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Coordination of Retailer-Led Closed Loop Supply Chain Considering Corporate Social Responsibility

Huanhuan Zhao [1], Yong Liu [2,*] and Zhiyang Liu [2]

1 School of Management, Wuxi Institute of Technology, Wuxi 214121, China
2 School of Business, Jiangnan University, Wuxi 214122, China
* Correspondence: liuy@jiangnan.edu.cn; Tel.: +86-150-6188-9051

Abstract: As resource waste and environmental degradation become increasingly serious, closed-loop supply chains have attracted more and more attention from the public. How to effectively undertake corporate social responsibility (CSR) and alleviate the conflict of closed-loop supply chains has become an urgent task for managers and researchers to resolve. With respect to the coordination problems of closed-loop supply chains led by retailers, based on Stackelberg game theory, we establish some models considering manufacturer's CSR, exploit them to compare the optimal decisions under centralized and decentralized decisions, explore the impact of CSR on supply-chain decisions, then design a coordination mechanism through two-step pricing.

Keywords: closed-loop supply chain; corporate social responsibility (CSR); two-step pricing; coordination mechanism

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Citation: Zhao, H.; Liu, Y.; Liu, Z. Coordination of Retailer-Led Closed Loop Supply Chain Considering Corporate Social Responsibility. *Sustainability* **2022**, *14*, 10851. https://doi.org/10.3390/su141710851

Academic Editors: Huixiang Zeng, Zhifang Zhou, Zongfeng Xiu and Youliang Jin

Received: 23 July 2022
Accepted: 16 August 2022
Published: 31 August 2022

1. Introduction

With the rapid development of the economy, problems such as the large amount of greenhouse gas emissions, frequent haze, environmental pollution and resource waste are becoming increasingly serious; more and more enterprises and consumers have begun to pay attention to CSR management. Undertaking CSR can help enterprises win a good social image and sustainable and better development. In a closed-loop supply chain, if the enterprises do not undertake CSR, then the supply chain will face a decline in consumers, and damage to the company's reputation and business operations. Meanwhile, there exists a variety of pricing conflict in a closed-loop supply chain. To deal with the decision and conflict problems of closed-loop supply chains considering corporate social responsibility, we exploit the Stackelberg game to analyze the optimal decisions and explore the impact of CSR on supply-chain decisions, and then we design a coordination mechanism.

The rest of this paper is organized as follows: Section 2 summarizes the related literature. In Section 3, we establish some basic models considering CSR, and exploit them to analyze the optimal decisions under decentralized and centralized decisions, and then we design a pricing coordination mechanism; Section 4 gives a number analysis, and in Section 5 conclusions are given.

2. Literature Review

Faced with increasingly serious environmental pollution, the wasting of resources and other problems, more and more researchers have been paying attention to CSR management problems. The early studies tended to analyze CSR from the empirical point of view. Becker-Olsen et al. analyzed the impact of CSR on consumers' purchasing motivation and behavior [1]. Korschun et al. used empirical methods to analyze the impact of CSR on the work efficiency of front-line employees [2]. Huang and Chen studied the relationship between foreign investment, supply-chain pressure and CSR in China [3]. In recent years, frequent CSR failure events have led to the continuous emergence of various social problems. With this background, people have gradually realized that CSR management is not only the behavior of a single enterprise, but also a manifestation of the relationship between the upstream and downstream enterprises in the supply chain that influence and restrict each other. Some scholars have subsequently discussed CSR management in the supply chain.

Regarding traditional supply-chain management, many researchers have discussed how to use contracts to improve the efficiency of supply chains. Ni and Li et al. analyzed the internal relationship between the CSR investment behaviors of upstream and downstream enterprises in a two-level supply chain and discussed the impact of CSR investment behaviors on supply-chain performance [4]. Liang et al. analyzed the competition strategy of a dual-channel supply chain in the context of product differentiation, and pointed out that retailers' CSR behavior would lead to decreases in the degree of market competition [5]. Ma Peng et al. studied the CSR coordination problem of a two-level supply chain under the condition of asymmetric cost information, proving that two billing contracts could achieve the coordination of supply chains with symmetric and asymmetric information, respectively, and improve the CSR performance of the supply chain [6]. On the other hand, closed-loop supply-chain management is fully in line with the basic needs of protecting the environment and building a resource-conserving society, which has attracted widespread attention in recent years. Scholars have studied the selection of recycling channels [7–9], pricing and coordination decisions [10–15] and whether to conduct remanufacturing [16–18].

With respect to the problems of the closed-loop supply chain with CSR, some scholars have been analyzing their decisions and effects. Among these, Panda et al. analyzed the impact of the manufacturer's fulfillment of CSR for product recycling and remanufacturing in a closed-loop supply chain, and introduced revenue-sharing contracts to encourage retailers to recycle waste products and achieve channel coordination in the closed-loop supply chain [18]. Gao et al. explored the optimal pricing and CSR-level decisions of a closed-loop supply chain when manufacturers and retailers undertake CSR at the same time, and proposed two contracts, revenue sharing and two-part pricing, to coordinate the decentralized closed-loop supply chain [19]. Zheng get al. discussed the game equilibrium and two-part pricing contract design of a closed-loop supply chain under four situations: manufacturer fulfilling CSR, retailer fulfilling CSR, both parties failing to fulfill CSR and both parties fulfilling CSR at the same time [20]. Shu et al. considered the government's carbon-emission reward and punishment mechanism and advertising strategy based on the analysis of CSR investment decisions [21]. In order to explore how to improve the quality of recycled products to dispel buyers' doubts, Li and Xu built a model from the perspectives of CSR and quality, and analyzed the best way to improve the quality of recycled products [22]. Modak et al. took donations (as CSR activities) into consideration and developed the best closed solutions for three decentralized and centralized channel structures [23]. Yao and others discussed the closed-loop supply chain considering CSR from three different recycling modes, and explored the best recycling channel [24]. In addition, considering that the undertaking of CSR pays corresponding costs, three different cost-bearing modes are proposed to explore the optimal undertaking mode [25]. Liu et al. discussed the cost in the implementation of CSR, assuming a commitment to a retailer-led situation [26]. Liu et al. explored how government subsidies affect retailers' CSR level, and how the government balanced the relationship between corporate profits and social welfare by subsidizing retailers, which not only promoted enterprises to assume social responsibility, but also did no harm to social welfare [27]. Zhang et al. considered the impact of CSR on the environmental benefits of the supply chain through carbon tax policy and subsidy policy in the context of carbon regulatory policy [28]. Through the analysis of the power structure of the three channels, Wang et al. found that the profits of channel leaders in the supply chain always increase with increases in the level of CSR [29]. Similarly, Liu et al. proved retailers considering CSR can effectively improve their own profits in a closed-loop supply chain by analyzing three supply-chain investment models [30]. Mondal et al. studied how government subsidies and second-hand product mobile phone strategies affect the level of CSR, and analyzed the impact of the level of corporate social responsibility of retailers on supply-chain decision making [31].

According to the above discussion and analysis, many interesting conclusions have been found, but there are still the following shortcomings: (1) The existing literature on CSR mainly focuses on the manufacturer as the dominant player. With the development of e-commerce, retailers are becoming more and more powerful. The analysis of the retailer-led supply-chain model is conducive to reflecting the real situation. (2) Previous literature on closed-loop supply chains seldom takes the impact of CSR into consideration, but the inclusion of the impact of CSR is only consistent with reality. In order to solve the above shortcomings, by introducing the influence of CSR into a retailer-led closed-loop supply chain, we establish some models to explore and discuss the optimal decisions under centralized and decentralized decision making, and then we propose a coordination mechanism.

3. Decision Analysis and Coordination Mechanism of Supply Chain Considering CSR

In real life, with the increasing awareness of environmental protection, consumers are paying more and more attention to CSR; many enterprises have implemented the manufacturing–sales–recovery–remanufacturing closed-loop supply chain. For example, Suning, Gome and other well-known retailers have cooperated with TCL Group to recycle waste household appliances through the implementation of a closed-loop supply chain, which has not only obtained rich profits, but also achieved sustainable and better development. However, CSR input will lead to conflicts between manufacturers and retailers. Taking electronic product waste as an example in a retailer-led closed-loop supply chain, manufacturers delivered products to retailers at a wholesale price, and retailers sold products to customers at a retail price according to market conditions, and then manufacturers recycled waste products with relevant means and methods [28]. To deal with the conflicts of the supply chain, we will establish decision models of the retailer-led closed-loop supply chain considering CSR, discuss their optional decisions and the impact of CSR on manufacturers and retailers' pricing decisions, and then design a coordination mechanism in this section.

3.1. Basic Model

To construct the decision models of the retailer-led closed-loop supply chain considering CSR, some assumptions are made, as follows.

Hypothesis 1. *Suppose there exists a closed-loop supply chain consisting of a single manufacturer and a dominant retailer, which is risk neutral. In this supply chain, the manufacturer is responsible for producing new products and recycling waste for remanufacturing, and the retailer is responsible for selling products produced by manufacturers. Let w, p stand for the wholesale price of the manufacturer and the retail price of the product of the retailer, respectively.*

Hypothesis 2. *Assume that the market demand is positive. According to economic knowledge and the relevant literature of Modak [29], Panda [30] and Hsueh et al. [31], let D express the market-demand function, and which satisfies $D = a - bp$ where a, b represent the market size and price sensitivity coefficient, respectively. Let C_L, ξ, β represent the recovery scale parameter, CSR cost coefficient and the sensitivity coefficient, respectively, and $a > 0, b > 0, C_L > 0, \xi > 0, \beta > 0$, a is far greater than other parameters, and $C_L > \frac{\Delta^2 b}{2}, b > \beta$.*

Hypothesis 3. *Let $c_m, c_r, \tau, \Delta, C(\tau), C_L$express the unit cost of the new product, the unit cost of the remanufactured product, recovery rate of waste products, average production cost of manufacturers, recovery cost and scale parameter, respectively, and satisfy $0 \leq \tau \leq 1$ $\Delta = c_m - c_r, \widetilde{c} = c_m - \Delta\tau$ [20], $C(\tau) = \frac{1}{2}C_L\tau^2$, and $C_L > \frac{\Delta b\xi(a+b\Delta-bc_m)}{2(b\xi-\beta^2)}$.*

Hypothesis 4. *CSR includes employee welfare and social welfare, etc. In order to describe the manufacturer's input behavior of CSR in a closed-loop supply chain, similar to the assumptions in the existing literature [32], assume that only the manufacturer undertakes CSR. Generally speaking, the input cost of CSR is a quadratic function of CSR. Let* $e_m, C(e_m)$ *stand for the input level of CSR and input cost of CSR, respectively, and then* $C(e_m) = \frac{1}{2}\xi {e_m}^2$*. Considering investment in CSR, the original demand function will be written as* $D = a - bp + \beta e_m$*.*

Hypothesis 5. *Assume that there is only a single-cycle operation in this paper. That is, the reverse and forward processes are completed in one cycle.*

Let π_m, π_r, π, represent the profits of the manufacturer, retailer and supply chain, respectively, according to the above hypothesis and analysis; their expressions can be written as follows:

$$\pi_m = (w - \tilde{c})D - C(e_m) - C(\tau) = (w - c_m + \Delta\tau)(a - bp + \beta e_m) - \frac{1}{2}\xi {e_m}^2 - \frac{1}{2}C_L\tau^2 \quad (1)$$

$$\pi_r = (p - w)D = (p - w)(a - bp + \beta e_m) \quad (2)$$

$$\pi = [a - bp + \beta e_m](p - c_m + \Delta\tau) - \frac{1}{2}\xi {e_m}^2 - \frac{1}{2}C_L\tau^2 \quad (3)$$

For convenience, let the variable superscripts $N, c, d, *$ indicate the cases of decision without considering CSR, centralized decision, decentralized decision, and optimal decision, respectively.

3.2. Decision Analysis of Closed-Loop Supply Chain without CSR

In this section, we mainly discuss decisions in the supply chain without CSR (namely, $e_m = 0$) under the decentralized and centralized conditions, and determine their optimal pricing strategies.

3.2.1. The Optimal Pricing Strategies without CSR under Decentralized Decisions (Model A)

In the supply chain without CSR, the dominant retailer gives the retail price of a product according to its own profits, and then the manufacturer will determine wholesale price w and recovery rate τ according to the measures taken by the retailer. They follow the Stackelberg game. Under decentralized decisions, both retailer and manufacturer make relevant decisions with the goal of maximizing their own profits. In this section, let $m = p - w$ indicate sales unit profits.

Theorem 1. *In the supply chain without CSR, let* $w_N^{d*}, \tau_N^{d*}, p_N^{d*}$ *represent the optimal wholesale price, manufacturer's recovery rate under decentralized decision making and retail channel price, respectively, if there exist unique values to maximize the supply chain members, then it will hold true as follows:*

$$w_N^{d*} = \frac{1}{2}\frac{\Delta^2 b^2 c_m + \Delta^2 ab - 3bC_L c_m - aC_L}{b(\Delta^2 b - 2C_L)} \quad (4)$$

$$\tau_N^{d*} = -\frac{\Delta(a - bc_m)}{2\Delta^2 b - 4C_L} \quad (5)$$

$$p_N^{d*} = \frac{1}{2}\frac{2\Delta^2 ab - bC_L c_m - 3aC_L}{b(\Delta^2 b - 2C_L)} \quad (6)$$

Proof. By computing the second-order partial derivative of the Equation (2) with respect to m, we can obtain $\frac{\partial^2 \pi_r^N}{\partial m^2} = \frac{2C_L b}{\Delta^2 b - 2C_L} < 0$, and there exists an optimal value. Based on a Hessian matrix, the results are as follows:

$$H_{\substack{4.1.1 \\ (w,\tau)}} = \begin{vmatrix} \frac{\partial^2 \pi_m}{\partial w^2} & \frac{\partial^2 \pi_m}{\partial w \partial \tau} \\ \frac{\partial^2 \pi_m}{\partial \tau \partial w} & \frac{\partial^2 \pi_m}{\partial \tau^2} \end{vmatrix} = \begin{vmatrix} -2b & -\Delta b \\ -\Delta b & -C_L \end{vmatrix} = 2bC_L - \Delta^2 b^2 \tag{7}$$

When $2bC_L - \Delta^2 b^2 > 0$, the manufacturer will have an optimal value. By using the backward induction method and solving $\frac{\partial \pi_m^N}{\partial w} = 0$ and $\frac{\partial \pi_m^N}{\partial \tau} = 0$, we can obtain the following expressions.

$$\frac{\partial \pi_m^N}{\partial \tau} = \Delta(a - b(m + w)) - C_L \tau = 0 \tag{8}$$

$$\frac{\partial \pi_m^N}{\partial w} = a - b(m + w) - (\Delta \tau + w - c_m)b = 0 \tag{9}$$

By combining the above expressions, we can obtain the following results:

$$\tau(m) = -\frac{\Delta(a - bm - bc_m)}{\Delta^2 b - 2C_L} \tag{10}$$

$$w(m) = \frac{-\Delta^2 b^2 m + \Delta^2 ab + bmC_L - bC_L c_m - aC_L}{(\Delta^2 b - 2C_L)b} \tag{11}$$

According to the decision variables of different subjects, by substituting (10) and (11) into (2) and solving $\frac{\partial \pi_r^N}{\partial m} = 0$, we can determine the following result.

$$m = \frac{1}{2} \frac{a - bc_m}{b} \tag{12}$$

By putting Equation (12) into Equations (10) and (11), respectively, we can obtain the results of Theorem 1. □

Let $D_N^{d^*}$ express the optimal demand, by putting the above results into the original demand function; we obtain $D_N^{d^*} = -\frac{C_L(a - bc_m)}{2\Delta^2 b - 4C_L}$. Let $\pi_m^{Nd*}, \pi_r^{Nd*}, \pi^{Nd*}$ express the optimal profits of the manufacturer, retailer and closed-loop supply chain. By putting the above optimal values into the Equations (1)–(3), respectively, we can obtain their maximization profits as follows:

$$\begin{cases} \pi_m^{Nd*} = -\frac{1}{8} \frac{C_L(a - bc_m)^2}{b(\Delta^2 b - 2C_L)} \\ \pi_r^{Nd*} = -\frac{1}{4} \frac{C_L(a - bc_m)^2}{b(\Delta^2 b - 2C_L)} \\ \pi^{Nd*} = -\frac{3}{8} \frac{C_L(a - bc_m)^2}{b(\Delta^2 b - 2C_L)} \end{cases} \tag{13}$$

According to the calculation results, $\pi_m^{Nd^*} > \pi_r^{Nd^*}$, which indicates that the manufacturer gains more profit while the retailer gains less under decentralized decision making.

3.2.2. The Optimal Pricing Strategies without CSR under Centralized Decision Making (Model B)

Theorem 2. *Assume that $p_N^{c^*}, \tau_N^{c^*}$ represent the optimal retail price and recovery rate under centralized decision making in the supply chain without CSR, respectively. If there exist unique values to maximize the supply chain, then they will hold true as follows*:

$$p_N^{c*} = \frac{\Delta^2 ab - bC_L c_m - aC_L}{b(\Delta^2 b - 2C_L)} \tag{14}$$

$$\tau_N^{c*} = -\frac{\Delta(a - bc_m)}{\Delta^2 b - 2C_L} \tag{15}$$

Proof. By computing the second-order partial derivative of Equation (2) with respect to p and τ, respectively, we can determine the Hessian matrix $H_{4.1.2} = \begin{bmatrix} -2b & -\Delta b \\ -\Delta b & -C_L \end{bmatrix}$ of the supply-chain profit function. If and only if $2bC_L - \Delta^2 b^2 > 0$, namely $C_L > \frac{\Delta^2 b}{2}$, it is a negative definite matrix, thus there exists an optimal value.

By solving $\frac{\partial \pi_m^N}{\partial p} = 0$ and $\frac{\partial \pi_m^N}{\partial \tau} = 0$, we can obtain the following expressions:

$$\frac{\partial \max \pi^N}{\partial p} = -bp + a - (\Delta\tau + p - c_m)b = 0 \tag{16}$$

$$\frac{\partial \max \pi^N}{\partial \tau} = \Delta(a - bp) - C_L \tau = 0 \tag{17}$$

By combining the above two expressions, we can obtain $p_N^{c^*}, \tau_N^{c^*}$.

Theorem 2 is proved. □

According to the theorem, let D_N^{c*} stand for the optimal demand; by putting the above two optimal solutions into the demand function, we can determine its value as follows:

$$D_N^{c*} = -\frac{C_L(a - bc_m)}{\Delta^2 b - 2C_L}. \tag{18}$$

By putting Equations (14) and (15) into (3), we can obtain the optimal profits of the closed-loop supply chain without CSR under centralized decisions as follows:

$$\pi_N^{c*} = -\frac{1}{2} \frac{(b^2 c_m{}^2 - 2abc_m + a^2)C_L}{b(\Delta^2 b - 2C_L)}. \tag{19}$$

3.3. Decision Analysis of Closed-Loop Supply Chain with CSR

In this section, we mainly discuss decisions in the supply chain considering CSR under decentralized and centralized conditions, and determine their optimal decisions.

3.3.1. The Optimal Pricing Strategies with CSR under Decentralized Decision Making (Model C)

In the supply chain with CSR, the dominant retailer and manufacturer follow the Stackelberg game. Let π_m^d, π_r^d stand for the profits of the manufacturer and retailer, respectively. By putting $p = m + w$ into the manufacturer's profit function, we can obtain their expressions as follows:

$$\pi_m^d = (w - c_m + \Delta\tau)(a - b(m + w) + \beta e_m) - \frac{1}{2}\xi e_m{}^2 - \frac{1}{2}C_L \tau^2 \tag{20}$$

$$\pi_r^d = (p - w)D - C(e_r) = m[a - b(m + w) + \beta e_m]. \tag{21}$$

Theorem 3. *In the supply chain with CSR, let $w^{d^*}, e_m{}^{d^*}, \tau^{d^*}, p^{d^*}$ represent the wholesale price, CSR level under decentralized decision making, retail product recovery rate and optimal retail price, respectively. If there exist unique values to maximize the supply chain member, then they will hold true as follows*:

$$w^{d*} = \frac{[(a + bc_m)b\Delta^2 - (a + 3bc_m)C_L]\xi + 2\beta^2 C_L c_m}{(2\Delta^2 b^2 - 4bC_L)\xi + 2\beta^2 C_L} \tag{22}$$

$$e_m{}^{d*} = \frac{-(a - bc_m)\beta C_L}{(2\Delta^2 b^2 - 4bC_L)\xi + 2\beta^2 C_L} \tag{23}$$

$$\tau^{d*} = -\frac{(a-bc_m)\xi b\Delta}{(2\Delta^2 b^2 - 4bC_L)\xi + 2\beta^2 C_L} \tag{24}$$

$$p^{d*} = \frac{1}{2}\frac{2\Delta^2 ab^2\xi - b^2\xi C_L c_m + b\beta^2 C_L c_m - 3ab\xi C_L + a\beta^2 C_L}{(\Delta^2 b^2\xi - 2b\xi C_L + \beta^2 C_L)b} \tag{25}$$

Proof. By taking the second derivative of Formula (21), we can obtain $\frac{\partial^2 \pi_r^d}{\partial m^2} = -2b < 0$, that is, there exits an optimal value for the retailer. Following this, by calculating the Hessian matrix, we can determine

$$\underset{(w,e_m,\tau)}{H}{}_{4.4.2} = \begin{vmatrix} \frac{\partial^2 \pi_m}{\partial w^2} & \frac{\partial^2 \pi_m}{\partial w \partial e_m} & \frac{\partial^2 \pi_m}{\partial w \partial \tau} \\ \frac{\partial^2 \pi_m}{\partial e_m \partial w} & \frac{\partial^2 \pi_m}{\partial {e_m}^2} & \frac{\partial^2 \pi_m}{\partial e_m \partial \tau} \\ \frac{\partial^2 \pi_m}{\partial \tau \partial w} & \frac{\partial^2 \pi_m}{\partial \tau \partial e_m} & \frac{\partial^2 \pi_m}{\partial \tau^2} \end{vmatrix} = \begin{vmatrix} -b & \frac{1}{2}\beta & -\frac{1}{2}\mathrm{b}\Delta \\ \frac{1}{2}\beta & -\xi & \frac{1}{2}\beta\Delta \\ -\frac{1}{2}\mathrm{b}\Delta & \frac{1}{2}\beta\Delta & -C_L \end{vmatrix} = -b\xi C_L + \frac{1}{4}\beta^2 C_L + \frac{1}{4}\Delta^2 b^2 \xi \tag{26}$$

From the above formula, π_r^d is the joint concave function of (w, τ, e_m) with an optimal value when $\Delta^2 b^2 \xi - 4b\xi C_L + \beta^2 C_L < 0$.

According to the backward induction method, by solving $\frac{\partial \pi_m^d}{\partial w} = 0$, $\frac{\partial \pi_m^d}{\partial e_m} = 0$ and $\frac{\partial \pi_m^d}{\partial \tau} = 0$, we can obtain the following results:

$$\begin{cases} w = \frac{a-(\Delta\tau+m-c_m)b+\beta e_m}{2b} \\ e_m = \frac{(\Delta\tau+m-c_m)\beta}{\xi} \\ \tau = \frac{(a-(m+w)b+\beta e_m)\Delta}{C_L} \end{cases} \tag{27}$$

By putting the three results into Equation (21), we obtain $\pi_r^d = -\frac{mC_L\xi[a-(m+c_m)b]b}{\Delta^2 b\xi - 2b\xi C_L + \beta^2 C_L}$, then solving $\frac{\partial \pi_r^d}{\partial m} = 0$, we can obtain the following results:

$$m^{d*} = \frac{a-bc_m}{2b} \tag{28}$$

By combining (27) and (28), we can obtain $w^{d^*}, \tau^{d^*}, {e_m}^{d^*}$, and then we can obtain the optimal price p^{d*}.

Theorem 3 is proved. □

According to Theorem 3, in the supply chain with CSR, by putting the above optimal solutions into the demand function under decentralized decision making, we can determine the optimal demand as follows

$$D^{d*} = -\frac{b\xi C_L(a-bc_m)}{2\Delta^2 b^2\xi - 4b\xi C_L + 2\beta^2 C_L} \tag{29}$$

Let $\pi_m^{RS*}, \pi_r^{RS*}, \pi^{d*}$ express the optimal profits of manufacturer, retailer and closed-loop supply chain under decentralized decisions, respectively. Consequently, we can obtain their values as follows:

$$\pi_m^{RS*} = -\frac{1}{8}\frac{\xi C_L(a-bc_m)^2}{(\Delta^2 b^2 - 2bC_L)\xi + \beta^2 C_L} \tag{30}$$

$$\pi_r^{RS*} = -\frac{1}{4}\frac{\xi C_L(a-bc_m)^2}{(\Delta^2 b^2 - 2bC_L)\xi + \beta^2 C_L} \tag{31}$$

$$\pi^{d*} = -\frac{1}{8}\frac{3\xi C_L(a-bc_m)^2}{[(\Delta^2 b^2 - 2bC_L)\xi + \beta^2 C_L]} \tag{32}$$

By comparison, $\pi_m^{d^*} < \pi_r^{d^*}$.

Theorem 4. *In the supply chain with CSR, if C_L is much larger than $\frac{\Delta^2 b}{2}$, and $\frac{\Delta^2 b^2 \xi + C_L \beta^2}{4} - b\xi C_L < 0$, then the optimal product sales price p^{d^*} is negatively correlated with the CSR cost coefficient ξ.*

Proof. By solving $\frac{\partial p^{d*}}{\partial \xi} = 0$, we obtain: $\frac{\partial p^{d*}}{\partial \xi} = \frac{1}{2}\frac{\beta^2 C_L(\Delta^2 b - C_L)(a - bc_m)}{(\Delta^2 b^2 \xi - 2b\xi C_L + \beta^2 C_L)^2}$. According to $C_L > \frac{\Delta^2 b}{2}$, $\frac{\Delta^2 b^2 \xi + C_L \beta^2}{4} - b\xi C_L < 0$, $a - bc_m > 0$, and C_L is much larger than $\frac{\Delta^2 b}{2}$, we can obtain $\frac{\partial p^{d^*}}{\partial \xi} < 0$, indicating that retailer's optimal price p^{d^*} is negatively correlated with CSR cost coefficient ξ.□

According to Theorem 4, when the manufacturer invests more to undertake CSR, the sales price of the product set by the retailer will be correspondingly reduced.

Theorem 5. *In the supply chain with CSR, there exits the following results.*

(1) If $2bC_L - \Delta^2 b^2 > 0$, and $\frac{\Delta^2 b^2 \xi + C_L \beta^2}{4} - b\xi C_L < 0$, then the manufacturer's product recovery τ will decrease with the increasing CSR cost coefficient ξ.

(2) If $\frac{\Delta^2 b}{2} < C_L < \Delta^2 b$, and $\frac{\Delta^2 b^2 \xi + C_L \beta^2}{4} - b\xi C_L < 0$, then the manufacturer's optimal wholesale price w^{d^*} will increase with the increasing of CSR cost coefficient ξ. If $C_L > \Delta^2 b$, then w^{d^*} will decrease with the increasing of ξ.

Proof. By solving $\frac{\partial \tau^{d*}}{\partial \xi} = 0$, we can obtain: $\frac{\partial \tau^{d*}}{\partial \xi} = -\frac{1}{2}\frac{\beta^2 C_L \Delta b(a - bc_m)}{(\Delta^2 b^2 \xi - 2b\xi C_L + \beta^2 C_L)^2}$, because $a - bc_m > 0$, therefore $\frac{\partial \tau^{d*}}{\partial \xi} < 0$. Similarly, by solving $\frac{\partial w^{d*}}{\partial \xi} = 0$ we can get $\frac{\partial w^{d*}}{\partial \xi} = \frac{1}{2}\frac{\beta^2 C_L(\Delta^2 b - C_L)(a - bc_m)}{(\Delta^2 b^2 \xi - 2b\xi C_L + \beta^2 C_L)^2}$, because $\Delta^2 b - 2C_L < 0$, according to the actual situation, $C_L > \Delta^2 b$, and then $\frac{\partial w^{d*}}{\partial \xi} < 0$, which indicates that w^{d^*} decreases with the increase in ξ.□

From Theorem 5, the more the manufacturer inputs in CSR, the product recovery rate will be reduced. As the CSR cost coefficient increases, the manufacturer's optimal wholesale price increases first and then decreases.

Theorem 6. *In the supply chain with CSR, if $C_L > \frac{\Delta^2 b}{2}$ and $C_L > \frac{\Delta b \xi (a + b\Delta - bc_m)}{2b\xi - \beta^2}$, then the profits of manufacturer, retailer and supply chain will decrease with the increasing of CSR cost coefficient.*

Proof. By solving $\frac{\partial \pi_m^{d*}}{\partial \xi} = 0$, $\frac{\partial \pi_r^{d*}}{\partial \xi} = 0$ and $\frac{\partial \pi^{d*}}{\partial \xi} = 0$, we can obtain $\frac{\partial {\pi_m}^{d*}}{\partial \xi} = -\frac{1}{8}\frac{(a - bc_m)^2 \beta^2 {C_L}^2}{(\Delta^2 b^2 \xi - 2b\xi C_L + \beta^2 C_L)^2}$, $\frac{\partial {\pi_r}^{d*}}{\partial \xi} = -\frac{1}{4}\frac{(a - bc_m)^2 \beta^2 {C_L}^2}{(\Delta^2 b^2 \xi - 2b\xi C_L + \beta^2 C_L)^2}$, $\frac{\partial \pi^{d*}}{\partial \xi} = -\frac{3}{8}\frac{{C_L}^2 (a - bc_m)^2 \beta^2}{(\Delta^2 b^2 \xi - 2b\xi C_L + \beta^2 C_L)^2}$.

Due to $\Delta^2 b - 2C_L < 0$, and $\Delta^2 b^2 \xi - 4b\xi C_L + \beta^2 C_L < 0$, $a - bc_m > 0$, we obtain $\frac{\partial \pi_m^{d^*}}{\partial \xi} < 0$, $\frac{\partial \pi_r^{d^*}}{\partial \xi} < 0$, $\frac{\partial \pi^{d^*}}{\partial \xi} < 0$. □

According to Theorem 6, under decentralized decisions, the profits of supply chain members will decrease with an increasing CSR input.

3.3.2. The Optimal Pricing Strategies with CSR under Centralized Decision Making (Model D)

Theorem 7. *In the supply chain with CSR, let $p^{c*}, \tau^{c*}, e_m^{c*}$represent the optimal retail price, recovery rate and CSR input level under centralized decision making, respectively. If there exist unique values to maximize the supply chain, then it will hold true as follows*:

$$p^{c*} = \frac{\Delta^2 ab\xi - b\xi C_L c_m + \beta^2 C_L c_m - a\xi C_L}{\Delta^2 b^2 \xi - 2b\xi C_L + \beta^2 C_L} \quad (33)$$

$$\tau^{c*} = -\frac{\Delta b\xi(a - bc_m)}{\Delta^2 b^2 \xi - 2b\xi C_L + \beta^2 C_L} \tag{34}$$

$$e_m{}^{c*} = -\frac{\beta C_L(a - bc_m)}{\Delta^2 b^2 \xi - 2b\xi C_L + \beta^2 C_L} \tag{35}$$

Proof. Similar to Theorem 2 (omitted). □

According to Theorem 7, let D^{c^*} and π^{c^*} represent the optimal demand and profits of supply chain, we can obtain their values as follows

$$D^{c^*} = -\frac{b\xi C_L(a - bc_m)}{\Delta^2 b^2 \xi - 2b\xi C_L + \beta^2 C_L} \tag{36}$$

$$\pi^{c*} = -\frac{1}{2}\frac{\xi C_L(a - bc_m)^2}{[(-2b\xi + \beta^2)C_L + \Delta^2 b^2 \xi]} \tag{37}$$

3.3.3. Comparative Analysis under Different Decision Situations

Theorem 8. *Let Δ_τ^c represent the difference of the manufacturer's optimal recovery rate when introducing CSR and not introducing CSR in the closed-loop supply chain; from this, we can obtain:*

$$\Delta_\tau^c = \frac{\Delta(a - bc_m)\beta^2 C_L}{(\Delta^2 b^2 \xi - 2b\xi C_L + \beta^2 C_L)(\Delta^2 b - 2C_L)} > 0 \tag{38}$$

Proof. According to $\Delta_\tau^c = \tau^{c*} - \tau_N^{c*}$, we can obtain the result. □

In the closed-loop supply chain, when CSR is introduced, the product recovery rate of the manufacturer is higher than that without CSR. Firstly, due to the introduction of the concept of CSR, supply chain members' awareness of resource conservation and environmental protection in the whole closed-loop supply chain process is improved, which makes the manufacturer pay more attention to product recycling. Secondly, the introduction of CSR is also directly and indirectly conducive to promoting the image and honor of enterprises, which necessitates the manufacturers of products to fulfill their due responsibilities and reduce the negative impact of products at the terminal.

Theorem 9. *Let Δ_p^c represent the difference of retailer's optimal selling price when introducing CSR and not introducing CSR in the closed-loop supply chain, which can be obtained by calculation:*

$$\Delta_p^c = p^{c*} - p_N^{c*} = -\frac{(\Delta^2 b - C_L)(a - bc_m)\beta^2 C_L}{(\Delta^2 b^2 \xi - 2b\xi C_L + \beta^2 C_L)(\Delta^2 b - 2C_L)b} > 0 \tag{39}$$

Proof. Because $C_L > \frac{\Delta^2 b}{2}$, there are two situations facing $\Delta^2 b - C_L$. (1) when $\frac{\Delta^2 b}{2} < C_L < \Delta^2 b$; (2) $C_L > \Delta^2 b$. Based on the previous assumptions, $C_L > \frac{\Delta b\xi(a - bc_m + \Delta b)}{2b\xi - \beta^2}$, if $\frac{\Delta^2 b}{2} < C_L < \Delta^2 b$ is selected, it will not meet the conditions, which will destroy the conditions for obtaining the optimal value in this paper, so $C_L > \Delta^2 b$ at this time. Therefore, we can get:

$$\Delta_p^c = p^{c*} - p_N^{c*} = -\frac{(\Delta^2 b - C_L)(a - bc_m)\beta^2 C_L}{(\Delta^2 b^2 \xi - 2b\xi C_L + \beta^2 C_L)(\Delta^2 b - 2C_L)b} > 0 \tag{40}$$

□

From the results of the above formula, when the upstream and downstream enterprises consider the overall interests, the results will be very different from those obtained under decentralized decision. Considering CSR, the retail price of products is higher than that without considering it, which shows that after considering CSR, corresponding costs will inevitably be invested. From the perspective of enterprise profits, the vast majority of enterprises will share these extra costs equally among various links, such as sales price. From another angle, it also shows whether to invest, whether to invest heavily in CSR, and how much investment is appropriate, which has reference and guidance significance for enterprise managers.

Theorem 10. *In the closed-loop supply chain, let* Δ_π^c, Δ_D^c *represent the difference of the overall profits and sales volume of the closed-loop supply chain with and without CSR, respectively, and then we can determine as follows:*

$$\Delta_\pi^c = \frac{1}{2}\frac{(a-bc_m)^2\beta^2{C_L}^2}{(\Delta^2b^2\xi - 2b\xi C_L + \beta^2C_L)(\Delta^2b - 2C_L)b} > 0 \tag{41}$$

$$\Delta_D^c = \frac{(a-bc_m)\beta^2{C_L}^2}{(\Delta^2b^2\xi - 2b\xi C_L + \beta^2C_L)(\Delta^2b - 2C_L)} > 0 \tag{42}$$

Proof. According to $\Delta_\pi^c = \max\pi^{c*} - \max\pi_N^{c*}$ and $\Delta_D^c = D^{c*} - D_N^{c*}$, we can obtain the results. □

As mentioned above, when the upstream and downstream enterprises consider their overall interests, when one party actively fulfills its CSR, the profits and other relevant decisions obtained are quite different from those obtained when it fails to fulfill its corresponding CSR. On the one hand, the implementation of CSR standardizes the adoption among supply chain members of more legal and compliant production methods and codes of conduct, and affects the wholesale price, sales price and even product recovery rate, but produces positive effects, leading to a great increase in product sales and the overall income of the supply chain. On the other hand, the introduction of CSR has also strengthened the attention to environmental protection and resource conservation of products, and led consumers to love the products of enterprises with good CSR, which led to the change in consumption structure, resulting in a large increase in product sales of enterprises with a good reputation for CSR. Overall, the introduction of corporate society is bound to optimize and improve the structural adjustment of production, sales and recycling of upstream and downstream products in the supply chain, and will also promote the supervision of CSR undertaken by the whole society, which will be more conducive to the development and operation of the social economy.

Theorem 11. *In a closed-loop supply chain, let* Δ_τ *represent the difference between the product recovery rate of manufacturers in the closed-loop supply chain with and without CSR, and then the following will hold true:*

$$\Delta_\tau = \frac{1}{2}\frac{\Delta(a-bc_m)\beta^2C_L}{(\Delta^2b^2 - 2bC_L + \beta^2C_L)(\Delta^2b - 2C_L)} > 0 \tag{43}$$

Proof. According to $\Delta_\tau = \tau^{d*} - \tau_N^{d*}$, we can obtain the result. □

The above results indicate that in the retailer-led closed-loop supply chain, the manufacturer's product recovery rate in the whole supply chain after the introduction of CSR is higher than that without the introduction of CSR. When CSR is introduced into the closed-loop supply chain, both enterprises and consumers pay attention to CSR, which will change business strategy and the production mode for enterprises, and improve the product recovery rate, which is one of the results of such a business strategy and production-mode change.

Theorem 12. *In the closed-loop supply chain, let Δ_p, Δ_w represent the difference between the retailer's product sales price and the manufacturer's product wholesale price in a closed-loop supply chain with and without CSR, respectively. The results are shown as follows:*

$$\Delta_p = -\frac{1}{2}\frac{\beta^2 C_L(\Delta^2 b - C_L)(a - bc_m)}{(\Delta^2 b^2 \xi - 2b\xi C_L + \beta^2 C_L)b(\Delta^2 b - 2C_L)} > 0 \tag{44}$$

$$\Delta_w = -\frac{1}{2}\frac{\beta^2 C_L(\Delta^2 b - C_L)(a - bc_m)}{(\Delta^2 b^2 \xi - 2bC_L\xi + \beta^2 C_L)b(\Delta^2 b - 2C_L)} > 0 \tag{45}$$

Proof. According to $\Delta_p = p^{d*} - p_N^{d*}$ and $\Delta_w = w^{d*} - w_N^{d*}$, we can obtain the results. □

According to Theorem 12, the product selling price of the retailer and the wholesale price of the manufacturer after the introduction of CSR through the closed-loop supply chain are higher than the product selling price and wholesale price without considering CSR. From the perspective of the pricing mechanism of the supply chain, the wholesale price of the manufacturer's products will increase after the introduction of CSR, so the sales price of the products will inevitably increase. After the introduction of CSR, because the introduction of CSR has improved product production and processing to better meet environmental protection requirements and resource conservation, the manufacturer's wholesale price will have increased. Therefore, from the perspective of cost, the manufacturer will increase their wholesale prices of products.

Theorem 13. *Let $\Delta_{\pi_r}, \Delta_{\pi_m}, \Delta_D$ represent the difference between the profits of the retailer, manufacturer and the demand of the market in a closed-loop supply chain with and without CSR, respectively. The results are as follows:*

$$\Delta_D = \frac{1}{2}\frac{\beta^2 {C_L}^2(a - bc_m)}{(\Delta^2 b^2 \xi - 2b\xi C_L + \beta^2 C_L)(\Delta^2 b - 2C_L)} > 0 \tag{46}$$

$$\Delta_{\pi_r} = \frac{1}{4}\frac{\beta^2 {C_L}^2(a - bc_m)^2}{(\Delta^2 b^2 \xi - 2b\xi C_L + \beta^2 C_L)b(\Delta^2 b - 2C_L)} > 0 \tag{47}$$

$$\Delta_{\pi_m} = \frac{1}{8}\frac{\beta^2 {C_L}^2(a - bc_m)^2}{(\Delta^2 b^2 \xi - 2b\xi C_L + \beta^2 C_L)b(\Delta^2 b - 2C_L)} > 0 \tag{48}$$

Proof. According to $\Delta_D = D^{d*} - D_N^{d*}$, $\Delta_{\pi_r} = {\pi_r}^{d*} - {\pi_r}^{Nd*}$ and $\Delta_{\pi_m} = {\pi_m}^{d*} - {\pi_m}^{Nd*}$, we can obtain the results. □

According to Theorem 13, after the introduction of CSR, product sales will be improved. From the perspective of corporate profits, after the introduction of CSR, the wholesale price, sales price and sales volume of products will have increased, leading to an increase in the profits of both retailers and manufacturers compared with the situation in which CSR was not taken into account. Once again, the above results must be clearly expounded in daily operation and employee rights as enterprises shoulder CSR. This has a positive effect not only upon the enterprise's sustainable development and corporate image, but also has social utility in the healthy sustainable development of the economy and society.

3.4. A Coordination Mechanism

According to the above analysis, various decisions and overall profits of the closed-loop supply chain under centralized decision making are better than those under decentralized decision making. However, some supply-chain members often face loss, and then the supply chain produces conflicts. In this retailer-led closed-loop supply chain, the retailer often takes some measures such as subsidy in order to stimulate the retailer to undertake CSR and improve the level and efficiency of supply-chain operation. Based on relevant literature (Zhao [32]; Ai et al. [33]; Liu et al. [34]), from the angle of the wholesale price negotiation and subsidy, we will propose a two-step pricing contract mechanism in this section. Its basic idea is as follows: firstly, the retailer sets the retail price p according to its own profits, and then the manufacturer sets w and τ. Secondly, the retailer offers a wholesale price w at unit cost and a transfer fee F, and the manufacturer can accept or reject according to its profits; they reach a contract through discussion and negotiation. Generally speaking, if the manufacturer's profits after negotiation and coordination are bigger than those under decentralized decision making, the manufacturer will accept the retailer's contract. Therefore, we take the manufacturer's profits under decentralized decision making as a deterrent point to design a two-step pricing coordination mechanism.

Firstly, the backward method is used to solve the retail price of the retailer. According to the retailer's response function and the resulting retail price p, the manufacturer will determine the optimal CSR e_m, the wholesale price and the recycling rate of waste products τ. Finally, based on the manufacturer's CSR level e_m, the retailer will determine the optimal w_{RS}^{TPT} and F_{RS}^{TPT}, where the superscript "TPT" represents the condition of two step pricing mechanism and the subscript "RS" represents retailer-led.

According to the above discussion and analysis, let $\pi_{(RS)_m}^{TPT}$, $\pi_{(RS)_r}^{TPT}$ stand for the profit function of the manufacturer and retailer after the coordination, respectively; their expressions can be determined as follows:

$$\pi_{(RS)_m}^{TPT} = (w_{RS}^{TPT} - c_m + \Delta\tau)(a - bp + \beta e_m) - \frac{1}{2}\xi {e_m}^2 - \frac{1}{2}C_L\tau^2 - F_{RS}^{TPT} \tag{49}$$

$$\pi_{(RS)_r}^{TPT} = (p - w_{RS}^{TPT})(a - bp + \beta e_m) + F_{RS}^{TPT} \tag{50}$$

Conclusion 1. *According to the cooperative pricing mechanisms provided by the retailer, we can obtain the corresponding optimal* $(F_{RS}^{TPT}, w_{RS}^{TPT})$ *as follows:*

$$w_{RS}^{TPT} = \frac{1}{4}\frac{(-8\xi^2b^2c_m + 2\beta^4c_m - 4\xi a\beta^2){C_L}^2 + 4[\xi(\frac{1}{2}\beta^2c_m + \xi a)b - \frac{1}{4}\beta^4c_m + \frac{1}{2}\xi a\beta^2)]\Delta^2bC_L - \Delta^4\xi ab^2\beta^2}{(\Delta^2b - 2C_L)[(\xi^2b^2 + \frac{1}{2}b\xi\beta^2 - \frac{1}{4}\beta^4)C_L - \frac{1}{4}b^2\xi\beta^2\Delta^2]} \tag{51}$$

$$F_{RS}^{TPT} = \frac{[(a - bw_{RS}^{TPT})\xi - \frac{1}{4}\beta^2(c_m - w_{RS}^{TPT})](c_w - w_{RS}^{TPT})C_L + \xi\Delta^2(a - bw_{RS}^{TPT})^2}{-2\Delta^2\beta^2 + 8\xi C_L} - (-\frac{1}{8}\frac{\xi C_L(a - bc_m)^2}{(\Delta^2b^2 - 2bC_L)\xi + \beta^2C_L}) \tag{52}$$

Proof. By taking the first-order derivative of the retailer's profit function with respect to p, we can obtain $\frac{\partial \pi_{(RS)_r}^{TPT}}{\partial p} = -bp + \beta e_m + a - (p - w_{RS}^{TPT})b$. Let $\frac{\partial \pi_r}{\partial p} = 0$; we can obtain $p = \frac{1}{2}\frac{bw_{RS}^{TPT} + \beta e_m + a}{b}$. Thus, we can obtain the following expression:

$$\begin{aligned}\pi_{(RS)_m}^{TPT} = &-\tfrac{1}{2}bw_{RS}^{TPT2} + \tfrac{1}{2}(a - \Delta b\tau + bc_m + \beta e_m)w_{RS}^{TPT} - \tfrac{1}{2}\xi {e_m}^2 + \tfrac{1}{2}\beta(\Delta\tau - c_m)e_m + \tfrac{1}{2}\Delta a\tau - \\ &\tfrac{1}{2}C_L\tau^2 - \tfrac{1}{2}ac_m - F_{RS}^{TPT}\end{aligned} \tag{53}$$

By then taking the first derivative of $\pi_{(RS)_m}^{TPT}$ with respect to τ and e_m, respectively, the results are as follows:

$$\frac{\partial \pi_{(RS)_m}^{TPT}}{\partial e_m} = \frac{1}{2}\Delta\beta\tau - \xi e_m - \frac{1}{2}\beta c_m + \frac{1}{2}\beta w_{RS}^{TPT} = 0 \tag{54}$$

$$\frac{\partial \pi_{(RS)_m}^{TPT}}{\partial \tau} = -\frac{1}{2}\Delta b w_{RS}^{TPT} + \frac{1}{2}\Delta \beta e_m + \frac{1}{2}\Delta a - C_L \tau = 0 \tag{55}$$

Based on this, we can obtain:

$$\tau = -\frac{\Delta[(a - bw)2\xi + \beta^2(w - c_m)]}{\Delta^2\beta^2 - 4\xi C_L} \tag{56}$$

$$e_m = -\frac{\beta[(a - bw)\Delta^2 + 2C_L(w - c_m)]}{\Delta^2\beta^2 - 4\xi C_L} \tag{57}$$

At this time, the retailer's problem can be expressed as:

$$\begin{aligned} \pi_{\substack{(RS)_r \\ (w^{TPT}, F^{TPT})}}^{TPT} &= \frac{4{C_L}^2[(a - bw_{RS}^{TPT})\xi - \frac{1}{2}\beta^2(c_m - w_{RS}^{TPT})]^2}{b(4\xi C_L - \Delta^2\beta^2)^2} + F_{RS}^{TPT} \\ s.t.\ \pi_{(RS)_m}^{TPT} &= \frac{[(a - bw_{RS}^{TPT})\xi - \frac{1}{4}\beta^2(c_m - w_{RS}^{TPT})](c_w - w_{RS}^{TPT})C_L + \xi\Delta^2(a - bw_{RS}^{TPT})^2}{-2\Delta^2\beta^2 + 8\xi C_L} - F_{RS}^{TPT} \geq \pi_m^{d*}(RS) \end{aligned} \tag{58}$$

From this passage, $\pi_m^{d*}(RS) = -\frac{1}{8}\frac{\xi C_L(a - bc_m)^2}{(\Delta^2 b^2 - 2bC_L)\xi + \beta^2 C_L}$. When the profits of the manufacturer exceed the profits under decentralized decision making, the manufacturer will accept it and make it 0. It can be found that:

$$F_{RS}^{TPT} = \frac{[(a - bw_{RS}^{TPT})\xi - \frac{1}{4}\beta^2(c_m - w_{RS}^{TPT})](c_w - w_{RS}^{TPT})C_L + \xi\Delta^2(a - bw)^2}{-2\Delta^2\beta^2 + 8\xi C_L} - (-\frac{1}{8}\frac{\xi C_L(a - bc_m)^2}{(\Delta^2 b^2 - 2bC_L)\xi + \beta^2 C_L}) \tag{59}$$

By putting this into the profit function of $\pi_{(RS)_r}^{TPT}$, and taking the first-order derivative and second-order derivative of it, we can obtain the following results.

$$\begin{aligned} \frac{\partial \pi_{(RS)_r}^{TPT}}{\partial w_{RS}^{TPT}} = &\frac{8{C_L}^2[(a - bw_{RS}^{TPT})\xi - \frac{1}{2}\beta^2(c_m - w_{RS}^{TPT})](-b\xi + \frac{1}{2}\beta^2)}{b(-\Delta^2\beta^2 + 4\xi C_L)^2} + \\ &\frac{-b[(-4c_m + 4w_{RS}^{TPT})C_L + \Delta^2(-bw + a)]\xi + (-bw_{RS}^{TPT} + a)(-\Delta^2 b + 4C_L)\xi - 2\beta^2 C_L(c_m - w_{RS}^{TPT})}{-2\Delta^2\beta^2 + 8\xi C_L} \end{aligned} \tag{60}$$

$$\frac{\partial^2 \pi_{(RS)_r}^{TPT}}{\partial w_{RS}^{TPT2}} = \frac{(\Delta^2 b - 2C_L)(-\Delta^2\xi b^2\beta^2 + 4\xi^2 b^2 C_L + 2\xi b\beta^2 C_L - \beta^4 C_L)}{b(-\Delta^2\beta^2 + 4\xi C_L)^2} \tag{61}$$

According to $\Delta^2 b^2\xi - 4b\xi C_L + \beta^2 C_L < 0$, $\Delta^2 b - 2C_L < 0$, from this we can obtain $\frac{\partial^2 \pi_{(RS)}^{TPT}}{\partial w^2} < 0$; therefore, there is an optimal value w_{RS}^{TPT} for the retailer, which can be obtained by solving the first-order derivative function:

$$w_{RS}^{TPT} = \frac{1}{4}\frac{(-8\xi^2 b^2 c_m + 2\beta^4 c_m - 4\xi a\beta^2){C_L}^2 + 4[\xi(\frac{1}{2}\beta^2 c_m + \xi a)b - \frac{1}{4}\beta^4 c_m + \frac{1}{2}\xi a\beta^2)]\Delta^2 bC_L - \Delta^4\xi ab^2\beta^2}{(\Delta^2 b - 2C_L)[(\xi^2 b^2 + \frac{1}{2}b\xi\beta^2 - \frac{1}{4}\beta^4)C_L - \frac{1}{4}b^2\xi\beta^2\Delta^2]} \tag{62}$$

$$F_{RS}^{TPT} = \frac{[(a - bw_{RS}^{TPT})\xi - \frac{1}{4}\beta^2(c_m - w_{RS}^{TPT})](c_w - w_{RS}^{TPT})C_L + \xi\Delta^2(a - bw_{RS}^{TPT})^2}{-2\Delta^2\beta^2 + 8\xi C_L} - (-\frac{1}{8}\frac{\xi C_L(a - bc_m)^2}{(\Delta^2 b^2 - 2bC_L)\xi + \beta^2 C_L}) \tag{63}$$

□

Similarly, we can obtain the following expressions:

$$\tau_{RS}^{TPT} = -\frac{\Delta[(a - bw_{RS}^{TPT})2\xi + \beta^2(w_{RS}^{TPT} - c_m)]}{\Delta^2\beta^2 - 4\xi C_L} \tag{64}$$

$$e_{m_{RS}}^{TPT} = -\frac{\beta[(a - bw_{RS}^{TPT})\Delta^2 + 2C_L(w_{RS}^{TPT} - c_m)]}{\Delta^2\beta^2 - 4\xi C_L} \tag{65}$$

$$p_{RS}^{TPT} = \frac{[(-c_m + w_{RS}^{TPT})\beta^2 + 2\xi(bw_{RS}^{TPT} + a)]C_L - \Delta^2 b\beta^2 w_{RS}^{TPT}}{(-\Delta^2 b\beta^2 + 4\xi bC_L)} \quad (66)$$

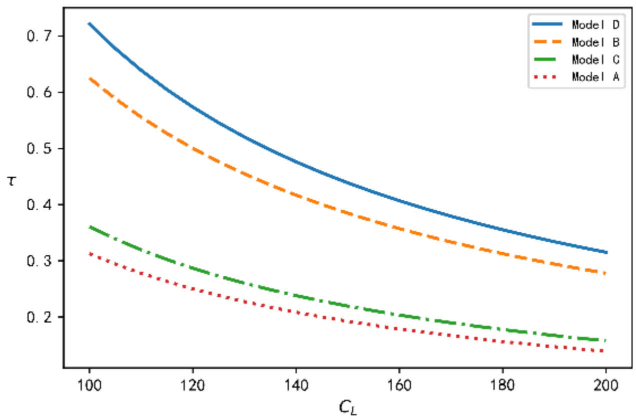
4. Numerical Illustration

Many enterprises do not know how to manage and implement CSR. In this chapter, we will show the changes of CSR input to the decision of each member of the closed-loop supply chain and the hidden management inspiration behind it through examples. Market demand not only depends on the change in product price, but also the result of the interaction of price and CSR input. If an enterprise's products are affordable, but it is poor in its treatment of employees and social reputation, this will also have an impact on the upstream and downstream enterprises and cause unnecessary losses. According to the actual situation and calculation difficulty, this paper considers the decision model that the manufacturer takes as the recycler and invests in CSR. Considering the practical significance of the model comprehensively, combined with relevant literature, such as the research of Ma et al. [6], Zheng et al. [35], Song and Gao et al. [36], Zhao et al. and Ai et al. [37,38] all the parameters need to meet the requirements of $C_L\beta^2 + \Delta^2 b^2\xi - 2b\xi C_L < 0$, $\frac{\Delta^2 b^2\xi + C_L\beta^2}{4} - b\xi C_L < 0, 2bC_L - \Delta^2 b^2 > 0$, $C_L > \frac{\Delta b\xi(a + bc_m + \Delta b)}{2b\xi - \beta^2}$ to be of practical significance. After some processing, the experimental data are selected as follows: $a = 100, c_m = 5, \Delta = 2, \xi = 30, b = 10, \beta = 8$. When C_L is a variable, $\xi = 30$; when ξ is the variable, $C_L = 150$. As a variable, the value of C_L is between $[100, 200]$, and the value of ξ is between $[7, 30]$.

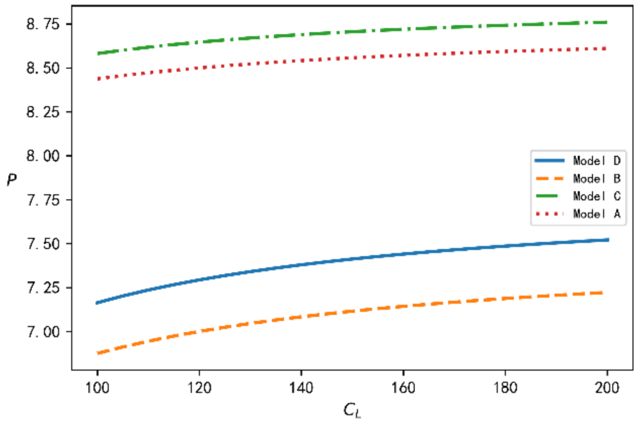
4.1. The Influence of Enterprise Recovery Scale Parameter on Various Variables

According to the above analysis process and conclusion, in order to make a more clear and intuitive comparison, this section describes the change curve of each enterprise decision variable with or without CSR. It is to analyze the impact of the introduction of CSR on the decision of members of the supply chain. For better comparative analysis, C_L is taken as an independent variable. On the one hand, C_L is involved in all models, and on the other hand, it reflects the recovery cost of the closed-loop supply chain. The C_L value here is $C_L > 80$.

According to the established models in this paper and the above data, we can determine the changing trend of each variable with the change in C_L under four different modes (A,B,C,D), which is shown in Figures 1–3.

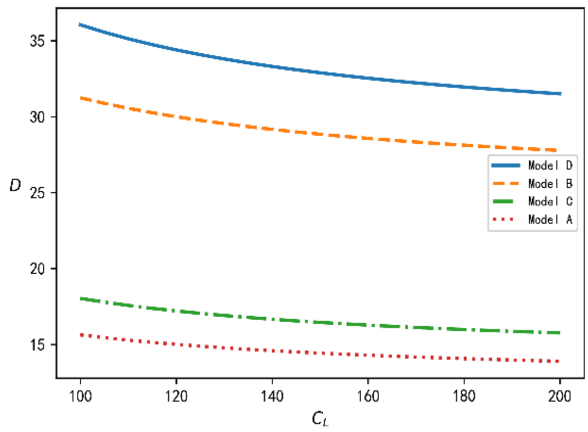


(**a**)

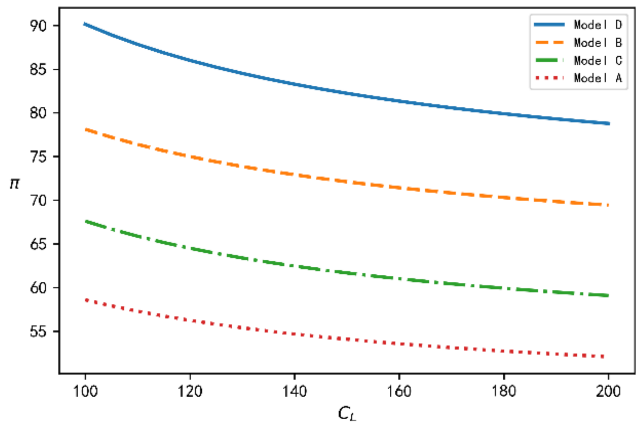


(**b**)

Figure 1. The changing of the recovery rate and retail price with C_L (**a**,**b**).

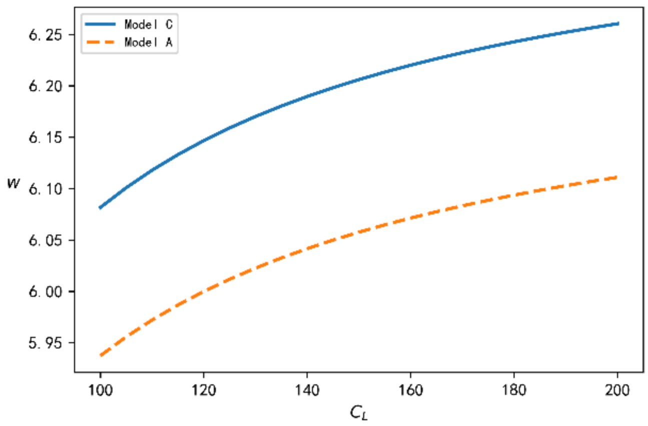


(**a**)

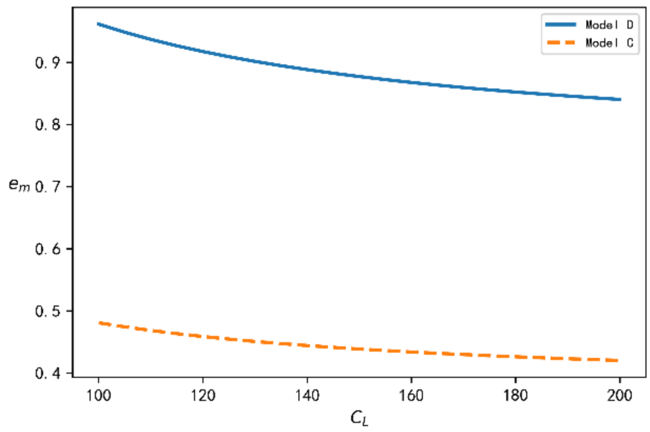


(**b**)

Figure 2. The changing of the demand and total profits of closed-loop supply chain with C_L (**a**,**b**).



(**a**)



(**b**)

Figure 3. The changing of the wholesale price and CSR level with C_L (**a**,**b**).

According to Figures 1–3, the recovery rate, market demand and total profits of the supply chain under centralized decision making are bigger than those under decentralized decision making, while the product price under centralized decision making is lower, which shows that the overall situation of the closed-loop supply chain under centralized decision making, including factors such as the overall profits, product pricing, product recovery rate and market demand, is optimal and better than that under decentralized decision making. However, in reality, due to the consideration of their own profits, each supply-chain member will often make decisions on the basis of their own profits, so that the situation under centralized decision making becomes the ideal goal that each enterprise and the closed-loop supply chain system want to achieve or approach. In terms of product recovery, the greater the input required for recovery, the lower the overall product recovery.

Parallel to the changes of the results under centralized and decentralized decision making, there are differences in the supply chain with and without CSR. The results show that the effect is best when the upstream and downstream partners make the centralized decisions, and the effect is also better when they join in CSR. It also suggests that centralization decisions have a wider impact. In terms of market demand and CSR, with the increase in recovery cost, the market demand and CSR gradually show a downward trend. Similarly, the overall level under centralized decision making is still better than that under decentralized decision making. With increases in recovery cost, the wholesale price of products is bound to increase, as shown in Figure 3a. As a result, the retail price of products will increase, while the demand, the level of CSR (Figure 3b) and the overall profits of the closed-loop supply chain will show a downward trend (Figure 2b). according to the above analysis, the decisions and overall profits of the closed-loop supply chain under centralized decision making are better than those under decentralized decision making, and the supply-chain members should cooperate more and consider problems from the perspective of overall profits.

4.2. The Influence of CSR Cost Coefficient on Various Variables

According to the established models in this paper and the above data, we can determine the influence of the parameter ξ on the equilibrium retail price, recovery rate and CSR input level in the closed-loop supply chain, shown in Figure 4. According to Figure 4, with a slow increase in CSR cost, the equilibrium price, CSR input level, recovery rate and wholesale price of the closed-loop supply chain all decrease. However, with a gradual increase in ξ, the equilibrium price of the closed-loop supply chain, the level of CSR input, the recovery rate and the wholesale price of products will all stabilize around a value and not change. The reasons for this are that the increasing of CSR cost level will lead to increases in enterprise investment and product wholesale price and retail price, while good CSR can promote product sales. However, product prices are of great guidance, so the cost increasing will lead to a rise in prices and lower demand. From the perspective of the influence of CSR input on consumers and the social environment, the retail price of product is the highest, which is the most unfavorable to consumers. However, the input level of CSR, the recovery rate of waste products, market demand and overall profits of the supply chain with CSR are all higher than those without CSR, and thus it is most beneficial to improve the environmental performance. On the whole, CSR investment in a closed-loop supply chain cannot always improve consumer surplus, environmental performance and social welfare. When it is in a suitable range, it will achieve the best effect. The results provide guidance for enterprises and governments to improve profits or the level of social welfare. Therefore, we should comprehensively pay attention to closed-loop supply chain decisions in CSR.

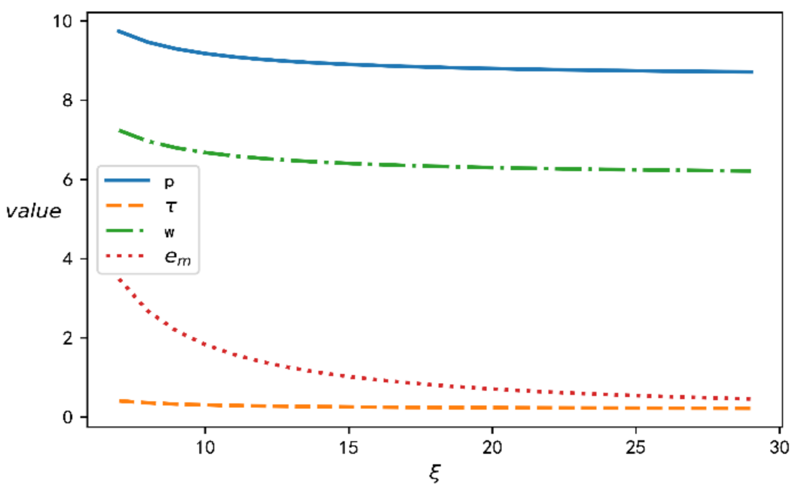


Figure 4. The influence of ξ on closed-loop supply-chain decisions.

4.3. Coordination Result Analysis

Based on the above analysis, this section will explore the effect of closed-loop supply chain coordination mechanisms. After comprehensive consideration of the practical significance of the model and related literature, the following initial experimental data were selected after certain processing: $a = 100, c_m = 5, \Delta = 2, C_L = 150, b = 10, \beta = 8$; among them, ξ is the independent variable, indicating the cost coefficient of responsibility. According to the established coordination mechanism, by computing, we can obtain the coordination results shown in Table 1. Specifically, the recovery rate changed before and after coordination, and increased after coordination. However, product prices, wholesale prices and CSR effort level decreased after coordination. The reduction in product wholesale price and product retail price is predictable, because this paper adopts cooperative pricing contract mechanisms. In the implementation process, ξ is reduced, and then a fee F is transferred. In this case, we can see that although the level of CSR has declined, the decline is not significant, and the input cost has been indirectly reduced without loss of effect.

Table 1. The results before and after coordination.

ξ	τ	e_m	p	w	π_m	π_r	τ^{TPT}	${e_m}^{TPT}$	p^{TPT}	w^{TPT}	${\pi_m}^{TPT}$	${\pi_r}^{TPT}$
6	0.500	5.000	10.250	7.750	46.875	93.750	0.483	4.986	10.366	7.774	46.875	90.448
10	0.305	1.829	9.177	6.677	28.528	57.165	0.362	1.693	8.643	6.393	28.528	65.677
20	0.236	0.708	8.797	6.297	22.111	44.222	0.337	0.490	7.864	5.552	22.111	56.416
30	0.219	0.439	8.706	6.206	20.599	41.118	0.341	0.240	7.632	5.218	20.599	54.695
40	0.212	0.318	8.665	6.165	19.862	39.725	0.347	0.143	7.514	5.024	19.862	54.130
50	0.208	0.249	8.642	6.142	19.466	38.933	0.351	0.096	7.441	4.895	19.466	53.901
60	0.205	0.205	8.627	6.127	19.211	38.422	0.355	0.068	7.391	4.803	19.211	53.800

According to the established coordination mechanism in this paper and the above data, we can determine the profit changes of the supply chain and manufacturer and retailer before and after coordination, which are shown in Figures 5 and 6. From Figure 5, the overall profits of the closed-loop supply chain after coordination are greater than those before coordination. The total system profits of the closed-loop supply chain decrease obviously with the increasing of ξ before coordination, but the trend of rapid decline is alleviated after coordination, and the profit loss is reduced. From Figure 6, the profits of the manufacturer are guaranteed to remain unchanged at least after coordination, while the overall profits of the retailer are significantly improved to a higher level after coordination. Therefore, the proposed pricing coordination mechanism can further improve and guarantee the profits of the entire closed-loop supply chain of products, and make the decision level reach or approach the best level under centralized decision making.

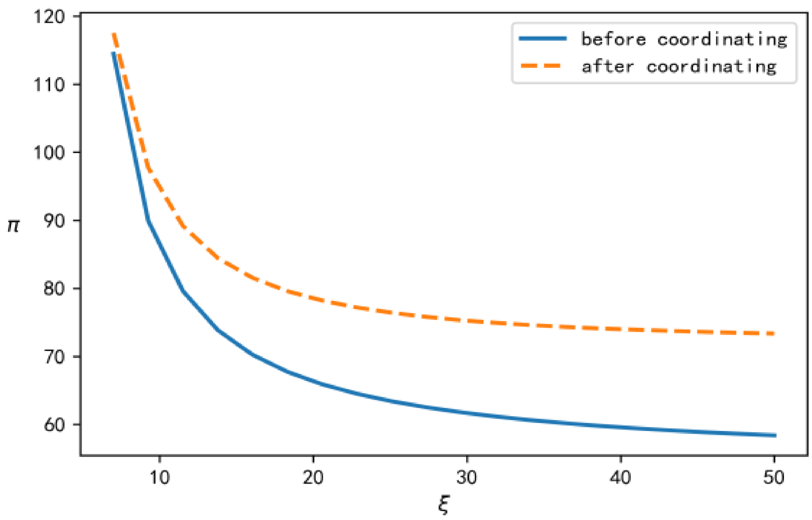


Figure 5. The profit changing of the supply chain before and after coordination.

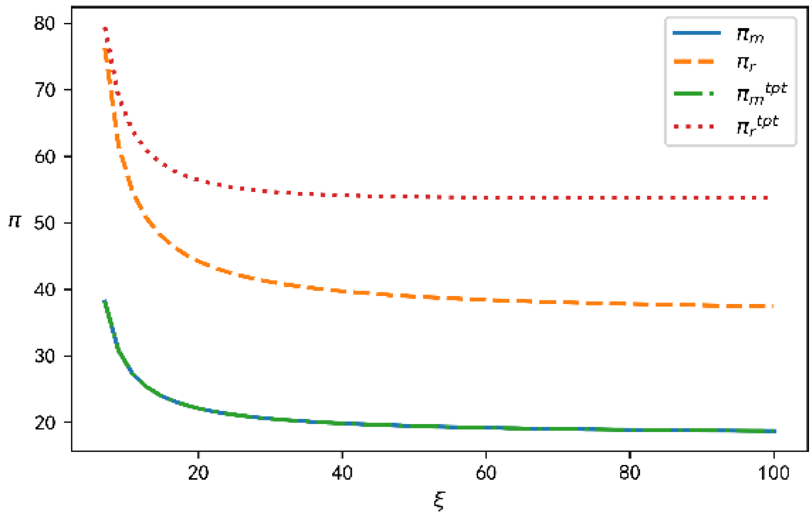


Figure 6. The changing profits of the supply-chain members before and after coordination.

5. Conclusions

In this paper, we discuss the impact of CSR on supply-chain decisions and analyze the optional decisions under different situations, and then propose a coordinated mechanism. According to the analysis of the models, some conclusions are as follows:

(1) CSR input can promote the overall development of a closed-loop supply chain. The product recovery rate, product pricing, CSR, consumer product demand and other aspects have been strengthened, and the overall revenue has been improved. However, the investment in CSR must be within a certain range, that is, the investment in CSR should be moderate, in line with the development of the enterprise itself, in order to play a greater effect. Too little or too much CSR investment cannot greatly promote the development of a closed-loop supply chain.

(2) With a slow increase in CSR cost, the equilibrium price, CSR input level, recovery rate and wholesale price of the closed-loop supply chain all decrease. However, with a gradual increase in the equilibrium price of the closed-loop supply chain, the CSR input level, the recovery rate and the wholesale price of products will all stabilize around a value and will not change. In summary, CSR input in a closed-loop supply chain cannot always improve consumer surplus and environmental performance, or social welfare. The optional decisions will be realized when they happen to be within an appropriate range.

(3) The proposed pricing coordination mechanism can improve the profits of the manufacturer, retailer and the whole closed-loop supply chain, and make the decision level reach or approach to the best level under centralized decision making. The above conclusions indicate that in the daily operation of enterprises, it is necessary to strengthen the cooperation between upstream and downstream and share information, and at the same time, appropriately consider the implementation of CSR in the daily operation of the company. Secondly, in the daily operation of the supply chain, considering the retailer as the leader is conducive to obtaining the demand of the market, consumers and customers, and is conducive to timely communication with the manufacturer, so as to promote an increase in enterprise efficiency and income.

However, this paper still has some shortcomings: we only studied retailer-led closed-loop supply chains without considering the influence of CSR on supply-chain decision-making under different channel power structures; we did not conduct research on uncertain information, production costs and attitude preferences that may have an impact on closed-loop supply chain decision making, and the deterministic model designed in this article cannot address this point. In the future, we could study the impact of CSR on the decision making of closed-loop supply chains with asymmetric information under different power structures.

Author Contributions: H.Z. and Z.L. designed and wrote this article. Y.L. examined the article and revised the article format. All authors have read and agreed to the published version of the manuscript.

Funding: This work is partially funded by the National Natural Science Foundation of China (72074100; 71503103); National Social Science Foundation of China (19FGLB031); National Vocational College Teachers' Teaching Innovation Team Construction Systematic Research Project(TX20200801); Outstanding Youth in Social Sciences of Jiangsu Province; Qinglan Project of Jiangsu Province, Fundamental Research Funds for the Central Universities (JUSRP321016), and the Tender Project from Wuxi Federation of Philosophy and Social Sciences (WXSK22-A-03); Jiangsu Province Social Science Fund Project (22GLB041), and the opinions, findings and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the Foundation.

Data Availability Statement: The data used to support the findings of this study will be considered by the corresponding author. Email: liuy@jiangnan.edu.cn.

Acknowledgments: We would like to thank the anonymous reviewers for their helpful remarks, and we thank the academic editors for their useful feedback that improved this paper.

Conflicts of Interest: The authors declare no conflict of interest.

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