

Steps of the fuzzy AHP approach and calculation of criteria weights

I. The steps of the fuzzy AHP approach

Step 1: Pair-wise comparisons of criteria

Based on the hierarchy structure developed in the semi-structured interviews and modified Delphi technique, a questionnaire was formulated and participants made pair-wise comparisons of the importance between each pair of criteria. Consider a hierarchy with n criteria, a total of $n(n-1)/2$ pair-wise comparisons should be made. The comparison is in the form of linguistic variables. A linguistic variable is a variable whose values are words or sentences in a natural or artificial language (Zadeh, 1975). In this paper, the linguistic variables are expressed as 'equally important', 'weakly important', 'strongly important', 'very strongly important', 'absolutely important'.

Step 2: Establishing comparison matrices

The comparison matrix for each expert was established. The linguistic variables, obtained from experts, were converted into a triangular fuzzy number, as shown in Table S1 (Kahraman et al., 2006). Illustratively, as for the k th expert, he/she considers criterion i to be very strongly important compared to criterion j ; he/she may set $\tilde{a}_{ij} = (6,7,8)$. The comparison matrix for the k th expert is $\tilde{A} = [\tilde{a}_{ij}]$, which is represented as:

$$\tilde{A}^K = \begin{bmatrix} 1 & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & & \tilde{a}_{2n} \\ \vdots & & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \dots & 1 \end{bmatrix} \quad (1)$$

The matrix has reciprocal properties, which are

$$a_{ij} = \frac{1}{a_{ji}}, \forall i, j = 1, 2, 3 \dots n$$

Step 3: Calculating the consistency index and the consistency ratio of the comparison matrix

To assure the consistent judgments of experts, the consistency of an evaluation was analyzed. Saaty (1980) proposed a consistency index (CI) to measure consistency of a comparison matrix. It is computed with the following equation:

$$CI = \frac{\lambda_{\max} - n}{n-1} \quad (2)$$

where λ_{\max} is the largest eigenvalue of the comparison matrix and n is the dimension of the matrix.

The consistency ratio (CR) can be defined as a ratio between the consistency of a given evaluation matrix and consistency of a random matrix.

$$CR = \frac{CI}{RI} \quad (3)$$

where RI is the random consistency index that depends on n. Table S1 shows the value of RI from matrices of order 1 to 10 as suggested by Saaty (1980). If CR is equal or less than 0.1, then the comparison is acceptable. When CR is greater 0.1, the value is indicative of inconsistent judgment. In such case, the expert is encouraged to reconsider and revise the original values in the pair-wise comparisons (Saaty, 1980). In this step, the MATLAB package can be used to calculate the eigenvalue of all comparison matrices.

Step 4: Aggregating experts' opinions and establishing a group judgment matrix.

Each individual judgment matrix represents the opinion of one expert. The geometric mean method was employed to aggregate the experts' opinions (Buckley, 1985). The element in the group judgment matrix is represented as:

$$a_{ij}^{gm} = \sqrt[k]{\prod_{k=1}^K a_{ij}^k} \quad (4)$$

where a_{ij}^k is an element of matrix \tilde{A} of an individual expert k (k=1, 2, ..., K) and a_{ij}^{gm} is the geometric mean of all experts a_{ij}^k .

Step5: Calculating the local weight of each criterion

Suppose that there is a comparison matrix at the j level, the fuzzy weight of each criterion at the j level is calculated as:

$$w_i = \frac{(\prod_{j=1}^n a_{ij})^{\frac{1}{n}}}{\sum (\prod_{j=1}^n a_{ij})^{\frac{1}{n}}} \text{ for } i, j = 1, 2, \dots, n \quad (5)$$

Suppose $w_i = [a, b, c]$, then the weight of each criterion at a specific level by employing the defuzzification procedure proposed by Lee and Li (1998) is presented as:

$$DF_i = \frac{a+b+c}{3} \quad (6)$$

Finally, the weight of each criterion was normalized as follows:

$$NW_i = \frac{DF_i}{\sum_{i=1}^n DF_i} \quad (7)$$

Step 6: Calculating the global weight of each attribute

When the local weights at different levels are obtained, the global weight of the attribute is computed by multiplying the local weight of the attribute with the local weight of the criterion to which it belongs.

II. Calculation of weights of aspects and their associated items

To acquire the weights of aspects and items, the questionnaires were delivered to the participants. As seen in Appendix, the questionnaire consisted of two parts: Part 1 included the pair-wise comparisons between aspects; Part 2 focused on the pair-wise comparisons between items under each aspect. For example, participants were asked: with respect to 'safe angling sites' or 'resource use', which is more important? The verbal judgment from equal importance to absolute importance was then translated into the corresponding number in the relative importance scale (as seen in Table S2).

By employing Equation (1), the fuzzy judgment matrices were formed based on the obtained pair-wise data comparisons. To assure the consistent judgments of experts, the consistency ratio of each matrix was calculated by employing equations (2) and (3). All CR values are lower than 0.1, indicating all the judgments are consistent (see below Table S3). Subsequently, following step 4, 5 and 6, the weights of four dimensions and their associated items were calculated.

The global weight of the item was then calculated by multiplying the local weight of the item with the local weight of the dimension to which it belongs. As an illustration, the procedure in calculating the weights of three items under the dimension of angling sites was presented.

Suppose the comparison matrix of one participant is:

$$A = \begin{vmatrix} 1 & 5 & 2 \\ 1/5 & 1 & 1/3 \\ 1/2 & 3 & 1 \end{vmatrix}$$

According to Table S2 and employing equation (2), this matrix is transferred as:

$$\tilde{A} = \begin{vmatrix} (1,1,1) & (4,5,6) & (1,2,3) \\ (\frac{1}{6}, \frac{1}{5}, \frac{1}{4}) & (1,1,1) & (\frac{1}{4}, \frac{1}{3}, \frac{1}{2}) \\ (\frac{1}{3}, \frac{1}{2}, 1) & (2,3,4) & (1,1,1) \end{vmatrix}$$

By employing equation (4), the geometric mean was applied to get the representative comparison matrix of the group (consisting of 28 respondents):

$$\tilde{\tilde{A}} = \begin{vmatrix} (1,1,1) & (1.100, 1.386, 2.015) & (0.670, 0.862, 1.304) \\ (0.496, 0.721, 0.909) & (1,1,1) & (0.489, 0.643, 1.002) \\ (0.767, 1.160, 1.492) & (0.998, 1.556, 2.046) & (1,1,1) \end{vmatrix}$$

By employing equation (5), the fuzzy weight of each item pertaining to the dimension of angling sites was determined in the following:

First, we calculated the geometric of triangular fuzzy number of each item.

$$\text{Let } \tilde{r}_i = (\tilde{a}_{i1} \otimes \tilde{a}_{i2} \otimes \dots \otimes \tilde{a}_{in})^{\frac{1}{n}}$$

$$\tilde{r}_1 = (\tilde{a}_{11} \otimes \tilde{a}_{12} \otimes \tilde{a}_{13})^{1/3} = \left((1 * 1.100 * 0.670)^{1/3}, (1 * 1.386 * 0.862)^{1/3}, (1 * 2.015 * 1.304)^{1/3} \right) = (0.903, 1.061, 1.380)$$

Similarly, we obtained \tilde{r}_2 and \tilde{r}_3 .

$$\tilde{r}_2 = (0.624, 0.774, 0.969)$$

$$\tilde{r}_3 = (0.915, 1.218, 1.451)$$

$$\text{Then, we obtained } (\tilde{r}_1 + \tilde{r}_2 + \tilde{r}_3)^{-1} = (0.263, 0.328, 0.410).$$

Subsequently, the fuzzy weight of each item within the *angling site* is:

$$\tilde{w}_1 = \tilde{r}_1(\tilde{r}_1 \oplus \tilde{r}_2 \oplus \tilde{r}_3)^{-1} = (0.238, 0.348, 0.566)$$

$$\tilde{w}_2 = \tilde{r}_2(\tilde{r}_1 \oplus \tilde{r}_2 \oplus \tilde{r}_3)^{-1} = (0.164, 0.254, 0.397)$$

$$\tilde{w}_3 = \tilde{r}_3(\tilde{r}_1 \oplus \tilde{r}_2 \oplus \tilde{r}_3)^{-1} = (0.241, 0.399, 0.595)$$

By employing equation (5) and (7) to defuzzify the fuzzy weight and then normalize the weight, the weights of the three items are 0.356, 0.256 and 0.388, respectively.

Similarly, the weights of the remaining three dimensions and their associated items were calculated. The global weight of the item was then calculated by multiplying its local weight with the local weight of the dimension to which it belongs.

Table S1 Random inconsistency indices (RI).

N	1	2	3	4	5	6	7	8	9	10
RI	0.00	0.00	0.58	0.9	1.12	1.24	1.32	1.41	1.46	1.49

Source: Saaty (1980).

Table S2 Linguistic scales and fuzzy scales for importance.

Linguistic scales for importance	Quantitative value	Triangular fuzzy scales (l, m, u) ^a
Equal important (EI)	1	(1,1,2)
Intermediate between the two judgments	2	(1,2,3)
Weakly important (WI)	3	(2,3,4)
Intermediate between the two judgments	4	(3,4,5)
Strongly important (SI)	5	(4,5,6)

Intermediate between the two judgments	6	(5,6,7)
Very strongly important (VSI)	7	(6,7,8)
Intermediate between the two judgments	8	(7,8,9)
Absolute important (AI)	9	(8,9,9)

^a (l, m, u) denotes a triangular fuzzy number.

Source: Kahraman et al. (2006).

Table S3 Consistency tests for items relating to angling management.

Level	Consistence ratio	Consistence test
Goal	0.035	Accepted
Dimension		
Angling sites	0.035	Accepted
Resource use	0.047	Accepted
Eligibility of angling	0.052	Accepted
Education	0.076	Accepted

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