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The Nonlinear Model of Intersectoral Linkages of Kazakhstan for Macroeconomic Decision-Making Processes in Sustainable Supply Chain Management

Seyit Kerimkhulle [1,*,†], Natalia Obrosova [2,3,4,*,†], Alexander Shananin [2,3,4,†] and Gulmira Azieva [1,†]

1 Department of Information Systems, L.N. Gumilyov Eurasian National University, Satpayev Street 2, Astana 010008, Kazakhstan
2 Moscow Center for Fundamental and Applied Mathematics, Lomonosov Moscow State University, GSP-1, Leninskie Gory, Moscow 119991, Russia
3 Federal Research Center Computer Science and Control of Russian Academy of Sciences, Vavilov Street 44/2, Moscow 119333, Russia
4 Institute of Physics and Technology, Dolgoprudny, Moscow Oblast, Moscow 141701, Russia
* Correspondence: kerimkhulle@gmail.com (S.K.); nobrosova@yandex.ru (N.O.); Tel.: +7-(701)-391-72-52 (S.K.); +7-(916)-255-30-57 (N.O.)
† These authors contributed equally to this work.

Abstract: We provide a nonlinear model of intersectoral balance with constant elasticity of substitution (CES) production functions of industries and CES utility function of final consumer for the study of intersectoral linkages in the Kazakhstan economy. The model is formalized in terms of the primal problem of resource allocation and the corresponding Fenchel dual problem which solution gives costs of inputs of industries in a supply network. We identify the model with the actual data of the Input-Output tables of Kazakhstan and estimate the elasticity of substitution of production factors for the aggregated industry complexes. With the help of developed framework, we evaluate the inter-industry financial flows in the aggregated supply network for the period 2013–2020 and compare the results with the actual data of Kazakhstan. The developed framework can be used to support decision-making processes in sustainable supply chain management in a situation of the government economic policy change and external shocks. Using the developed framework, we evaluate the risks for Kazakhstan's supply chains in scenario of sharp weakening of the national currency.

Keywords: resource allocation problem; Fenchel duality; Young transform; CES production function; elasticity of substitution; input–output tables; sustainable supply chain management; shock; scenario calculations

Citation: Kerimkhulle, S.; Obrosova, N.; Shananin, A.; Azieva, G. The Nonlinear Model of Intersectoral Linkages of Kazakhstan for Macroeconomic Decision-Making Processes in Sustainable Supply Chain Management. *Sustainability* **2022**, *14*, 14375. https://doi.org/10.3390/su142114375

Academic Editors: Antonella Moretto, Antonio Messeni Petruzzelli and Federico Frattini

Received: 30 July 2022
Accepted: 28 October 2022
Published: 2 November 2022

1. Introduction

Interindustry analysis is one of the most widely applied methods to estimate the intersectoral connections, economy growth characteristics and the sustainability of intersectoral linkages in a particular economic area (a nation, a region, a state, a group of regions, etc.) [1]. The fundamental base of the interindustry analysis is the linear input–output model developed by Prof. Wassily Leontief in the 1930s, for which he was awarded the Nobel Prize in Economic Science in 1973 [2,3]. The Leontief model provides the interindustry connections of m pure industries of the economy as an m-dimensional system of linear equations [4]. Each linear equation describes the distribution of an industry's product throughout the other industries and final consumers of the economy. The main hypothesis of the linear input–output model of Leontief is the constancy in the time of the technology matrix, that coefficients a_{ij} equal to the norm of the material cost of the goodof industry i for a unit of output of industry j. The simplicity of the model allows us to derive the key summary analytical measures that are known as economic multipliers. The most used economic multipliers estimate the macroeconomic effects of exogenous changes on new outputs of industries, income of households as a result of new outputs, employment generated from new outputs and the value added generated by production [4].

Since the middle of the last century, the set of extensions of the Leontief input–output model has been developed and used by scientists to analyze the structure of the economies of different countries, calculate economic multipliers, identify sectors that are drivers of economic growth and analyze the impact of the external economic environment on the intersectoral linkages; for example, see [4–10]. Symmetric input–output tables, as well as Leontief's technology matrices, are the part of the system of national accounts of many developed countries as well as international databases (for example, see [11]).

Against the backdrop of the crises of the late 20th and early 21st centuries, interest in interindustry analysis in the world has grown [12–15]. The paper [16] presents review and discusses how different disaster modeling aspects have been incorporated in various known input–output techniques.

Despite the wide use and applications of the Leontief-type models, their theoretical base is limited by some basic assumptions that become critical in modern, complicated economies. The main restricted assumptions are the linearity and the fixed technology coefficients. Since the 1980s of the 20th century, developed countries, when using the Leontief method, have encountered difficulties associated with an increase in the variety of goods and services in the cost structure of producers. In this situation, the input substitution in the sectors of the economy increased, and the main hypothesis of the Leontief model about the constancy of the technology coefficients violated.

In the global scientific literature, new methods for analyzing structural imbalances in the economy have started to develop [17–23]. The methods make it possible to analyze the propagation of shocks in complex production networks, taking into account the substitution of production factors.

In a whole series of empirical and theoretical works, the intersectoral linkages were described on the base of nonlinear dependencies. The main purpose of such studies was to analyze the aggregate macroeconomic fluctuations as a result of random firm-level (idiosyncratic) shocks in supply networks. The baseline model was presented in papers [17,18] and was modified in [19–23]. In that cycle of studies, the authors replaced the main Leontief hypothesis with a weaker hypothesis about the constancy of the proportions of financial costs in intersectoral linkages. From a mathematical point of view, such a hypothesis corresponds to the use of the Cobb–Douglas production function and leads to nonlinear models of input–output balance. More specifically, the papers [17–20] provide a simple network model of perfect competition with Cobb–Douglas production functions and utility function with constant proportions of consumption. In terms of the framework, the authors studied the influence of the shock multiplier for various examples of the topology of the economic network and discuss the question of how the macroeconomic consequences of shocks are connected with the importance of the role of the shocked sector in the supply network. The conditions of the underlying supply network of the economy are studied, and relatively small shocks can create cascade effects and, as a result, the standard central limit theorems need not hold.

The generalization of the results from [17–20] to the class of constant elasticity of substitution (CES) utility functions, including the models with imperfect competition, is considered in [21–23].

Methods for assessing the stability of financial networks of different topologies involved in shock events are considered in [24].

On the base of ideas from [17,18], in this paper we provide an interpretable nonlinear input–output framework that has some advantages compared to the linear Leontief-type models and is much more general than the models from [17–23]. Our framework takes into account the substitution of inputs and describes an open economy with imported input flows and the flows of export of industries. Due to the clear identification and verification process, we show that our model can be used for estimation of economic multipliers on the base of the actual input-output data of the economy. Compared to the results of [17–23], we build a more general model of economic equilibrium in the production network that allows us to estimate the dynamics of prices of goods in a supply network. The advantage of our model is that we describe not only the production and optimal allocation of resources in the economy (as a solution of the corresponding extremal problem) but also the dynamics of costs of inputs when the external to the production network characteristics (for example, final demand, prices for primary resources) are altered. Such effects we obtain in terms of the dual extremal task to the resource allocation problem.

This article continues the cycle of our (with co-authors) research [25–30] related to the development and application of methods of nonlinear interindustry analysis, taking into account the substitution of production factors.

In the papers [25,26,31], the mathematical foundations of models of a nonlinear intersectoral balance with CES technologies are developed. The results provided a basis for the development of methodology for the medium-term analysis of intersectoral linkages that take into account the substitution of production factors. The model of nonlinear input–output balance is formalized as a problem of optimal resource allocation with production functions and utility function of the final consumer, which are concave and positively homogeneous. The balance constraints in the model are nonlinear. The analysis of the problem is based on the construction of the Fenchel dual problem and the Young transforms of the original production functions. This makes it possible to construct a dual description of production technologies in terms of prices for production factors and aggregated price indices. The possibilities of the calibration and aggregation of the obtained model were studied in [26]. The nonlinear intersectoral balance models allow us to solve the problems of analysis and forecasting of intersectoral relations at a new level that is adequate to the current complexity of economic networks.

In paper [27], the model of nonlinear intersectoral balance with Cobb–Douglas production technologies and the traditional linear Leontief's model are used to assess the degree of "centrality" of industries in the modern supply network of Kazakhstan. The comparison of the evaluation possibilities of macroeconomic characteristics for the model with Cobb–Douglas technologies and for the linear Leontief model is represented in [28] using the example of large economies at different stages of economic development. In paper [29], the nonlinear intersectoral model with Cobb–Douglas technologies is identified according to Russia's data. The aggregation of the intersectoral balance in terms of t2e nonlinear model with CES technologies on the example of Russia's statistics is considered in [30]. Note that the actual information about the intersectoral financial flows is published in Russia once every 5 years. This makes it difficult to calibrate the model and evaluate the substitution of input coefficients.

In this paper, we provide the interpretable nonlinear input-output framework with CES production technologies, which is based on mathematical methods developed in [25,26]. In contrast to the linear Leontief scheme, we take into account substitution of inputs and present the new method of elasticities of substitution assessments in terms of the model. The advantage of the model is the possibility of clear identification and verification processes on the base of actual input–output statistics of a state. The model can be applied for the intersectoral analysis and allows us to evaluate macroeconomic effects from external and internal shocks.

The rest of the paper is organized as follows. In Section 2.1, we present the general framework. We formulate the problem of optimal resource allocation and on the basis of the Young transform technique construct the dual problem that gives the optimal prices of inputs in the model. We show that that the optimal mechanisms of the allocation of intermediate inputs in our framework are the equilibrium market-type mechanisms. In this sense, our model is close to the class of computable general equilibrium (CGE) models that, on the basis of the actual economic data, allows us to estimate the macro characteristics of an economy's reactions to changes in external factors and are a very utilized tool for development planning and macro policy analysis [32]. One of the major criticisms related to the CGE models comes down to the large number exogenous variables that define results [33]. Note that our framework does not have this drawback.

In Section 2.2, we construct a closed-form solution of the inverse problem of identification of our model in the class of CES production and utility functions. The input data for the solution are the symmetric input-output tables that are usually available from the official national accounts statistics of a state.

In Section 2.3, we provide the method of IO balances forecasting and scenario calculations that is based on the solutions of the resource allocation problem and the dual problem of price formation.

In Section 3, we apply the developed nonlinear model of intersectoral balance with CES technologies for the interindustry analysis in the Kazakhstan economy. We start Section 3.1 from the detailed analysis of the industries of Kazakhstan because we need to take into account the main features of the regional economy. That allows us to justify the method of verification of the model. In Section 3, we apply the developed framework to the aggregated input–output statistics. We consider the four basic industry complexes of Kazakhstan that are defined by their involvement in the export–import operations due to high heterogeneity of the economy in relation to external trade processes. We solve the inverse identification problem on the basis of the data of the symmetric input–output tables of Kazakhstan for 2013–2020, which are published annually as part of the national accounts' statistics. As a result of verification of the model, we evaluate elasticities of substitution for the aggregated complexes of industries in Section 3.1 and estimate their stability by comparing the predicted values of the model with the actual data (Section 3.2). In Section 3.2, we evaluate the interindustry financial flows in the aggregated supply network of Kazakhstan for the period 2013–2020.

In Section 4.1, we consider an example of scenario calculations. Within the model we evaluate the risks for Kazakhstan's supply chains in the case of the national currency's (tenge) sharp weakening as a result of external shocks.

2. Materials and Methods

2.1. Baseline Model

Consider the network economy with m pure industrial sectors. The output of each sector can be either used as intermediate input of other sectors or consumed by final consumers of the product. Let X_i^j is the amount (in prices of the base year) of commodity of sector i used as input of sector j, i.e., $X^j = \left(X_1^j, \ldots, X_m^j\right)$ is the vector of intermediate domestic inputs of the sector j. Note that $X_i^j, i, j = 1, \ldots, m$ is the financial flow in prices of the base year from the pure industrial sector j to the pure industrial sector i. Each sector j has a production function $F_j\left(X^j, l^j\right)$ depending on domestic intermediate inputs X^j and n primary production factors (primary inputs) $l^j = \left(l_1^j, \ldots, l_n^j\right)$ which are not produced by the sectors $j = 1, \ldots, m$. Let $X^0 = \left(X_1^0, \ldots, X_m^0\right)$ be the vector of final consumption of products $1, \ldots, m$. We denote the class of functions (with k arguments) with neoclassical properties by ϕ_k, assuming that the functions from ϕ_k are concave, continuous, monotonically nondecreasing with arguments from $R_{\geq 0}^{m+n}$, positively homogeneous of degree one and vanish at the origin. Assume that $F_j\left(X^j, l^j\right) \in \phi_{m+n}$ and the final consumption is described by the utility function $F_0\left(X^0\right) \in \phi_m$. Let the total intermediate input of primary production factors $l^j = \left(l_1^j, \ldots, l_n^j\right)$ be bounded from above by a vector $l = (l_1, \ldots, l_n) \geq 0$. The problem is to find an optimal allocation of domestic and primary inputs in the production network maximizing the utility of final consumption in the case of balance constraints on factors (primary inputs) and outputs of sectors.

$$F_0\left(X^0\right) \to max, \tag{1}$$

$$F_j\left(X^j, l^j\right) \geq \sum_{i=0}^{m} X_j^i, j = 1, \ldots, m, \tag{2}$$

$$\sum_{j=1}^{m} l^j \leq l, \tag{3}$$

$$X^0 \geq 0, X^1 \geq 0, \ldots, X^m \geq 0, l^1 \geq 0, \ldots, l^m \geq 0. \tag{4}$$

We impose the following assumptions on the production network.

Assumption 1. *The group of sectors* $1, \ldots, m$ *is productive, i.e., there exist* $\hat{X}^1 \geq 0, \ldots, \hat{X}^m \geq 0, \hat{l}^1 \geq 0, \ldots, \hat{l}^m \geq 0$ *such that*

$$F_j\left(\hat{X}^j, \hat{l}^j\right) > \sum_{i=1}^{m} \hat{X}_j^i, j = 1, \ldots, m.$$

Assumption 2. *There exists* $\hat{l} \in int\mathbb{R}^n_{>0}$ *such that the set* $A\left(\hat{l}\right)$ *is bounded, where*

$$A\left(\hat{l}\right) = \left\{ X^0 = \left(X_1^0, \ldots, X_m^0\right) \geq 0 \middle| X_j^0 \leq F_j\left(X^j, l^j\right) - \sum_{i=1}^{m} X_j^i, j = 1, \ldots, m; \right. \\ \left. \sum_{j=1}^{m} l^j \leq \hat{l}, X^1 \geq 0, \ldots, X^m \geq 0, l^1 \geq 0, \ldots, l^m \geq 0 \right\}.$$

Assumptions 1 and 2 guarantee that the Slater condition holds for the optimization problem (1)–(4) and the set $A(l)$ is bounded, convex and closed for any $l \in \mathbb{R}^n_{\geq 0}$. Therefore, the limited optimal solution of (1) exists [27].

Proposition 1 ([26])**.** *A set of vectors* $\left\{\hat{X}^0, \hat{X}^1, \ldots, \hat{X}^m, \hat{l}^1, \ldots, \hat{l}^m\right\}$*, satisfying constraints (2)–(4), is a solution of the optimization problem (1)–(4) if and only if there exist Lagrange multipliers* $p_0 > 0$*,* $p = (p_1, \ldots, p_m) \geq 0$ *and* $s = (s_1, \ldots, s_n) \geq 0$ *such that*

$$\left(\hat{X}^j, \hat{l}^j\right) \in Argmax\left\{p_j F_j\left(X^j, l^j\right) - pX^j - sl^j \middle| X^j \geq 0,\, l^j \geq 0\right\}, j = 1, \ldots, m, \tag{5}$$

$$p_j\left(F_j\left(\hat{X}^j, \hat{l}^j\right) - \hat{X}_j^0 - \sum_{i=1}^{m} \hat{X}_j^i\right) = 0, j = 1, \ldots, m, \tag{6}$$

$$s_k\left(l_k - \sum_{j=1}^{m} \hat{l}_k^j\right) = 0, k = 1, \ldots, n, \tag{7}$$

$$\hat{X}^0 \in Argmax\left\{p_0 F_0\left(X^0\right) - pX^0 \middle| X^0 \geq 0\right\}. \tag{8}$$

The Lagrange multipliers $p = (p_1, \ldots, p_m)$ to the balance constraints on outputs (2) can be interpreted as prices of the outputs. The Lagrange multipliers $s = (s_1, \ldots, s_n)$ to the balance constraints on factors (3) we interpret as the prices of factors.

It follows from Proposition 1 that the competitive equilibrium is optimal mechanism of the allocation of inputs in the production network.

We introduce the dual description of the technology $j = 1, \ldots, m$ as the cost function $q_j(p, s)$ that is the Young transform of the production function $F_j\left(X^j, l^j\right)$.

$$q_j(p, s) = inf\left\{\frac{pX^j + sl^j}{F_j\left(X^j, l^j\right)} \middle| X^j \geq 0, l^j \geq 0, F_j\left(X^j, l^j\right) > 0\right\}. \tag{9}$$

The cost function $q_j(p,s)$ corresponds to the cost of a unit of production of the industry $j = 1,\ldots,m$ with prices of inputs (p,s).

The dual description of the utility function $F_0(X^0)$ is the consumer price index.

$$q_0(q) = inf\left\{ \frac{qX^0}{F_0(X^0)} \middle| X^0 \geq 0, F_0\left(X^0\right) > 0 \right\}. \tag{10}$$

It follows from (9), (10) that $q_0(p) \in \phi_m$, $q_j(p,s) \in \phi_{m+n}$ $(j = 1,\ldots,m)$. The Young transform is involution, so we obtain

$$F_0\left(X^0\right) = inf\left\{ \frac{qX^0}{q_0(q)} | q \geq 0, q_0(q) > 0 \right\}, \tag{11}$$

$$F_j\left(X^j, l^j\right) = inf\left\{ \frac{pX^j + sl^j}{q_j(p,s)} | p \geq 0, s \geq 0, q_j(p,s) > 0 \right\}. \tag{12}$$

For the details, see [31,34,35]. We introduce the aggregate production function $F^A(l) \in \phi_n$ that equals the optimal value of (1) for any vector $l = (l_1,\ldots,l_n) \geq 0$ (see the right part of (3)).

Correspondingly, the aggregate cost function $q_A(s)$ is the Young transform of the aggregate production function $F^A(l)$, i.e.,

$$q_A(s) = inf\left\{ \frac{sl}{F^A(l)} \middle| l \geq 0, F^A(l) > 0 \right\}.$$

We have $q_A(s) \in \phi_n$ and

$$F^A(l) = inf\left\{ \frac{sl}{q_A(s)} | s \geq 0, q_A(s) > 0 \right\}.$$

Proposition 2 ([25,26])**.** *If Lagrange multipliers $\hat{p} = (\hat{p}_1,\ldots,\hat{p}_m) \geq 0$, $\hat{s} = (\hat{s}_1,\ldots,\hat{s}_n) \geq 0$ to the constraints of the problem (1)–(4) satisfy to (5)–(8), then $\hat{p} = (\hat{p}_1,\ldots,\hat{p}_m) \geq 0$ is the solution of the following problem.*

$$q_0(p) \to \max_p, \tag{13}$$

$$q_j(\hat{s},p) \geq p_j, j = 1,\ldots,m, \tag{14}$$

$$p \geq 0. \tag{15}$$

Moreover, the aggregate cost function $q_A(\hat{s}) = q_0(\hat{p}(\hat{s}))$.

The convex programming problem (13)–(15) is called the Young dual problem to the problem (1)–(4) [25].

The problem of identification of the provided framework is posed as an inverse problem. The input data for the solution of the inverse problem are the intersectoral financial flows data in the selected year (base year). Usually, this data are the part of the periodically publishing statistics. We give details about the initial data format in the next section. We solve the inverse problem of identification in the class of constant elasticity substitution (CES) production and utility functions. Thus, the inverse problem solution is reduced to a construction of the extremal problem (1)–(4), whose solution reproduces the intersectoral financial flows in the economy in the base year. The parameters of elasticity of substitution of technologies remain free parameters of the model in the solution. They can be evaluated by verification of the model on datasets observed by statistics. The verification process depends on the availability of annual statistics of the input–output tables. We assume that the production technologies do not change over the last several years to verify the model. Resolving the problem (1)–(4) and the dual problem (13)–(15) for the given initial data, we evaluate corresponding intersectoral financial flows with inputs substitution. We compare the calculation results with the actual data over last several years to evaluate the remaining parameters of the model and to check their stability.

Based on the results of identification and verification of the nonlinear intersectoral balance model, we provide the approach to intersectoral linkages analysis that takes into account the substitution of industrial inputs. This approach is more general than the classical Leontief scheme since it does not imply fixed proportions of the material costs of the production. In the next section, we focus on the format of the initial data of the model and solve the inverse identification problem in the case of CES technologies.

2.2. Identification Problem of the Nonlinear Intersectoral Model in the Class of CES Technologies

Assume that the output of each pure industry of the economy and the utility of the final consumer are defined by CES functions.

$$F_j\left(X^j, l^j\right) = \left(\sum_{i=1}^{m}\left(\frac{X_i^j}{w_i^j}\right)^{-\rho_j} + \sum_{k=1}^{n}\left(\frac{l_k^j}{w_{m+k}^j}\right)^{-\rho_j}\right)^{-\frac{1}{\rho_j}} j = 1, \ldots, n, \tag{16}$$

$$F_0\left(X^0\right) = \left(\sum_{i=1}^{m}\left(\frac{X_i^0}{w_i^0}\right)^{-\rho_0}\right)^{-\frac{1}{\rho_0}}, \tag{17}$$

with parameters $\rho_j, \rho_0 \in (-1, 0) \cup (0, +\infty)$, $w_1^j > 0, \ldots, w_{m+n}^j > 0$, $j = 1, \ldots, m$, $w_1^0 > 0, w_2^0 > 0, \ldots, w_m^0 > 0$. Note that the constant elasticity of substitution of industry j equals to $\sigma_j = \frac{1}{1+\rho_j}$, $j = 0, 1, \ldots, m$. We call the parameter ρ_j by the same term "elasticity of substitution" for simplification in this section.

Remark 1. *Consider the CES production function.*

$$f(x_1, \ldots, x_n) = \left(\left(\frac{x_1}{\omega_1}\right)^{-\rho} + \left(\frac{x_2}{\omega_2}\right)^{-\rho} + \ldots + \left(\frac{x_n}{\omega_n}\right)^{-\rho}\right)^{-\frac{1}{\rho}},$$

where $\rho \in (-1, 0) \cup (0, +\infty)$, $\omega_1 > 0, \ldots, \omega_n > 0$. *It follows from (12) that the Young dual cost function* $g(\lambda_1, \ldots, \lambda_n) \in \phi_n$, $(\lambda_1, \ldots, \lambda_n) \in R^n_{>0}$ *to the production function* $f(x_1, \ldots, x_n)$ *has the CES form as well:*

$$g(\lambda_1, \ldots, \lambda_n) = \left((\lambda_1\omega_1)^{\frac{\rho}{1+\rho}} + (\lambda_2\omega_2)^{\frac{\rho}{1+\rho}} + (\lambda_n\omega_n)^{\frac{\rho}{1+\rho}}\right)^{\frac{1+\rho}{\rho}}.$$

The initial data for the solution of the inverse problem of identification of the model is the symmetric input–output (IO) table. Symmetric IO tables are the part of national accounts that are published by state statistical services periodically. A symmetric IO table contains the annual data on financial flows in terms of pure industries (products) that reflect the generation of products and their allocation among the components of intermediate and final demand.

A symmetric input–output table of domestic products consists of three quadrants. The values $\|Z_i^j\|, i, j = 1, \ldots, m\ of\ the$ first quadrant reflect the intermediate domestic inputs of industries, i.e., Z_i^j denotes the sum of money that i received from j for the intermediate inputs produced by i and consumed by j. The second quadrant of the table consists of the column vectors of final consumption of products $j = 1, \ldots, m$. The second quadrant reflects the final consumption of agents of the economy, such as households, public administration bodies, export flows, non-profit organizations serving households (NPOs), gross fixed capital formation, etc. Denote the elements of the second quadrant $\|Z_i^j\|$, $i = 1, \ldots, m, j = m+1, \ldots, m+k$. The third quadrant of the table, with elements

$\|Z_i^j\|, i = m+1, \ldots, m+n, j = 1, \ldots, m$, collects rows with net taxes on products, gross value added components and imported inputs of industries. Thus, the third quadrant contains the information on factors (primary inputs) used by the economy. We consider the economy with n factors (primary inputs). Recall that we consider the aggregate final consumer with the utility function $F_0(X^0)$ in the model. Therefore, we aggregate the final consumption into one column vector $Z^0 = (Z_1^0, \ldots, Z_m^0)^*$ with

$$Z_i^0 = \sum_{j=m+1}^{m+k} Z_i^j, i = 1, \ldots, m. \tag{18}$$

Denote

$$A_j = \sum_{i=1}^{m+n} Z_i^j, j = 1, \ldots, m.$$

Values A_j correspond to the total inputs (intermediate and primary), consumed by the pure industry $j = 1, \ldots, m$ and are usually presented as the last row of the initial symmetric input–output table. Due to the symmetry of the IO table, the value A_j equals the total consumption of product $j = 1, \ldots, m$ in the economy and gives us the last column of the initial symmetric IO table, i.e., $A_j = \sum_{i=1}^{m+k} Z_j^i, j = 1, \ldots, m$ and $\sum_{j=1}^{m} \sum_{i=m+1}^{m+n} Z_i^j = \sum_{j=1}^{m} \sum_{i=m+1}^{m+k} Z_j^i$. We illustrate the scheme of the symmetric IO table in the Table 1.

Table 1. Symmetric IO table for an Economy.

		Processing Sectors			**Final Demand**			**Total Output**
		1	...	m				
Processing Sectors	1	Z_1^1	...	Z_1^m	Z_1^{m+1}	...	Z_1^{m+k}	A_1
	⋮	⋮	⋱	⋮	⋮	⋮	⋮	⋮
	m	Z_m^1	...	Z_m^m	Z_m^{m+1}	...	Z_m^{m+k}	A_m
Payments Sectors	Value Added	Z_{m+1}^1	...	Z_{m+1}^m				
	⋮	⋮	⋮	⋮				
	Imports	Z_{m+n}^1	...	Z_{m+n}^m				
Total Outlays		A_1	...	A_m				

Proposition 3. *Given the IO table Z for the base year (in prices of the base year too), we define the parameters of the production functions (16) and the utility function (17) as follows:*

$$w_i^j = \left(Z_i^j\right)^{\frac{1+\rho_j}{\rho_j}} \left(\sum_{k=1}^{m+n} Z_k^j\right)^{-\frac{1+\rho_j}{\rho_j}}, i = 1, \ldots, m+n; j = 1, \ldots, m, \tag{19}$$

$$w_i^0 = \left(Z_i^0\right)^{\frac{1+\rho_0}{\rho_0}} \left(\sum_{k=1}^{m} Z_k^0\right)^{-\frac{1+\rho_0}{\rho_0}}, i = 1, \ldots, m, \tag{20}$$

and the vector of supply of primary inputs is defined by

$$l = (l_1, \ldots, l_n), l_i = \sum_{j=1}^{m} Z_{m+i}^j, i = 1, \ldots, n. \tag{21}$$

Then the set of values of variables

$$\left\{\hat{X}_i^0 = Z_i^0, \hat{X}_i^j = Z_i^j, \hat{l}_t^j = Z_{m+t}^j, i = 1, \ldots, m; j = 1, \ldots, m, t = 1, \ldots, n\right\}, \tag{22}$$

is the solution of the convex programming problem (1)–(4) with the production functions (16) and the utility function (17).

Proof. Obviously, we have

$$\sum_{i=1}^{m} \left(w_i^0\right)^{\frac{\rho_0}{1+\rho_0}} = 1, w_i^0 \geq 0, i = 1, \ldots, m,$$
$$\sum_{i=1}^{m+n} \left(w_i^j\right)^{\frac{\rho_j}{1+\rho_j}} = 1, w_i^j \geq 0, j = 1, \ldots, m; i = 1, \ldots, m+n.$$

Substitute the values $p_1 = 1, \ldots, p_m = 1, s_1 = 1, \ldots, s_n = 1$ of the Lagrange multipliers to the constraints of the problem (1)–(4) and $p_0 = q_0(p)$, where $q_0(p)$ is the consumer price index (10). □

Note that the set of vectors (22) satisfies the conditions (5)–(8) for the functions (16) and (17) with parameters (18)–(20). Proposition 1 implies that the set (22) is the solution of the convex programming problem (1)–(4).

Thus, the constructed problem (1)–(4) with conditions (16), (17), (19), (20) explains the first quadrant $\|Z_i^j\|, i, j = 1, \ldots, m$ and the third quadrant $\|Z_i^j\|, i = m+1, \ldots, m+n$, $j = 1, \ldots, m$ of the symmetric IO table in the base year.

Remark 2. *Proposition 3 implies that the parameters of elasticity of substitution $\rho_j, \rho_0 \in (-1, 0) \cup (0, +\infty)$, $j = 1, \ldots, m$ remain free parameters of the model in the solution of the inverse problem of identification.*

2.3. Forecasts of Symmetric IO Tables in the Model with CES Technologies, the Problem of Estimating the Elasticity of Substitution of Inputs

Let the base year be fixed and identify the parameters of the CES production functions and utility function by (18) and (19) for any $\rho_j, \rho_0 \in -1, 0) \cup 0, +\infty$. Assume that we know the price indexes on primary inputs $s = (s_1, \ldots, s_n)$ for the projected year t related to the base year and the aggregate vector of the spending of final consumers $\hat{Z}^0 = \left(\hat{Z}_1^0, \ldots, \hat{Z}_m^0\right)$ in prices of year t. If so, we can evaluate the first quadrant $\|\hat{Z}_i^j\|, i, j = 1, \ldots, m$ and the third quadrant $\|\hat{Z}_i^j\|, i = m+1, \ldots, m+n, j = 1, \ldots, m$ of the symmetric IO table for the projected year t. Note that $\hat{Z}_i^j = p_j X_i^j$ in terms of our model.

It follows from Remark 1 that the dual description of the utility function and of the j-th technology (9) and (10) have the form

$$q_0(p) = \left(\sum_{i=1}^{m} \left(w_i^0 p_i\right)^{\frac{\rho_0}{1+\rho_0}}\right)^{\frac{1+\rho_0}{\rho_0}}, \tag{23}$$

$$q_j(p, s) = \left(\sum_{i=1}^{m} \left(w_i^j p_i\right)^{\frac{\rho_j}{1+\rho_j}} + \sum_{k=1}^{n} \left(w_{m+k}^j s_k\right)^{\frac{\rho_j}{1+\rho_j}}\right)^{\frac{1+\rho_j}{\rho_j}}, j = 1, \ldots, m, \tag{24}$$

where the parameters w_{m+k}^j, w_i^j, w_i^0, $i, j = 1, \ldots, m$, $k = 1, \ldots, n$ are defined by (19) and (20).

Denote

$$a_{ij} = \left(w_i^j\right)^{\frac{\rho_j}{1+\rho_j}} = \frac{Z_i^j}{\sum_{k=1}^{m+n} Z_k^j}, i = 1, \ldots, m, j = 1, \ldots, m, \tag{25}$$

$$b_{kj} = \left(w_{m+k}^j\right)^{\frac{\rho_j}{1+\rho_j}} = \frac{Z_{m+k}^j}{\sum_{k=1}^{m+n} Z_k^j}, \; i = 1, \ldots, m, j = 1, \ldots, m, k = 1, \ldots, n, \tag{26}$$

$A = \|a_{ij}\| - (m \times m)$ matrix, $B = \|b_{kj}\| - (n \times m)$ matrix, $E - (m \times m)$ identity matrix, where Z is the symmetric IO table for the base year. Note that A corresponds to the classical Leontief matrix of direct costs that does not depend on ρ_j.

Proposition 4. *The vector* $p = (p_1, \ldots, p_m) \geq 0$ *is the solution of the dual problem (13)–(15) with the CES functions (23) and (24) for the given vectors* $= (s_1, \ldots, s_n)$ *only if* $p = (p_1, \ldots, p_m)$ *is the solution of the following nonlinear system of equations:*

$$\left(\sum_{i=1}^{m} a_{ij} (p_i)^{\frac{\rho_j}{1+\rho_j}} + \sum_{k=1}^{n} b_{kj} (s_k)^{\frac{\rho_j}{1+\rho_j}} \right)^{\frac{1+\rho_j}{\rho_j}} = p_j,\ s = (s_1, \ldots, s_n), j = 1, \ldots, m. \tag{27}$$

Proof. The inequalities (14) are equivalent to the inequalities

(i) $$\begin{aligned} &\left(1 - a_{jj}\right)\left(p_j\right)^{\frac{\rho_j}{1+\rho_j}} - \sum_{\substack{i=1 \\ i \neq j}}^{m} a_{ij} (p_i)^{\frac{\rho_j}{1+\rho_j}} \\ &\leq \sum_{k=1}^{n} b_{kj} (s_k)^{\frac{\rho_j}{1+\rho_j}} \end{aligned} \quad \text{if} \quad \begin{aligned} &\rho_j \in (0, +\infty), \\ &j = 1, \ldots, m. \end{aligned}$$

or to the inequalities

(ii) $$\begin{aligned} &-\left(1 - a_{jj}\right)\left(p_j\right)^{\frac{\rho_j}{1+\rho_j}} + \sum_{\substack{i=1 \\ i \neq j}}^{m} a_{ij} (p_i)^{\frac{\rho_j}{1+\rho_j}} \\ &\leq -\sum_{k=1}^{n} b_{kj} (s_k)^{\frac{\rho_j}{1+\rho_j}}, \end{aligned} \quad \text{if} \quad \begin{aligned} &\rho_j \in (-1, 0), \\ &j = 1, \ldots, m. \end{aligned}$$

It follows from (25) that $a_{ij} < 1$, then $1 - a_{jj} > 0$, so the left part of the of the j-th inequality (i) or (ii) is increasing on $p_j, j = 1, \ldots, m$. The function (13) is nondecreasing on $p_i, i = 1, \ldots, m$. Obviously, the solution $p = (p_1, \ldots, p_m)$ of the problem (13)–(15) satisfies to the system of equalities (27). □

Proposition 4 implies the price indexes of goods $p_j(s)$, $j = 1, \ldots, m$ in the supply network can be found for any given values of primary price indexes $s = (s_1, \ldots, s_n)$ of the projected year and the fixed value of ρ_j, $j = 1, \ldots, m$.

We assume that the CES technologies are stable over several years to evaluate the first and third quadrant of the symmetric IO table in the projected year. We solve the resource allocation problem (1)–(4) with CES technologies (16) and CES utility function (17) for the given vector of final consumption $\hat{Z}^0 = \left(\hat{Z}_1^0, \ldots, \hat{Z}_m^0\right)$ of the projection year t. Obviously, from (5) for the given values of $s = (s_1, \ldots, s_n)$ in the projection year t we have

$$p_j(s) \frac{\partial F_j\left(X_1^j, \ldots, X_m^j, l_1^j, \ldots, l_n^j\right)}{\partial X_i^j} = p_i(s),\ i = 1, \ldots, m, j = 1, \ldots, m, \tag{28}$$

$$p_j(s) \frac{\partial F_j\left(X_1^j, \ldots, X_m^j, l_1^j, \ldots, l_n^j\right)}{\partial l_i^j} = s_i,\ i = 1, \ldots, n, j = 1, \ldots, m, \tag{29}$$

where $p_j(s)$, $j = 1, \ldots, m$ we calculate from the system (27).

Additionally, it follows from (6) that on the solution of the problem the following equality holds:

$$p_j F_j\left(X_1^j, \ldots, X_m^j, l_1^j, \ldots, l_n^j\right) = \hat{Z}_j^0 + \sum_{i=1}^{m} \hat{Z}_j^i,\ j = 1, \ldots, m. \tag{30}$$

Denote the output of the j-th pure industry

$$\hat{Y}_j = p_j F_j\left(X_1^j, \ldots, X_m^j, l_1^j, \ldots, l_n^j\right),\ j = 1, \ldots, m. \tag{31}$$

Due to the form of the CES function (16) we obviously have

$$\frac{\partial F_j\left(X_1^j,\dots,X_m^j,l_1^j,\dots,l_k^j\right)}{\partial X_i^j}=\left(\frac{X_i^j}{w_i^j}\right)^{-(1+\rho_j)}\frac{1}{w_i^j}\left(F_j\left(X_1^j,\dots,X_m^j,l_1^j,\dots,l_k^j\right)\right)^{1+\rho_j},\quad(32)$$

$$\frac{\partial F_j\left(X_1^j,\dots,X_m^j,l_1^j,\dots,l_k^j\right)}{\partial l_i^j}=\left(\frac{l_i^j}{w_{m+i}^j}\right)^{-(1+\rho_j)}\frac{1}{w_{m+i}^j}\left(F_j\left(X_1^j,\dots,X_m^j,l_1^j,\dots,l_k^j\right)\right)^{1+\rho_j}.\quad(33)$$

Then (28), (31), (32) imply that

$$\hat{Z}_i^j=\hat{Y}_j\left(w_i^j\frac{p_i(s)}{p_j(s)}\right)^{\frac{\rho_j}{1+\rho_j}}=a_{ij}\hat{Y}_j\left(\frac{p_i(s)}{p_j(s)}\right)^{\frac{\rho_j}{1+\rho_j}},i=1,\dots,m,j=1,\dots,m.\quad(34)$$

Similarly, (29), (31), (33) imply that

$$\begin{gathered}\hat{Z}_{m+k}^j=\hat{Y}_j\left(w_{m+k}^j\tfrac{s_k}{p_j(s)}\right)^{\frac{\rho_j}{1+\rho_j}}=b_{k,j}\hat{Y}_j\left(\tfrac{s_k}{p_j(s)}\right)^{\frac{\rho_j}{1+\rho_j}},\\ k=1,\dots,m,j=1,\dots,m.\end{gathered}\quad(35)$$

Obviously, we obtain the following balance equalities from (30), (34), (35):

$$\hat{Y}_i=\hat{Z}_i^0+\sum_{j=1}^{m}a_{ij}\hat{Y}_j\left(\frac{p_i(s)}{p_j(s)}\right)^{\frac{\rho_j}{1+\rho_j}},\ i=1,\dots,m,j=1,\dots,m,$$

i.e., in the vector form

$$\hat{Y}=\Lambda\hat{Y}+\hat{Z}^0,\quad(36)$$

where $\Lambda=\|\lambda_{ij}\|$ is $(m\times m)$-matrix with the elements

$$\lambda_{ij}=a_{ij}\left(\frac{p_i(s)}{p_j(s)}\right)^{\frac{\rho_j}{1+\rho_j}},i=1,\dots,m,j=1,\dots,m,\quad(37)$$

and $p_j(s), j=1,\dots,m$ are defined from (27).

We assume that the matrix $\Lambda=\|\lambda_{ij}\|$ is productive. Then the inverse matrix $(E-\Lambda)^{-1}\geq 0$ exists. We evaluate the output vector $\hat{Y}$ of the projection year t from the balance (36)

$$\hat{Y}=(E-\Lambda)^{-1}\hat{Z}^0.\quad(38)$$

Thus, the equalities (34) and (35) with the output vector $\hat{Y}$ defined by (38) give us the elements of the quadrants I and III of the symmetric IO matrix for the projection year t.

Recall that the all obtained results are correct formally for any fixed values of elasticities of substitution $\rho_j\in(-1,0)\cup(0,+\infty)$, $j=1,\dots,m$. The evaluation of the parameters ρ_j $j=1,\dots,m$ is based on the comparison of the results of the calculation with the actual statistics of IO tables in the year t. The quality of the model predictions depends on the sustainability of the elasticities over a several years. Therefore, the evaluation of elasticities requires the official statistics of IO tables of the economy over several years. The base of further calculations are the symmetric IO tables of Kazakhstan from 2013–2020, which were annually published in the same nomenclature of pure industries. In the next section, we verify the model based on the given sequence of IO tables from 2013–2020. As a result, we evaluate the elasticities of substitution of inputs for four large complexes of Kazakhstan's economy.

3. Results

3.1. Analysis of the Economic Structure of Kazakhstan, Evaluation of the Elasticities of Substitution of the Inputs of the Industrial Complexes of Kazakhstan

The database for the analysis includes IO tables of the economy of Kazakhstan from 2013–2020 published as the part of National Accounts by the Agency for Strategic Planning and Reforms (https://stat.gov.kz, accessed on 20 April 2022).

Based on the data, we estimate the elasticities of substitution in terms of the model for the industrial complexes of Kazakhstan's economy and analyze their stability over the period 2013–2020.

We aggregate the initial nomenclature of pure industries and specify the primary inputs in terms of the model taking into account the main structural characteristics of the Kazakhstan economy.

The key middle-term problem of the economy in Kazakhstan is to reduce its dependency on raw material exports and to provide innovative development of the manufacturing sector. Macro indicators in Figure 1 show the essential dependency of the economy on the situation in the raw materials markets. The export of natural resources generates large economic rent that is the main source of revenue for the government. The export volume is 36.4% of GDP according to 2019. At the same time, the share of processed product is 29% of the total exports and 93% of the total imports of the products in Kazakhstan (www.damu.kz, 2019, accessed on 20 April 2022). The output of the labor-intensive manufacturing sector (11.4% of GDP, 2019) depends on the imported inputs. The products of manufacturing sector compete in the domestic market with imported counterparts. More than half of GDP comes from the service sector (55.4% of GDP, 2019). In [27], we analyzed the modern sectoral structure of the economy of Kazakhstan using the Leontief approach and the nonlinear model of input-output balance with Cobb–Douglas technologies. Based on the calculation of various measures of centrality, the dominant sectors in the economy were identified. It is shown in [27] that the export-oriented oil and gas complex and the connected export-oriented wholesale trade sector are the dominant sectors in the economy of Kazakhstan. Thus, the economy of Kazakhstan is highly heterogeneous in relation to export–import operations, which is an important feature in the current conditions of restructuring global markets.

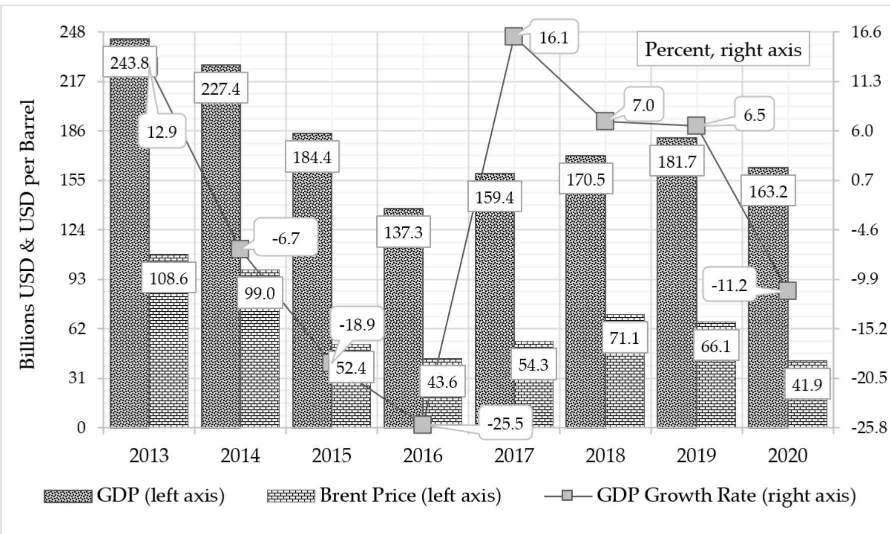

Figure 1. Kazakhstan GDP value, GDP growth and Brent world price, 2013–2020.

Subject to the above conditions, we consider the two types of primary inputs in the model: import and labor.

Recall the notation in Section 2.2: we construct the three quadrants of the initial symmetric IO table Z on the basis of given symmetric tables "use of domestic goods" and "use of imported goods" in basic prices (the part of IO tables of Kazakhstan). The use tables are symmetric commodity-by-commodity matrixes with 68 products (pure industries). The notation of the industries has no changes in 2013–2020. We build the raw "intermediate use of imported goods" in the third quadrant of the table $\|Z_i^j\|$ from the data of the table "use of imported goods" that is published part of the system of national accounts. We aggregate the third quadrant of the symmetric IO table $\|Z_i^j\|, i = m+1, \ldots, m+n, j = 1, \ldots, m$ in two rows: intermediate use of import and gross value added that we roughly interpret as labor supply, so $n = 2, m = 68$. The first quadrant $\|Z_i^j\|, i, j = 1, \ldots, 68$ is the same with the first quadrant of the symmetric table "use of domestic goods". The second quadrant has one column $Z^0 = \left(Z_1^0, \ldots, Z_m^0\right)^*, m = 68$ and is identical to the sum of final consumption of each product (see (18)). As a result, the symmetric IO table that is the input of our model has a desired structure.

In this paper, in terms of the constructed framework, we evaluate elasticities of substitution for the aggregated complexes of the industries of Kazakhstan's economy. The quality of the model estimates depends on the splitting of the set of pure industries into groups. Based on the analysis of the main features of the Kazakhstan economy we consider the four main industrial complexes, which are defined by their involvement in the export–import operations: manufacturing, exporting, infrastructure and services. Manufacturing complex aggregates the pure industries that have high dependence on imported inputs. Their final products compete with import analogues in the domestic market. The exporting complex covers industries with a big share of export and includes mainly oil-and-gas-related industries (including services) as well as some other industries with significant export shares in their value-added products. The infrastructure complex aggregates electric power, transport and communication, which have a stable position in the economy and do not compete with imports. The fourth complex, named services, collects pure industries of trade and service with the exception of oil-and-gas services (for ex., "wholesale trade"). More detailed splitting into complexes is presented in Appendix A. Note that the splitting into complexes for the model purposes differs from the standards of the official statistics. Figures 2 and 3 illustrate a few macroeconomic statistics of the industrial complexes specified in the model that substantiate the considered aggregates. Thus, in notation of the model $m = 4, n = 2, k = 1$.

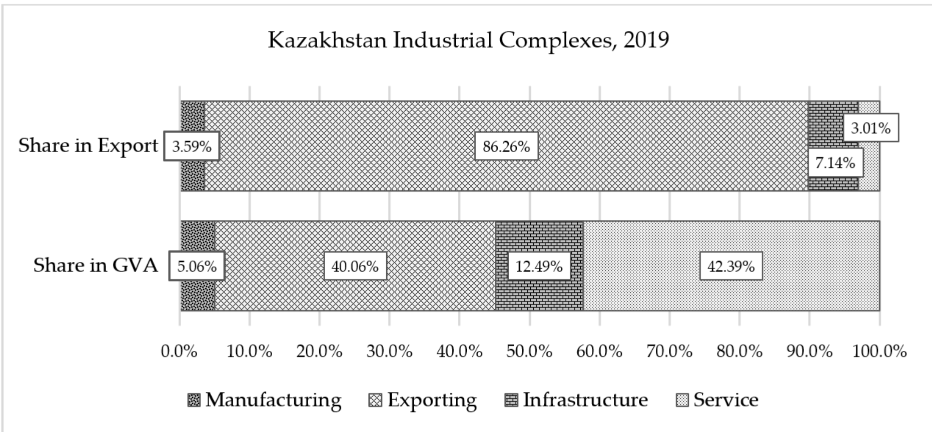

Figure 2. Macroeconomic characteristics of industrial complexes of Kazakhstan (statistics).

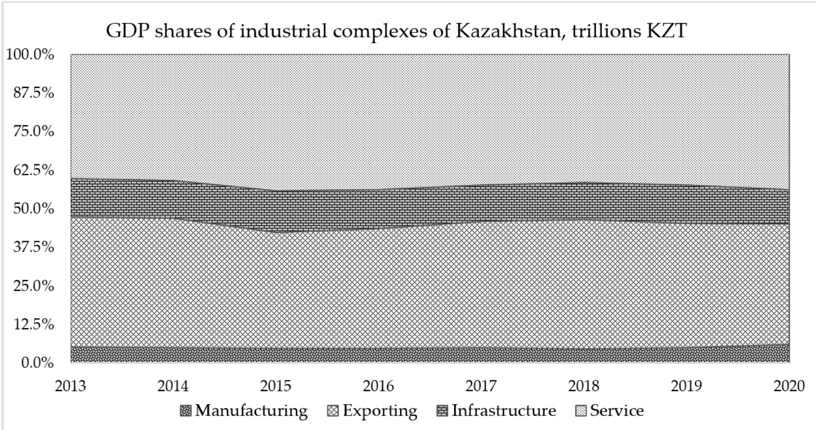

Figure 3. Shares of industrial complexes in gross domestic product of Kazakhstan (statistics).

We set 2013 as the base year in the model and solve the inverse problem of identification based on the results of the Section 2.2. For the projection of symmetric IO table to the year t with the model we consider the following block of initial data (see Section 2.3).

1. Price indexes $s(t) = (s_1(t), s_2(t))$ on the primary inputs (labor and import) in a projection year against the base year t. The row $s_1(t)$ corresponds to the consumer price index of Kazakhstan 2014–2020 against the base year (2013). The row $s_2(t)$ corresponds to KZT–USD exchange rate index 2014–2020 against 2013. These rows are from the official statistics of Kazakhstan (https://stat.gov.kz/official/dynamic, accessed on 20 April 2022). The dynamics of the price indexes are shown in Figure 4.

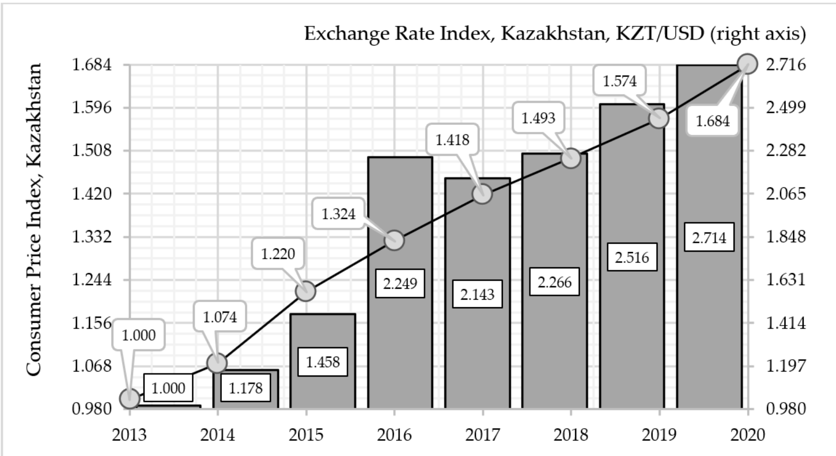

Figure 4. Price indexes of primary resources and KZT–USD exchange rate index, 2013–2020 (statistics, 2013 = 1).

2. The vector of total final consumption (in current prices) for the base year (2013) and for the projection year t. Let $Z^0(t) = \left(Z_1^0(t), \ldots, Z_m^0(t)\right)$, $m = 4$ is the column vector of final consumption for the year t, which equals the sum of columns of the second quadrant of the symmetric IO table $\|Z_i^j(t)\|, i = 1, \ldots, m, j = m+1, \ldots, m+k, m = 4$.

We turn now to the verification of the model that allows us to evaluate the elasticities of inputs. Recall that the rows of the third quadrant of the symmetric IO table have the following meaning:

$Z_{m+2}^j(t)$—statistics of import intermediate inputs, in current prices ("import used") in complex $j = 1,\ 2,\ 3,\ 4$ in the year t.

$Z_{m+1}^j(t)$—statistics of gross value added, in current prices (GVA) in complex $j = 1,\ 2,\ 3,\ 4$ in the year t.

The modern period of the evolution of Kazakhstan's economy is essentially connected with the dynamics of export–import financial flows. Therefore, the suitable criterion for the estimation of the parameters of elasticity of substitution ρ_j, $j = 1, 2, 3, 4$ seems to be the ratio between "import used" and value added for each industrial complex. From the projection of the rows of the primary inputs for the year t, we calculate from (35):

$$\hat{Z}^j_{m+2} = \hat{Z}^j_{m+2}(\rho_j, t),\ \hat{Z}^j_{m+1} = \hat{Z}^j_{m+1}(\rho_j, t),\ j = 1, 2, 3, 4.$$

Thus, we find the value of ρ_j from the following equation:

$$\frac{\hat{Z}^j_{m+2}(\rho_j, t)}{\hat{Z}^j_{m+1}(\rho_j, t)} = \frac{Z^j_{m+2}(t)}{Z^j_{m+1}(t)},\ \ j = 1, 2, 3, 4. \tag{39}$$

Obviously, from (35) and (39) we have

$$\rho_j(t) = \frac{ln\left(\frac{b_{2,j}}{b_{1,j}}\frac{Z^j_{m+1}(t)}{Z^j_{m+2}(t)}\right)}{ln\left(\frac{s_2(t)}{s_1(t)}\frac{b_{1,j}}{b_{2,j}}\frac{Z^j_{m+2}(t)}{Z^j_{m+1}(t)}\right)},\ \ j = 1, 2, 3, 4.$$

We define the elasticity of substitution parameter of each industrial complex in our model as the mean value of the results of the evaluations for the years $t = 2014, \ldots, 2020$

$$\rho_j = \frac{1}{7}\sum_{t=2014}^{2020}\rho_j(t),\ \ j = 1, 2, 3, 4.$$

Recall that we should turn to the initial sense of the parameters (see (16)), i.e., the elasticity of substitution is defined by

$$\sigma_j = \frac{1}{1+\rho_j},\ j = 1, 2, 3, 4.$$

Table 2 presents the results of evaluations of parameters of CES technologies ρ_j and the corresponding elasticities of substitution for the industrial complexes of Kazakhstan. Note, that the parameters ρ_j are internal parameters that we estimate by verification of the model on the basis of IO statistics of national accounts.

Table 2. Evaluation of elasticities of substitution.

Estimation/Industrial Complex	Manufacturing	Exporting	Infrastructure	Services
Parameter of CES technology, ρ_j	−0.51	0.42	0.98	−0.29
Elasticity of substitution, σ_j	2.03	0.70	0.51	1.41

The results in Table 2 complete the solution of the inverse identification problem and verification of the model. Thus, the nonlinear model of intersectoral balance with CES technologies is constructed, which allows one to perform scenario calculations and evaluate the symmetric IO table for a projection year. We summarize inputs and outputs of the model in Table 3.

Table 3. Inputs and outputs of the model.

Input Parameters (for a Projection Year *t*)	Output Variables (for a Projection Year *t*)	
Price indexes on the primary inputs (labor and import against the base year t) : $s(t) = (s_1(t), s_2(t))$	Symmetric *IO table (quadrant I)*	$\hat{Z}_i^j, i, j = 1, \ldots, m$
	Value Added of sectors	$\hat{Z}_{m+1}^j j = 1, \ldots, m$
Total final consumption (in current prices) $Z^0(t) = \left(Z_1^0(t), \ldots, Z_m^0(t)\right)$	*Import Used* in sectors	$\hat{Z}_{m+2}^j \quad j = 1, \ldots, m$
	Output of sectors	$\hat{Y}_i, \quad i = 1, \ldots, m$

In Section 3.2 , we discuss the quality of evaluations and limitations of the framework.

3.2. Model Validation

Still considering 2013 as the base year, we fix the parameters ρ_j, $j = 1, 2, 3, 4$ with values from Table 2. With the fixed parameters ρ_j, $j = 1, 2, 3, 4$ (similar to the previous section) we evaluate the symmetric IO tables for 2014–2020. To analyze the quality of evaluations we compare the evaluated values with the corresponding statistics of the macroeconomic characteristics of industrial complexes in 2014–2020. We observe the following macroeconomic characteristics of each complex: gross value added (GVA), total output, intermediate use of imported goods ("import used"). Figure 5 presents the results of evaluation of the macroeconomic parameters of the whole economy as well as the corresponding statistics (in current prices). The same results for the industrial complexes are shown in Appendix B (Figures A1–A4). Relative forecast errors ((forecast-actual)/actual, 2014–2020) for each industry complex are shown in Figure 6.

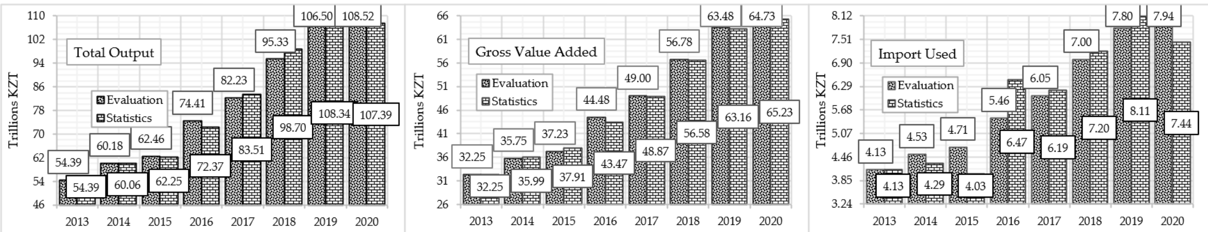

Figure 5. Macroeconomic characteristics of industrial complexes 2013–2020.

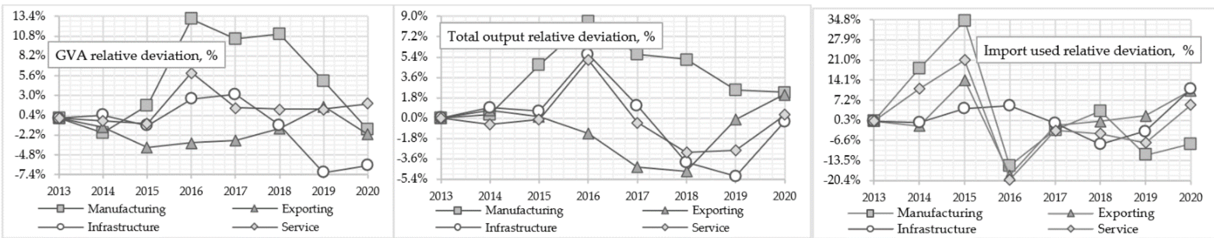

Figure 6. Evaluation errors of macroeconomic characteristics of industrial complexes.

Comparing the predicted values of the model with the actual data indicates that the largest deviation is observed in 2015–2016. This may be attributed to a set of factors of instability of Kazakhstan's economy in the period, which is confirmed by the official data (Figure 1). It was the recession in the economy in the period 2015–2016, which had been coupled with the decrease in GDP. The drop in world oil prices led to the essential decline in export volumes. The consumer price index rose more than 20% over that in 2015–2016 due to a sharp depreciation of the national currency (Figure 4) that resulted in a significant decline in external trade and led to significant instability of elasticities of substitution of products in 2015–2016. Therefore, the input–output model with CES technologies does not seem appropriate for the period 2015–2016.

The forecasts during non-recession years are more precise, with small systematic errors (not more than 10%, except "import used" in the early recession period in 2014). Note that the model gives the successful forecasts of gross value added (GVA) and total output of complexes during the COVID-19 pandemic year 2020. The drop of "import used" 2020 does not fit to the model because the global trade value plummeted in 2020 when many countries implemented lockdowns. Note that the manufacturing complex has the worst indicators for the forecast in comparison to the other complexes. Additionally, the manufacturing complex is the main consumer of imported goods. It could mean that the processes of import substitution should be specified in the model. That subject needs further investigation.

Thus, the comparison of the model assessments with the retrospective input–output statistics for several years allows us to calibrate the model and evaluate the elasticities of inputs. In addition to that, we can see that the years with a high currency exchange rate volatility limit the possibilities of identification of the model. However, if the initial dataset of IO tables demonstrates periods of sustainable economic growth, then we can estimate the elasticities of inputs that allows us to evaluate these tables with appropriate accuracy. Note, that the solution of the inverse problem of identification gives us the precise values of the input-output flows in the base year (see Proposition 3) for any values of elasticities.

If the initial dataset of IO tables is enough for elasticities of inputs assessment, then on the basis of the model, we obtain a useful tool for the interindustry connections' medium-term analysis and forecasting in scenarios of government program implementation or exogenous shocks. By changing of the structure and volume of final demand and prices for primary resources, we can construct the input of the model in scenarios that are connected to the economic policy and the various kinds of external shock effects on the economy (crises, pandemic, other factors). Recall that the advantage of the model is that it takes into account the substitution of inputs in a supply network.

In the Section 4.1 we present an example of scenario calculations on the basis of the model to analyze the limits of the sustainability of Kazakhstan's interindustry connections in the case of currency exchange rate shock.

4. Discussion

4.1. An Example of Scenario Calculation, the Analysis of Medium-Term Macroeconomic Risks of Kazakhstan's Economy Caused by External Shocks

Kazakhstan is integrated into the system of world economic ties, so Kazakhstan's economy suffers indirectly from global challenges and shocks. In this section we discuss the macroeconomic consequences of foreign currency exchange shocks that may be caused by the external shocks. It is known that the Kazakh tenge (KZT) is tied to regional currencies of the Chinese yuan (CNY) and the Russian ruble (RUB), and at the same time, the KZT is tied to the USD due to the big oil-and-gas export volumes. In order to mitigate the indirect effects of external shocks the medium-term government economic policy may reorient the main financial flows from the foreign currency zone to the dollar zone that affects the KZT-USD pair. A possible change in exchange rate makes imports more expensive. In the case of the KZT weakening, the model allows us to analyze the middle-scale macroeconomic characteristics of the import substitution possibilities of the economy.

Since 2020 was the year of the pandemic collapse, we consider the more stable 2019 for the scenario calculations. We suggest that the modern interindustry connections in the Kazakhstan economy change slowly in relation to 2019. The exchange rate shock is modelled by a two-times weakening of the KZT to the USD. In terms of the model, we double the exchange rate index $s_2(2019)$. Empirical data from Kazakhstan shows that the total volume of exports (in current domestic prices) of the four industrial complexes repeats with a close amplitude the currency exchange fluctuations in periods of currency rate instability (2015–

2017). That confirms our assumption about a proportional growing of export volumes in current prices with the rate of KZT to USD growth. Therefore, we calculate the export volumes of each complex from 2019 in new prices and form a new vector of final consumption $Z^0(2019) = \left(Z_1^0(2019), \ldots, Z_m^0(2019)\right)$ according to the new export volumes and the share of the export of each complex in the final consumption of its products.

In Figure 7, we present the results of scenario calculations and the actual 2019 data for the industrial complexes of Kazakhstan that are highlighted in the model. The percentage above the results of scenario calculations (columns "Shocked Forecast", Figure 7) show the change of the corresponding characteristic. As we can see, the two-time weakening of the KZT to the USD does not produce a recession in Kazakhstan's economy. The shock of exchange rate results in the export-led economic growth of Kazakhstan. The growth of export income leads to growth in the total output and GVA for industrial complexes despite of the increasing costs of imported inputs of industries. Note that the volumes of "import used" decline in manufacturing and service while the GVA and total output demonstrate growth at the same time. This result shows that the Kazakhstan economy has a potential for import substitution to overcome the shocks of import flows on the macroeconomic level. However, the export-led economic growth strengthens the heterogeneity of the Kazakhstan economy. Recall that we divide the economy of Kazakhstan into industrial complexes depending on their foreign trade intensity. Scenario calculation shows that the sharp weakening of the KZT to the USD course results in disproportional growth of the exporting complex in comparison to the manufacturing complex. That means increasing of the Kazakhstan economy's dependence on resource export. The exporting complex is capital-intensive and not labor-intensive. At the same time, the manufacturing complex is labor-intensive. Thus, the further heterogeneity of the economy is accompanied by risks of social problems. The situation can be remedied through the industrial diversification of export and the reorientation of oil-export revenue to the strategic investments in the manufacturing complex. These processes depend on the sensible economic policy that should be guided by middle- and long-term growth objectives.

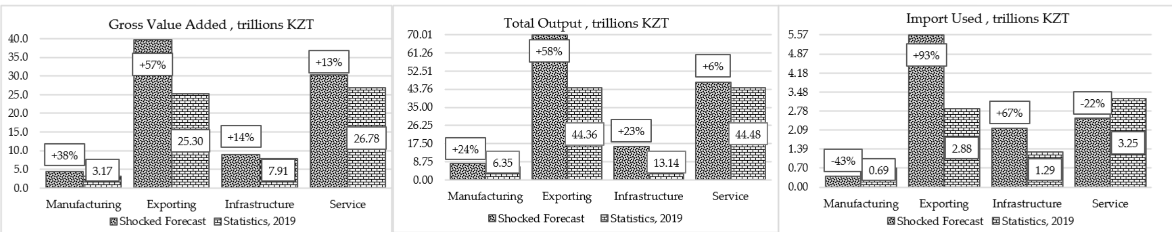

Figure 7. The macroeconomic response to the two-time exchange rate weakening (KZT/USD).

5. Conclusions

Economic globalization stimulates decision-making support system technology development based on information on the intersectoral flows of goods and services to increase government economic policy efficiency for achieving higher and more sustainable growth. It is an especially relevant task for so-called "catching-up economies", and the Kazakhstan economy is one of them. Decision-making support systems should take into account the features of sectoral structure of a state economy and can help analyze and improve efficiency of government projects of sustainable development. Mathematical models as a part of such systems allow us to investigate hidden causal links and data consistency and therefore are relevant tools for decision-making support system technologies development.

This paper presents the nonlinear interindustry balance model with CES production that is clearly interpretable in terms of the official statistics of national accounts. Note that the goal of the present research was to develop a clear useful tool for the analysis of risks for catching-up, heterogeneous economies in the case of external shocks, taking into account the features of the regional economy and the substitution of inputs. The preliminary analysis and assessments show that the framework can be successfully used as a part of a policy decision-making support system for measurements of the macroeconomic effects of global shocks (such as economic crises, external trade falls, pandemics, etc.).

An important aspect of the model implementation is the potential possibility of interindustry analysis for the detailed structure of industries in the economy and the estimation of elasticities of inputs for the large numbers of industries in complicated supply-networks. Formally, the inverse problem of the identification of the model in this case is reduced to a global optimization problem. We intend to analyze such problems in our further studies.

Author Contributions: Conceptualization, S.K., N.O. and A.S.; methodology, S.K., N.O. and A.S.; software, S.K., N.O. and G.A.; validation, S.K., N.O. and G.A.; formal analysis, S.K., N.O., A.S. and G.A.; investigation, S.K., N.O., A.S. and G.A.; resources, S.K., N.O., A.S. and G.A.; writing—original draft preparation, S.K., N.O., A.S. and G.A.; writing—review and editing, S.K., N.O., A.S. and G.A.; visualization, S.K., N.O. and G.A.; supervision, S.K. and N.O.; project administration, S.K. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP09259435).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: We express gratitude to the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan for its support through the realization of this research (Grant No. AP09259435).

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Table A1. Aggregate complexes of pure industries of Kazakhstan.

Manufacturing	Exporting	Infrastructure	Services
Fishing and aquaculture Food products, beverages Wearing apparel Manufacture of coke Manufacture of basic pharmaceutical products and pharmaceutical preparations Manufacture of rubber and plastic products Manufacture of other non-metallic mineral products Manufacture of fabricated metal products, except machinery and equipment Manufacture of motor vehicles, trailers and semi-trailers Manufacture of other transport equipment Manufacture of furniture Manufacture of furniture; other manufacturing Repair and installation of machinery and equipment	Crop and animal production, hunting and related service activities Forestry and logging Mining and quarrying Tobacco products Manufacture of textiles Leather products Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials Manufacture of paper and paper products Refined petroleum products Manufacture of chemicals and chemical products Manufacture of basic metals Manufacture of fabricated metal products, except machinery and equipment Manufacture of computer, electronic and optical products Manufacture of electrical equipment Manufacture of machinery and equipment n.e.c. Wholesale trade, except of motor vehicles and motorcycles	Electricity Land transport and transport via pipelines Water transport Air transport Warehousing and support activities for transportation Information services Telecommunications	Gas distribution services Steam and air conditioning supply Water collection, treatment and supply Construction Wholesale and retail trade and repair of motor vehicles and motorcycles Retail trade, except of motor vehicles and motorcycles Postal and courier activities Accommodation and food service activities Food products and beverages supply services Financial service activities, except insurance and pension funding Insurance, reinsurance and pension funding, except compulsory social security Activities auxiliary to financial services and insurance activities Real estate activities Scientific research and development, other professional, scientific and technical activities; Administrative and support service activities Public administration and defense; compulsory social security Education Human health and social work activities Legal and accounting activities; activities of head offices; management consultancy activities Entertainments Other service activities Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use

Appendix B

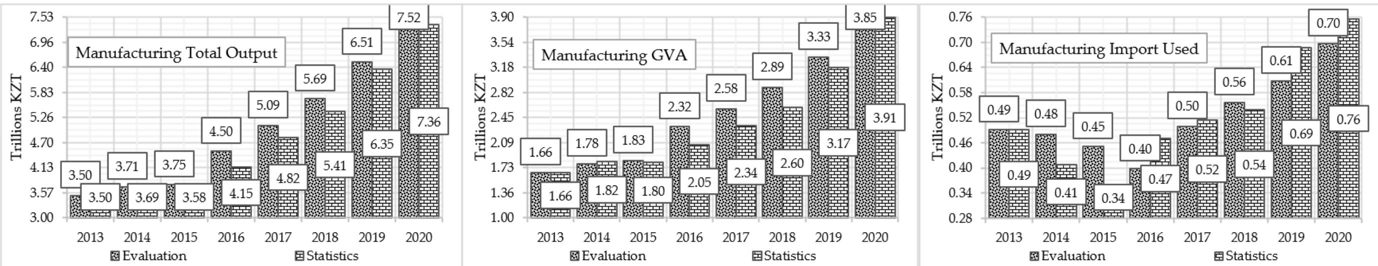

Figure A1. Evaluation of the main macroeconomic characteristics of industrial complexes of Kazakhstan by the nonlinear model of IO balances with CES technologies: Manufacturing complex.

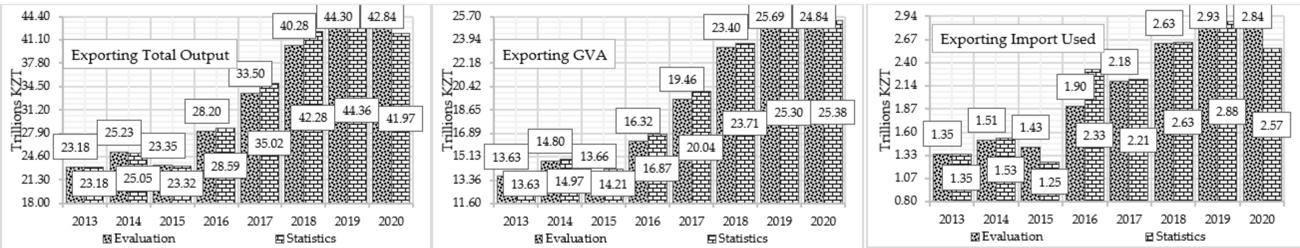

Figure A2. Evaluation of the main macroeconomic characteristics of industrial complexes of Kazakhstan by the nonlinear model of IO balances with CES technologies: Exporting complex.

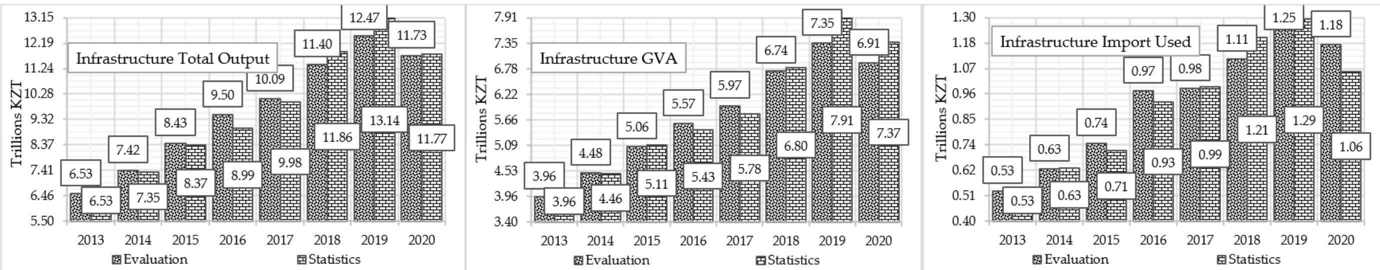

Figure A3. Evaluation of the main macroeconomic characteristics of industrial complexes of Kazakhstan by the nonlinear model of IO balances with CES technologies: Infrastructure complex.

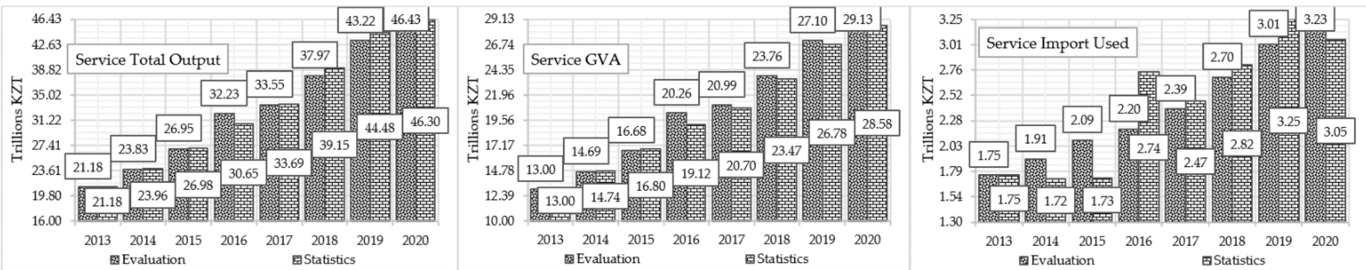

Figure A4. Evaluation of the main macroeconomic characteristics of industrial complexes of Kazakhstan by the nonlinear model of IO balances with CES technologies: Service complex.

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