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Cost Optimization of Prestressed U-Shaped Simply Supported Girder Using Box Complex Method

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Abstract: The use of U-shaped girders has become increasingly popular in advanced projects such as metro rail systems due to their ability to provide greater vertical clearance beneath bridges. These girders, characterized by two webs and a bottom flange, contribute essential longitudinal stiffness and strength to the overall structure while effectively countering torsional forces in curved bridges. However, the design and construction of U-shaped girders present challenges, including their relatively higher self-weight compared to other girder types. Consequently, cost optimization has become a crucial focus in structural design studies. This research aims to develop an optimization model for prestressed U-shaped girders using the AASHTO LRFD bridge design specifications. The model is based on the Box complex method, with necessary modifications and improvements to achieve an optimal design. The objective is to minimize the total cost of materials, including concrete, steel reinforcement, and prestressing strands, while satisfying explicit and implicit design constraints. To facilitate the analysis, design, and optimization processes, a program is developed using Visual Studio 2010 and implemented in Visual Basic (VB.NET). The program incorporates separate subroutines for analysis, design, and optimization of the prestressed U-shaped girder, which are integrated to produce the desired results. When running the program, the optimization process required 229 iterations to converge to the optimal cost function value. The results demonstrate that the developed algorithm efficiently explores economically and structurally effective solutions, resulting in cost savings compared to the initial design. The convergence rate of the moment capacity constraint is identified as a key factor in achieving the optimal design. This research makes a significant contribution to the field of civil engineering by applying the classical Box complex method to the optimization of girders, an area where its utilization has been limited. Furthermore, this study specifically addresses the optimization of prestressed U-shaped girders in metro rail projects, where they serve as both the deck and support structure for train loading. By employing the Box complex method, this research aims to fill the research gap and provide valuable insights into the optimization of U-shaped girders. This approach offers a fresh perspective on designing these girders, considering their unique role in supporting metro rail loads. By leveraging the benefits of the Box complex method, researchers can explore new possibilities and uncover optimal design solutions for U-shaped girders in metro rail applications.

Keywords: prestressed girder; optimization technique; Box complex method; VB.Net; AASHTO LRFD bridge design specifications; global optimization; U-shaped girder

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1. Introduction

Construction of bridges has significantly increased over the last decade [1]. After the end of World War II, in North America, the basic worries of durability and economy resulted in the introduction of prestressed concrete [2]. Globally, most of the bridges built are prestressed girder bridges [3]. According to the survey, about 50% of total bridges built in the U.S. are prestressed concrete bridges [4]. When prestressed girders older than

25 years are tested, the result shows that these girders have maintained good structural strength and ductility demand [5]. The prestressed girder has become a popular and cost-effective solution in more than 30 states of the U.S. [6]. In many states of the U.S., AASHTO Bridge Design LRFD Specifications [7] have been used for many years instead of AASHTO Standard Specifications for Highway Bridges (AASHTO 1996) [8], which eventually results in how the bridges will be designed in future [9].

From the mid-1950s to the mid-1970s, a large number of bridges were constructed with non-prestressed channel beams as the superstructure of the bridge to resist both shear and flexure [10]. The U-shaped viaduct concept was initially proposed by SYSTRA in 1992–93 for Santiago Chile [11]. The U-shaped bridge (also known as a half-through or channel bridge) is made of reinforced concrete and prestressed in the longitudinal direction [12,13]. Furthermore, due to economic considerations, state Departments of Transportation (DOTs) have shown a preference for reducing the number of girders. Consequently, the U-shaped prestressed girder has emerged as a viable alternative in this context [14]. The U-shaped viaduct has many advantages, such as the level of rail, which can be reduced due to the distance between the bottom of the flange and the rail, which results in savings in the foundation and station design. The visual effect is in the side webs acting as a noise barrier and the capability of retaining the rail on the bridge in case of such unfavorable conditions as severe earthquakes [11,15–18]. A prestressed U-shaped girder is an innovative and creative concept in metro rail bridge structure design. If the bridge is exposed to twisting/torsional forces, which are produced due to the curve in the girder, the second web in a prestressed U-shaped girder provides added strength and also resists such type of torsional forces, which increase the demand on the girder. In bridges having any important curve in them, the prestressed U-shaped girder is the best choice compared to other conventional types of bridges. Prestressed U-shaped girders are also used for aesthetic purposes [15,18–21]. The prestressed U-shaped girder design was determined to provide superior resistance to collapse compared with multiple-plate girders because of its lower temperature rise and a higher moment resistance [22]. The schematic diagram of a U-shaped girder supporting a single track is shown in Figure 1.

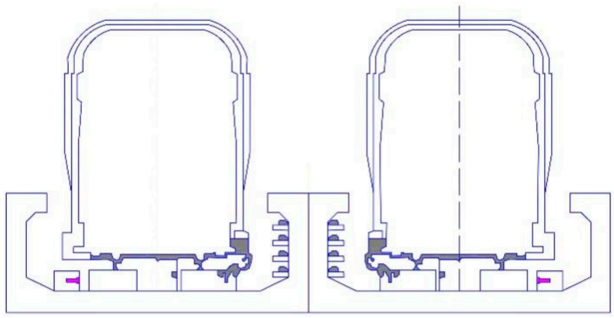

Figure 1. U-shaped girder supporting single track.

In recent years, much research has been published on structure optimization, in which the most used method is the minimum weight design [23–26]. It does not necessarily mean that the minimum weight design will be cost-effective [27]. The minimum stress design is also not cost-effective [28]. An efficient and reliable computational tool has been developed to analyze and achieve the optimal design of pre-stressed box girder bridges for high-speed railways. The optimal design is obtained by selecting appropriate design variables based on the constraint of the deflection limit [29]. Similarly, The RBF neural network algorithm has been trained using a mixed orthogonal experimental table to establish the relationship between design variables, maximum stress, and deformation. Subsequently, the design scheme of the RBF neural network is optimized using the APSO algorithm. The ultimate result is a new layout form for the box girder, ensuring that the total mass remains unchanged [30].

In the area of structural design, cost optimization has been the subject under consideration in various studies [31]. The objective of structure designers is to guarantee safety [32] and to find a design that is optimal for the structure. By optimal design, we mean the structure which is cost-effective without disturbing the practical purposes of the structure under attention [33]. The total expense of the structure is mostly the summation of the cost of its distinct component materials, which include concrete, steel reinforcement, and prestressing strands in the case of a prestressed girder [27,34]. The computation of prestressing losses also plays a crucial role in optimization problems, especially in the design of PC girder bridges. Overestimation can lead to an unreasonable camber and uneconomical design. A novel method has been successfully implemented to accurately identify the residual prestressing force in simply supported PC girder bridges. This is achieved by measuring the vertical deflection at a quarter or midspan of the PC girder bridge [35]. Similarly, non-linear discrete modeling has been used in conjunction with system identification and optimization to determine the dynamic properties of PRC beams while considering the influence of prestressing level [36]. Using the modified harmony search algorithm, new optimized sections for prestressed concrete bridge girders have been developed having various heights to facilitate the more economical design of bridges and having various geometric properties [37].

The differences in optimization of prestressed concrete girders from each other exist in the fixed parameters, design variables, design constraints, the objective (cost) function, and solution procedure algorithms [38]. With the advancement in technology and computer science, the structure optimization technique has been of great importance in order to obtain economical, lightweight, and high-performance structures [39]. The optimization method converts the conventional design procedure of trial and error to a proper, organized, and digital computer-aided programming process that yields the best design in the designer-specified limit [40]. Comparatively, the solution to the general problem has been assumed as a non-linear problem in many studies of prestressed concrete beams [41–43]. Generally, the prestressed concrete design optimization problems are nonlinear because the objective function and most of the constraints are nonlinear functions of the design variables; thus, nonlinear programming procedures are required to be used. Therefore, the Box complex method, which is a non-linear constraint optimization technique, is used in this study [44]. However, a number of techniques have been developed to accomplish the optimization objective, among which heuristic and gradient-based methods could be good examples for solution [45,46].

Most of the studies are related to the weight minimization problem, but very limited studies have been carried out on the cost optimization of structures [27]. However, classical methods, such as the Box complex method, have seen limited utilization in the field of civil engineering, particularly regarding the optimization of prestressed U-shaped girders. The optimization method employed in this study offers a systematic and digital approach, replacing the conventional trial-and-error design procedure. It effectively addresses the non-linear nature of prestressed concrete design optimization problems, where the objective function and constraints are typically non-linear functions of the design variables. To tackle these complexities, the Box complex method, a non-linear constraint optimization technique, is utilized in this research. The Box complex method is a powerful approach for solving nonlinear global optimization problems with multiple design constraints. Its utilization in this study aims to demonstrate its effectiveness as a tool for optimization. The objective is to validate whether the Box complex method remains a viable and efficient approach for addressing optimization problems. Through this validation, the hypothesis can be confirmed or supported, providing valuable insights into the method's continued relevance and applicability in solving optimization problems. This research aims to bridge this gap by employing the Box complex method and providing valuable insights into the optimization of prestressed U-shaped girders, particularly in the context of metro rail projects where these girders serve as both the deck and support structure for train loading. By leveraging the advantages of the Box complex method, this study offers a fresh perspective on designing prestressed U-shaped girders, considering their unique role in supporting metro rail loads. The proposed optimization model, based on the Box complex method and aligned with AASHTO LRFD bridge design specifications, provides a platform for finding optimal designs for prestressed U-shaped girders. The objective function in this model is the cost of materials, including concrete, steel reinforcement, and prestressing strands. The optimization of prestressed U-shaped girders presents a significant opportunity for cost savings. However, it is crucial to ensure the safety of the design throughout the optimization process. Therefore, in addition to cost considerations, this study incorporates checks for flexural stresses, moment capacity, shear capacity, and other design criteria to ensure a balanced and safe design that is both economical and structurally sound. By integrating the Box complex method into the optimization process, researchers can explore new possibilities and uncover optimal design solutions for prestressed U-shaped girders in metro rail applications. This research contributes to the advancement of structural design practices in the field of civil engineering, providing engineers with a valuable tool for achieving safe and cost-effective designs for prestressed U-shaped girders in metro rail projects [38,40,41].

2. Problem Formulation and Definition

The mathematical optimization method provides an organized process by which cost-effective designs can be found with significantly less effort and time. The proper formulation of a cost-effective design problem needs proper identifications of design variables to define the structure, an objective function to find the advantages of alternative designs, and feasible design constraints [23]. In problems related to optimization, different parameters are involved, including fixed parameters, design variables, design constraints, and an objective function formulation [38,41,47]. Most of the time, the objective function is the cost-reducing function in structure design problems which are directly or indirectly related to the design variables [27].

To have a safe and economical design, about 50% of all the effort is required by proper formulation in the optimization model in order to obtain the minimum cost design. The technique used for such problems and the minimum value of the function also depends on the proper formulation of design and other formulae. Optimization model formulation needs a proper declaration of fixed parameters, design variables, constraints, and cost function [38].

2.1. Fixed Parameters

All the parameters which remain constant throughout an optimization model are known as fixed parameters, which include many parameters that depend on the nature of the problem under consideration. In the case of a simply supported girder, all those material properties related to strength, such as compressive strength of concrete, yield strength of steel, elastic modulus, and loading intensity, are taken as fixed parameters. The unit weight and cost of each material, such as concrete, steel, and prestressing strands, are also taken as fixed parameters [40].

2.2. Design Variables

All the parameters which vary or change during the whole optimization model are called the design variables [48]. So, these are the parameters that control the objective function value [3]. In this study, these variables control the overall material cost of a prestressed U-shaped girder, so it is necessary to carefully choose the design variables for the optimization model [40]. All the variables need to be chosen which are independent and do not depend on other variables [49]. That is why choosing design variables is the first step in the optimization model. This research has taken into account nine variables as design variables, as shown in Figure 2.

1. X1 = Width of the top flange;
2. X2 = Depth of the top flange;
3. X3 = Thickness of the web;

4. X4 = Height of the web;
5. X5 = Width of the inclined portion;
6. X6 = Height of the inclined portion;
7. X7 = Thickness of the bottom flange;
8. X8 = Number of the steel bars;
9. X9 = Number of the prestressing strands.

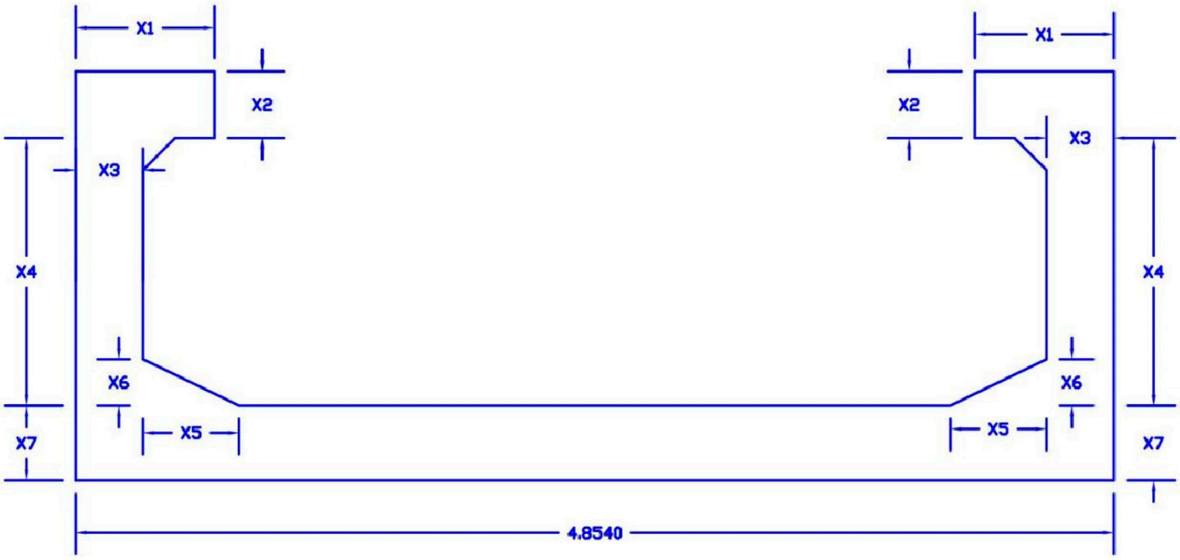

Figure 2. Cross-section of prestressed U-shaped girder.

2.3. Design Constraints

After carefully choosing the design variables, the next step is to identify the design constraint depending on the design variables. Design constraint is a restriction imposed during the optimization model to obtain values within these ranges [48]. The optimization model finds the best possible design within these limits in order to have a safe and economical design [3]. In the prestressed U-shaped girder optimization model, the restrictions are based on AASHTO LRFD bridge design specifications.

There are two types of design constraints involved in this optimization model, i.e., explicit and implicit constraints.

2.3.1. Explicit Constraints

The restrictions on design variables are known as the explicit constraints [50]. These are identified restrictions (lower or upper limits) on design variables, which originate from geometric requirements, the least appropriate dimensions for construction and code limitations, etc. [40].

The restrictions are defined as

$$x^L \le x_i \le x^U \; i = 1, 2, 3, \ldots\ldots\ldots\ldots\ldots\ldots, n$$

where x = Design variable;

x^L = Lower limiting value of the design variable;

x^U = Upper limiting value of the design variable.

2.3.2. Implicit Constraints

$$G_j \le 0 \; j = 1, 2, 3, \ldots\ldots\ldots\ldots\ldots\ldots, m$$

m = Number of implicit constraints.

Whereas G represents implicit constraints which are either constants or dependent functions of the explicit independent variables $x_1, x_2, x_3, x_4 \ldots\ldots x_n$.

The equations of implicit constraints for the U-shaped girder are based on AASHTO LRFD Bridge design specifications [7,51–53]. They consist of stresses, deflection, ultimate strength, cracking moment, and shear strength [34,38] as follows:

1. The constraint of flexural stresses at the top of the girder at transfer (G_1);
2. The constraint of flexural stresses at the bottom of the girder at transfer (G_2);
3. The constraint of flexural stresses at the top of the girder at service (G_3);
4. The constraint of flexural stresses at the bottom of the girder at service (G_4);
5. The constraint of deflection (G_5);
6. The constraint of moment capacity (G_6);
7. The constraint of minimum reinforcement limit (G_7);
8. The constraint of shear capacity (G_8);

1. The constraint of flexural stresses at the top of the girder at transfer (G_1):

$$G_1 = \left| \frac{f_{t\_transfer}}{\sigma_{transfer\_comp}} \right| - 1 \tag{1}$$

Whereas

$$f_{\ ttransfer} = \left[-\left(\frac{P_i}{A_g} \right) + \frac{P_i e_c}{S_t} - \frac{M_{dl}}{S_t} \right] \tag{2}$$

$$\sigma_{transfer\_comp} = 0.6 f_{ci} \tag{3}$$

2. The constraint of flexural stresses at the bottom of the girder at transfer (G_2):

$$G_2 = \left| \begin{matrix} \left[\!\left[\left(\frac{f_{b\_transfer}}{\sigma_{transfer\_tensile}} \right) - 1 \right]\!\right] \text{ if } \left(\frac{f_{b\_transfer}}{\sigma_{transfer\_tensile}} \right) < 0 \\ \left(\left| \frac{f_{b\_transfer}}{\sigma_{transfer\_tensile}} \right| - 1 \right) \text{ otherwise} \end{matrix} \right. \tag{4}$$

Whereas

$$f_{b\_transfer} = -\left(\frac{P_i}{A_g} \right) - \frac{P_i e_c}{S_b} + \frac{M_{dl}}{S_b} \tag{5}$$

$$\sigma_{transfer\_tensile} = 1.38 \text{ MPa} \tag{6}$$

3. The constraint of flexural stresses at the top of the girder at service (G_3):

$$G_3 = \left| \frac{f_{t\_Service}}{\sigma_{service\_comp}} \right| - 1 \tag{7}$$

Whereas

$$f_{t\_service} = -\left(\frac{P_e}{A_g} \right) + \frac{P_e e_c}{S_t} - \frac{M_T}{S_t} \tag{8}$$

$$\sigma_{service\_comp} = 0.45\ f_c \tag{9}$$

4. The constraint of flexural stresses at the bottom of the girder at service (G_4):

$$G_4 = \left| \begin{matrix} \left[\!\left[\left(\frac{f_{b\_service}}{\sigma_{service\_ten}} \right) - 1 \right]\!\right] \text{ if } \left(\frac{f_{b\_service}}{\sigma_{service\_ten}} \right) < 0 \\ \left(\left| \frac{f_{b\_service}}{\sigma_{service\_ten}} \right| - 1 \right) \text{ otherwise} \end{matrix} \right. \tag{10}$$

Whereas

$$f_{b\_service} = -\left(\frac{P_e}{A_g} \right) - \left(\frac{P_e e_c}{S_b} \right) + \left(\frac{M_{dl} + M_{sdl} + 0.8 M_L}{S_b} \right) \tag{11}$$

$$\sigma_{service\_ten} = 0.48\sqrt{f_c} \tag{12}$$

5 The constraint of deflection (G_5):

$$G_5 = -\left|\frac{\Delta_{LL}}{\Delta_{Limit}}\right| + 1 \tag{13}$$

Whereas

$$\Delta_{LL} = 1.33\left(\frac{PL^3}{48E_cI_g}\right) \tag{14}$$

$$\Delta_{Limit} = \frac{L}{800} \tag{15}$$

6. The constraint of moment capacity (G_6):

$$G_6 = \left|\frac{M_{U\_Strength\_1}}{M_n}\right| - 1 \tag{16}$$

Whereas

$$M_{u\_Strength\_1} = 1.25M_{dl} + 1.5M_{sdl} + 1.75 \times 1.33M_L \text{ (Strength limit state 1)} \tag{17}$$

M_n depends on the location of the neutral axis, as shown in Figure 3.

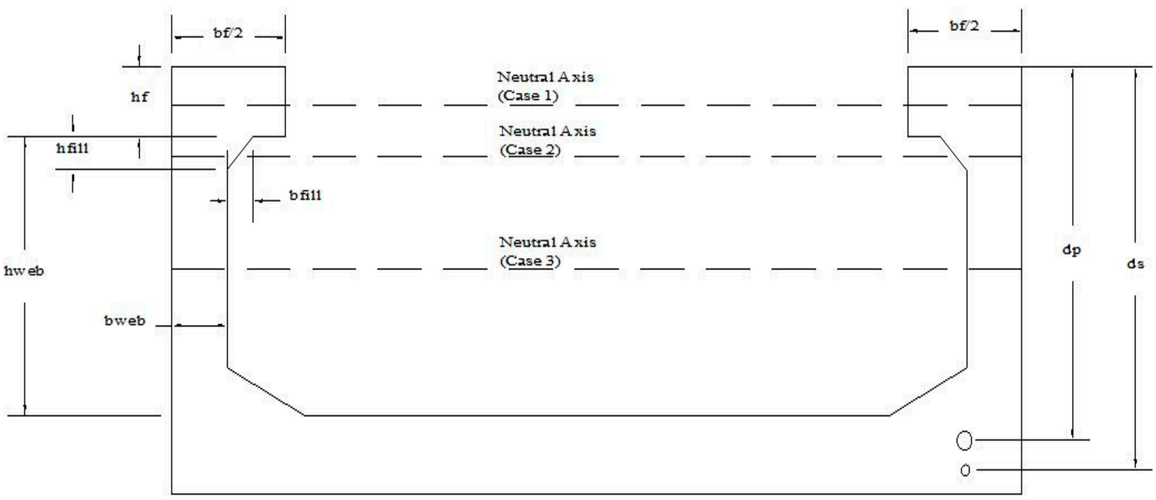

Figure 3. Neutral axis location.

Rectangular Section behavior (Case 1).

When the neutral axis falls within the girder's top flange, the reduced nominal strength of the girder can be found using the following equation:

$$M_{n\_rect} = \varnothing_{tension}\left[A_{ps}f_{ps}\left(d_p - \frac{h_f}{2}\right) + A_{steel}f_y\left(d_s - \frac{h_f}{2}\right)\right] \tag{18}$$

When the neutral axis lies within the inclined portion below the top flange (Case 2).

When the neutral axis falls within the girder's inclined portion below the top flange, the reduced nominal strength of the girder can be found using the following equation:

$$M_{n\_fill} = \varnothing_{tension}\left[A_{ps}f_{ps}\left(d_p - \frac{h_f}{2}\right) + A_{steel}f_y\left(d_s - \frac{h_f}{2}\right) - 0.85f_c\beta_{girder}(b_{web} + b_{fill})\left(h_{fill}\frac{h_f}{2}\right)\right] \tag{19}$$

When the neutral axis lies within the girder web below the inclined portion (Case 3).

When the neutral axis falls within the girder web below the inclined portion, the reduced nominal strength of the girder can be found using the following equation.

$$M_{n\_rect} = \varnothing_{tension}\begin{bmatrix} A_{ps}f_{ps}\left(d_p - \frac{h_f}{2}\right) + A_{steel}f_y\left(d_s - \frac{h_f}{2}\right) \\ -0.85f_c\beta_{girder}(b_{web} + b_{fill})h_{fill}(0.5h_{fill} + 0.5h_f) \\ -0.85f_c\beta_{girder}b_{web}(c\_web - h_f - h_{fill})(0.5h_{web} + h_{fill} + 0.5h_f) \end{bmatrix} \quad (20)$$

7. Constraint of minimum reinforcement limit (G_7):

$$G_7 = \left|\frac{M\_Limit}{M_n}\right| - 1 \quad (21)$$

Whereas

$$M\_limit = \min\left(1.2M_{cr}, 1.33M_{u\_strength\_1}\right) \quad (22)$$

$$M_{cr} = S_b(f_r + f_{cpe}) - M_{dl}\left(\frac{S_b}{S_b} - 1\right) \quad (23)$$

8. Constraint of shear capacity (G_8):

$$G_8 = \left|\frac{V_u}{V_n}\right| - 1 \quad (24)$$

Whereas

$$V_n = V_c + V_s + V_p \quad (25)$$

$$V_u = \varnothing_{shear}(1.25V_{dl} + 1.5V_{sdl} + 1.75 \times 0.33V_{LL}) \quad (26)$$

$$V_c = 0.083\beta\sqrt{f_c}b_v d_{v\_critical} \quad (27)$$

$$V_s = \frac{(V_u - \varnothing V_c)}{\varnothing} \quad (28)$$

2.4. Objective Function

In the optimization method utilized in this study, the objective function plays a crucial role. It is defined based on the design variables and the constraints imposed on the problem [48]. In the case of the prestressed U-shaped girder, the objective function is chosen as the cost function, specifically, the cost of materials, which encompasses concrete, steel reinforcement, and prestressing strands [40]. The nature of the objective function, whether it is a minimization or maximization problem, depends on the specific problem being addressed [54]. In engineering problems, the choice of the objective function depends on the specific requirements of the problem at hand. For instance, it can be based on minimizing the weight or minimizing the cost. In the case of cost optimization, the objective function considers the costs associated with various materials, including concrete, steel reinforcement, prestressing strands, and the overall framework. The cost of the prestressed U-shaped girder, along with the initial cost of the modified design, are evaluated and compared to determine the cost-effectiveness of the proposed approach in this research.

The objective function in this work is basically the cost function, and it is a minimization problem.

Function F (material cost) is

$$F(\text{materialcost}) = q_{conc} \times C_{concrete} + q_{steel} \times C_{steel} + q_{strand} \times C_{strand} \quad (29)$$

where

q_{conc} is the volume of concrete in mm^3;
$C_{concrete}$ is the unit concrete cost in Pkr/mm^3;
q_{steel} is the weight of steel in tons;
C_{steel} is the unit steel cost in Pkr/ton;
q_{strand} is the weight of cable in tons;
C_{strand} is the unit cable cost in Pkr/ton.

3. Methodology

A computer program has been developed using Visual Studio 2010 to integrate two main sub-procedures. The first sub-procedure involves a customized version of the Box complex method, which is used for solving non-linear constrained optimization problems. The second sub-procedure encompasses an algorithm designed to aid in the design and analysis of a prestressed U-shaped girder, specifically adhering to AASHTO LRFD specifications. These two sub-procedures are combined within the program to generate the desired outcomes. The program has been implemented using the VB.NET programming language.

3.1. Box Complex Method

The Box complex method was initially proposed by Box [47,55]. The technique is derived from the Simplex Method with the introduction of constraint, hence called Complex Method, and advanced from the Simplex Method [47,55–57]. The key dissimilarity between these two methods is that more points are generated during Complex Method, and, therefore, it uses more points during the search process. This technique attempts to locate a point,
where x = $x_{.i}$

$$X_{i,k} = x\ i = 1, 2, 3, \ldots\ldots\ldots\ldots\ldots., n$$

Such that to optimize k = 2n
$F(x_i)$
Subjected to n explicit constraints

$$x^L \leq x \leq x^U\ i = 1, 2, 3, \ldots\ldots\ldots\ldots\ldots., n$$

And m implicit constraints

$$G_j(X) \leq 0\ j = 1, 2, 3, 4, \ldots\ldots\ldots\ldots\ldots.., m$$

whereas

n represents the number of design variables;
m represents the number of implicit constraints;
xL represents the lower limits on the variables;
xU represents the upper limits on the variables.

The main objective is to reduce the cost function F(x). The algorithm works on the principle of replacing the worst design point with a new point called a reflecting point, which is found by reflecting the worst design point having maximum objective function through the centroid of the leftover design points in the complex [47,49,55–60]. The Logic diagram of the Box complex method is shown in Figure 4.

There are two steps involved in this optimization model: the generation of initial design points of $2n - 1$ in the complex and then the optimization phase.

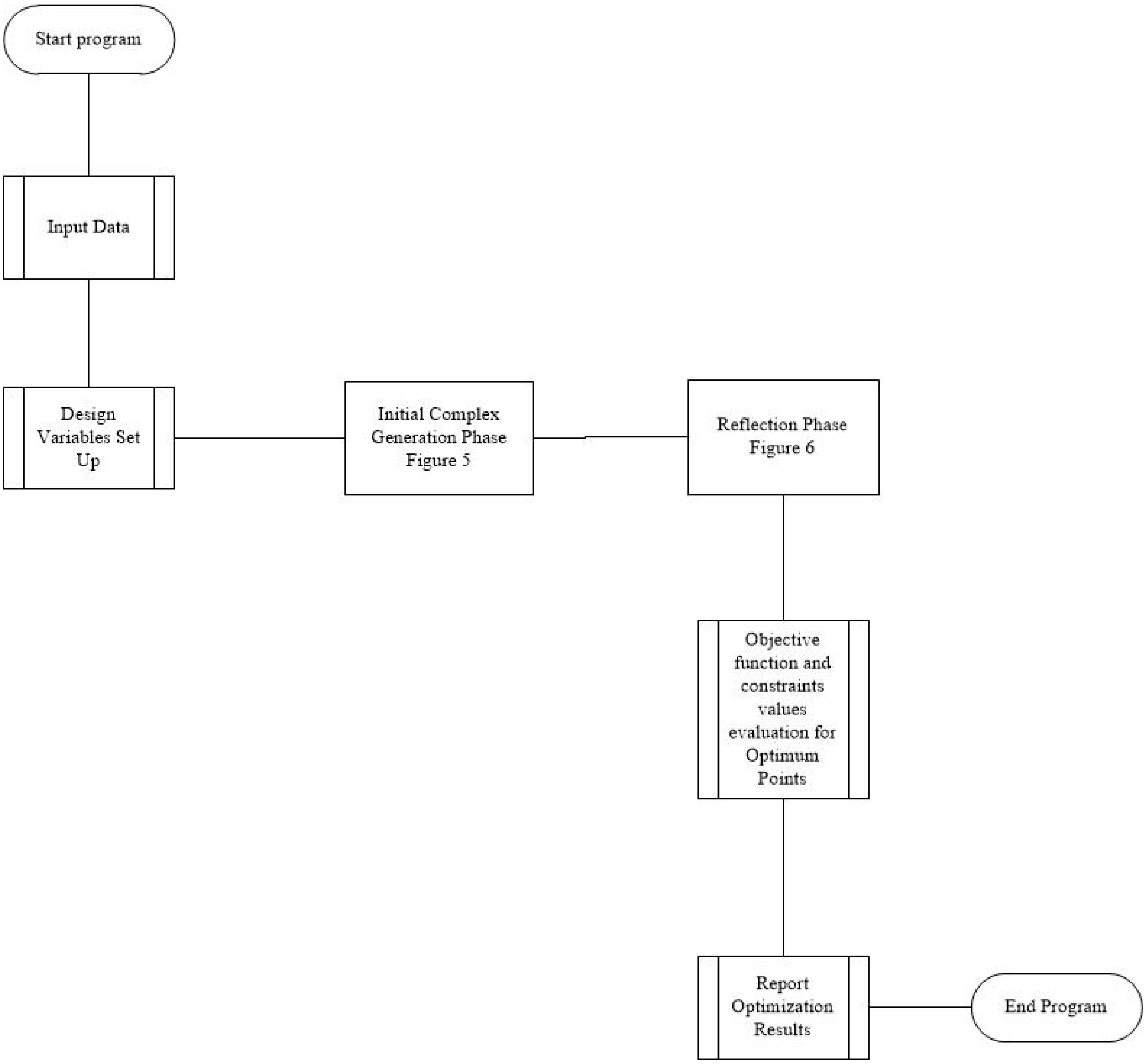

Figure 4. Box complex method logic diagram.

3.1.1. Generation of 2n − 1 Points in a Complex

Initially, 2n − 1 design points are generated, which are achieved by adding a random number increment of the range between the bounds of lower and upper limits to the lower limit of the design variables [47,50,55–57,60].

$$X_{i,j} = x^L + r\left(x^U - x^L\right) \tag{30}$$

where

i = 1, 2, 3, 4. . . . . .n;
j = 1, 2, 3, 4. . . . . ..2n;
r = Random number (between 0 and 1);
xL = Lower limit of design variables;
xU = Upper limit of design variables.

The above equation will find a new design point, and it will always be valid, which means it will be between the range of lower and upper limit bounds, but it is not essential that this point will be feasible. The initial design points will be generated until we find a design point that is feasible, which means it satisfies both the explicit and implicit constraints (in the case of our optimization model, one can skip the time-consuming step of finding the initial feasible design point only by self-introducing the first design point which is feasible) [47,55,60].

Every time generation of a design point can take benefit of an already available feasible design point if the newly generated point violates any of the implicit constraints; then, it will start retraction of half the distance toward the centroid of already available feasible design points. The process of retraction continues until a new feasible design point is located. This whole procedure continues until 2n number of feasible design points have been generated in a complex [47,50,55,56,60]. The logic diagram of the initial complex generation phase is shown in Figure 5.

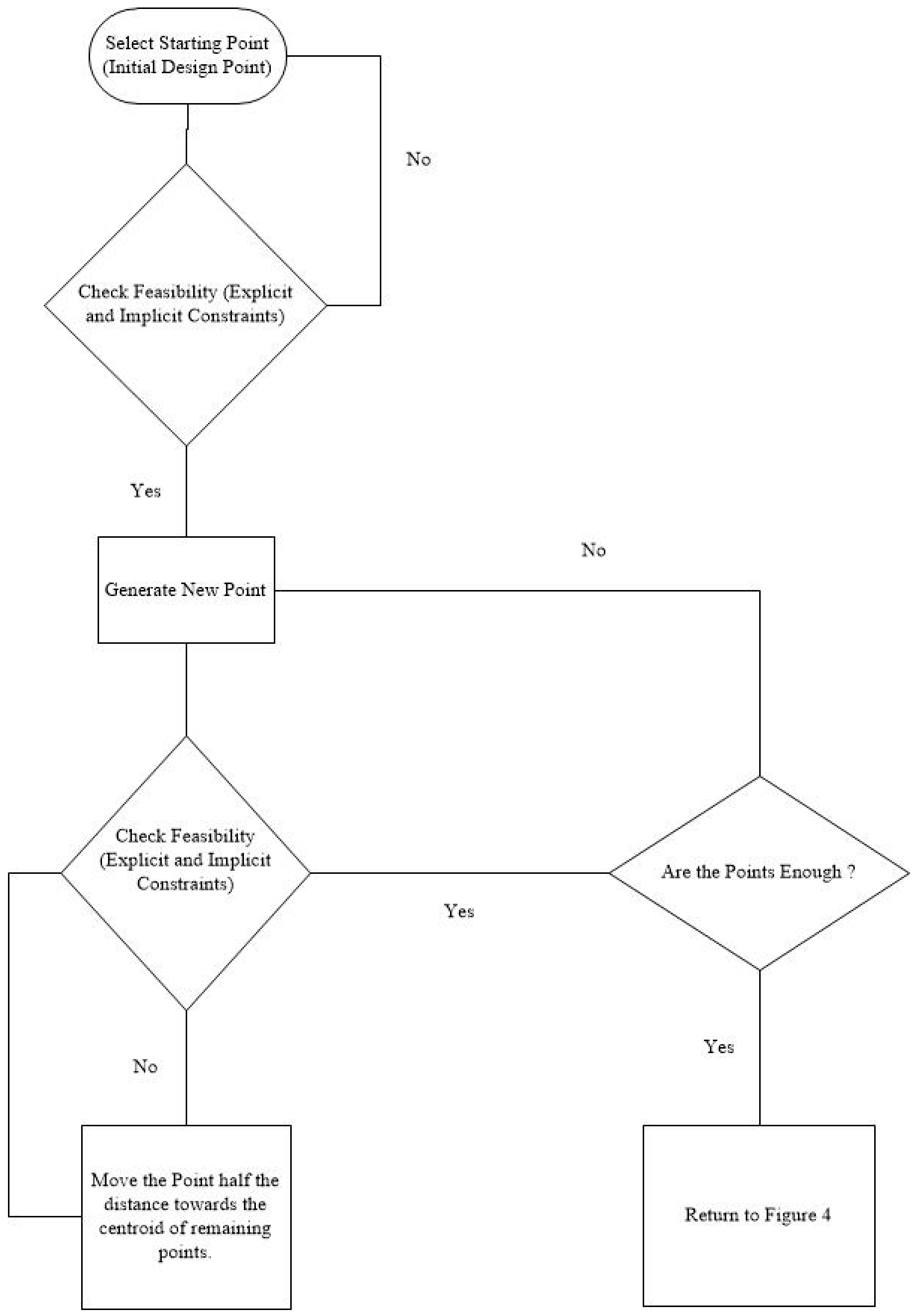

Figure 5. Initial complex generation phase.

The new point moving half the distance toward the centroid is

$$X_{i.J} = {}^{1} / {}_{2} (X_{i.J} + C_i) \tag{31}$$

The centroid Ci of the feasible design points can be calculated as follows [47,56,57,60]:

$$C_i = \frac{\sum X_{ij}}{\sum K_c} \tag{32}$$

3.1.2. Reflection Phase

After the generation of 2n feasible design points, the algorithm moves toward the next step, which is the optimization phase. In the reflection process, the algorithm search to modify the design point, which is the maximum cost function called the worst design (point). The worst design is reflected in this step toward the centroid of the leftover design points in order to find a new reflected point [47,50,59,60].

The new reflected point in the reflection process is defined by

$$X_i^R = C_i + \alpha(C_i - W_i) \tag{33}$$

where XiR represents the reflected design point and α is known as the over reflection parameter which is taken as 1.3 [50,56], whereas the centroid of the leftover points, apart from the worst point, is defined by

$$C_i = \frac{\sum X_{ij} - W_i}{\sum K_c - 1} \tag{34}$$

where

W_i = The worst design point;

K_c = Number of successfully generated vertices in the construction of the complex.

During the process of reflection, if the new reflected point violates any one of the bound of the design variable, the new design point needs to be reset to the value which is violated.

1. If $X_i{}^R < x_i{}^L$ then $X_i{}^R = x_i{}^L$;
2. If $X_i{}^R > x_i{}^U$ then $X_i{}^R = x_i{}^U$.

Once the new reflected point has been generated, the cost function and the design constraints are estimated, which leads to the following three possibilities [47,55–57,59,60]:

1. If $F_R < F_H$, the worst design is replaced by the new point, and the reflection process continues; finding the worst point within these design points continues, which is then replaced by the new point, and the process continues until the termination criteria in order to stop the program when it meets the set limit;
2. If $F_R > F_H$, the retraction process is repeated until $F_R < F_H$;
3. If the implicit constraint of the new point is violated, retract half the distance toward the centroid until a new feasible design point is generated,

where

F_R = Cost of the new reflected point;

F_H = Cost of the worst point.

The logic diagram of the reflection phase is shown in Figure 6.

3.2. Modification Involved in Box Complex Method

The modification steps involved in Box complex method are summarized as follows.

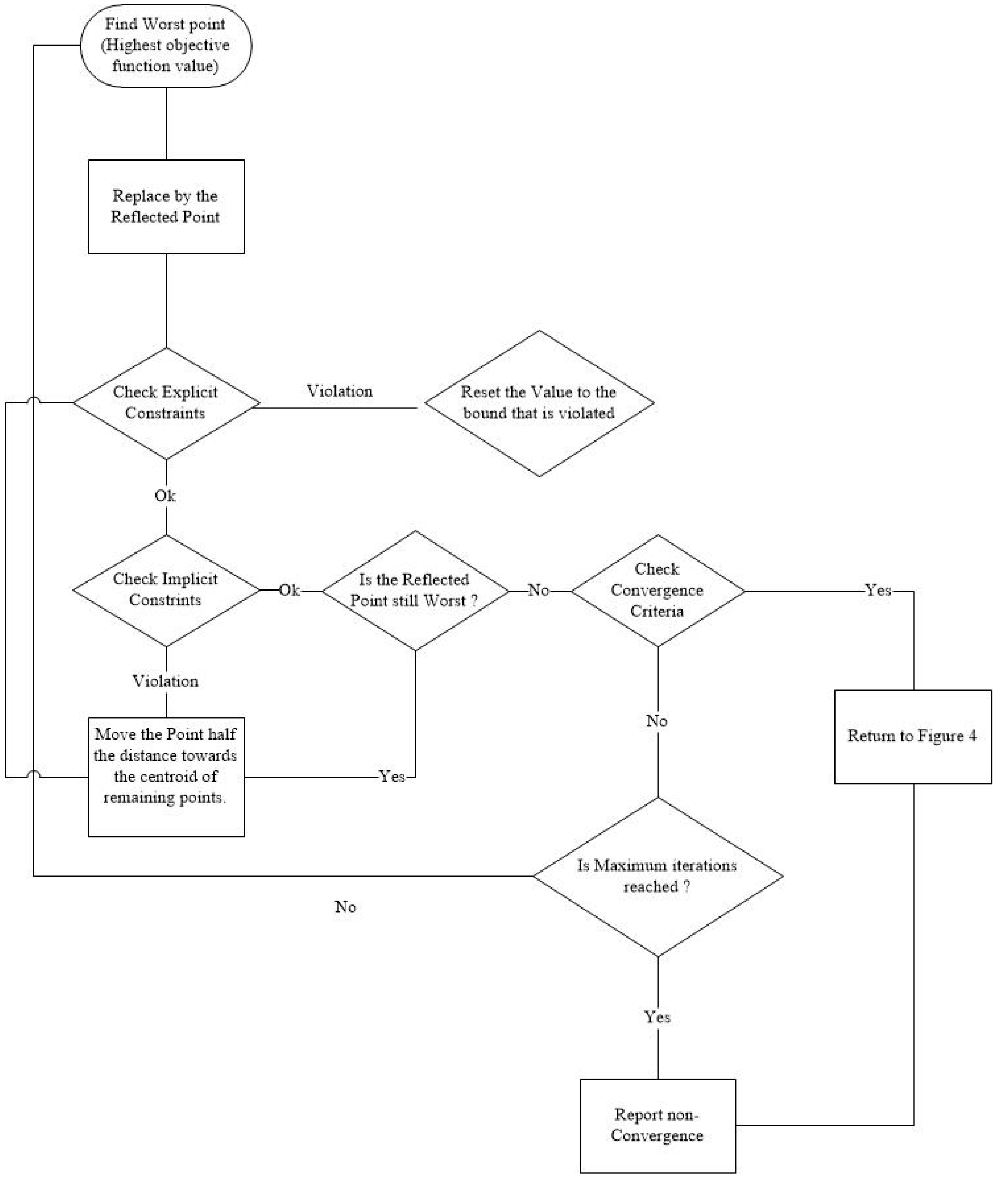

Figure 6. Reflection Phase.

3.2.1. Improvement in the Complex

In this procedure, when all the design points are generated, and the optimization phase is started, the worst design point is moved to the process of reflection through the centroid of the leftover design points to a new reflected design point in each iteration. After that, the feasibility of this new reflected point is checked, and the objective function is calculated and then compared with that of the worst design (point). If the objective function of the new design (point) is less than that of the worst point, then it is accepted as a new, improved design point in the design space. After that, the criteria for termination are checked [47,49,50,55,60]. On the other hand, if the objective function of the new design point is greater than that of the worst point, then instead of being halved continuously α times, it is halved thrice during the whole procedure. If it is still greater than that of the worst design point, then the centroid of the leftover design points is considered a new, improved design (point). If the objective function of this new point is still greater than that of the worst design point, then a new point is located at a line joining the centroid of the best design point. Furthermore, if it is greater than the worst design point, then the best design is considered a new point in a complex [49,50,55–57,59].

3.2.2. Termination Criteria Checks

The above procedure is continued until a criterion for termination is advanced. Two different criteria for termination are set in this optimization model. The first one is related to the objective function value of all the points (design) in the complex. If the difference in the maximum cost value (Fmax) and minimum cost value (Fmin) is divided by the maximum objective value (Fmax), and then the result is compared with a user-defined value ε, if it is less than the value ε, the termination criterion will be met; otherwise, the iteration will be continued until the second criterion of termination is reached [49,50,55].

$$\frac{F_{max} - F_{min}}{F_{max}} \leq \varepsilon \tag{35}$$

where ε is a very small positive numeral. The second termination criterion is based on the maximum number of iterations that need to be reached. A constant value for a maximum number of iterations is set in the program. The program will stop if any of the termination criteria is reached, and the result is evaluated [49,50,54,55].

4. Design Example

The continuous rise of computing power, along with the advancement of the new program, nowadays provides massive capabilities to engineers and architects [61]. In this research study, a computer program is established in Visual Studio 2010 version [62] using Visual Basic (VB) programming language for the cost optimization of a prestressed U-Shaped girder for the metro train using AASHTO LRFD bridge design specifications. The established program can be used in any newest version of Microsoft Windows (i.e., Windows 11) with a .NET extension. The program is developed in two different modules, which are then integrated to find the desired solution. In the first module, a modified Box complex method is developed, whereas, in the second module, a structural analysis and design procedure for a prestressed U-shaped girder is developed. The modified Box complex method is developed using the guidance of R. Belegundu and D. Chandrupatla [58] and J. Kuester and P. Mize [57]. Software is developed to easily obtain the desired results when needed and also efficiently make changes in the prestressed design without much disturbing the code. The aim of this whole procedure is to optimize the material cost of a prestressed U-shaped girder by selecting appropriate dimensions and other design variables while satisfying all the design constraints based on AASHTO-LRFD specifications restrictions, which are specified in the design procedure. The data used in designing a prestressed U-shaped girder are as follows.

The girder is assumed to be simply supported. Analysis and design are based on AASHTO LRFD bridge design specifications [7,51–53]. Only straight-strand tendons are used; neither draping/harping nor debonding/shielding of strands are considered. Table 1 represents the material properties used in this example. A 30-m-length is specified for the girder, whereas the girder comprises 25 kN/m super-imposed dead load in addition to its own self-weight, and the axle live load of the metro rail is shown in Figure 7.

Table 1. Material Properties.

Compressive strength of Concrete (fc′)	45 MPa
Compressive strength of concrete at release (fci′)	35 MPa
Unit weight of concrete (wc)	23.56 kN/m^3
Yield stress (fy)	420 MPa
Modulus of elasticity of steel (Es)	200,000 MPa
Diameter of strand (d_strand)	15.24 mm
Area of strand (A_strand)	140 mm^2
Modulus of elasticity of strand (Eps)	196,500 MPa
Ultimate tensile strength of strand	1860 MPa
Unit cost of concrete	19,500 Pkr/m^3
Unit cost of steel	108,800 Pkr/ton
Unit cost of strand	266,180 Pkr/ton

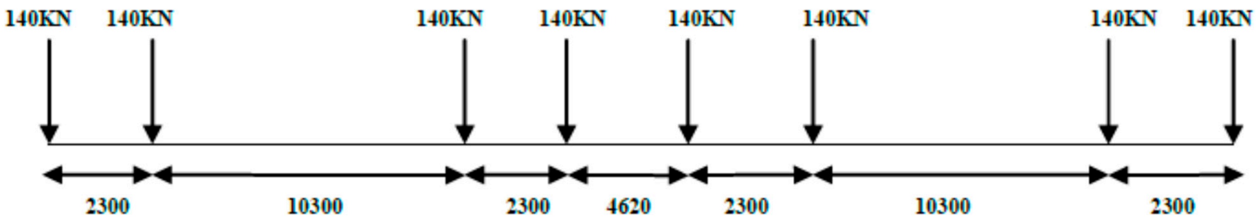

Figure 7. Axle live load (kN, mm).

5. Results and Discussion

After executing the program, the optimization process for the prestressed U-shaped girder required 229 iterations to reach the optimal value of the cost function. The developed algorithm efficiently explores economically and structurally effective solutions. Similar outcomes have been observed in the existing literature [63]. Table 2 represents the initial feasible design values of a prestressed U-shaped girder in order to start the program. Table 3 represents the optimum values of a prestressed U-shaped girder in a complex. The cost of the initial design is 15.50% higher than that of the best design achieved through the optimization process. Comparable findings can be seen in the literature [32,40,41,48,64]. The cost of a U-shaped girder during the first 20 iterations in the minimum cost curve in Figure 8 is 9.70% greater than the cost of the best design. Similarly, the cost of the worst design in Figure 8 is 19.65% higher than the cost of the best design. The graph of the history of the minimum and maximum cost of a U-shaped girder in a complex versus iteration is shown in Figure 8. The histories of flexural stresses, deflection, moment capacity, minimum reinforcement limit, and shear capacity are plotted in Figures 9–16, respectively. From these figures, one can easily depict that the design points are within limits throughout the iteration, and none of the design constraints violate the design limits. From Figure 14, one can easily depict that the moment capacity is an effective constraint that shows the rate of convergence in the complex. Similar results are reported in the literature where moment capacity is the active criterion in the final design solution [41]. In the cost-effective design, the number of tendons and steel bars is reduced compared to the initial design. Similar outcomes have been reported in the literature [40,65]. While various optimization techniques have been employed for different types of girders, the optimization of the prestressed U-shaped girders for metro rail has not been extensively discussed. This program offers valuable assistance to structural designers in effectively achieving the optimal design for prestressed U-shaped girders with minimal effort.

Table 2. Initial Feasible Design.

Initial Design	**Value**
Width of top flange (X1)	690 mm
Depth of top flange (X2)	345 mm
Thickness of web (X3)	345 mm
Height of web (X4)	1190 mm
Width of inclined portion (X5)	440 mm
Height of inclined portion (X6)	245 mm
Thickness of bottom flange (X7)	350 mm
Number of steel bars (X8)	34 no's
Number of prestressing strands (X9)	84 no's
Function value	2.71 million Pkr
Implicit constraints values	$G_1 = -0.653$
	$G_2 = -3.869$
	$G_3 = -0.287$
	$G_4 = -1.020$
	$G_5 = -0.540$
	$G_6 = -0.137$
	$G_7 = -0.014$
	$G_8 = -0.100$

Table 3. Optimized results.

Optimized Design	**Value**
Width of top flange (X1)	620 mm
Depth of top flange (X2)	343 mm
Thickness of web (X3)	280 mm
Height of web (X4)	1183 mm
Width of inclined portion (X5)	345 mm
Height of inclined portion (X6)	155 mm
Thickness of bottom flange (X7)	275 mm
Number of steel bars (X8)	33 no's
Number of prestressing strands (X9)	56 no's
Minimum function value	2.29 million Pkr
Number of function evaluation	262
Number of iterations	229
Implicit constraints values	$G_1 = -0.63967$
	$G_2 = -2.88573$
	$G_3 = -0.19768$
	$G_4 = -0.44934$
	$G_5 = -0.60625$
	$G_6 = -0.00053$
	$G_7 = -0.07823$
	$G_8 = -0.10000$

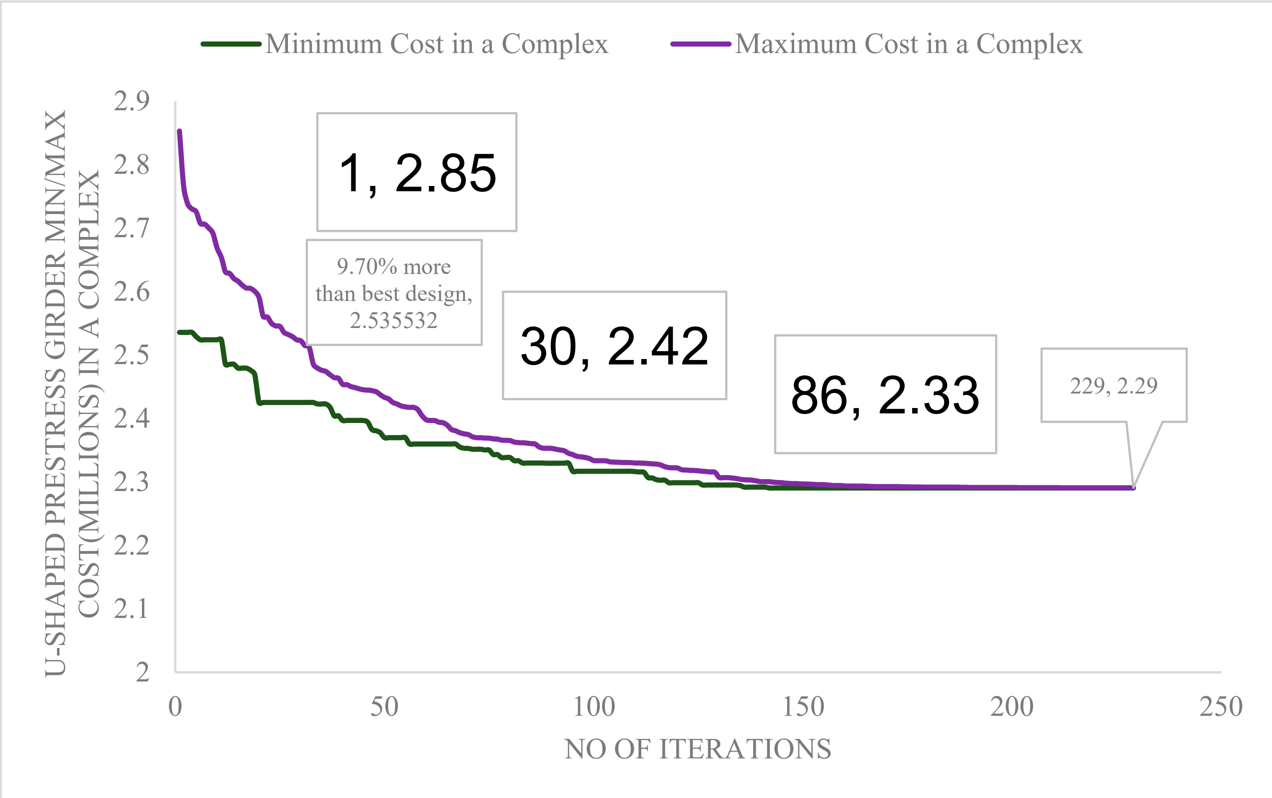

Figure 8. History of cost in a complex (max and min).

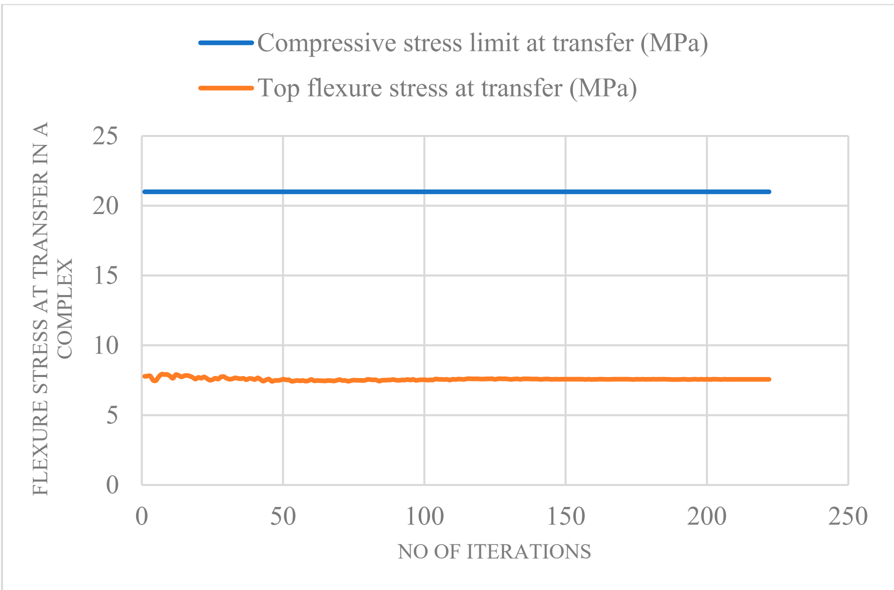

Figure 9. History of top flexural stress at transfer in a complex.

The results of the optimization process, including the number of iterations, cost reduction, adherence to design constraints, and the role of moment capacity, provide evidence of the algorithm's effectiveness and the significance of the findings. Furthermore, the comparison with the existing literature and the discussion of the cost curves, design constraints, and previous research support the conclusions and highlight the contribution of the research to the field. The use of classical methods, such as the Box complex method, in civil engineering has been limited. This is especially true when it comes to optimizing

U-shaped girders, which are integral components in metro rail projects, serving as both the deck and support structure for train loading. Recognizing the significance of this research gap, the present study provides valuable insights into the design of these girders, particularly in the context of their crucial role in supporting metro rail loads. The adoption of the Box complex method offers a fresh and innovative approach, opening up new possibilities for researchers to discover optimal design solutions for U-shaped girders in metro rail applications.

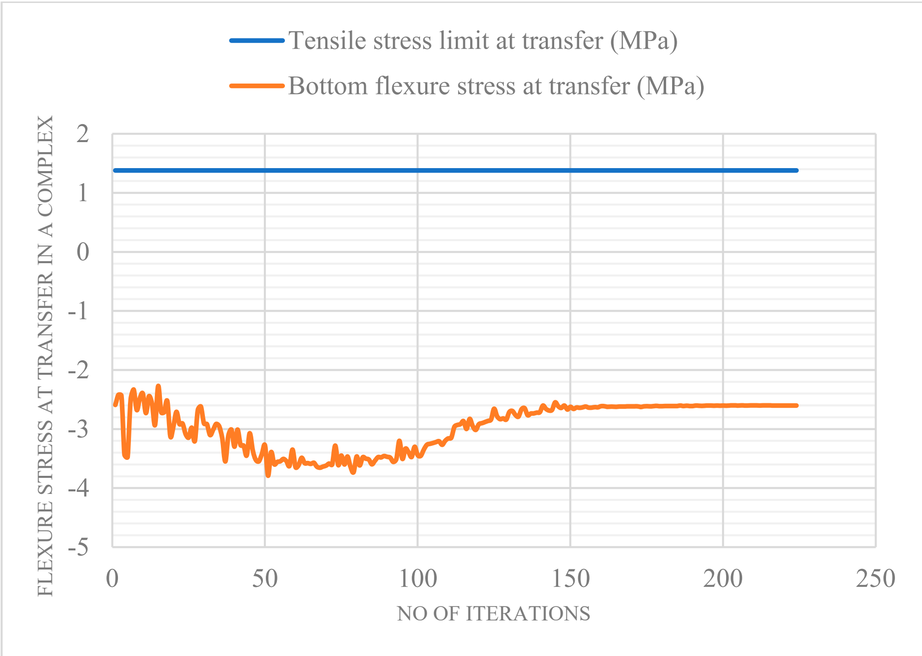

Figure 10. History of bottom flexural stress at transfer in a complex.

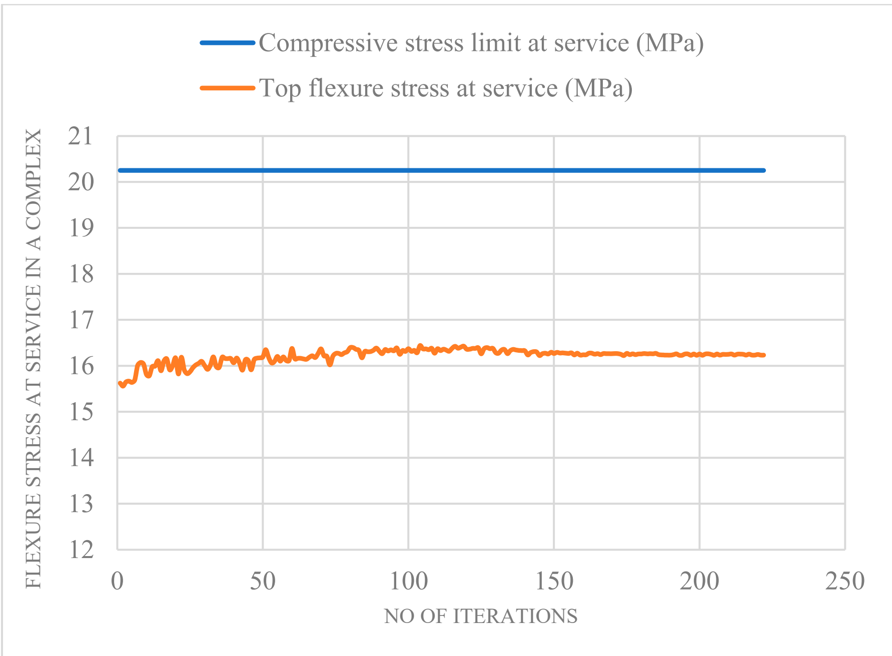

Figure 11. History of top flexural stress at service in a complex.

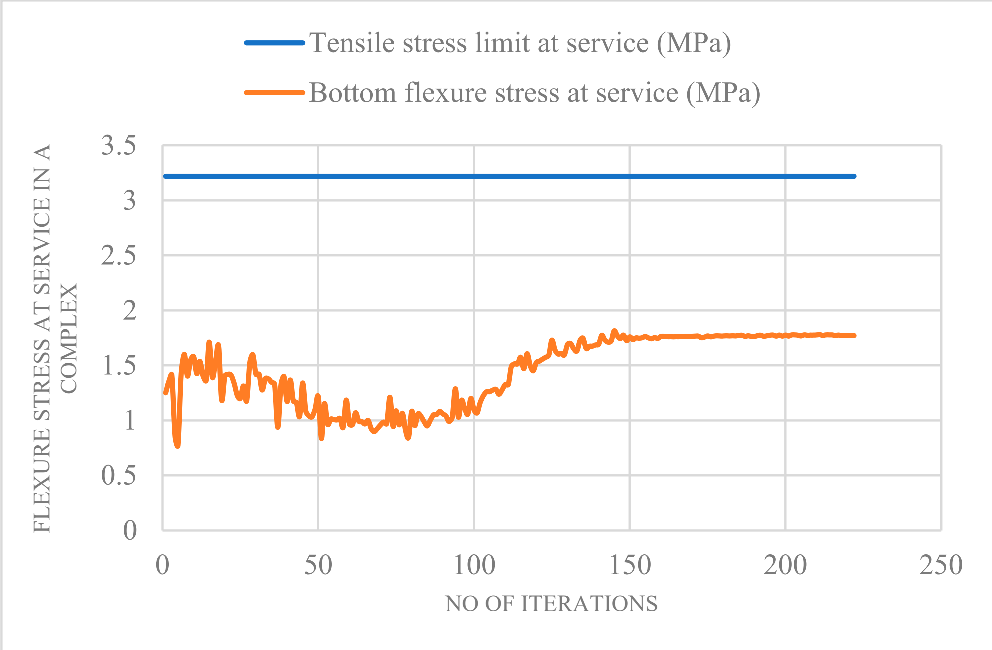

Figure 12. History of bottom flexural stress at service in a complex.

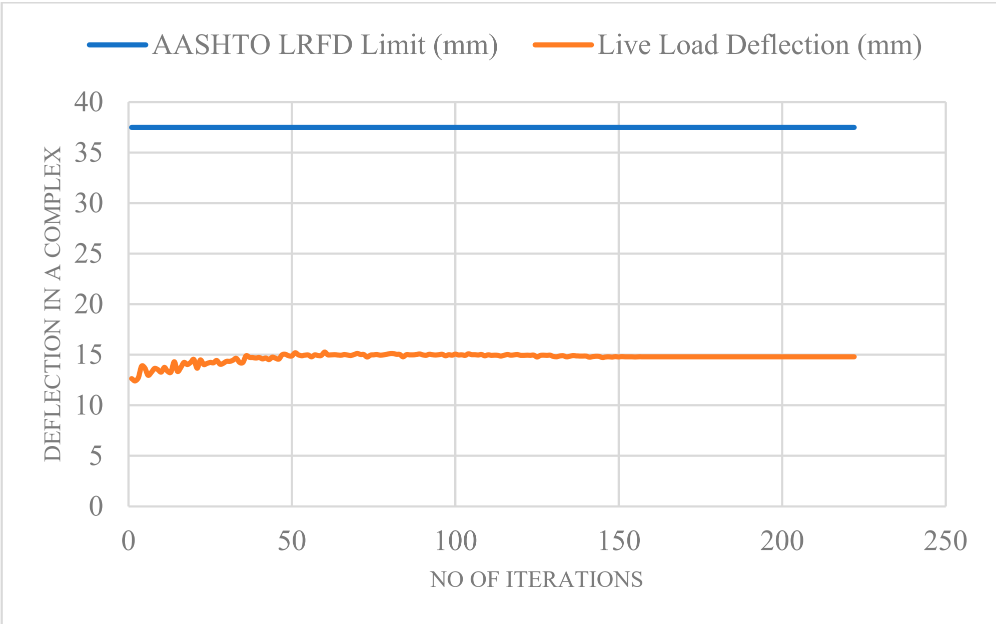

Figure 13. History of live-load deflection in a complex.

Similarly, A VB.NET software application has also been created to enable the integrated analysis, design, and cost optimization of prestressed U-shaped girders. The application includes multiple tabs, as depicted in Supplementary Materials from Figures S1–S5, providing a user-friendly interface for inputting parameters and obtaining optimal designs. It leverages the power of Visual Studio and VB.NET to streamline the entire process, enhancing the efficiency and effectiveness of prestressed U-shaped girder projects.

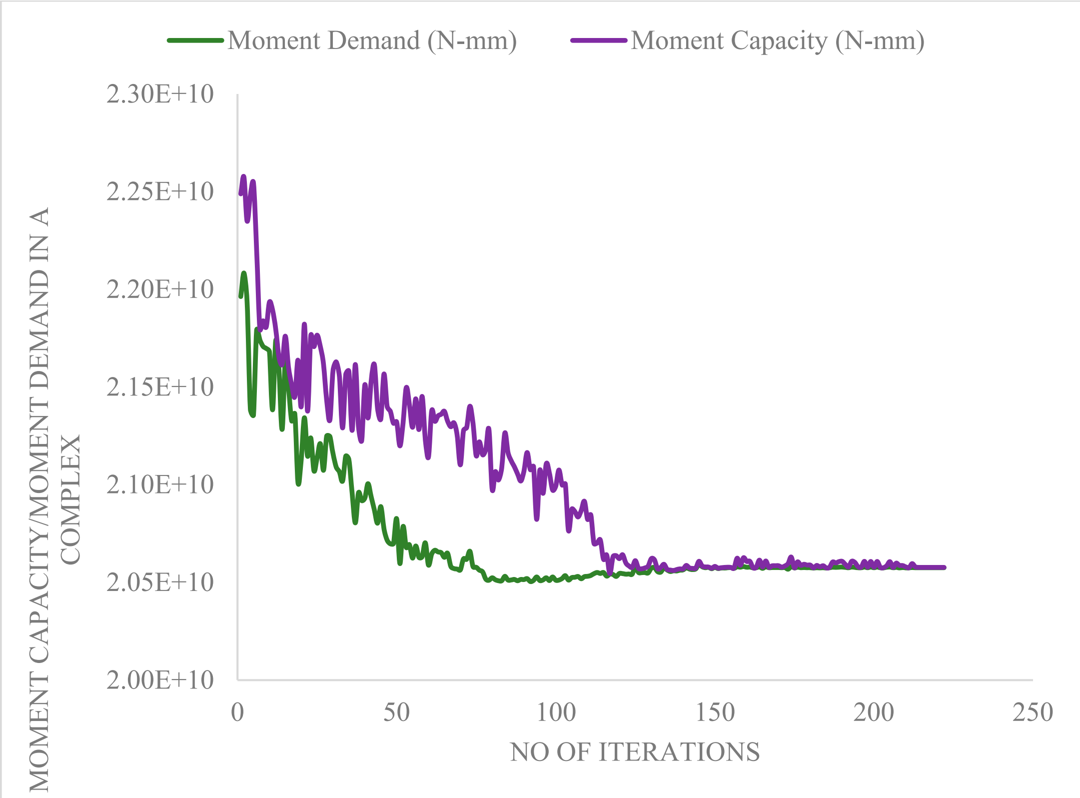

Figure 14. History of moment capacity/moment demand in a complex.

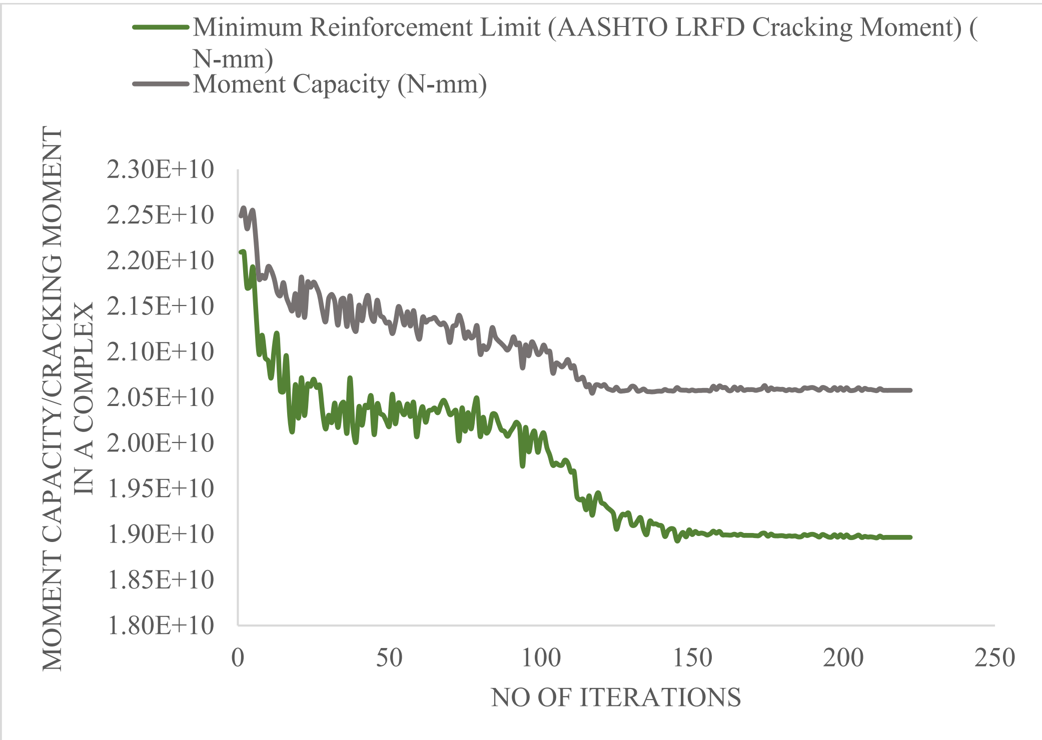

Figure 15. History of moment capacity/cracking moment in a complex.

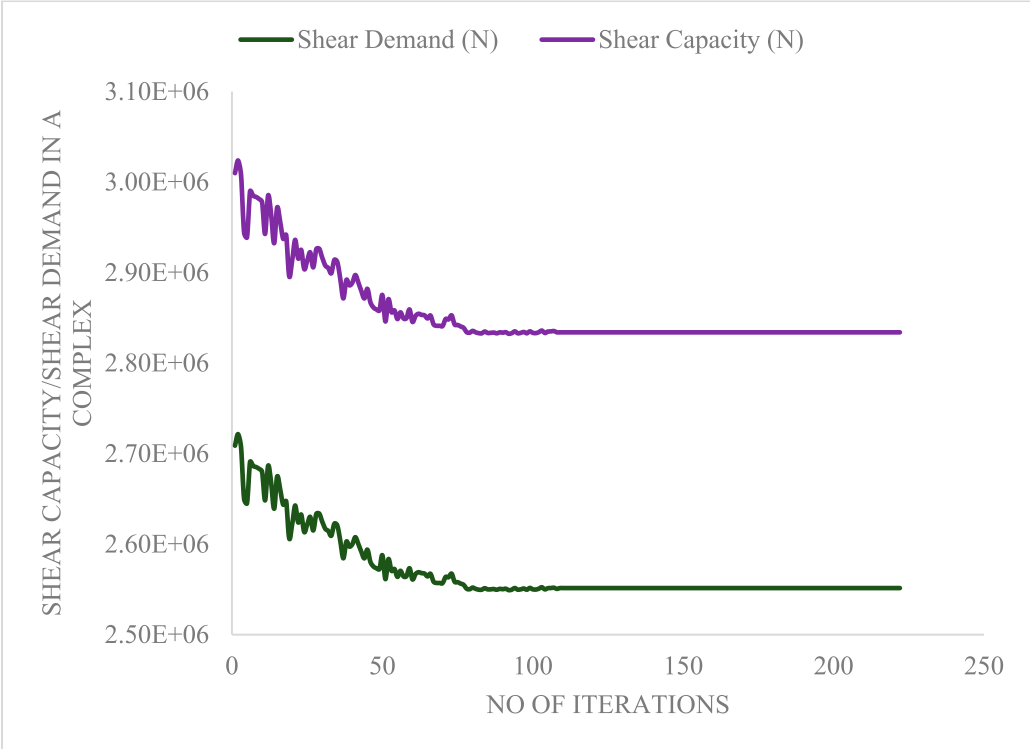

Figure 16. History of shear capacity/shear demand in a complex.

6. Conclusions and Recommendations

This algorithm efficiently explores economically and structurally effective solutions. The comparison of initial and optimum design values demonstrates a significant reduction in cost, further supported by the analysis of the minimum and maximum cost curves. The plotted histories of various design parameters confirm that all constraints are satisfied throughout the iteration process. The moment capacity emerges as an effective constraint, indicating the rate of convergence in achieving the optimal design. The cost-effective design achieved through the optimization process showcases a reduction in the number of tendons and steel bars compared to the initial design. While the optimization of prestressed U-shaped girders for metro rail projects has not received extensive attention, this program serves as a valuable tool for structural designers, enabling them to efficiently obtain optimal designs for prestressed U-shaped girders with minimal effort. This approach eliminates the need for iterative design processes, as the optimization software handles them.

The Box complex method is a robust approach that enables the exploration of the entire design space within the explicit constraint limits, offering a higher likelihood of finding the global optimum compared to other methods. It is particularly beneficial for large structures with numerous structural members, leading to significant cost savings. The outcomes of this study are of great value to practicing engineers. The proposed model provides a straightforward and accurate nonlinear solution for optimizing prestressed simply supported U-shaped girders without relying on approximations. It yields optimal design values and demonstrates cost savings compared to manual, unoptimized solutions. As part of future development, this work aims to extend the model to consider all section parameters as design variables and explore different methods for solving the nonlinear model.

The formulation of the design problem aligns well with the Box complex method, making it suitable for this approach. It can be easily applied to problems with multiple constraints. This method follows a step-by-step process that can be efficiently handled and modified when a simulation algorithmic model is available. This procedure is effective and does not require significant time investment. By integrating the analysis and design algorithms into a program, an optimal design can be generated with minimal effort.

The Box complex method is a powerful tool regardless of the explicit constraint limit. Once an initial feasible design is obtained, it can explore the design space within the specified constraints to find the optimum design. In the design example, the rate of convergence is demonstrated by the moment capacity. Constraints that have no impact on the optimization problem can be easily removed. This study addresses the limited utilization of classical methods, such as the Box complex method, in civil engineering research. It identifies the optimization of U-shaped girders as an unexplored research area, particularly in the context of metro rail applications. By adopting the Box complex method, the research bridges this gap and explores the optimization potential of U-shaped girders.

The developed algorithm and program offer a novel approach to designing prestressed U-shaped girders. By incorporating the Box complex method, the research introduces a fresh perspective and explores new possibilities for optimizing the girder design. The optimization process significantly improves the cost efficiency of the girder, reducing material usage and overall construction expenses. Additionally, the program ensures that the optimized designs satisfy all relevant design constraints, ensuring structural integrity and performance. The findings of this study hold promising applications in the field of metro rail infrastructure development. Optimal designs for prestressed U-shaped girders can enhance the cost-effectiveness and structural efficiency of metro rail projects. By reducing material costs and improving structural performance, the optimized designs contribute to the development of more sustainable and reliable transportation systems.

The optimization process yields valuable results encompassing several key aspects, such as the number of iterations, cost reduction, adherence to design constraints, and the influential role of moment capacity. These outcomes serve as compelling evidence for the efficacy of the algorithm and underscore the significance of the findings. Additionally, a comprehensive analysis comparing the results with the existing literature, along with a thorough examination of cost curves, design constraints, and prior research, further substantiates the conclusions drawn and emphasizes the research's notable contribution to the field.

The procedure utilized in this research can be expanded to encompass other types of prestressed girders with minor modifications to the analysis and design procedures. Similarly, the same procedure can be applied to prestressed U-shaped girders while considering different code specifications. Furthermore, this method has the potential to be applied to various engineering problems with minimal effort required to develop the necessary analysis and design procedures. It would be valuable to explore the application of other optimization methods, such as genetic algorithms, to these problems and compare the results. Additionally, future research can involve testing the procedure for different values of α and extending the study to incorporate various values of fixed parameters.

Supplementary Materials: The following supporting information can be downloaded at: https://www.mdpi.com/article/10.3390/su151411457/s1, Figure S1. General view of developed software. Figure S2. Tab for Initial design variables and explicit constraints. Figure S3. Button for initial design analysis and results. Figure S4. Different buttons for initial design checks. Figure S5. Optimum design result in the developed Software.

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Abbreviations

The following are the symbols which are used in this paper:

A.g	Gross area of girder, mm^2;
A.steel	Total area of steel reinforcement in flexure, mm^2;
A.ps	Total area of prestressing strands, mm^2;
B	Tensile stress factor;
b.v	Thickness of the web, mm;
b.web	Thickness of the web, mm;
b.fill	Width of the inclined portion below the top flange, mm;
c_web	Distance between the neutral axis and compressive face, if the neutral axis lies in girder web, mm;
d.s	Distance from the top compressive fiber to the centroid of steel reinforcement, mm;
d.p	Distance from the top compressive fiber to the centroid of prestressing strands. Mm;
E.c	Modulus of elasticity of the concrete, Mpa;
e.c	Eccentricity at the center, mm;
f.b_transfer	Flexure stress at the bottom of the girder at transfer, Mpa;
f.t_transfer	Flexure stress at the top of the girder at transfer, Mpa;
f.b_service	Flexure stress at the bottom of the girder at service, Mpa;
f.t_service	Flexure stress at the top of the girder at service, Mpa;
f.ci	Concrete compressive strength at the time of initial prestress, Mpa;
f.c	Concrete compressive strength, Mpa;
f.ps	Stress in prestressing steel, Mpa;
f.r	Modulus of rupture, Mpa;
f.cpe	Compressive stress in a concrete section due to effective prestress force only at the outer, most extreme fiber, where tensile stress is caused by externally applied loads, Mpa;
h.f	Depth of the top flange, mm;
h.fill	Thickness of the inclined portion below the top flange, mm;
h.web	Height of the web, mm;
I.g	Moment of inertia, mm^4;
k	Factor taken as 0.28 for a low-relaxation strand;
L	Length of the girder, mm;
M.dl	Dead-load moment, N-mm;
M.T	Total moment due to service load, N-mm;
M.sdl	Super-imposed dead-load moment, N-mm;
M.L	Live-load moment, N-mm;
M.cr	Cracking moment, N-mm;
P.i	Initial prestress force before losses, N;
P.e	Effective prestress force after all losses, N;
S.b	Section modulus at the bottom of the girder, mm^3;
S.t	Section modulus at the top of girder, mm^3;
V.dl	Shear force due to deal load, N;
V.sdl	Shear force due to a super-imposed deal load, N;
V.LL	Shear force due to a live load, N;
V.u	Factored shear at a critical section, N;
V.c	Concrete contribution in shear, N;
d.v_critical	Effective shear depth, mm;
V.s	Steel contribution in shear, N;

V.p	Contribution of vertical component of prestressing strand in shear, N;
V.n	Nominal shear resistance;
β.girder	Parameter of a stress block for the girder;
ϕ	Resistance factor for the shear;
Φ.tension	Resistance factor for the flexure;
ϕ.shear	Strength reduction factor for the shear;
σ.transfer_comp	Compressive stress limit at transfer, Mpa;
σ.transfer_tensile	Tensile stress limit at transfer, Mpa;
σ.service_comp	Compressive stress limit at service, Mpa;
σ.service_ten	Tensile stress limit at service, Mpa.

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