

Article

All-Day Energy Harvesting Power System Utilizing a Thermoelectric Generator with Water-Based Heat Storage

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Abstract: Solar thermal systems, especially solar hot water household heating/storage systems, are considered the most cost-effective alternatives to fossil fuel hot water heating energy systems. Recently, solar hot water systems are combined with a thermoelectric generator, forming hybrid systems. However, these hybrid systems described in the literature cannot generate electricity from sunset to sunrise, or at night, when residential consumers use the most electricity. In this paper, an all-day energy harvesting power system utilizing a thermoelectric generator with water-based heat storage is presented to generate electricity all-day and also produce warm water. The experimental and theoretical analyses were conducted to evaluate and verify the performance of the systems. In the case study, the scaled-up system shows potential to provide 198.9 L of warm water per day, 0.912 kWh of electricity in the daytime, and 0.0332 kWh of electricity at nighttime for a typical house with 6.34 m² of available surface area in Tokyo, Japan. Although the electric power at night is low, this novel lab-scale system shows the potential to be a viable source of electricity and warm water throughout the day, without emitting any greenhouse gas.

Keywords: thermoelectric generator; solar hot water system; heat pipe; Fresnel lens; all-day energy harvesting

1. Introduction

Solar thermal systems, especially solar hot water household systems, are considered the most cost-effective alternatives to fossil fuel hot water heating energy systems [1]. Since more than 60% of the end-use energy is used for space heating and water heating in residential buildings in many countries (nearly two thirds of residential energy consumption in US [2,3], about 70% home energy use in Japan and China [3,4], and around 80% household energy usage in EU and UK [3,5]), reducing their energy use is necessary for the future low-carbon society. Biao and Bernier [6] examined four alternative water heating technologies for two climates (Montréal and Los Angeles). Those four technologies are (i) standard electric water heating, (ii) the desuperheater with electric backup, (iii) the solar water heating system with electric backup, and (iv) a heat pump water heater system. Their research proved that the solar water heating system with an electric backup can minimize the required power for heating hot water.

Recently, solar hot water systems are combined with a thermoelectric generator (TEG), and they are considered a promising system. A TEG is a device which can convert the thermal energy into electrical energy by using the Seebeck effect. It only needs a temperature difference between its two surfaces to generate electricity. When compared to other solar heat driven mechanical power

generation methods, TEGs are much more suitable for small-scale applications, as they are reliable, light and compact, and have no mechanical moving parts. Therefore, the combination systems can maintain their high cost-effectiveness and compactness, while producing heat and also generating electricity. Though the relatively low conversion efficiency is a drawback, the solar heat driven TEG technology will become more attractive with decreasing prices of thermoelectric materials [7]. Regarding the electricity generation, the combination systems have one unique feature, that there is a delayed thermal response resulting from the system heat capacitance. This enables the continuation of the power output even when the sun is partially blocked by clouds. It is an attractive feature, since temporary cloud coverage can cause large power fluctuations for Photovoltaics (PV) systems [8].

Ashwin et al. [9] developed such a combination system utilizing a heat pipe and concentrated solar power technologies while reporting on both theoretical analysis and experimental results of the transient behavior of the proposed system. From the study, it was proven that the combination system has the potential to heat 300 L of water to 76 °C, and to generate 0.756 kW h of electricity during the peak sun hours for a typical house with 5 m² of available surface area in Melbourne. Wei et al. [10] developed the combination system utilizing the heat pipe with glass evacuated tube, and it has been studied through simulation and by experimental work. The simulation results indicate that for the solar irradiation of larger than 600 W/m², the combination system may have a thermal efficiency of about 55% and, meanwhile, have an electrical efficiency above 1%. Guiqiang et al. [11] developed the combination system, utilizing a novel micro-channel heat pipe with an evacuated tube, and conducted theoretical analysis. The results show that the combination system has a high thermal performance with electricity production. All of the combination systems shown have the potential to both produce hot water and generate electricity. However, all of them cannot generate electricity during the night (from sunset to sunrise) when residential consumers use the most electricity [12–14]. Since the combination systems can generate electricity by temperature differences, the stored heat can be used to generate electricity in the house at nighttime.

In this paper, an all-day energy harvesting lab-scale power system utilizing TEG with water-based heat storage was developed. This system is a combination system of TEG and a solar hot water system, and was derived based on Ashwin's model [9]. The system includes the TEG, Fresnel lens, and heat pipes. The Fresnel lens is used to create a higher heat flux desirable for better performance of the TEG from the solar light, and also achieve high temperature on the hot side surface of the TEG. Heat pipes are used due to their high heat flux carrying capacity, and are useful to transfer the heat from the heat source. Water is used as a sensible heat storage medium, and the thermal energy will be stored from the solar light during the daytime. The novelty of the proposed system is that during the nighttime, the heat stored as hot water is used for TEG to provide a high temperature sink, and thereafter, it is stored as water in order to use it for various applications in a model house. A mathematical model based upon the energy balance was used to analyze the system's potential. The experimental study was conducted to verify the mathematical model. The feasibility of the proposed lab-scale system was analyzed through comparison with Ashwin's model in regard to the efficiency of systems. Moreover, a case study was conducted to investigate the potential of a scaled-up system in a practical case.

2. Experimental Approach

2.1. Design of the System

The design of the system is shown in Figure 1. Two flat aluminum heat pipes (flat aluminum heat pipe 1, 2) are used, and each of them has a thermal couple attached to the TEG's upper and lower sides by thermally conductive tapes (3M Japan Limited, Tokyo, thermal conductivity of 1.5 W/m K). Moreover, these flat aluminum heat pipes are connected with water tank 1, 2 by the waterproof adhesive (CEMEDINE CO., LTD., Tokyo, Japan). The wick structure is used in the heat pipes so the system can easily shift the heat transfer direction. Metal weights (3087.5 g) are placed on top of the

TEG and flat aluminum heat pipe 1, 2 in order to reduce the thermal resistance between TEG and flat aluminum heat pipe 1, and between TEG and flat aluminum heat pipe 2.

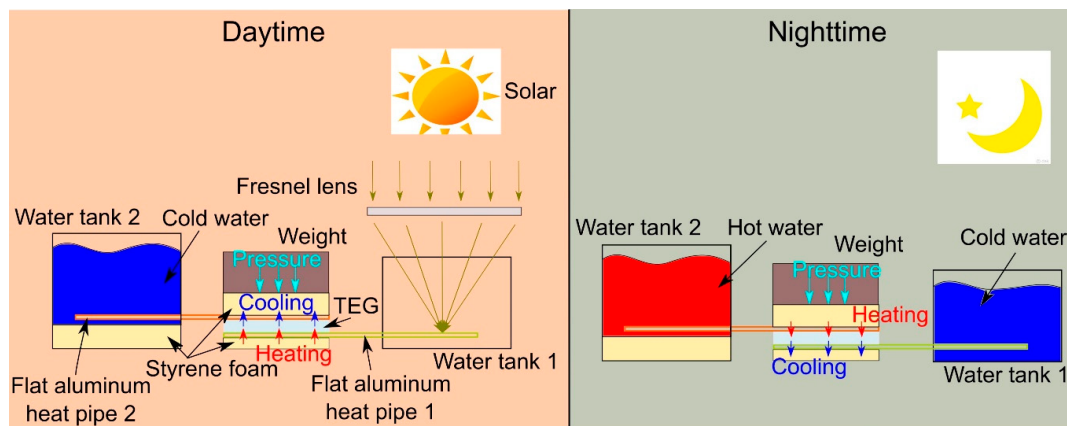


Figure 1. Schematic of the system (solar light and cold water during daytime, hot water and cold water during nighttime). TEG = thermoelectric generator.

During the daytime, solar radiation (light) passes through the Fresnel lens, which focuses the light on a portion of flat aluminum heat pipe 1, which is covered by a black absorbing paint (ICHINEN TASCO CO., LTD., Osaka, Japan, emissivity of 0.94). The heat transfers from the concentrated solar radiation to cold water (thermal energy transfers to the water tank 2). The TEG can generate electricity due to the temperature differences resulting from the concentrated solar radiation below and contact with the cold water on the top.

During the nighttime, cold water (tap water) at room temperature is poured into water tank 1. Since the water in tank 2 was heated during the daytime, it is relatively hot by nighttime. Therefore, the heat flow is reversed in the system. The heat transfers from hot water to cold water via thermoelectric generator (thermal energy transfers to the water tank 1). The TEG generates electricity due to temperature differences on its surface by being in contact with hot water and cold water tanks.

Figure 2 shows a photograph of the system. Although there is no inclusion of the metal weights and Fresnel lens in the picture, these are incorporated into the system. A portion of the heat pipe (which is called “adiabatic section”) and the TEG are covered by glass wool (MASUDA CORPORATION, Nagano, Japan, heatproof temperature of 200 °C) in order to reduce the heat loss from the system.

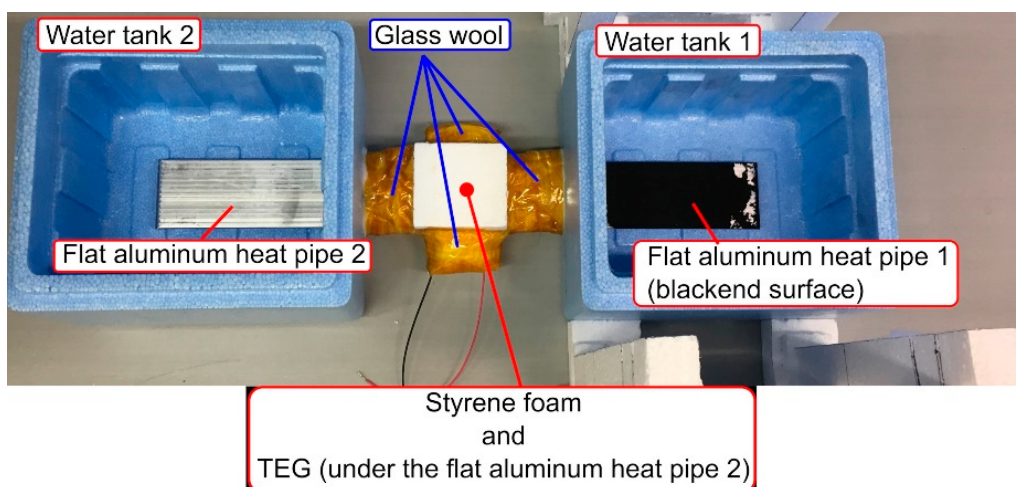


Figure 2. Photograph of the system showing empty water tanks 1 and 2.

2.2. Experimental Method

The schematic diagram of the daytime experiment is shown in Figure 3. The purpose of this daytime experiment is to test the light-to-heat conversion efficiency of the system during the day. In the daytime experiment, a halogen lamp was used to supply the light. It was chosen since the light supplied by that lamp has a similar wavelength as solar radiation. The power of the halogen lamp was set to 75 W and 100 W (maximum power of the halogen lamp is 110 W). However, due to the heat loss or the other losses from the halogen lamp, the power supplied to the Fresnel lens becomes smaller. Moreover, the optical efficiency of the Fresnel lens and absorptivity of the black absorbing film must be taken into account in the system so the power concentrated on the flat aluminum heat pipe becomes much lower than the initial power. Specifically, the initial power supplied by the halogen lamp decreased from 75 W (electrical power) to 14.84 W (light intensity), and 100 W to 21.17 W, respectively. The values of light intensity were measured by a solar radiation meter. The temperature of the halogen lamp became very hot during the operation, so a small fan was used to cool the lamp. Two Fresnel lenses were used in the experiment since the halogen lamp only produces scattered light. The first Fresnel lens parallelized the scattered light from the lamp, and the second Fresnel lens was used to focus the parallel light. A temperature data logger with J-type grounded thermocouple was used to measure the temperature of cold water. A voltage data logger was used to measure the open-circuit voltage of TEG generated by the Seebeck effect. Cold water of masses of 1000 g and 2000 g was used in the experiments.

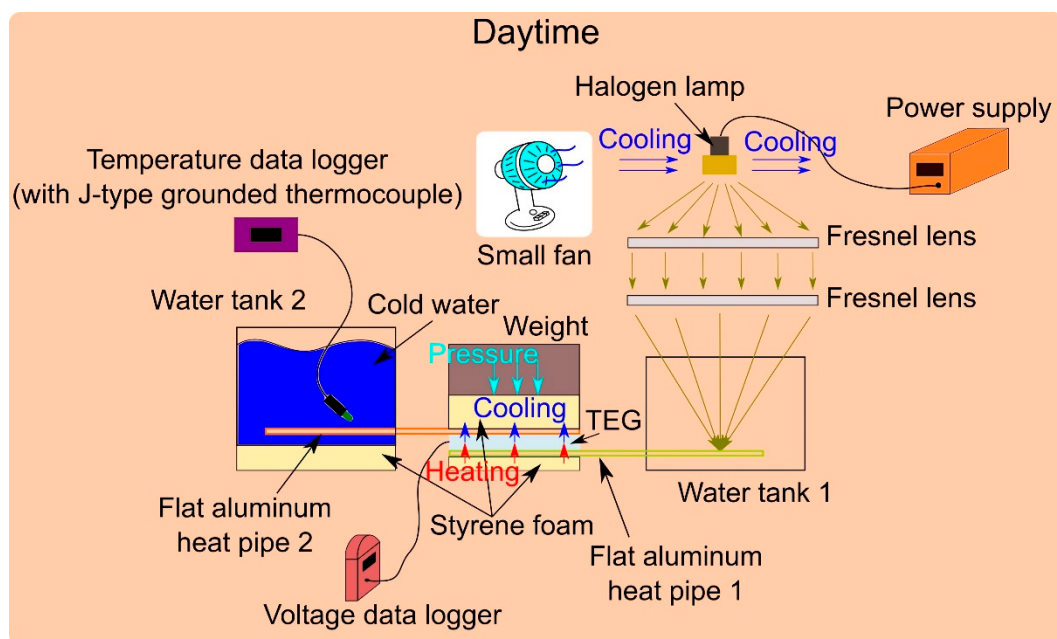


Figure 3. Schematic diagram of the daytime experiment.

The schematic diagram of the nighttime experiment is shown in Figure 4. The purpose of this nighttime experiment is to check the reverse operation of the heating efficiency of the system at night. For the nighttime experiment, the halogen lamp is turned off, since there is no sunlight during the nighttime. Hot water (80–90 °C) was provided by heating water using an electric kettle since the halogen lamp did not have sufficient power to make hot water in a short period of time for this experiment. A temperature data logger with J-type grounded thermocouple and thermometer were used to measure the temperature of hot water and cold water, respectively. As is the case in the daytime experiment, the voltage data logger was used to measure the open-circuit voltage of TEG. Cold water amounts of 2000 g and 3000 g, and hot water amounts of 1000 g and 2000 g, were used in the experiments.

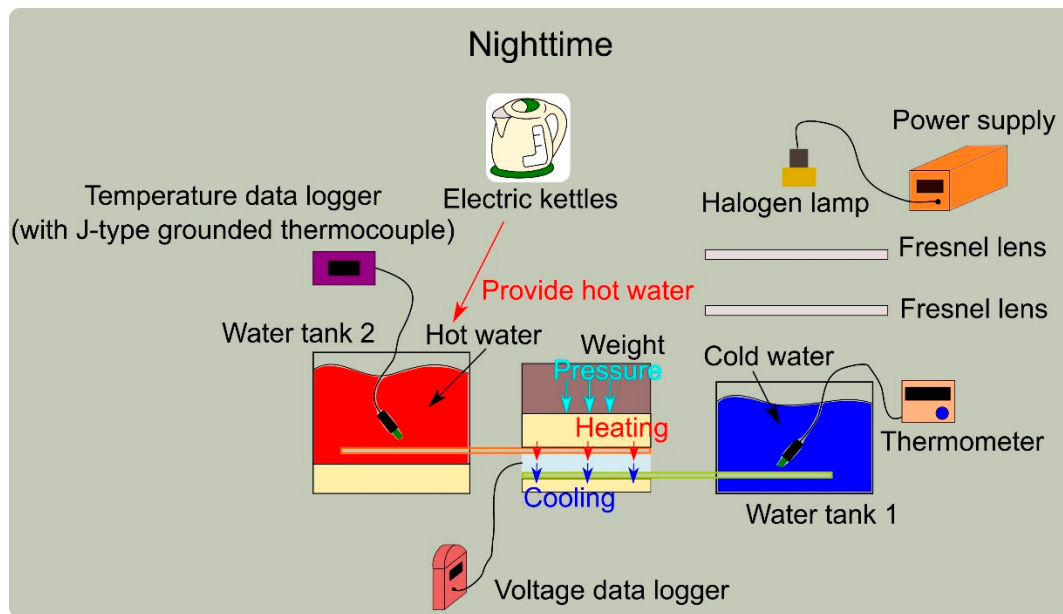


Figure 4. Schematic diagram of the nighttime experiment.

2.3. Parameters

2.3.1. TEG (Thermoelectric Generator)

The TEG (size: $50.8 \times 50.8 \times 3.73 \pm 0.10$ mm, internal resistance: $1.90 \Omega \pm 10\%$ at $T_h = 25^\circ\text{C}$) was supplied from VICS CO., LTD., Tokyo, Japan. The load and hot side maximum temperature of the TEG are 4–6 kgf (uniform load) and 150°C , respectively. Here, in the experiment, low input power (about 40% of the solar radiation of 1000 W/m^2) is used so the hot side temperature of TEG does not exceed 150°C . However, in the practical case using focused sunlight, the hot side temperature can easily exceed 150°C . Most commercially available generators have a limiting temperature within the range of 150 – 300°C [9]. Therefore, the TEG that can withstand a higher temperature (more than 200°C) will be used in reality. The thermal resistance of TEG was determined by the simple experiment, and the value is $R_{\text{TEG}} = 2.6 \text{ [K/W]}$.

2.3.2. Heat Pipes and Fresnel Lens

The flat aluminum heat pipes (size: $300 \times 50 \times 2.5$ mm, material: A1050, thermal resistance: $<0.4 \text{ K/W}$, wick structure: grooved) were supplied from SIMOTEC Co., Ltd., Osaka, Japan. The flat shape was adopted since it is easy to attach those heat pipes to the TEG's upper and lower sides. The evaporator and condenser area of the heat pipe is $150 \times 50 \text{ mm}^2$ or $50 \times 50 \text{ mm}^2$. These areas will switch in the daytime and nighttime operations.

A Fresnel lens (size: $250 \times 250 \times 2$ mm, focal length: 200 mm, material: PMMA, effective diameter: 354 mm) was provided from NTKJ Co., Ltd., Tokyo, Japan.

2.3.3. Instruments

A thermometer (precision: ± 1 (0 – 40) $^\circ\text{C}$, ± 2 (otherwise) $^\circ\text{C}$) was supplied from Shinwa Rules Co., Ltd., Niigata, Japan. A temperature data logger with J-type grounded thermocouple (precision: $\pm 2.2^\circ\text{C}$ or $\pm 0.75\%$) was provided from National Instruments Corporation, Tokyo, Japan. A voltage data logger (precision: $\pm 0.5\% + 5\text{dgt.}$ in the environment of $20^\circ\text{C} \pm 10^\circ\text{C}$) was supplied from T&D CORPORATION, Nagano, Japan. A solar radiation meter (precision: $\pm (5\% \text{ of reading} + 5 \text{ W})$) was used from EKO INSTRUMENTS CO., LTD., Tokyo, Japan.

2.3.4. Heat Loss from the Water Tank

The water tanks used in these experiments were made of styrene foam (outer size: $183 \times 248 \times 173$ mm, inner size: $130 \times 198 \times 125$ mm). This type of water tank is used in order to reduce the water's heat loss. The thermal conductivity of the styrene foam is very low, and a lid is put on the water tank so the temperature inside the tank is maintained constantly for a long time. However, the water tank is not completely sealed, and a hole is made in the lid of the water tank in order to measure the water temperature by the thermocouple connected to a display. Although the hole is covered by glass wool in the experiment, it is not enough to prevent the movement of the air. Therefore, the heat loss from the water tank must be considered in the experiment. Since the thermal conductivity of the styrene foam is very low and the temperature of the water in the experiment is not very high, it is assumed that the convective heat loss is the dominant heat loss mechanism.

The equation of the convective heat loss from the water tank is expressed as follows.

$$Q_{wloss} = h_{wcon} \cdot (T_w - T_a) \quad (1)$$

Here, h_{wcon} is the rate of heat loss from the water tank per unit temperature difference, T_w is the temperature of water in the tank, T_a is the ambient temperature. The value of h_{wcon} with respect to the water temperature T_w are determined by the simple experiment and by using Equation (1). In the simple experiment, the hot water is poured into the water tank (the heat pipes are not connected) and the hot water temperature change (heat loss from the water tank) is measured. Q_{wloss} is calculated for every 5 °C (e.g., 80–85 °C, 75–80 °C, 70–75 °C) by using the measured values. The value of Q_{wloss} and the average temperature at each temperature range are substituted into Equation (1) so that the value of h_{wcon} is determined. The results are shown in Figure 5.

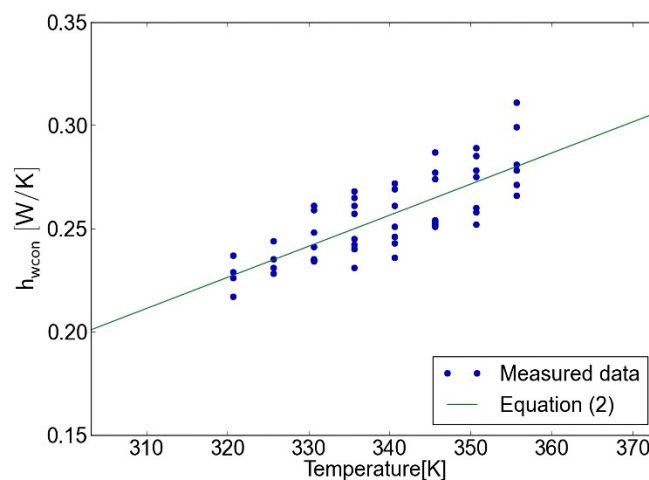


Figure 5. Values of h_{wcon} with respect to temperature of water T_w (K).

The linear least squares regression method is used to determine the equation of the relationship between h_{wcon} and T_w (K). From the measured data and the least squares method, the equation derived is as follows,

$$h_{wcon} = 0.00151 \cdot T_w - 0.255 \quad (2)$$

Equation (2) is expressed as green line in Figure 5. The value of h_{wcon} used in the mathematical model of the system is calculated by Equation (2).

3. Mathematical Modeling of the System

The mathematical model of the system is derived based on the energy balance of the system. The following assumptions were made to simplify the analysis of the complex heat transfer process in the system.

- All energy balance equations are one-dimension and based on the steady-state condition, so the thermal capacity of any parts of the system is neglected (resistance analogy instead of RC analogy is used in the mathematical model).
- The “adiabatic section” of flat aluminum heat pipe 1 and 2, and TEG are covered by glass wool, so the heat loss in these parts are neglected.
- The electric power from the TEG (W_{TEG}) is very low when compared to the thermal energy transferred to the water, so it is neglected in the heat balance equation.
- The black absorbing plate is surrounded by the wall (water tank 1), and it can be considered that the effect of the wind to this plate is very small. Therefore, the wind heat transfer coefficient is neglected in the heat balance equation.
- When the internal electrical resistance is equal to the external electrical resistance, the TEG attains the maximum output power, and this maximum output power is used to calculate the electrical efficiency of the system.
- The heat loss factors (heat loss from the water tank (Q_{wloss} , Q_{w1loss} , Q_{w2loss}) and the black absorbing plate (Q_{lrc})) are treated as a constant value in the heat balance equation to derive the equation of water temperature change easily. These factors will be calculated separately and substituted into the equation of water temperature change for every unit of time (one minute). In other words, the water temperature will be calculated for every unit of time (one minute). The initial value of the heat loss factors can be determined by using the initial water temperature.

These assumptions may cause a difference between the experimental and theoretical results. Specifically, the theoretical results may show better performance than the experimental results.

3.1. Daytime Operation

Figure 6 shows the heat balance diagram of the system during the daytime.

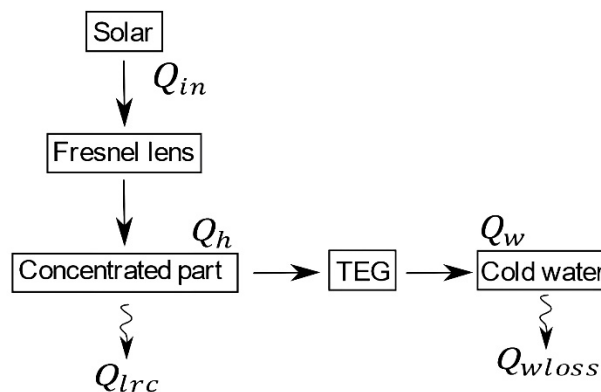


Figure 6. Heat balance diagram of the system during daytime operation.

During the daytime, solar radiation passes through the Fresnel lens and is concentrated on the black absorbing film. In the experiment, the halogen lamp is used instead of solar light. The heat concentrated on the black absorbing film can be expressed as follows,

$$Q_h = Q_{in} \times A_{lens} \times \eta_{lens} \times \varepsilon_{ab} \quad (3)$$

Here, Q_{in} is the solar radiation flux incident of the Fresnel lens, A_{lens} is the area of the lens, η_{lens} is the optical efficiency of the lens, and ε_{ab} is the absorptivity of the black absorbing film.

Applying the conservation of energy principle, the heat balance equation of the system can be expressed as follows,

$$Q_h = Q_w + Q_{wloss} + Q_{lrc} \quad (4)$$

Here, Q_w is the heat input to cold water in the storage tank, Q_{wloss} is the heat loss rate from the water storage tank, and Q_{lrc} is the radiation and convection heat loss rate from the black absorbing film.

Sensible heat absorbed by cold water can be expressed as,

$$Q_w = mc_p \frac{dT_w}{dt} \quad (5)$$

Here, T_w is the temperature of water in the tank, m is the mass of water in the storage tank, c_p is the specific heat of water, and t is the time. Substituting Equation (5) into Equation (4),

$$Q_h = mc_p \frac{dT_w}{dt} + Q_{wloss} + Q_{lrc} \quad (6)$$

The above equation is a first order, non-homogeneous, linear differential equation. The solution of the above equation will be as follows,

$$T_w = \left(\frac{Q_h - Q_{wloss} - Q_{lrc}}{mc_p} \right) t + A \quad (7)$$

Here, A is a constant. Using the initial conditions (when $t = 0$, $T_w = T_i$), the value of the constant A will be as follows,

$$A = T_i \quad (8)$$

Substituting that constant A into Equation (7),

$$T_w = \left(\frac{Q_h - Q_{wloss} - Q_{lrc}}{mc_p} \right) t + T_i \quad (9)$$

Here, T_i is the initial temperature of water in the tank. Q_{wloss} can be calculated from Equation (1) and Equation (2). Q_{lrc} can be calculated by the following equation [15],

$$Q_{lrc} = U_t A_c (T_p - T_a) \quad (10)$$

Here, U_t is the top heat loss factor, A_c is the gross area of the black absorbing plate, and T_p is the temperature of the black absorbing plate. U_t can be calculated by using the following equation,

$$U_t = h_{nat} + h_r \quad (11)$$

Here, h_{nat} is the natural convection heat transfer coefficient, and h_r is the radiation heat transfer coefficient. The natural convection heat transfer coefficient can be expressed as follows [16,17],

$$h_{nat} = \frac{Nu \cdot k}{L_c} = 1.61 (T_p - T_a)^{1/3} \quad (12)$$

Here, Nu is the Nusselt number, k is the thermal conductivity of air, and L_c is the length of the black absorbing plate. The Nusselt number (Nu) as shown below is used to derive the above equation.

$$Nu = 0.135 (Ra)^{1/3} \quad (13)$$

Here, Ra is a Rayleigh number. The above equation is valid for $10^3 < Gr$ (Grashof Number) $< 10^9$, Pr (Prandtl Number) = 0.72 (which means that the working fluid is air).

h_r can be calculated as follows [18,19],

$$h_r = \sigma \varepsilon_{ab} (T_p + T_a) (T_p^2 + T_a^2) \quad (14)$$

Here, σ is the Stefan-Boltzmann constant. The temperature of the black absorbing plate can be derived by using the resistance analogy. Figure 7 illustrates the system thermal resistance circuit during daytime operation derived from the resistance analogy.

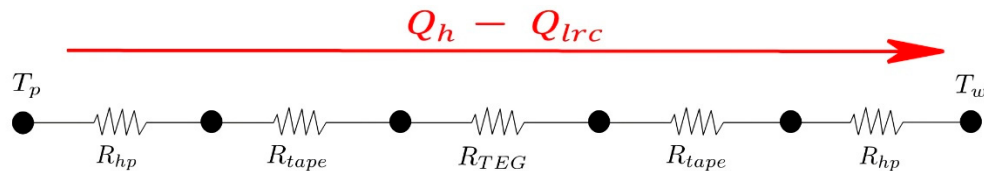


Figure 7. Thermal resistance circuit of the system during daytime operation.

The equation of the thermal resistance circuit can be expressed as follows,

$$T_p = T_w + (Q_h - Q_{lrc}) \times R \quad (15)$$

Here, total thermal resistance R can be expressed as follows,

$$R = \frac{R_{TEG}}{N_{TEG}} + 2R_{tape} + 2R_{hp} \quad (16)$$

R_{TEG} is the thermal resistance of TEG (≈ 2.6 [K/W]), N_{TEG} is the number of TEG, R_{tape} is the thermal resistance of the thermally conductive tape (≈ 0.0513 [K/W]), and R_{hp} is the thermal resistance of the heat pipe (≈ 0.2 [K/W]).

The transient behavior of the temperature of water in the tank can be predicted by using Equation (9).

The equations of the thermal and electrical efficiency can be expressed as follows,

$$\eta_{thermal, daytime} = \frac{Q_w}{Q_h} \quad (17)$$

$$\eta_{electrical, daytime} = \frac{W_{TEG}}{Q_h} \quad (18)$$

W_{TEG} can be calculated by the following equation,

$$W_{TEG} = \frac{V_0^2}{4r} \quad (19)$$

Here, V_0 is the open-circuit voltage of TEG, and r is the internal resistance of TEG.

3.2. Nighttime Operation

Figure 8 shows the heat balance diagram of the system during the nighttime.

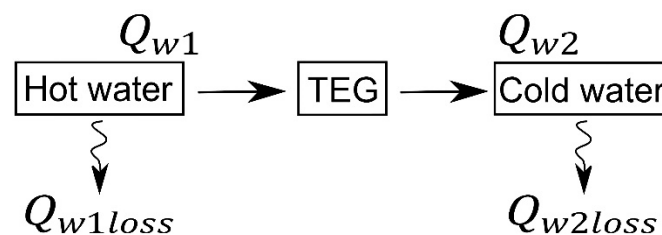


Figure 8. Heat balance diagram of the system during nighttime operation.

The heat balance equation of the system can be expressed as follows,

$$Q_{w1} = Q_{w1loss} + Q_{w2} + Q_{w2loss} \quad (20)$$

Here, Q_{w1} is the heat output from the hot water in the storage tank, Q_{w2} is the heat input to the cold water in the storage tank, Q_{w1loss} is the heat loss rate from hot water, and Q_{w2loss} is the heat loss rate from cold water.

Sensible heat released by hot water can be expressed as follows,

$$Q_{w1} = -m_1 c_p \frac{dT_{w1}}{dt} \quad (21)$$

Here, m_1 is the mass of hot water in the storage tank, and T_{w1} is the temperature of hot water in the tank. Sensible heat absorbed by cold water can be expressed as follows,

$$Q_{w2} = m_2 c_p \frac{dT_{w2}}{dt} \quad (22)$$

Here, m_2 is the mass of cold water in the storage tank, T_{w2} is the temperature of cold water in the tank. Substituting Equation (21) and Equation (22) into Equation (20),

$$-m_1 c_p \frac{dT_{w1}}{dt} = m_2 c_p \frac{dT_{w2}}{dt} + Q_{wloss} \quad (23)$$

Here,

$$Q_{wloss} = Q_{w1loss} + Q_{w2loss} \quad (24)$$

In order to derive the equation of T_{w1} or T_{w2} , resistance analogy is used. Figure 9 illustrates the system thermal resistance circuit during nighttime operation derived from the resistance analogy.

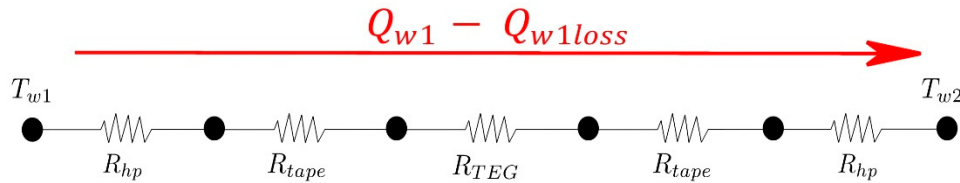


Figure 9. Thermal resistance circuit of the system during nighttime operation.

The equation of the thermal resistance circuit can be expressed as follows,

$$T_{w2} = T_{w1} - (Q_{w1} - Q_{w1loss}) \times R \quad (25)$$

Here, $R = (R_{TEG}/N_{TEG} + 2R_{tape} + 2R_{hp})$. Substituting Equation (21) into the above equation,

$$T_{w2} = T_{w1} - \left(-m_1 c_p \frac{dT_{w1}}{dt} - Q_{w1loss} \right) \times R \quad (26)$$

The heat loss rate from the hot water and the cold water can be expressed as follows,

$$Q_{w1loss} = h_{wcon1} \cdot (T_{w1} - T_a) \quad (27)$$

$$Q_{w2loss} = h_{wcon2} \cdot (T_{w2} - T_a) \quad (28)$$

Here, h_{wcon1} is the rate of heat loss from the water tank (hot water) per unit temperature difference, h_{wcon2} is the rate of heat loss from the water tank (cold water) per unit temperature difference. Substituting Equation (27) into Equation (26),

$$T_{w2} = T_{w1} - \left\{ -m_1 c_p \frac{dT_{w1}}{dt} - h_{wcon1} \cdot (T_{w1} - T_a) \right\} \times R \quad (29)$$

Substituting Equation (29) into the Equation (23),

$$\alpha \frac{d^2 T_{w1}}{dt^2} + \beta \frac{dT_{w1}}{dt} + Q_{wloss} = 0 \quad (30)$$

Here, $\alpha = m_1 m_2 c_p^2 R$, $\beta = \{(m_1 + m_2)c_p + m_2 c_p R h_{wcon1}\}$. The above equation is a second order, non-homogeneous, linear differential equation. Q_{wloss} can be calculated from Equation (2), Equation (27), and Equation (28) (The same water tank is used, so the coefficients in Equation (2) are the same in h_{wcon1} and h_{wcon2}).

The final equation of T_{w1} becomes as follows,

$$T_{w1} = T_{i1} + \frac{\alpha}{\beta} \left\{ 1 - \exp\left(-\frac{\beta}{\alpha} t\right) \right\} \left(K_1 + \frac{Q_{wloss}}{\beta} \right) - \frac{Q_{wloss}}{\beta} t \quad (31)$$

where K_1 is,

$$K_1 = -\frac{T_{i1} - T_{i2} + R h_{wcon1} (T_{i1} - T_a)}{m_1 c_p R} \quad (32)$$

Here, T_{i1} is the initial temperature of hot water in the storage tank, and T_{i2} is the initial temperature of cold water in the storage tank. The transient behavior of the temperature of the hot water in the storage tank can be predicted by using this Equation (31).

The final equation of T_{w2} becomes as follows,

$$T_{w2} = T_{i2} - \frac{G_2}{G_1} \left(1 + R h_{wcon1} - \frac{\beta}{\alpha} m_1 c_p R \right) \left\{ 1 - \exp\left(-\frac{\beta}{\alpha} t\right) \right\} - (1 + R h_{wcon1}) \frac{Q_{wloss}}{\beta} t \quad (33)$$

where G_1 and G_2 are,

$$G_1 = \left(\frac{\beta^2}{\alpha^2} \right) m_1 c_p R - \frac{\beta}{\alpha} - \frac{\beta}{\alpha} R h_{wcon1} \quad (34)$$

$$G_2 = \frac{T_{i1} - T_{i2} - R h_{wcon2} (T_{i2} - T_a)}{m_2 c_p R} + \frac{Q_{wloss}}{\beta} + \frac{Q_{wloss}}{\beta} R h_{wcon1} \quad (35)$$

The transient behavior of the temperature of cold water in the storage tank can be calculated by using Equation (33). The detailed calculation process to derive the final equation of T_{w1} and T_{w2} is shown in Appendix A.

The equations of the thermal and electrical efficiency can be expressed as follows,

$$\eta_{thermal,nighttime} = \frac{Q_{w2}}{Q_{w1}} \quad (36)$$

$$\eta_{electrical,nighttime} = \frac{W_{TEG}}{Q_{w1}} \quad (37)$$

4. Results and Discussion

4.1. Water Temperature T_w (Daytime Operation)

The power of the halogen lamp is set to 14.84 W and 21.17 W based upon irradiance measurements. The mass of cold water is set to 1000 g and 2000 g. Totally, four patterns are calculated and measured. The total measurement time is one hour. In this section, only the result when $(Q_h, m) = (21.17 \text{ W}, 1000 \text{ g})$, $(21.17 \text{ W}, 2000 \text{ g})$ are shown.

Figure 10 shows the experimental and theoretical results of T_w . T_{w_Theory} , which is represented as a solid line, was calculated from the mathematical model derived in Section 3.1. It considers the heat loss from the water and the black absorbing plate. In the figure, it is observed that the theoretical results have good agreement with the experimental results in both conditions (when $(Q_h, m) = (21.17 \text{ W}, 1000 \text{ g})$ and $(21.17 \text{ W}, 2000 \text{ g})$). The experimental and theoretical results when $(Q_h, m) = (21.17 \text{ W}, 1000 \text{ g})$ have a slight difference when compared to the results when $(Q_h, m) = (21.17 \text{ W}, 2000 \text{ g})$. This can be considered as a difference in the effect of natural heat convection of water. The larger mass of water causes the higher water surface in the water tank. Since the heat pipe is placed at the bottom of the water tank, the effect of the natural heat convection of water increases with a larger mass of water,

and the heat transfer from the heat pipe to water becomes more efficient. The same trend was observed under the other two conditions (when $(Q_h, m) = (14.84 \text{ W}, 1000 \text{ g})$ and $(14.84 \text{ W}, 2000 \text{ g})$).

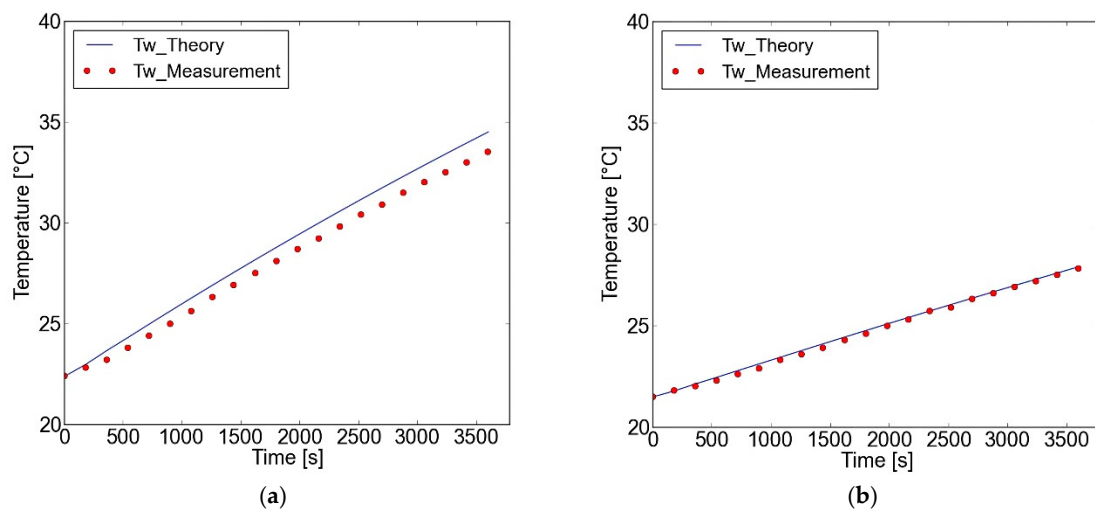


Figure 10. Experimental and theoretical results of T_w ; (a) when $Q_h = 21.17 \text{ W}$, $m = 1000 \text{ g}$; (b) when $Q_h = 21.17 \text{ W}$, $m = 2000 \text{ g}$.

Figure 11 shows the theoretical results (considering the heat loss from water and black absorbing plate) when $Q_h = 56.88 \text{ W}$, $m = 1000 \text{ g}$ and 2000 g . $Q_h = 56.88 \text{ W}$ corresponds to the solar radiation of 1000 W/m^2 . The ambient temperature and the initial water temperature are set to 28°C and 25°C , respectively. No wind is considered in this simulation. The time range is set to three hours.

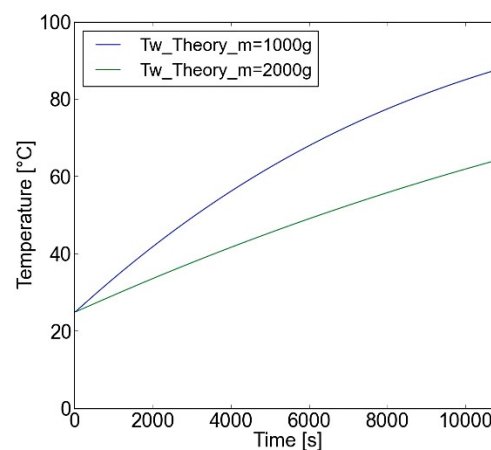


Figure 11. Theoretical results of T_w , When $Q_h = 56.88 \text{ W}$, $m = 1000 \text{ g}$ and 2000 g .

From the figure, it is observed that the system can raise the water temperature to about 87.8°C after three hours when the mass of water is 1000 g . However, the actual system can only raise the water temperature to about 64.4°C after three hours when the mass of water is 2000 g . In order to provide more heat and create a larger temperature difference for the TEG at night, the larger mass of water is better. However, the solar radiation set in this simulation is the maximum value in Japan [20]. Therefore, the system needs to reduce the heat loss to raise the water temperature to 70 or 80°C when the mass of water is more than 2000 g . From the analysis of the results, it is confirmed that a large amount of heat loss is due to the heat loss from the black absorbing plate. In order to make the system to raise the water temperature to 70 or 80°C in a larger mass of water, it is necessary to reduce a large amount of heat loss from this black absorbing plate.

4.2. Hot Water Temperature T_{w1} and Cold Water Temperature T_{w2} (Nighttime Operation)

For nighttime operation, the mass of hot water m_1 is set to 1000 g and 2000 g. The mass of cold water m_2 is set to 2000 g and 3000 g. Totally, four patterns are calculated and measured. The total measurement time is three hours. In this section, only the results when $(m_1, m_2) = (1000 \text{ g}, 2000 \text{ g})$, $(2000 \text{ g}, 2000 \text{ g})$ are shown.

Figure 12 shows the experimental and theoretical results of the system with respect to T_{w1} and T_{w2} . T_{w1_Theory} is the hot water temperature calculated by Equation (31), T_{w2_Theory} is the cold water temperature calculated by Equation (33). In Figure 12a, it is observed that T_{w1_Theory} and T_{w2_Theory} are in good agreement with the experimental results. On the other hand, in Figure 12b, it is observed that there is a slight difference between the theoretical values and the experimental values. T_{w1} decreases faster than T_{w1_Theory} in the experiment so that T_{w2} does not increase like T_{w2_Theory} . The same trend was observed under the other two conditions $((m_1, m_2) = (1000 \text{ g}, 3000 \text{ g}), (2000 \text{ g}, 3000 \text{ g}))$. In the system, the flat aluminum heat pipe is installed at the bottom of the water tank. When the hot water is poured into the water tank, hotter water will rise up and colder water circulates downward due to natural heat convection. Therefore, it can be considered that the natural heat convection in the water tank is causing a loss of energy during the heat transfer at nighttime operation. The larger mass of hot water (or the higher hot water surface) causes greater natural heat convection so that it affects the measured values more when m_1 became larger.

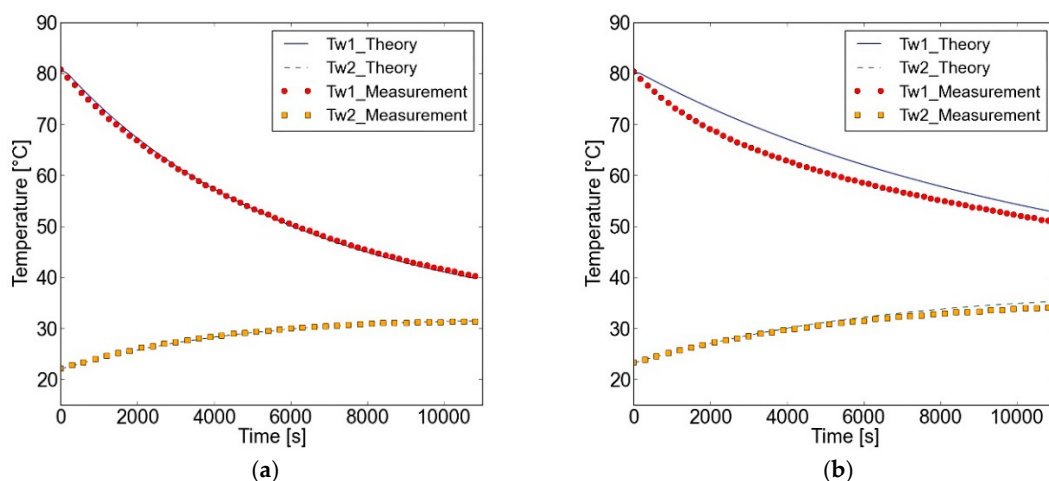


Figure 12. Experimental and theoretical results of T_{w1} and T_{w2} . (a) When $m_1 = 1000 \text{ g}$, $m_2 = 2000 \text{ g}$. (b) When $m_1 = 2000 \text{ g}$, $m_2 = 2000 \text{ g}$.

From the results shown in this section, it is obvious that a larger mass of hot water is better to maintain a larger temperature difference. The larger temperature difference is needed for TEG to generate larger electrical power. However, the larger mass of hot water causes a loss of energy during the heat transfer due to the natural heat convection in hot water in the tank. A different mechanism or system design is necessary to more effectively use the larger mass of hot water.

4.3. Thermal and Electrical Efficiency (Daytime and Nighttime Operation)

In this section, the results when $Q_h = 14.84 \text{ W}$, 21.17 W (daytime) and $m_1 = 1000 \text{ g}$, 2000 g (nighttime) are shown. Here, both the thermal and electrical efficiency are not shown separately by the mass of cold water since it has no significant effect on the results.

During the daytime, the thermal efficiency is calculated by Equation (17), and the electrical efficiency is calculated by Equation (18). When $Q_h = 14.84 \text{ W}$, the electrical and thermal efficiency of the system becomes 0.680% and 65.7%, respectively. When $Q_h = 21.17 \text{ W}$, the electrical and thermal efficiency of the system becomes 0.994% and 65.7%, respectively.

During the nighttime, the thermal efficiency is calculated by Equation (36), and the electrical efficiency is calculated by Equation (37). Since the temperature difference and the voltage in the system decrease over time, the thermal and electrical efficiency vary over time so that they are shown as time averaged values. When $m_1 = 1000$ g, the time averaged values of thermal and electrical efficiency for the hot water temperature change from 80 °C to 40 °C and become 0.574% and 49.2%, respectively. The average power generated by TEG becomes 0.090 W. When $m_1 = 2000$ g, the time averaged values of thermal and electrical efficiency for the hot water temperature change from 80 °C to 50 °C and become 0.499% and 40.0%, respectively. The average power generated by TEG becomes 0.116 W.

From the time averaged values of thermal efficiency during the nighttime operation shown in this section, it is observed that the system has a large amount of heat loss. From the calculation, it is confirmed that the majority of the heat loss is from the water (Q_{w1loss} and Q_{w2loss}). Therefore, the water tank needs countermeasures, such as greater insulation to reduce the heat loss from water. From the results, it is also observed that the time averaged values of thermal and electrical efficiency become lower when the mass of hot water becomes larger. This can be accounted for by the same reason as mentioned in Section 4.2. A larger mass of hot water causes increased heat convection in the water tank.

4.4. Comparison with Ashwin's Model

In this section, the thermal and electrical efficiency of each system are compared. The result of the comparison is shown in Table 1. Here, the thermal and electrical efficiency of the proposed system at nighttime are the time averaged values for the hot water temperature change from 80 °C to 50 °C (when $m_1 = 2000$ g) or from 80 °C to 40 °C ($m_1 = 1000$ g).

Table 1. Comparison of the results.

Results	Proposed System	Ashwin's Model [9]
Electrical efficiency (Daytime)	0.680% (14.84 W)	2.1% (60 W)
	0.994% (21.17 W)	1.94% (180 W)
Thermal efficiency (Daytime)	65.7% (14.84 W)	57.6% (20 W)
	65.7% (21.17 W)	55.0% (40 W)
Electrical efficiency (Nighttime)	0.574% ($m_1 = 1000$ g)	-
	0.499% ($m_1 = 2000$ g)	-
Thermal efficiency (Nighttime)	49.2% ($m_1 = 1000$ g)	-
	40.0% ($m_1 = 2000$ g)	-
Overall efficiency	33.2%	59.5%
	(21.17 W, $m = 1000$ g)	(20 W, $m = 450$ g)

In the above table, it is observed that Ashwin's model has a higher electrical efficiency than the proposed system during the daytime. However, the input power is totally different in the comparison to the electrical efficiency. Therefore, a system which has the same mechanism as Ashwin's model was built in the study for comparison. The same experiment was conducted, and the results show that the electrical efficiency of such a system becomes 1.29% when $Q_h = 14.84$ W and 1.87% when $Q_h = 21.17$ W, respectively. From the results, it is observed that the mechanism of Ashwin's model has more advantages than the mechanism of the proposed system to generate electricity during the daytime. In terms of thermal efficiency during the daytime, it is observed that the proposed system has higher thermal efficiency than Ashwin's model. It can be considered that the heat loss in the proposed system is smaller than the heat loss in Ashwin's model. Since Ashwin's model does not store heat using water for nighttime operation, the thermal and electrical efficiency of the proposed system during the nighttime are only shown in Table 1. In terms of electrical efficiency during the nighttime, it is observed that the value of the proposed system is lower than the value in the daytime. One of the main reasons for this is the decrease in electrical efficiency over time. It is necessary to make the temperature decrease much more slowly using more thermal insulation or to make the temperature difference constant in order to make the electrical efficiency much higher. The latter could be accomplished by using hot spring water, which is readily available in Japan in selected areas, or by using the phase

change materials as latent heat storage mediums. In terms of thermal efficiency during the nighttime, it is observed that the value of the proposed system is lower when compared to the value during the daytime. The heat loss must be reduced in order to make the value of thermal efficiency higher.

Not only each efficiency, but also the overall efficiency, are compared. Here, the input power of 21.17 W and the mass of water of 1000 g are applied in the proposed system. On the other hand, the input power of 20 W and the mass of water of 450 g are applied in Ashwin's model. Since the electrical efficiency during the daytime related to the input power of 20 W is not mentioned in Ashwin's model, the electrical efficiency of the system, which was built in the study and has the same mechanism as Ashwin's model (1.87%), is used as the overall efficiency. From this comparison, it is observed that the overall efficiency of Ashwin's model is about two times higher than the overall efficiency of the proposed system. In order to make the overall efficiency higher than the Ashwin's model, both the daytime and nighttime efficiency should be about 80%.

5. Case Study

This section presents a case study of the proposed system for the standard house in Tokyo, Japan for its domestic hot water needs and part of its electric demand. The case study will consider the daily household hot water demand, as well as the electricity demand and the size of the system to satisfy these demands.

Before sizing the system, the case study of the proposed lab-scale system is presented. Figure 13 shows the hot and cold side water temperature change, ambient temperature change, and the change of solar radiation in a day. In this case study, the date is set to April 1, a sunny day. The data of the solar radiation and the ambient temperature was taken from NEDO's solar radiation databases [21]. Here, it is assumed that the solar radiation and the ambient temperature are constant for every one hour. At each interval, the average values are applied. The initial hot side water temperature is set to 17.6 °C. This value is the average tap water temperature in Tokyo in April [22]. Here, the hot side water is the water heated by the solar light in the daytime.

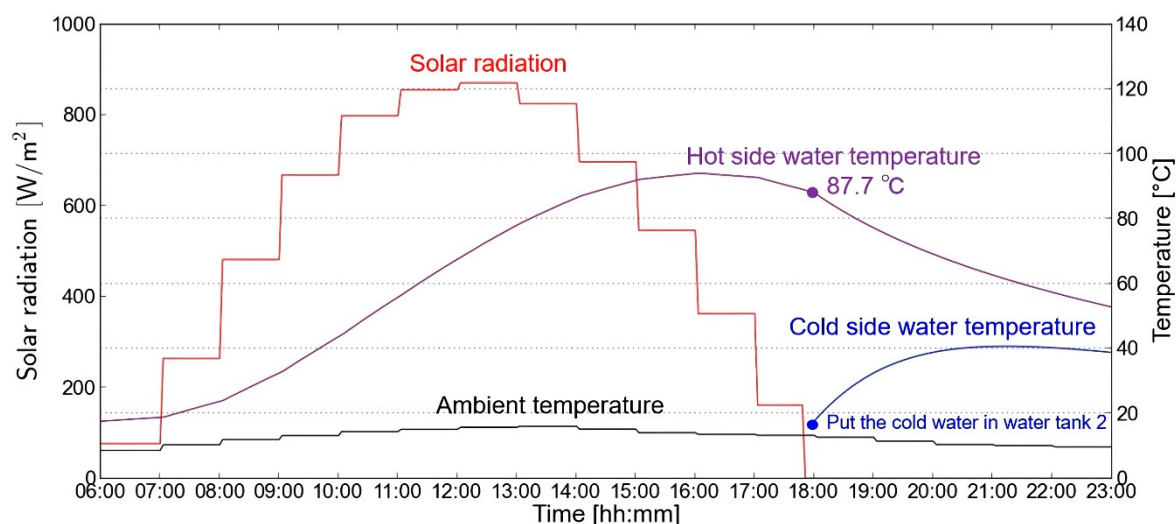


Figure 13. Predicted transient behavior of hot and cold side water temperature in a day (1 April, case study).

In the case study, the time range is set from 6:00 to 23:00. The mass of hot and cold side water is set to 3000 g and 900 g, respectively. The size of the Fresnel lens is changed from 250 × 250 mm to 350 × 350 mm in the simulation in order to heat the hot side water more than 80 °C in the daytime operation. The sun will go down at 17:48, and the cold water will be put in water tank 2 at 18:00. In other words, the nighttime operation will start from 18:00 in this case study. The initial cold side water temperature is set to 17.6 °C. From the figure, it is observed that the solar radiation becomes

the highest at noon. The hot side water temperature will exceed $80\text{ }^{\circ}\text{C}$ at about 13:15. The hot side water temperature at 18:00 is $87.7\text{ }^{\circ}\text{C}$, and this hot water will transfer the heat to the cold water at the nighttime operation. It can be seen from the figure that the cold side water temperature will exceed $38.0\text{ }^{\circ}\text{C}$ at 19:48. This means that the cold side water will be ready to use as warm water for showering or the other applications at 19:48 PM. If the warm water is needed before 19:48, the system can provide it by using the part of hot and cold side water. We can mix the hot side water and cold side water to make the warm water and supply it. The remaining warm water can be used for the next morning. If all of the cold side water is used, the new cold side water can be supplied in water tank 2 to make the temperature difference. The TEG will generate electricity as long as there is a temperature difference in the system. In the daytime operation, the TEG can generate 2.98 W for 6 h (from 9:00 to 15:00). This value is an average value for 6 hours, and that will be equivalent to 17.88 Wh . In the nighttime operation, the TEG can generate 0.22 W for 3 h (from 18:00 to 21:00). This value is an average value for 3 h when no water is taken out from the tank, and that will be equivalent to 0.65 Wh .

The sizing of the system is conducted based on the result of the case study. The total daily hot water consumption and the total daily electricity usage in the typical house in Tokyo are 400 L [23] and 11.2 kWh [24], respectively. The available roof surface area (non-sloped) for the system is 6.34 m^2 [25,26]. In this available roof surface area, we can utilize the solar collector equivalent of about 51 Fresnel lenses at most, which means that the system equivalent of about 51 lab-scale systems can be used. Here, not only one, but multiple Fresnel lenses with cumulative collector aperture area of 6.34 m^2 will be used to make the structural design simple and flat [9].

Figure 14 shows the image of the scaled-up system, which is set up on the non-sloped roof. In the scaling up process, water tank 1 and 2 will be scaled up. The heat pipes will be connected with the water tank 1 and 2 as with the lab-scale system, and they will be placed in parallel with each other, as shown in Figure 14. Moreover, the aluminum plate will be attached to the heat pipes in water tank 1 in order to connect them with each other. The solar light will be collected on this aluminum plate by utilizing the Fresnel lenses. The TEGs will be placed between the two heat pipes as with the lab-scale system, and each TEG will be placed in parallel as shown in Figure 14.

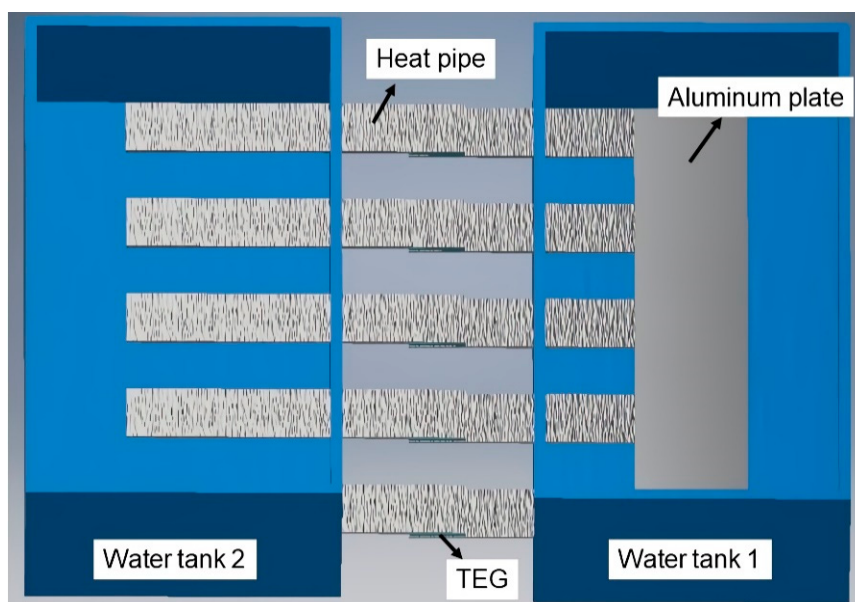


Figure 14. Image of the scaled-up system which is set up on the non-sloped roof (Fresnel lenses are used to collect the solar light to the aluminum plate).

The scaled-up system is equivalent to 51 lab-scale systems. Therefore, we can assume that the total amount of hot and cold side water and the total electricity generation of the scaled-up system in

the case study will become 51 times larger. The total amount of hot and cold side water will become 153 L and 45.9 L, respectively. As mentioned in the case study of the lab-scale system, both the hot side water and the cold side water can be used to provide warm water. In other words, 198.9 L of warm water can be provided by using this scaled-up system. The total electricity generation in the daytime and the nighttime operation will become 152 W for 6 h and 11.1 W for 3 h, respectively. Those will be equivalent to 0.912 kWh and 0.0332 kWh.

From the case study and sizing, it is proven that the scaled-up system has the potential to provide 198.9 L of warm water per day. Moreover, it is proven that the system has the potential to provide 0.912 kWh of electricity in the daytime and 0.0332 kWh of electricity at the nighttime. However, when compared to the total demand (400 L, 11.2 kWh), the warm water and electricity are not enough, and the system design/construction needs to be improved. In order to achieve this, the system needs to reduce heat loss and to maintain the hot side temperature almost constant at night.

6. Conclusions

The all-day energy harvesting power system utilizing the TEG with water-based heat storage has been studied through simulation and tested experimentally. From the study, it has been proven that the proposed system can generate electricity not only during the daytime, but also at nighttime by utilizing the stored heat. Moreover, it has been proven that the system can produce warm water after using the stored heat for generating electricity. From the case study, it has been proven that the scaled-up system has the potential to provide 198.9 L of warm water per day, 0.912 kWh of electricity in the daytime, and 0.0332 kWh of electricity at the nighttime for a typical house with 6.34 m² of available surface area in Tokyo. Although the overall efficiency is less than Ashwin's system and the electric power at night is low, this novel lab-scale system shows the potential to be a viable source of electricity and warm water throughout the day without emitting any greenhouse gas. In order to make the system more attractive and satisfy the potential consumers in the household more, the system needs to reduce a large amount of heat loss and make the overall efficiency higher. During the daytime, the heat loss from the flat aluminum heat pipe is significant. In order to make the thermal and electrical efficiency much higher and to raise the water temperature to more than 70 °C, different mechanisms for capturing the solar light should be considered. During the nighttime, the heat loss from the water is significant. Moreover, the TEG's power and the heat flow decreases over time due to the decrease in temperature difference in the system. Since these are causing the low thermal and electrical efficiency, the system needs to reduce the heat loss from water and to make the temperature difference constant. The latter could be accomplished by using the phase change materials as latent heat storage mediums, and this is considered as a future plan to increase the system's efficiency. The shape-stabilized phase change materials [27,28] will be used to store and release the heat without any metallic capsule, and they will be used with water [29,30].

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Nomenclature

$\eta_{\text{electrical, daytime}}$	= electrical efficiency of the system, daytime
$\eta_{\text{electrical, nighttime}}$	= electrical efficiency of the system, nighttime
η_{lens}	= optical efficiency of the lens
$\eta_{\text{thermal, daytime}}$	= thermal efficiency of the system, daytime
$\eta_{\text{thermal, nighttime}}$	= thermal efficiency of the system, nighttime
ε_{ab}	= absorptivity of the black absorbing film
σ	= Stefan–Boltzmann constant ($\text{W/m}^2 \text{K}^4$)
A_c	= gross area of the black absorbing plate (m^2)
A_{lens}	= area of the lens (m^2)
c_p	= specific heat of water (J/g K)
Gr	= Grashof Number
h_{nat}	= Natural convection heat transfer coefficient ($\text{W/m}^2 \text{K}$)
h_r	= Radiation heat transfer coefficient ($\text{W/m}^2 \text{K}$)
h_{wcon}	= rate of heat loss from the water storage tank per unit temperature difference, daytime (W/K)
h_{wcon1}	= rate of heat loss from the water storage tank (hot water) per unit temperature difference, nighttime (W/K)
h_{wcon2}	= rate of heat loss from the water storage tank (cold water) per unit temperature difference, nighttime (W/K)
k	= thermal conductivity of air (W/m K)
L_c	= length of the black absorbing plate (m)
m	= mass of water in the storage tank, daytime (g)
m_1	= mass of hot water in the storage tank, nighttime (g)
m_2	= mass of cold water in the storage tank, nighttime (g)
N_{TEG}	= number of TEG
Nu	= Nusselt number
Pr	= Prandtl Number
Q_h	= concentrated solar radiation on the target (W)
Q_{in}	= solar radiation flux incident of the Fresnel lens (W)
Q_{lrc}	= radiation and convection heat loss rate from the black absorbing film (W)
Q_{w1loss}	= heat loss rate from the hot water in the storage tank, nighttime (W)
Q_{w1}	= heat output from the hot water in the storage tank, nighttime (W)
Q_{w2loss}	= heat loss rate from the cold water in the storage tank, nighttime (W)
Q_{w2}	= heat input to the cold water in the storage tank, nighttime (W)
Q_{wloss}	= heat loss rate from the water storage tank, daytime (W)
Q_w	= heat input to the cold water in the storage tank, daytime (W)
r	= internal resistance of TEG (Ω)
R	= total thermal resistance (K/W)
Ra	= Rayleigh number
R_{hp}	= thermal resistance of heat pipe (K/W)
R_{tape}	= thermal resistance of thermally conductive tape (K/W)
R_{TEG}	= thermal resistance of TEG (K/W)
t	= time (s)
T_a	= ambient temperature (K)
T_{i1}	= initial temperature of hot water in the storage tank, nighttime (K)
T_{i2}	= initial temperature of cold water in the storage tank, nighttime (K)
T_i	= initial temperature of water in the storage tank, daytime (K)
T_p	= temperature of the black absorbing plate (K)
T_{w1}	= temperature of hot water in the storage tank, nighttime (K)
T_{w2}	= temperature of cold water in the storage tank, nighttime (K)
T_w	= temperature of water in the tank, daytime (K)
U_t	= top heat loss factor ($\text{W/m}^2 \text{K}$)
V_0	= open-circuit voltage of TEG (V)
W_{TEG}	= power generated by TEG (W)

Appendix A Derivation of Equation of T_{w1} and T_{w2} (Nighttime)

Firstly, the equation of T_{w1} is derived. The solution of the homogeneous part of the Equation (30) will be as follows,

$$T_{w1h} = C_1 + C_2 e^{-\frac{\beta}{\alpha} t} \quad (A1)$$

with a particular solution of the non-homogeneous part of the equation, which will be as follows,

$$T_{w1p} = -\frac{Q_{wloss}}{\beta} t \quad (A2)$$

A general solution of the non-homogeneous equation is given by the sum of the solution for the homogeneous equation and the particular solution for the non-homogeneous equation. Therefore, the general solution is expressed as follows,

$$T_{w1} = T_{w1h} + T_{w1p} = C_1 + C_2 \exp\left(-\frac{\beta}{\alpha} t\right) - \frac{Q_{wloss}}{\beta} t \quad (A3)$$

Here, C_1 and C_2 are the constants. Initial conditions shown as below are used to determine the value of C_1 and C_2 .

$$\begin{cases} T_{w1} = T_{i1} \\ \frac{dT_{w1}}{dt} = -\frac{T_{i1} - T_{i2} + Rh_{wcon1}(T_{i1} - T_a)}{m_1 c_p R} \quad (\text{When } t = 0) \end{cases} \quad (A4)$$

Here, T_{i1} is the initial temperature of hot water in the storage tank, T_{i2} is the initial temperature of cold water in the storage tank. The second initial condition can be derived by using the equation of the thermal resistance circuit (Equation (25)). From these initial conditions, the value of C_1 and C_2 can be determined as follows,

$$C_1 = T_{i1} + \frac{\alpha}{\beta} \left(K_1 + \frac{Q_{wloss}}{\beta} \right) \quad (A5)$$

$$C_2 = -\frac{\alpha}{\beta} \left(K_1 + \frac{Q_{wloss}}{\beta} \right) \quad (A6)$$

where K_1 is,

$$K_1 = -\frac{T_{i1} - T_{i2} + Rh_{wcon1}(T_{i1} - T_a)}{m_1 c_p R} \quad (A7)$$

Substituting Equation (A5) and Equation (A6) into Equation (A3),

$$T_{w1} = T_{i1} + \frac{\alpha}{\beta} \left\{ 1 - \exp\left(-\frac{\beta}{\alpha} t\right) \right\} \left(K_1 + \frac{Q_{wloss}}{\beta} \right) - \frac{Q_{wloss}}{\beta} t \quad (A8)$$

Next, the equation of T_{w2} is derived. Substituting Equation (21) and Equation (27) into Equation (25),

$$T_{w2} = T_{w1} - R \left\{ -m_1 c_p \frac{dT_{w1}}{dt} - h_{wcon1} \cdot (T_{w1} - T_a) \right\} \quad (A9)$$

Substituting Equation (A3) into the above equation,

$$T_{w2} = D_1 (1 + Rh_{wcon1}) + D_2 \left(1 + Rh_{wcon1} - \frac{\beta}{\alpha} m_1 c_p R \right) \exp\left(-\frac{\beta}{\alpha} t\right) - (1 + Rh_{wcon1}) \frac{Q_{wloss}}{\beta} t - m_1 c_p R \frac{Q_{wloss}}{\beta} - Rh_{wcon1} T_a \quad (A10)$$

Here, constants in the equation of T_{w2} are different from the constants in the equation of T_{w1} , so the constants C_1 and C_2 in the equation of T_{w1} are changed to D_1 and D_2 , respectively. Differentiation of T_{w2} can be expressed as follows,

$$\frac{dT_{w2}}{dt} = -\frac{\beta}{\alpha} D_2 \left(1 + Rh_{wcon1} - \frac{\beta}{\alpha} m_1 c_p R \right) \exp\left(-\frac{\beta}{\alpha} t\right) - (1 + Rh_{wcon1}) \frac{Q_{wloss}}{\beta} \quad (A11)$$

Initial conditions of T_{w2} and $\frac{dT_{w2}}{dt}$ are expressed as follows,

$$\begin{cases} T_{w2} = T_{i2} \\ \frac{dT_{w2}}{dt} = \frac{T_{i1} - T_{i2} - Rh_{wcon2}(T_{i2} - T_a)}{m_2 c_p R} \quad (\text{When } t = 0) \end{cases} \quad (A12)$$

Here, when $T_{i2} < T_a$, $h_{wcon2} = 0$ in the second initial condition. From these initial conditions, the value of D_1 and D_2 can be determined as follows,

$$D_1 = \frac{T_{i2} + m_1 c_p R \frac{Q_{wloss}}{\beta} - \frac{C_2}{G_1} \left(1 - \frac{\beta}{\alpha} m_1 c_p R \right) - Rh_{wcon1} \left(\frac{G_2}{G_1} - T_a \right)}{(1 + Rh_{wcon1})} \quad (A13)$$

$$D_2 = \frac{G_2}{G_1} \quad (\text{A14})$$

where G_1 and G_2 are,

$$G_1 = \left(\frac{\beta^2}{\alpha^2} \right) m_1 c_p R - \frac{\beta}{\alpha} - \frac{\beta}{\alpha} Rh_{wcon1} \quad (\text{A15})$$

$$G_2 = \frac{T_{i1} - T_{i2} - Rh_{wcon2}(T_{i2} - T_a)}{m_2 c_p R} + \frac{Q_{wloss}}{\beta} + \frac{Q_{wloss}}{\beta} Rh_{wcon1} \quad (\text{A16})$$

Substituting Equation (A13) and Equation (A14) into Equation (A10),

$$T_{w2} = T_{i2} - \frac{G_2}{G_1} \left(1 + Rh_{wcon1} - \frac{\beta}{\alpha} m_1 c_p R \right) \left(1 - \exp\left(-\frac{\beta}{\alpha} t\right) \right) - (1 + Rh_{wcon1}) \frac{Q_{wloss}}{\beta} t \quad (\text{A17})$$

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