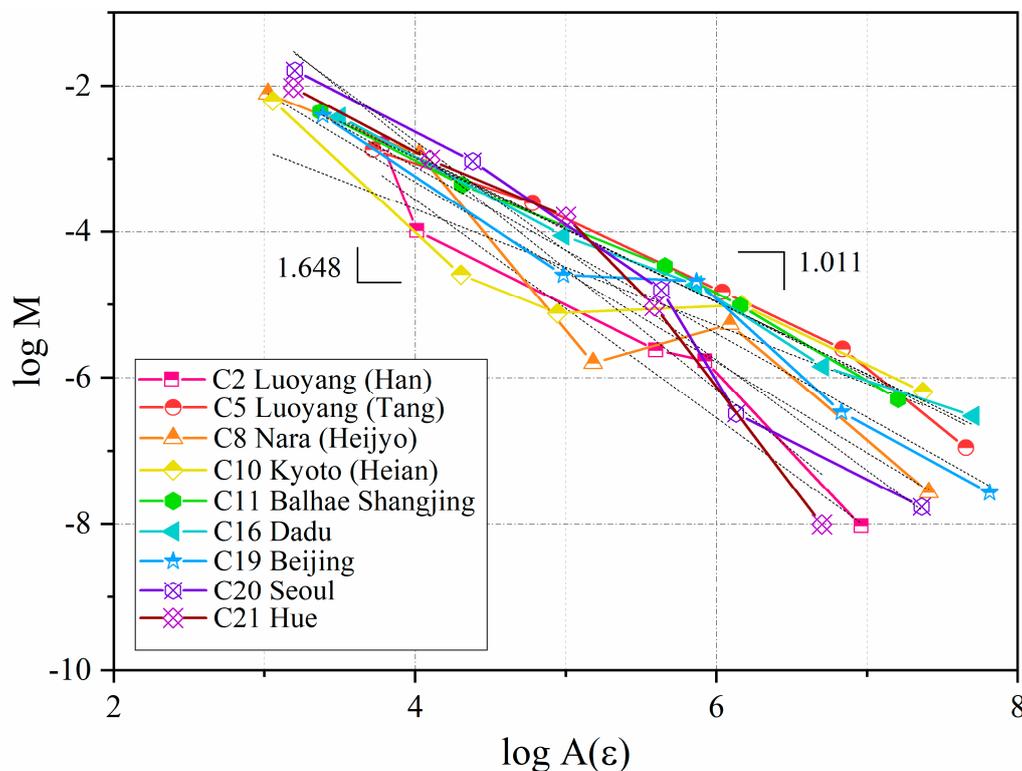


Figure 7. Double logarithmic relationship between radial profiles of least-squares fitting and agglomerate distribution embedding of the wall-enclosed boundary limit.

Consequently, after examining the overall historical pattern formation of Sino-influence walled cities, it can be found that their endogenous intra-urban process with multiple-wall enclosures shows similar scaling laws. As the initial positioning and manifestation of related dynamics to locale, the ordinary least-squares areas from localized urban-specific elements have established individual responses to the macro variables of the urban organism's adaptation. The overall tension generated (away from the growth center) can be evaluated by the continuity of walled boundary shifts (scaling ability) to assess the disturbance of the overall correlation dynamics. The wall-embedded city level can be considered a public facility that effectively organizes socioeconomic activity. It limits the development of the city in a self-sustaining manner within a reasonable range, and it ensures that the compactness is maintained in a relatively constant range and as a historical internal structural balance.

5. Summary and Discussion

Through the above practical analysis, regardless of whether the walled cities are based on the dispersion or aggregation of capital and information as a manifestation of the organic force of growth, the least-squares area based on the area-perimeter method estimate shows that these historical urban growth states exist with simple models. It is a response to the orderly induction of the relevant population size and the disorderly evolutionary process of social and economic activities. As a result of this research, the following questions have been clarified. First of all, owing to the influence of various driving forces in complexity dynamics, the agglomeration of cities in the current urban system is more likely to exhibit power-law-related allometric scaling, which is based on the double logarithmic relationship to the single logarithmic or even linear relationship [15,73–75]. In this study, because of

Table 7. Iteration schema for the wall-enclosed urban form extrusion and its correlated typologies and scaling.

Urban Spatial Rank	Spatial Types of Correlation			Representative Generic Typologies		
	Iteration Step	Stratification Division	Urban Dimensions	Introverted Urban Significant	Growth Rate $\Delta\lambda$	lnR (Local Average)
Rank 1	f(Xn)	Outer fortification	City	Multiple Wall-surrounding	6.293	80.27068
Rank 2	f(Xn-1)	Inner city wall	Metropolitan area	Square-planning	5.855	59.15584
Rank 3	f(Xn-2)	Palace enclosure	District	Ward-well	5.388	42.72298
Rank 4	f(Xn-3)	Courtyard enclosure	Neighborhood	quadrangle-layout	4.839	29.14126
Rank 5	f(Xn-4)	Hall building envelope	Dwelling	Brackets arches	4.18	18.41043

Third, the scaling trajectory of the city size with continuous wall enclosures leads to these walled cities' involvement in evolutionary economic geography, which focuses on how to relationally link individual growth in contingent spaces [48]. Through the fractal zooming above, a microscope-style visual zooming behavior can intuitively reproduce the evolution of the internal structure of the "location-related urban tissue" at the material level (city), where the scaling adapted from the quasi-least-squares area to each wall-bounded land consumption region serves as a cascading distribution that is morphologically correlated with the environmental and socioeconomic outcome through fractal abundance. If the quasi-least-squares area is located in and represented by the artificial urban element, and the element is ubiquitous in a texture that constitutes the urban material level, then the specificity of this cluster feature will replace the entire urban system to some extent, to serve as the statistics of clustering algorithms. In this study, the wall-enclosing space was considered to be suitable for this role in order to properly describe the spatial living pattern under different agglomeration scales and various wall enclosures.

In the existing data, five cities have retained their specific wall enclosure dimensions that reflect the city's fundamental element (building) size and also include the dimensions of this general feature in other urban dimensions. The fractal abundance is used here to classify the endogenized land consumption rate, whose start emerges at the initial location where the urban agglomeration is unfolded locally and is covered by local buildings. This hierarchical patrol system also reflects the traditional spatial life pattern in connection with political power, which can be regarded as the essence of the Sino-influenced urban structure [77]. For different scaling ranges of the historical built environment and socio-economic activities, the urban outlines of Sino-influenced walled cities are continuously and relationally depicted as a set of generally representative urban growth frameworks, corresponding to the structural elasticity of their agglomeration state, which is self-sustaining. Figure 9 portrays its general representative expansion and iterative steps of related dimensions.

This study did not explore historical cities with wall characteristics around the world, nor did we compare the current development status of the cities studied, which is considered to be the limitation of this study. The universal urban growth model derived from this study based on the laws of scaling and dimensional proportion of estimated walled cities cannot be fully adapted to all other historical cities with wall enclosures, since each city that builds its wall constructions is associated with different cultural and historical backgrounds. However, we can visit all these cities by applying the analytical methods in this study, because these cities in a historical walled environment can at least be regarded as spatial cells with complete boundaries that physically limit their normal growth. This means that the distribution probability of population density is artificially restricted by the historical texture of the city, whether in European cities (such as Paris, Nördlingen, Florence, or Milan) or other cities in Asia (such as Istanbul and Jerusalem): these cities were not included in this study. For the adaptation of the analytical model to different built environments, the characteristics of the wall enclosure can at least divide the historical city organization into three resolution levels and separate the objects from the complex urban texture through the correspondence of urban dimensions through microscopic zooming: specific public buildings (churches, city halls) are deployed as the initial agglomeration locales; inner city walls or city squares enclosed by buildings then provide the city's central places for the population to participate in social and economic activities and external fortifications at the threshold of the historical growth of the city. Therefore, the internal structure of walled cities in history

can be summarized through the common land consumption process, from the quasi-least-squares area enclosed by specific buildings to the boundary of the urban spatial organization embedded in the external defensive walls. Although these cities are located in different places and have their own geographical shapes and sizes, their proportional organizational levels and socioeconomic evolution can still be unified as the maintenance of an organism under the expression of homothetic growth.

Urban Scale	Organization hierarchy		Zoning type for different scaling stages with wall enclosure feature embedding				
	Iteration step	Scaling capacity (σ)	C8 Heijyo Kyo	C11 Balhae	C19 Beijing	C20 Seoul	C21 Hué
Dwelling scale (Level 5)	$f(X_{n-4})$	$\sigma \approx 4.18$					
Residential scale (Level 4)	$f(X_{n-3})$	$\sigma \approx 5.51$					
Community scale (Level 3)	$f(X_{n-2})$	$\sigma \approx 6.89$					
Ward scale (Level 2)	$f(X_{n-1})$	$\sigma \approx 7.80$					
City scale (Level 1)	$f(X_n)$	$\sigma \approx 9.15$					

Figure 9. Zoning type-dependent form iterations with existing walled city rank evolution and organizational tension.

For the current urban development profile, the area-perimeter method that we used can hardly estimate its fuzzy development boundary. This does not mean that the current boundary dispersion is not measured. Recently, many advantageous techniques have been used to deal with such probabilistic scenarios. For example, the maximum likelihood method is used to divide the grid that appears in the city [20,78]. The goal definition method is used to extract the state of social and economic activities from natural cities [79] and through the matching of cellular automaton models to simulate the natural evolution of cities [80]. However, the disadvantage is that there are few methods that can clearly define the urban ontology boundaries of the original cities between the increasingly diffuse and integrated urban metropolitan areas, and introducing new methods would involve too many tasks, which was beyond the scope of this study. However, this is very important for the current allometry estimation of the city’s radial effects and can help to assess its excessive growth rate (expressed by the spatiotemporal growth mechanism).

6. Conclusions

The socioeconomic evolution pattern and relational spatial property of 21 Sino-influenced walled capital cities were evaluated through the area-perimeter allometric estimate. The urban form enclosed by multiple walls was used to embed the historical population agglomeration into the endogenous growth mechanism of a specific location. In this simplified mechanism, the central-peripheral radial effect corresponds to the internal urban structure as an organization actualization. The control of its tightness comes from the constraints of the urban physical factor scale on growth. We compared the allometric scaling laws of these cities with the same scaling trajectory, where the trajectory was proportionally divided into the same cross-sections with walled urban profiles (ranges) as the organizational hierarchies. We also approached these enclosing frequencies (compactness of these cities' internal structures) by adapting the coherence scaling from the quasi-least-squares allometry. The area of this smallest cross-section comes from the area-perimeter estimate of the possibility of location growth. The fractal abundance herein appropriately characterized the material environment extrusion through single logarithmic degeneration and linearly traced the socioeconomic evolution.

Through empirical research, we first found a high correlation between the endogenization of the wall-bounded land consumption level and the divergence of relevant spatial living patterns' emanation, in which the city sizes in the same cross-section of the wall-bounded urban dimension maintain an approximate zooming capability in accordance with their boundary allometry. Then, the geometric properties of the aggregated city volume generated a series of walled scaling patterns that were observed as ubiquitous urban cluster features and are proportional to the whole city size integrated by some clustering algorithms, although these cities showed different scales and shapes from real geographic spatial observations. In addition, we proposed to adopt an allometric scaling-based urban zoning type that classifies the contours of the walled city size according to their possibility scaling trajectory from the individual urban elements of that locale to the overall aggregation environment. Additionally, regression from different allometric scaling with respect to cluster hierarchy and cross-section proportion can be attributed to a generic scaling rate of urban dimensions in homothetic unfolding (zooming). In the allometric description, the fractal cascade model can restore the evolution process by tracking the radial effect of endogenized land consumption. In this study, this central-peripheral mechanism helped to describe the internal structure compactness of actual urban growth under the constraints of multiple walls. As agglomerated self-regulation, it provided a stable growth regularity to maintain the relevant city's internal maintenance at a historical equilibrium.

The highlight of this study was our proposal of a quasi-allometric scale that approximates and replaces the logarithmic city size so that the initial location of the urban agglomeration center can be anchored to the heterogeneity of the city's internal structure, and the wall enclosures as the physical constraints of human activities are attributed to the natural way (bottom-up approach) of defining cities, which the initial constraint provides through the building envelope and with the differentiation at any introverted spatial type, marking the populated sites without incorporating any census data. We further set forth the dual roles of these traditional wall enclosures' compound features that are both available for the spatial living pattern's classification and for the socioeconomic hierarchies' quantification. The self-similarity of consistent spatial allometry was introduced to characterize the organic force fluctuations of the wall enclosures' urban organization with a homothetic cross-section.

This is the first fractal investigation of the Sino-influenced wall-bounded urban morphology. For cities with a complex driving force of growth, the specific wall enclosures' scaling trajectory from the local-global and central-peripheral radius enables a continuous growth force restriction that causes cities to form a regular evolution rate and prevents the fast expansion of land consumption, making cities the physical tissues of the overall self-sustaining agglomeration. Additionally, although it is difficult to capture the real growth singularity from the current urban structure, through this study, we at least found a historical city growth model which physically locates the monocentric tradition into centripetal walls that form an organizational structure and proportionally limit the city size's expansion and socioeconomic activity.

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Appendix A

Table A1. Data survey and list of the earlier empirical and theoretical literature on 21 historical East Asia cities enclosures form and size.

Abbr.	Name and Period	Literature and Reliable Data Basis Reference of Historical Surveys.	Scholar or Institute
C1	Chang'an (Han) 200 B.C-25	*Review on Han Chang'an city site (in Chinese), Archaeology, 2017-1, pp.9–16, http://www.kaogu.cn/uploads/soft/2017/20170906liuzhendong.pdf *Study on Conservation of Chang An City Wall Ruins in Han Dynasty, (in Chinese) https://www.doc88.com/p-1438749324768.html *Archaeological excavation and research on WeiYang palace, Chang'an city, Han dynasty. (in Chinese), Relics and Museology, 1995-3, pp.82–93 https://wenku.baidu.com/view/ddebe63a336c1eb91b375d15.html *A jinzhong in the Weiyang Palace of Han Chang'an Castle: A Study of the Domain (in Japanese), Gakushuin historical review (45), 35–62, 2007-03, URI: http://hdl.handle.net/10959/1004	1. Liu Zhendong 2. Hou Nan 3. Liu Yufang 4. Aoki Shunsuke
C2	Luo yang (Han) 25–190; 220–313	*Form and structures of capital cities in Wei, Jin, southern and Northern Dynasties (in Chinese), Historical studies of ancient and medieval China, 2015, Vol, No.35, pp 101–129. DOI: 10.15840/amch.2015.35.004 *New Discoveries and Research on the Structure in Han Wei Luoyang cheng Imperial Palace Area (in Japanese and Chinese), Hiraizumi studies 5, 1–11,13–22, 2017, NII Article ID (NAID):120006343917	1. Guo Xiang Qiang 2. Matsumoto Keita & Guo Xiang Qiang
C3	Ye City 213–580	*The System of Capital City in medieval China from an Archaeological Viewpoint (in Chinese and Korea), Middle ages studies of Chinese, 2015, vol., no.35, pp. 131–164, DOI: 10.15840/amch.2015.35.005 *Form and structures of capital cities in Wei, Jin, southern and Northern Dynasties (in Chinese), Historical studies of ancient and medieval China,2015, Vol, No.35, pp 101–129. DOI: 10.15840/amch.2015.35.004	1. Han Jianhua 2. Guo Xiang Qiang
C4	Jian kang 229–589	*Location of three cities in China: Jiankang, Chang'an, Luoyang, Natural and society (in Japanese), Study on the capital system (Nara Women's University Academic Information Center), 2015, Vol, No.9, pp.53–68, URL: http://hdl.handle.net/10935/4124	1. Seno, Tatsuhiko
C5	Luo yang (Tang) 605–938	*The multi capital system in the early Tang Dynasty (in Japanese), Osaka Museum of history, Summary of research, 2017.3, No.15, pp 1–18. http://www.mus-his.city.osaka.jp/education/publication/kenkyukiyo/pdf/no15/pdf15_01.pdf *The plane, structure and form on Qianyang palace and Qianyuan palace of Luoyang of Sui and Tang Dynasties (in Chinese), Journal of Chinese Architecture History, 2010, No.00, pp 97–141 https://wenku.baidu.com/view/ef5f24470066f5335a8121f5.html	1. Ken'ichi Muramoto 2. Wang Guixiang
C6	Chang'an (Tang) 582–940	*Planning techniques of CHANG'AN city in Sui and Tang Dynasty (in Chinese), City Planning Review, 2009, Vol 33 (06), pp 55–72, DOI: 10.3321/j.issn:1002-1329.2009.06.011 *The System of Capital City in medieval China from an Archaeological Viewpoint (in Chinese and Korea), Middle ages studies of Chinese, 2015, vol., no.35, pp. 131–164, DOI: 10.15840/amch.2015.35.005 *Investigation report on the basic design of the environmental restoration plan of Hanyuan Hall of Daming Palace, People Republic of China (in Japanese), 2002.1 http://open_jicareport.jica.go.jp/pdf/11774734.pdf	1. Wang Shusheng 2. Han Jianhua 3. Japan International cooperation Agency

Table A1. Cont.

Abbr.	Name and Period	Literature and Reliable Data Basis Reference of Historical Surveys.	Scholar or Institute
C7	Fujiwara Kyo 694–710	*Strip-block system in Fujihara Kyo-images and meaning (in Japanese), Report of 21 century COE program Nara Women's University Vol 16, pp.37–66, URL: http://hdl.handle.net/10935/2736 *ANNUAL BULLETIN of Nara National Cultural Properties Research Institute 1997-II(in Japanese), https://www.nabunken.go.jp/	1. Hayashibe, 2. Hitoshi Nara National Cultural Properties Research Institute
C8	Heijo Kyo 710–784	*ANNUAL BULLETIN of Nara National Cultural Properties Research Institute 1996 (in Japanese), https://www.nabunken.go.jp/ *Historical significance of the construction of ancient Japanese cities, in history of East Asia (in Japanese), Nichibunken Japanese studies series, Vol. 42, pp. 95–138, DOI: doi/10.15055/00005193 *The restoration model of Heijo Palace (in Japanese), ANNUAL BULLETIN of Nara National Cultural Properties Research Institute 1966, pp 23–26, URL: http://hdl.handle.net/11177/2629	1. Nara National Cultural Properties Research Institute 2. Inoue Kazuhito 3. Honmi Keizou
C9	Nagaoka Kyo 784–794	*Nagaoka Kyo and temples (in Japanese), Study on the capital system (Nara Women's University Academic Information Center),2014.3, Vol.8, pp. 67–86, URL: http://hdl.handle.net/10935/3643 *Japan-China ancient city atlas (in Japanese), 2002.8, ISBN 13: 9784878050121	1. Koga Masahiro 2. Nara National Cultural Properties Research Institute
C10	Heian Kyo 794–	*Japan-China ancient city atlas (in Japanese), 2002.8, ISBN 13: 9784878050121 *The designing of the roof for Fukko Shishin-den, restored ceremonial hall: study of the imperial palace in the Kansei era (4) (in Japanese), Journal of Architecture and Planning (Transactions of AIJ) 2011, Vol.76(669), pp. 2183–2190, DOI: 10.3130/aija.76.2183	1. Nara National Cultural Properties Research Institute 2. Kurimoto Yasuyo, Uematsu Kiyoshi, Iwama Kaori, Tani Naoki
C11	Balhae Shangjing 755–929	*A Restoration Study of the No.2 and the No.5 Building of the Balhae Shangjing Palace (in Chinese), Dissertation for the Master Degree, Harbin Institute of Technology, 2008.6, DOI: 10.7666/d.D255105 *Restoration Analysis on the Main Building Groups of Shangjing City in Balhae State (in Chinese), Dissertation for the Doctoral Degree, Harbin Institute of Technology, 2014.7, DOI: 10.7666/d.D751424 *Historical significance of the construction of ancient Japanese cities, in history of East Asia (in Japanese), Nichibunken Japanese studies series, Vol. 42, pp. 95–138, DOI: doi/10.15055/00005193	1. Liu Chuan 2. Sun Zhimin 3. Inoue Kazuhito
C12	Bianjing 907–1127	*Discussion on the Yingzao Chi of the Northern Song Dynasty Referring to the Archaeological Discovery of the Outer City of the Dongjing of the Northern Song Dynasty (in Chinese), Cultural Relics, 2018, Vol (2), http://www.kaogu.cn/uploads/soft/2018/20180327beisong.pdf *On the Old City of the Northern Song Capital of Kaifeng (in Japanese), Studies in Urban Culture, 2014, Vol. 16, pp. 79–91, DOI: info:doi/10.24544/ocu.20171213-053	1. Liu Chunying 2. Kazuo Kubota
C13	Lin'an 1129–1287	*Dream back to the Southern Song Dynasty, restoration the “underground imperial city” from historical obliterates (in Chinese), Hangzhou, 2017, Vol. 22, pp. 14–17, DOI: 10.16639/j.cnki.cn33-1361/d.2017.22.003 *The Study on the Spatial and Timing Structure of Festival Activities in Lin'an City during the Southern Song Period (in Chinese), Journal of Chinese Historical Geography, 2008.10, Vol. 23, No.4, pp. 5–22, http://www.doc88.com/p-7798909440293.html	1. Ren Riyin, Yao Minlv 2. Zhang Xiaohong, Mou Zhenyu, Chen Li, Ding Yannan
C14	Jin Zhongdu 1153–1251	*Research on the Urban Morphology and Foundations of Building Complexes in Yuan Dynasty Capital City (in Chinese), Dissertation for the Doctoral Degree, Tsing Hua University, 2007, http://kns.cnki.net/KCMS/detail/detail.aspx?dbcode=CDFD&filename=2008088115.nh *The History Study of Liao-Jin Yanjing City: Reviewing Urban Archaeology Methodology (in Chinese), Palace Museum Journal, 2016, Vol.02, pp. 77–97, DOI: 10.16319/j.cnki.0452-7402.2016.02.007	1. Jiang Dongcheng Liu Wei

Table A1. Cont.

Abbr.	Name and Period	Literature and Reliable Data Basis Reference of Historical Surveys.	Scholar or Institute
C15	Yuan Xanadu 1260–1267	*City Planning of Newly-built Cities of Mongol Empire (in Chinese), Research of China's Frontier Archaeology, 2015, Vol. 17, pp. 313–342, http://kg.lsxxy.ruc.edu.cn/uploadfile/2016/1218/20161218104217107.pdf	1. Liu Wei
C16	Dadu 1267–1421	*Research on the Urban Morphology and Foundations of Building Complexes in Yuan Dynasty Capital City (in Chinese), Dissertation for the Doctoral Degree, Tsing Hua University, 2007, http://kns.cnki.net/KCMS/detail/detail.aspx?dbcode=CDFD&filename=2008088115.nh *Study on the restoration of the palace in Dadu of the Yuan Dynasty (in Chinese), Acta Archaeologica Sinica, 1993, Vol.01, pp. 109–151, https://www.ixueshu.com/document/59300c641802d90d318947a18e7f9386.html *City Planning of Newly-built Cities of Mongol Empire (in Chinese), Research of China's Frontier Archaeology, 2015, Vol. 17, pp. 313–342, http://kg.lsxxy.ruc.edu.cn/uploadfile/2016/1218/20161218104217107.pdf	1. Jiang 2. Dongcheng 3. Fu Xinian Liu Wei
C17	Ming Zhongdu 1369–1375	*Form and Planning of Newly-Built Capital Cities in Late Imperial China: Archaeological restoration and comparative study of the central capitals of the Yuan and Ming dynasties (in Chinese), City Planning Review, 2018, Vol. 42(8), pp. 57–65, DOI: 10.11819/cpr20180810a	1. Chen Xiao, Sun Hua
C18	Nanjing 1375–1421	*Form and Planning of Newly-Built Capital Cities in Late Imperial China: Archaeological restoration and comparative study of the central capitals of the Yuan and Ming dynasties (in Chinese), City Planning Review, 2018, Vol. 42(8), pp. 57–65, DOI: 10.11819/cpr20180810a *Study on planning strategies of the Forbidden City of Ming Dynasty in Nanjing (in Chinese), Journal of Shandong University of Architecture and Engineering, 2006, Vol. 21(2), pp. 122–128, DOI: 10.3969/j.issn.1673-7644.2006.02.006	1. Chen Xiao, Sun Hua 2. Deng Shenghui, Yao Yifeng
C19	Beijing 1421–	*Form and Planning of Newly-Built Capital Cities in Late Imperial China: Archaeological restoration and comparative study of the central capitals of the Yuan and Ming dynasties (in Chinese), City Planning Review, 2018, Vol. 42(8), pp. 57–65, DOI: 10.11819/cpr20180810a *Peking und Umgebung (Map: col. ; 76 × 68 cm), Berlin: Reichsamt für Landesaufnahme, 1907, Digital ID: http://hdl.loc.gov/loc.gmd/g7824b.ct001944 *National Beiping Palace Museum plan (Map; 104.5 × 148 cm), 1948	1. Chen Xiao, Sun Hua 2. Library of Congress Geography and Map Division Washington, D.C. 20540-4650 USA dcu 3. Beiping Palace Museum
C20	Seoul 1394–	*A Study on the Designation in Korean Traditional Space design Text (in Korea), Korean Institute of Interior Design Journal, 2007, Vol. 16, No.4, pp. 31–38, UCI: G704-000249.2007.16.4.013 *Report on the basic restoration and repair plan of Jingfu Palace, 1994, http://116.67.83.213/NEW_PDF/EM017464.pdf	1. Park Kyung-Ae 2. Korean Cultural Heritage Administration
C21	Hué 1804–1945	*Reconstruction study for decorative painting at the 'Can Chanh Dien', main palace of the Nguyen dynasty (in Japanese), Dissertation for the Doctoral Degree, Waseda University, 2013, URL: http://id.ndl.go.jp/bib/024807810 *9005 Relation between Plans of Places and Bloc Plan: Studies on the Imperial Palace of Hue, Vietnam, in the Nguyen Dynasty (Part 154) (in Japanese), Proceeding of the architectural research meetings, Kanto Chapter, Architectural Institute of Japan, 2011, Vol. 81, pp. 475–478. https://www.aij.or.jp/paper/detail.html?productId=394004 *Analysis on the Measurement in Planning of Disposition (II): Studies on the Imperial Palace of Hue, Viet Nam, in the Nguyen Dynasty (Part 3) (in Japanese), Summaries of technical papers of Annual Meeting Architectural Institute of Japan. F-2, History and theory of architecture, 1995, pp. 531–532. https://www.aij.or.jp/paper/detail.html?productId=271246	1. Saito shiomi Kanayama Emiko, Nakagawa Takeshi, Kitani Kenta 2,3. Tsuchiya Takeshi, Nakagawa Takeshi, Nishimoto Shin-ichi, Takano Keiko, Nakazawa Shin-ichiro,

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