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Precautionary Saving and Liquidity Shortage

Guohua He and Zirun Hu *

Economics and Management School, Wuhan University, Wuhan 430072, China
* Correspondence: huzirun@whu.edu.cn

Abstract: Most of the canonical macroeconomic models simulate liquidity anomalies by changing the economic fundamentals or adding massive financial shock to firms' collateral constraints, but a few facts somehow tell a different story. Instead of relying on the exogenous shocks, we introduce uncertainty into an otherwise classical liquidity framework and try to answer what worsens the aggregate liquidity in the absence of exogenous simulations and what a firm dynamics and financing strategy would be. Our analysis shows that (1) uncertainty induces agents to make decisions under the worst-case scenario and hence generates a unique expectation threshold that drags market (or firms) liquidity from sufficiency to insufficiency even without any shock or economic changes. (2) Precautionary saving occurs before the real liquidity shortage as the expectation shifts, causing firms to secure external financing by raising the equity issuing price and hoarding liquid assets, such as fiat money, against liquidity tightening. (3) To achieve liquidity stability and sustainability, an extra mathematical constraint is supplemented for the uniqueness and the existence of equilibrium under uncertainty. Other properties of firms' intertemporal allocations, such as the bid-ask spread and return of holding of the illiquid asset, are derived. Moreover, some approaches for further empirical research are discussed.

Keywords: liquidity shortage; uncertainty; firm dynamics; precautionary saving

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1. Introduction

Liquidity, as the core factor in financial activities, has received a lot of academic attention, especially since the onset of the subprime crisis in 2008. The severe shortage of liquid assets during the subprime crisis prompted multiple authorities to inject a massive amount of liquidity into the market, in the forms of bailouts, quantitative easing, etc. The instability and unsustainability of liquidity has delivered a huge blow to the global economy and brought liquidity to the forefront of policy debate. To characterize and replicate the crisis theoretically, one of the pivotal studies, Kiyotaki and Moore [1], incorporates a firm's liquidity with the standard real business cycle model, opening the gate for macroeconomics to explore firm dynamics and financing strategy along with the change of aggregate liquidity status. Be that as it may, such a perfect model, similar to many other canonical macroeconomic models, blames exogenous shocks for the liquidity anomalies instead of agents' endogenous decision-making; therefore, it is limited in explaining some specific facts in reality. One robust example is the Knightian shares documented in Bachmann et al. [2]'s study: firms' uncertainty/Knightian shares in Greece spiked up shortly after the victory of the Syriza party on 25 January 2015, and peaked when the repayment to the IMF loan was overdue on June 30. Neither fundamental change nor prominent shock was found during this period, but uncertainty was reflected jointly in firm planning and observed risk premia in financial markets. With this in mind, we would like to ask two pivotal questions throughout the paper: What worsens the aggregate liquidity in the absence of exogenous simulation? How does a firm act and cause this effect under uncertainty? These two unresolved issues, according to our perspective, happen to be the primary elements to better understand the volatility of liquidity.

A few studies closely related to our research have anatomized the determinants of liquidity both theoretically and empirically. Among the theoretical literature, the early research concerning liquidity is mainly focusing on a firm's asset structure such as Bolton et al. [3], but the subprime crisis of 2008 was the turning point and changed the perspective from micro to macro. For example, Kiyotaki and Moore [1] survey the firm dynamics using the liquidity constraint in external financing, it turns out to be the solid proof of how agent's decision-making changes the aggregate liquidity status. Del Negro et al. [4] extend Kiyotaki and Moore [1]'s work by adding the nominal rigidity to a more comprehensive household sector and further examining the efficiency of quantitative easing around the zero lower bound. In contrast, Guerron-Quintana et al. [5] abstract Kiyotaki and Moore [1]'s setup to discuss the hysteresis/superhysteresis of the economic growth in the USA after the asset bubble burst. Similarly, Brunnermeier and Pedersen [6] point out that the optimistic belief accelerates the liquidity expansion and raises the associated asset risk, while the pessimistic belief is the final push to collapse; together, these two opposite components constitute financial cycles. Some other related examples in the literature include but are not limited to Shi [7,8], Ajello [9], Brunnermeier and Oehmke [10], Lorenzoni [11], Holmstrom and Tirole [12], Gertler et al. [13], Gertler and Karadi [14], and Geanakoplos [15].

For the empirical literature, Acharya and Merrouche [16] document that the liquidity demand of large settlement banks in the UK experienced a 30% increase before the subprime crisis and that strong precautionary nature cause the liquidity demand to rise on days of high payment activity and for banks with greater credit risk. Ashcraft et al. [17] show similar evidence but with the US data. Chiu et al. [18] examine the relationship between funding liquidity and equity liquidity during the subprime crisis by using the index and ETFs; the empirical results show that a higher degree of funding of illiquidity leads to an increase in bid–ask spread and a reduction in both market depth and net buying imbalance. In a more broad scope, Belke et al. [19] apply the liquidity shock to the open economy and emphasize that a global liquidity shock leads to a rise in consumer and global house prices, where the latter reaction is more pronounced. Some studies survey liquidity risk and its association with other factors. Cao and Petrasek [20] think that abnormal stock returns during liquidity crises are strongly negatively related to liquidity risk and that the degree of informational asymmetry and the ownership structure of the firm are the main reasons. The strategy under liquidity risk is also shown by Cao et al. [21,22], in which the authors explore the hedge-fund and mutual-fund managers' timing abilities by examining whether they can time market liquidity through adjusting their portfolios' market exposures as aggregate liquidity conditions change. In addition, Wegener et al. [23] examine the yields of traditional Pfandbriefe and Jumbo Pfandbriefe with different maturities where the yield spreads between these two types of German covered bonds can be considered as pure liquidity premia. The authors find that the yields are fractionally co-integrated before and after the crisis, but the degree of integration of the spread increases strongly during the crisis. Except for these macro level studies, a few specific types of research concerning firm level liquidity and firm dynamics are also worth noting. Under the typical agency problem, Lambrecht and Myers [24,25] show that managers tend to smooth payouts in consideration of smoothing the rents they draw from the firm, smoothing can either be performed by lending (capital expansion) or borrowing (leverage). The basic reason for this behavior is that managers are risk averse and the authors provide a few demonstrations of changing managers' preferences. Similarly, Hoang and Hoxha [26,27] provide a more solid proof of smoothing behavior using the empirical results of the US, China, and Taiwan; the authors find that firms use debt and investment to smooth a large fraction of shocks to the net income to keep payouts less variable.

Two comments can be presented from these related examples in the literature. First, the recent empirical results are putting forward evidence about the precautionary hoarding of liquidity in a way that most of the canonical macroeconomic models cannot explain. For example, the system in the macroeconomic model is staying at an efficient equilibrium (or steady state) if no perturbation occurs, implying agents will not suddenly and willingly choose to hoard liquidity for precaution in the absence of exogenous shocks. However, Bachmann et al. [2] and Acharya and Merrouche [16] depict a contradicted picture. Although a few studies in macro finance begin to emphasize the expectation-driven element, the associated modeling, often known as the sunspot, can still be regarded as an exogenous shock, such as the bank run settings in Gertler et al. [28]. Second, tractability could still be a main issue for macroeconomic models, especially the macro finance models, while concerning precautionary hoarding/saving. Using Maxted [29] as an example, the diagnostic expectation, an extrapolative expectation that overreacts to noise and may lead to a precautionary behavior, is introduced to He and Krishnamurthy [30]'s macro finance model, but the basic solution to the model is numerical computation. Despite the numerical method being a good and efficient way to solve the precautionary saving puzzle, the analytical solution is always indispensable for putting the mechanism in a more concrete perspective and helping us better understand the reason why firms adopt these strategy rules/choices. These two comments are in line with the two unresolved questions that were asked at the beginning.

This paper contributes to macro-finance theory; the questions of our topic are responded to with a modified liquidity framework where the investment chance is ambiguous. Specifically, we replace the exogenous shock in Kiyotaki and Moore [1] with the endogenous uncertainty and emphasize that the model's economy is without shocks and fundamentals changes. Meanwhile, we solve the modified liquidity framework analytically and collect all the necessary conditions to judge and characterize the firm dynamics and individual financing decisions and further anatomize why these factors can cause precautionary saving and liquidity shortage. We discover a few broad conclusions upon our specific settings: (1) Firms shift their expectations due to the existence of uncertainty and the model endogenously generates a unique expectation threshold that drags the aggregate liquidity from sufficiency to insufficiency, implying the market could spontaneously experience a liquidity shortage under uncertainty. This theoretical result matches Bachmann et al. [2]'s evidence that risk premia rose in Greece before the true debt crisis unfolded. (2) The shifted expectation induces firms to secure external financing by raising the equity issuing price and hoarding liquid assets, such as fiat money, as precautionary saving against liquidity tightening, which determines firms' precautionary saving behaviors prior to the real liquidity shortage. In echoing Acharya and Merrouche [16] and Ashcraft et al. [17], this mechanism explains the spikes of liquidity demand in the UK and the US before the subprime crisis to some extent. (3) Our theory also finds that the model generates a disequilibrium under some certain feasible conditions, where the aggregate liquidity is no longer stable and sustainable. To avoid this instability and unsustainability, an extra constraint is supplemented. Other properties about firms' intertemporal allocation, such as the bid-ask spread and return of holding of the illiquid asset, are derived.

2. Model Setup

To explore how precautionary saving and liquidity shortage rises in the absence of exogenous shocks, we introduce the uncertainty to Kiyotaki and Moore [1].

Basic framework. Assume an infinite-horizon discrete-time economy with firms and workers, the only commodity is produced by firms, and the inputs of this production include social capital and workers' labor. As for capital, it can be separated into two types according to the degree of liquidity: one is fully liquid fiat money, while the other is equity that is issued by firms for the sake of external financing. In summary, the firms' utility is provided by:

$$\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_s)$$

where $\mathbb{E}_t$ is the expectation operator, β is the objective discounter with $0 < \beta < 1$, and the single period utility has the form of $u(c_s) = \ln c_s$.

Moreover, the production function of firms is provided by:

$$y_t = A_t k_t^{\gamma} \ell_t^{1-\gamma}$$

where y_t, k_t, and ℓ_t are, respectively, the output, capital, and labor, A_t denotes for the industrial technology, and γ is input ratio between capital and labor. Obviously, due to the the capital held by firms, the firm's profit can be expressed as:

$$y_t - w_t \ell_t = r_t k_t.$$

Intuitively, $w_t \ell_t$ is the aggregate wage that firms need to pay to the labor force, while $r_t k_t$ is the aggregate investment revenue on production. One thing should be further noted is that capital depreciates over time:

$$k_{t+1} = \lambda k_t + i_t,$$

where we assume the depreciation rate is $1 - \lambda > 0$ and i_t is the new investment in capital per period.

In normal economic exchanges, firms are allowed to implement external financing for issuing new equities or reselling old ones, the borrowing constraint is valid if a firm decides to issue a certain amount of equities to the market: prepay the $1 - \theta$ ratio of capital for each unit of external financing as collateral and $\theta \in (0, 1)$. Moreover, the equity is supposed to be partially liquid, so that only a $\phi \in (0, 1)$ ratio of old equities can be resold in each period. In contrast, the fully liquid asset in our economy is fiat money that has a total amount of M and without any profit revenue by itself. Assume the asset portfolio of each firm at time t consists of money m_t (which can be 0 in equilibrium), the newly issued equities n_t^f, holdings of other firm's old equities n_t^o, and the total net worth n_t, then the balance sheet of an arbitrary firm is provided by $n_t = n_t^o + k_t - n_t^f$ and the corresponding liquidity constraints are:

$$n_{t+1} \geq (1 - \theta) i_t + (1 - \phi) \lambda n_t, \tag{1}$$

$$m_{t+1} \geq 0. \tag{2}$$

To obtain more of an insight, (1) can be treated as the liquidity constraint throughout asset accumulation: the LHS of this equation is the firm's net worth stock (saving or income of each period), while the RHS is the prepay plus unsold equities (expenditure of each period); the LHS should surpass the RHS. Moreover, (1) somehow reflects the market liquidity status depending on whether the equation is binding or not, that is, the excessive liquidity in each firm disappeared whenever the equation was binding, meaning that the firm has no extra asset to perform anything else. Likewise, the money holding m_t can be positive when firms face a liquidity shortage. To characterize these traits in detail, let the prices for equity and money as q_t, p_t, then the cash flow of each firm can be expressed as:

$$c_t + i_t + q_t(n_{t+1} - i_t - \lambda n_t) + p_t(m_{t+1} - m_t) = r_t n_t, \tag{3}$$

where $n_{t+1} - i_t - \lambda n_t$ and $m_{t+1} - m_t$ denote the change of assets.

Unlike the firms, workers have the utility such that

$$\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} U\left(c_s' - \frac{\varphi}{1+v} (\ell_s')^{1+v}\right),$$

where we use the prime symbol as the superscript to denote workers' variables, $\varphi > 0$ is the weight of labor and $1 + v > 0$ is the reverse of the elasticity of the labor supply. Compare the settings of the firms, the budget constraint of an arbitrary worker is provided by

$$c_t' + q_t(n_{t+1}' - \lambda n_t') + p_t(m_{t+1}' - m_t') = w_t \ell_t' + r_t n_t'.$$

Moreover, a worker becomes redundant at both the liquidity equilibrium and illiquidity equilibrium according to Kiyotaki and Moore [1]; thus, the rest of the paper no longer focuses on the behavior of workers.

Uncertainty/Ambiguity. To introduce uncertainty to the liquidity framework, we assume that an arbitrary firm in each period has the chance of π to restock its capital, i.e., invest its capital with i_t. Meanwhile, the agent has imperfect information about investment probability but infers it using the liquidity market status, implying that the investment probability π each firm faces depends on the finite state space and varyies across time. To a single firm, specify $\pi(j)_t = \Pr(\pi_{t+1} = j|n_t)$ as the conditional probability that the equity is n_t at t and the investment probability is j at $t+1$. Similarly, specify $\pi(j)^t_t = \Pr(\pi_t = j|n_t)$ as the conditional probability that the equity is n_t and investment probability is j at t. Using the Bayesian rule, it follows that

$$\pi(j)^{t+1}_{t+1} = \frac{f(n_{t+1} - n_t, j)\pi(j)_t}{\sum_i f(n_{t+1} - n_t, i)\pi(i)_t}$$

where

$$f(n_{t+1} - n_t, j) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(- \frac{(n_{j,t+1} - n_{j,t} - \mu)^2}{2\sigma^2} \right)$$

is the density of the net worth difference when it is at the state of j. Moreover, the transition path of the investment probability, $\pi(j)^t_t \to \pi(i)_t$, satisfies the Markov process such that

$$\pi(i)_{t+1} = \sum_{j=1}^{N} \Pi_{ji} \pi(j)^{t+1}_{t+1},$$

thus the posterior probability is provided by

$$\pi(i)_{t+1} = B_i(n_{t+1} - n_t, \pi_t) = \frac{\sum_{j=1}^{N} \Pi_{ji} f(n_{t+1} - n_t, j)\pi(j)_t}{\sum_i f(n_{t+1} - n_t, i)\pi(i)_t}.$$

Moreover, incorporating these settings with the firm's utility function yields the utility under uncertainty such that:

$$U_t = \ln c_t + \frac{\beta}{1-\eta} \ln \left\{ \mathbb{E}^{\pi} \exp\left((1-\eta)\mathbb{E}U_{t+1} \right) \right\}, \tag{4}$$

where $\mathbb{E}^{\pi}$ is the expectation over the distribution of π and η is the ambiguity aversion parameter. Note that, given $\eta \to 1$, the utility becomes $U_t = \ln c_t$ as the second term approaches 0; while, given $\eta \to \infty$, the utility is:

$$U_t = \ln c_t + \beta \min_{\pi} \mathbb{E}U_{t+1}, \tag{5}$$

that is, the classical ambiguity function where the agent makes their decision under the worst case scenario. To seek the analytical solutions to our model, we only focus on the case of (5) throughout the theoretical analysis.

Equilibrium conditions. According to Kiyotaki and Moore [1], the basic liquidity framework has two types of equilibrium, one is the liquidity equilibrium (LE) where fiat money serves no purpose, that is, $p_t = 0, q_t = 1$. The other one is the illiquidity equilibrium (IE) where firms need to preserve money to survive the time of liquidity shortage; it follows that $p_t > 0, q_t > 1$, as money becomes valuable. We begin with the LE conditions. First, note that the aggregate capital in the economy equals the summation of firms' saving/net worth

$$K_{t+1} - \lambda K_t = I_t = r_t K_t - C_t,$$

combining it with (1)–(3) under the condition of $p_t = 0, q_t = 1$, a little computation yields

$$\begin{aligned} r_t n_t - c_t &> (1-\theta)i_t - \phi_t \lambda n_t \\ \Rightarrow \pi(r_t K_t - C_t) = \pi I_t &> (1-\theta)I_t - \phi_t \lambda \pi K_t \\ \Rightarrow \pi(1-\lambda)K &> (1-\theta)(1-\lambda)K - \phi\lambda\pi K \\ \Rightarrow \pi(1-\lambda) &> (1-\theta)(1-\lambda) - \phi\lambda\pi, \end{aligned} \tag{6}$$

which is the core of the LE condition and also the criterion of the first best efficiency. To obtain the feasible policy recommendations for reality, we summarize the properties of (6) using Remark 1.

Remark 1. *The first best efficiency can be achieved by:*

(1) given $\phi^* \geq 0 > -\frac{1-\lambda}{\lambda}$ for $\theta = 1$ or $\phi^* > \frac{(1-\lambda)(1-\pi)}{\pi\lambda}$ for $\theta = 0$;
(2) given $\theta^* \geq 0 > 1 - \frac{\pi}{1-\lambda}$ for $\phi = 1$ or $\theta^* > 1 - \pi$ for $\phi = 0$.

Intuitively, Remark 1 characterizes the first best efficiency from both the resellablity constraint and the prepay collateral. For instance, given θ^* as the policy parameter, then, for any $\theta^* \in [0, 1]$, the LE will be realized whenever ϕ is sufficiently big (say $\phi = 1$). On the contrary, the firm may not resell enough old equities to back up its financing need if ϕ is sufficiently small, thus the only way to realize the LE is to lower the prepay/collateral ratio to $1 - \theta^* < \pi$. The reasoning and logic are very much the same at the perspective that ϕ^* is the policy parameter. To visualize these two efficiency boundaries, Figure 1 presents a numerical illustration.

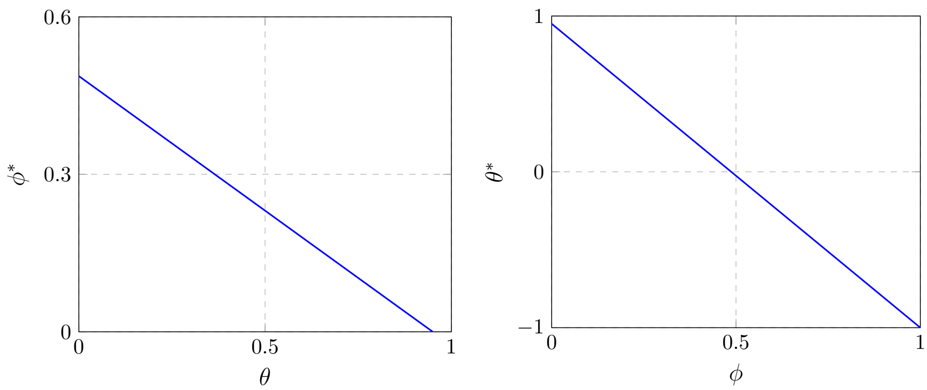


Figure 1. Efficiency boundaries.

We next discuss the IE conditions. The firm has the chance to invest but will face the binding liquidity constraint such that

$$n^i_{t+1} = (1-\theta)i_t + (1-\phi_t)\lambda n_t;$$

inserting this constraint into the cash flow, Equation (3) provides the new cash flow equation under IE:

$$c^i_t + q^R_t n^i_{t+1} = r_t n_t + [\phi_t q_t + (1-\phi_t)q^R_t]\lambda n_t + p_t m_t. \tag{7}$$

where $q^R_t = \frac{1-\theta q_t}{1-\theta}$ is the replacement cost and $q^R_t < 1$ for $q_t > 1$. According to Kiyotaki and Moore [1] and the log form of the firm's utility, the consumption c^i_t and investment i_t can be derived as

$$c_t^i = (1-\beta)\{r_t n_t + [\phi_t q_t + (1-\phi_t)q_t^R]\lambda n_t + p_t m_t\}$$
$$i_t = \frac{(r_t + \lambda\phi_t q_t)n_t + p_t m_t - c_t^i}{1-\theta q_t}.$$

For these firms that do not have the chance to invest, the liquidity constraint is not necessary and the investment equals 0, $i_t = 0$. Therefore the cash flow equation for this type of firm is provided by:

$$c_t^s + q_t n_{t+1}^s + p_t m_{t+1}^s = r_t n_t + q_t \lambda n_t + p_t m_t, \tag{8}$$

and the solution of consumption is $c_t^s = (1-\beta)(r_t n_t + q_t\lambda n_t + p_t m_t)$. Moreover, note that $c_t^i < c_t^s$ due to $q > 1 > q^R$.

These two cash flow equations can solved for the firms' intertemporal decisions using the Euler equations. In doing so, (4) is further specified as the expected utility containing both types of firms (or say a firm with two different states) such that:

$$U_t = \ln c_t + \frac{\beta}{1-\eta}\ln\Big\{\sum_j \pi(j)_t \exp\big((1-\eta)\mathbb{E}U_{t+1}(c_{t+1}^i)\big) + \sum_j (1-\pi(j)_t)\exp\big((1-\eta)\mathbb{E}U_{t+1}(c_{t+1}^s)\big)\Big\}.$$

Combining the last equation with (7) and (8) and using the first order condition with respect to n_t and m_t provides the Euler equations as follows:

$$u'(c_t) = \mathbb{E}_t\Bigg\{\frac{p_{t+1}}{p_t}\beta\frac{\pi(j)_t\exp\big((1-\eta)\mathbb{E}U_{t+1}(c_{t+1}^i)\big)u'(c_{t+1}^i)}{\sum_j \pi(j)_t\exp\big((1-\eta)\mathbb{E}U_{t+1}(c_{t+1}^i)\big) + \sum_j(1-\pi(j)_t)\exp\big((1-\eta)\mathbb{E}U_{t+1}(c_{t+1}^s)\big)}\Bigg\}$$
$$+\mathbb{E}_t\Bigg\{\frac{p_{t+1}}{p_t}\beta\frac{(1-\pi(j)_t)\exp\big((1-\eta)\mathbb{E}U_{t+1}(c_{t+1}^s)\big)u'(c_{t+1}^s)}{\sum_j \pi(j)_t\exp\big((1-\eta)\mathbb{E}U_{t+1}(c_{t+1}^i)\big) + \sum_j(1-\pi(j)_t)\exp\big((1-\eta)\mathbb{E}U_{t+1}(c_{t+1}^s)\big)}\Bigg\},$$

$$u'(c_t) = \mathbb{E}_t\Bigg\{\beta\frac{\overbrace{(r_{t+1}+\lambda q_{t+1})/q_t}^{R_{t+1}^s}\cdot\pi(j)_t\exp\big((1-\eta)\mathbb{E}U_{t+1}(c_{t+1}^i)\big)u'(c_{t+1}^i)}{\sum_j \pi(j)_t\exp\big((1-\eta)\mathbb{E}U_{t+1}(c_{t+1}^i)\big) + \sum_j(1-\pi(j)_t)\exp\big((1-\eta)\mathbb{E}U_{t+1}(c_{t+1}^s)\big)}\Bigg\}$$
$$+\mathbb{E}_t\Bigg\{\beta\frac{\overbrace{(r_{t+1}+\phi_{t+1}\lambda q_{t+1} + (1-\phi_{t+1})\lambda q_{t+1}^R)/q_t}^{R_{t+1}^i}\cdot(1-\pi(j)_t)\exp\big((1-\eta)\mathbb{E}U_{t+1}(c_{t+1}^s)\big)u'(c_{t+1}^s)}{\sum_j \pi(j)_t\exp\big((1-\eta)\mathbb{E}U_{t+1}(c_{t+1}^i)\big) + \sum_j(1-\pi(j)_t)\exp\big((1-\eta)\mathbb{E}U_{t+1}(c_{t+1}^s)\big)}\Bigg\}.$$

where R_{t+1}^i, R_{t+1}^s are the return rates of two different types of firms, with and without investment chance, respectively, for holding the equity from t to $t+1$. Moreover, the aggregate capital and market clearing are simply provided by:

$$(1-\theta q_t)I_t = \pi\{\beta[(r_t+\lambda\phi_t q_t)K_t + p_t M] - (1-\beta)(1-\phi_t)\lambda q_t^R K_t\}, \tag{9}$$

$$r_t K_t = I_t + (1-\beta)\Big\{\big[r_t + (1-\pi+\pi\phi_t)\lambda q_t + \pi(1-\phi_t)\lambda q_t^R\big]K_t + p_t M\Big\}. \tag{10}$$

3. Theoretical Analysis at Equilibrium

To facilitate the following analysis, it is useful to rewrite the liquidity constraint. In doing so, note that the original liquidity constraint has the form of

$$(1-\lambda)\theta + \lambda\pi\phi \le (1-\lambda)(1-\pi)$$

and the binding equation yields

$$\phi = \frac{(1-\lambda)(1-\pi-\theta)}{\lambda\pi},$$

then the superbinding equation for undersupplied liquidity can be expressed by

$$\phi' = \phi - \Delta.$$

One notable point is that the wedge $\Delta > 0$ separates these two types of liquidity constraints, so that Δ can also be considered as the gap between the liqudity demand and supply. Moreover, given $\phi \in [0, 1]$, the parameter condition for investment chance π is

$$(1 - \theta)(1 - \lambda) \leq \pi \leq (1 - \theta).$$

We now focus on the firm's behavior and how uncertainty affects the equilibrium. The aggregate condition (9) and (10) are solved for the risk free rate r_t and real money balances $l = pM/K$ such that:

$$\pi\beta r = (1 - \beta + \pi\beta)\kappa + (1 - \beta)[\lambda\pi\beta(1 - \pi)\eta - \widehat{\theta}]q, \tag{11}$$

$$\pi l = (1 - \pi)\kappa - [\lambda\pi - \lambda\pi(\beta + \pi - \pi\beta)\eta + \widehat{\theta}]q. \tag{12}$$

Note that fiat money becomes valuable whenever $l > 0$, corresponding to the binding liquidity constraint, indicating firms that have the investment chance may face liquidity shortage. Thus, using Equation (12) and $l > 0$, we can obtain the upper bound of the equity equilibrium price such that

$$1 < q < \frac{(1 - \pi)\kappa}{\lambda\pi - \lambda\pi(\beta + \pi - \pi\beta)\eta + \widehat{\theta}} \equiv \widehat{q}, \quad \forall l > 0,$$

where $\kappa, \eta, \widehat{\theta}$ are

$$\eta = \frac{\pi - (1 - \lambda)(1 - \theta)}{(1 - \theta)\lambda\pi}, \ \kappa = 1 - \lambda + \frac{(1 - \beta)[\pi - (1 - \lambda)(1 - \theta)]}{1 - \theta}, \ \widehat{\theta} = \frac{\theta}{1 - \theta}\pi.$$

The following content will show that the real money balances and the upper bound of the equity price can change from $l = 0, q = 1$ to $l > 0, q \in (1, \widehat{q})$ when firms are under uncertainty/ambiguity. Before anatomizing this effect, we need to propose a pivotal Lemma.

Lemma 1. *There is a unique expectation threshold that changes the status of the liquidity constraint.*

Proof. According to the claim of classical uncertainty theory, agents always make decisions under the worst case scenario, so we set $\underline{\pi} = \pi - \xi, \xi > 0$ as the investment chance for the worst case. Given the specification of liquidity demand ϕ, it is useful to define liquidity supply $\bar{\phi}$ such that

$$\bar{\phi} > \phi \equiv \frac{(1 - \theta - \pi)(1 - \lambda)}{\lambda\pi} > 0$$

and $\bar{\phi} < 1$. Moreover, the derivatives of ϕ to $\underline{\pi}$ further show that

$$\frac{\partial\phi}{\partial\underline{\pi}} = -\frac{(1 - \lambda)\lambda\underline{\pi} + \lambda(1 - \theta - \underline{\pi})(1 - \lambda)}{(\lambda\underline{\pi})^2} < 0,$$

$$\frac{\partial^2\phi}{\partial\underline{\pi}^2} = \frac{2\lambda^2\underline{\pi}\left[(1 - \lambda)\lambda\underline{\pi} + \lambda(1 - \theta - \underline{\pi})(1 - \lambda)\right]}{(\lambda\underline{\pi})^4} > 0,$$

indicating ϕ is strictly decreasing in $\underline{\pi}$. Meanwhile $\lim_{\underline{\pi} \to 0} \phi = +\infty, \lim_{\underline{\pi} \to (1-\theta)(1-\lambda)} \phi = 1$, saying that $\phi > \bar{\phi}$ holds for a unique $\underline{\pi} > 0$, corresponding to a unique ξ, that is, a unique expectation threshold. □

Intuitively, what Lemma 1 tries to convey is that firms tend to expand the size of external financing at the current period when they expect the future investment chance will drop and that this is the only best response according to their profit maximization. On the other hand, the liquidity market faced an intensified demand rise in the short run due to firms' "precautionary saving", resulting in liquidity shortage. Figure 2 provides a proper illustration of this effect.

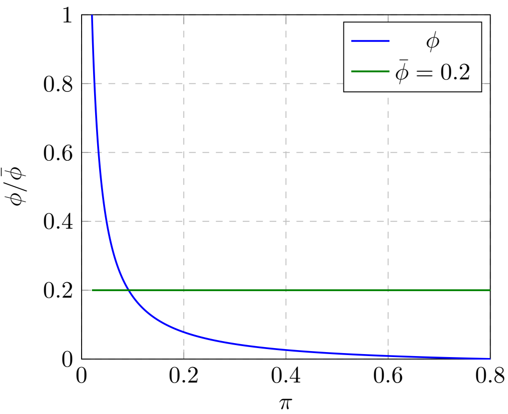


Figure 2. Liquidity shortage.

We next explore firms' financing strategy under uncertainty. The specification of R^s and R^i in the Euler equations provides:

$$R^s = \frac{r + \lambda q}{q}, \quad R^i = \frac{r + \phi\lambda q + (1-\phi)\lambda q^R}{q},$$

where R^s is the return rate of the firms that do not have the investment chance, while R^i denotes the return rate of the alternate. Note that for these firms that do not have the investment chance, R^s can be regarded as the interest rate of saving; this is because this type of firm can only buy the equities from the market rather than issue the new equities by themselves (see [1]). In contrast, R^i can be regarded as the interest rate of investing. Moreover, recall that $q^R < 1$ if $q > 1$ under liquidity shortage, which also implies $R^s > 1 > R^i$ by the last two expressions, suggesting the saving firms are more profitable than the investing firms. In contrast, $R^s = 1 = R^i$ should hold if model's economy is free of uncertainty. Therefore, the key to exploring firms' behaviors under uncertainty is to anatomize how R^s and R^i change against the expectation threshold in Lemma 1.

Reserve prices. In order to obtain enough external financing to offset the loss under uncertainty, it is reasonable for firms to set a minimum reserve price $\underline{q}$ (ask price) for their newly issued equities to meet the financing demand; the ask price is rooted in the expression of R^i. Similarly, to maximize the return from saving, firms can also set a maximum reserve price $\overline{q}$ (bid price) on buying equities; this factor is also rooted in R^s. Theoretically, these reserve prices help connect uncertainty with two return rates R^i, R^s; the following proposition characterizes this mechanism in detail.

Proposition 1. *The ask price $\underline{q}$ increases in the extent of uncertainty, while the bid price $\overline{q}$ is the opposite.*

Proof. Substituting the superbinding liquidity constraint into $R^i < 1$ provides the ask price as follows:

$$\underline{q} \equiv \frac{\left[1-\beta(1-\pi)\right]\left[\pi(1-\beta)+\beta(1-\lambda)(1-\theta)+\lambda\pi(1-\beta)\Delta\right]+\beta\pi-\beta(1-\lambda)(1-\theta)+\lambda\pi\beta\Delta}{\begin{pmatrix}\beta\left[\pi-(1-\pi)(1-\lambda)(1-\theta)+\lambda\Delta\pi\theta+\Delta\pi(1-\theta)\lambda\right] \\ -(1-\beta)\left\{\beta(1-\pi)[\pi-(1-\lambda)(1-\theta)]-\pi\theta+\Delta\lambda\pi\beta(1-\pi)-\Delta\pi\theta\lambda\right\}\end{pmatrix}}.$$

Likewise, the substitution of the superbinding liquidity constraint into $R^i < 1$ provides the bid price:

$$\overline{q} \equiv \frac{[1-(1-\pi)\beta]\{(1-\lambda)(1-\theta)+(1-\beta)[\pi-(1-\lambda)(1-\theta)]+\lambda\pi(1-\beta)\Delta\}}{(1-\lambda)(1-\theta)\pi\beta-(1-\beta)\{\beta(1-\pi)[\pi-(1-\lambda)(1-\theta)]-\theta\pi+\lambda\pi\beta(1-\pi)\Delta-\lambda\pi\theta\Delta\}}.$$

On the analytical solution of $\underline{q}$ and $\overline{q}$, a little computation shows their properties concerning the expectation threshold $\underline{\pi} = \pi - \xi$: the derivative of $\underline{q}$ to $\underline{\pi}$ is provided by

$$\frac{\partial \underline{q}}{\partial \underline{\pi}} \propto -(1-\beta)\underline{\pi}^2 - (\beta-\underline{\pi})(1-\lambda)(1-\theta) - (1-\beta)\lambda\underline{\pi}^2\Delta < 0.$$

This result proves that the ask price decreases to the extent of the uncertainty. However, the derivative of the bid price is a slightly complicated due to the indeterminacy of the sign:

$$\frac{\partial \overline{q}}{\partial \underline{\pi}} \propto [(1-\beta)(1-\theta)+\lambda(1-\beta)(1-\theta)\Delta][(1-\lambda)(1-\theta)-\underline{\pi}^2-\lambda\beta(1-\beta)\underline{\pi}^2\Delta].$$

To solve this problem formally, we first let $\Delta \to 0$, which provides $\partial\overline{q}/\partial\underline{\pi} \geq 0 \Leftrightarrow \underline{\pi} \leq \sqrt{(1-\lambda)(1-\theta)}$, thus a similar condition $\underline{\pi} \in [(1-\lambda)(1-\theta), \sqrt{(1-\lambda)(1-\theta)/[1+\lambda\beta(1-\beta)\Delta]}] \Leftrightarrow \partial\overline{q}/\partial\underline{\pi} \geq 0$ holds for $0 < \Delta < 1 < \frac{1-(1-\lambda)(1-\theta)}{\lambda\beta(1-\beta)(1-\lambda)(1-\theta)}$. Based on this analytical result and following all the parameterization and $\pi = 0.05$ in Kiyotaki and Moore [1], it follows that $\partial\overline{q}/\partial\underline{\pi} > 0, \forall\underline{\pi} = \pi - \xi < 0.05$, i.e., the bid price is increasing in the extent of uncertainty. $\square$

There are two important aspects of Proposition 1 worth noting. First, the reason we insert the superbinding liquidity constraint into inequality $R^i < 1$ and $R^s > 1$ is that the binding constraint also contains a scenario where the liquidity supply meets the demand perfectly so that the liquidity shortage never happens, which is rare in reality and very much deviated from our topic, thus the use of the superbinding condition is necessary. Second, the change of the reverse prices has a strong effect on the return rates. For instance, the ask price $\underline{q}$ would go up if $\pi \to \underline{\pi}$, inducing all those feasible prices q go up as well due to $\underline{q} = \inf q$. Meanwhile, the analytical expression of R^i indicates that $\partial R^i/\partial q < 0$, thus the uncertainty eventually drags down the level of R^i. Apply the same reasoning to the bid price, $\overline{q}$ goes down when $\pi \to \underline{\pi}$, and all the feasible prices q go down as well due to $\overline{q} = \sup q$. According to the analytical expression of R^s, its derivative $\partial R^s/\partial q < 0$ indicates that uncertainty raises R^s. This mechanism clearly explains how uncertainty affects the return rates and changes their property from $R^s = 1 = R^i$ to $R^s > 1 > R^i$. We use Figure 3 to describe the change of reserve prices over different pairs (π, Δ), where Δ is the wedge between the binding and superbinding liquidity constraint and can be regarded as a multiplier to the uncertainty effect.

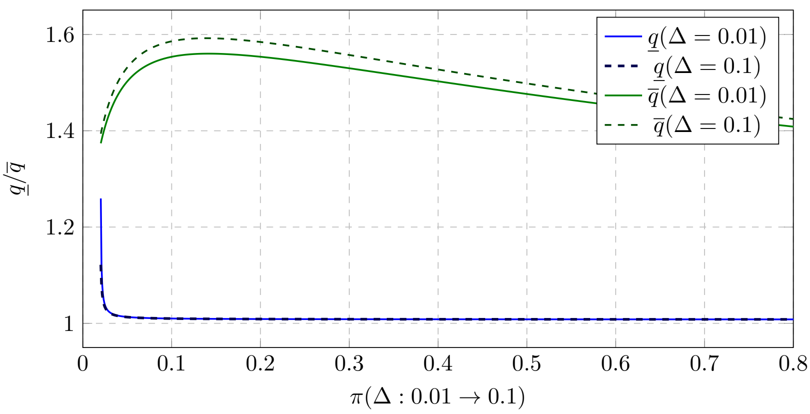


Figure 3. Reserve prices change over uncertainty.

One thing should be underlined here is that the reserve prices are only the boundaries of the equity price trading in the market rather than the equilibrium price for market clearing and the equilibrium price is still one of the pivotal elements throughout our analysis.

Equilibrium price. A simple way to obtain the equilibrium price under uncertainty is to substitute the superbinding condition $\phi' = \phi - \Delta$ into (9) and (10) and solve for a new expression of the upper bound of the equilibrium equity price such that

$$\widehat{q} \equiv \frac{(1-\pi)\big[(1-\beta)\pi + \beta(1-\lambda)(1-\theta) + \lambda\pi(1-\beta)\Delta\big]}{\lambda\pi(1-\theta) - (\beta+\pi-\pi\beta)\big[\pi-(1-\lambda)(1-\theta)\big] - \lambda\pi(\beta+\pi-\pi\beta)\Delta + \pi\theta + \lambda\pi\theta\Delta}$$

where the new parameters $\eta, \kappa, \widehat{\theta}$ are provided by

$$\begin{aligned}
\eta &= \frac{\pi-(1-\lambda)(1-\theta)}{(1-\theta)\lambda\pi} + \frac{\Delta}{1-\theta}, \\
\kappa &= 1-\lambda + \frac{(1-\beta)[\pi-(1-\lambda)(1-\theta)]}{1-\theta} + \lambda\pi(1-\beta)\frac{\Delta}{1-\theta}, \\
\widehat{\theta} &= \frac{\theta}{1-\theta}\pi + \lambda\pi\frac{\theta\Delta}{1-\theta}.
\end{aligned}$$

$\widehat{q}$ marks a certain region for the equity price at equilibrium, that is, one can always find equilibrium candidates within the region of $\widehat{q}$. However, this region can be significantly altered by uncertainty; this property is summarized by the following Proposition.

Proposition 2. *$\widehat{q}$ is increasing in the extent of uncertainty.*

Proof. In order to prove the statement in Proposition 2, it is useful to rewrite the expression of $\widehat{q}$ to $\widehat{q} = \frac{1+\varphi_1}{1+\varphi_2}$, where φ_1 and φ_2 are provided by:

$$\varphi_1 = \frac{\lambda\underline{\pi}(1-\beta)}{(1-\underline{\pi})\big[(1-\beta)\underline{\pi} + \beta(1-\lambda)(1-\theta)\big]} \cdot \Delta, \; \varphi_2 = \frac{\lambda\underline{\pi}\theta - \lambda\underline{\pi}\big[\beta + \underline{\pi}(1-\beta)\big]}{(1-\underline{\pi})\big[(1-\beta)\underline{\pi} + \beta(1-\lambda)(1-\theta)\big]} \cdot \Delta.$$

The derivative of φ_1 and φ_2 to $\underline{\pi}$ are

$$\frac{\partial\varphi_1}{\partial\underline{\pi}} \propto \lambda\underline{\pi}(1-\beta)^2 + \beta(1-\lambda)(1-\theta),$$

$$\begin{aligned}
\frac{\partial\varphi_2}{\partial\underline{\pi}} \propto & \big\{\lambda(1+\underline{\pi})[\theta-\beta-\underline{\pi}(1-\beta)] - \lambda\underline{\pi}(1-\beta)\big\}\big[(1-\beta)\underline{\pi} + \beta(1-\lambda)(1-\theta)\big] \\
& - \big[\lambda\underline{\pi}\theta - \lambda\underline{\pi}\beta - \lambda\underline{\pi}^2(1-\beta)\big](1-\underline{\pi})(1-\beta),
\end{aligned}$$

and one can conclude that $\partial\varphi_2/\partial\underline{\pi} < \partial\varphi_1/\partial\underline{\pi}$, according to the parameterization in Kiyotaki and Moore [1], that is, the change of φ_2 against $\underline{\pi}$ is smaller than the change of φ_1 against $\underline{\pi}$, implying that $\widehat{q}$ is increasing in $\underline{\pi}$ and that the upper bound of equilibrium price is increasing in the extent of uncertainty. □

Figure 4 plots the numerical result of Proposition 2. Recall the primitive result that is derived from (11) and (12), the real money balances and equity price are $l = 0, q = \widehat{q} = 1$ if the liquidity is sufficient. However, Proposition 2 reveals that uncertainty raises the value of $\widehat{q}$ and causes it to be greater than 1; this also pushes up the real money balance to positive, i.e., money starts being valuable. Therefore, the firm's dynamics or the equity price will always shift to the IE if the economy is imbued with uncertainty. Moreover, unlike Kiyotaki and Moore [1], with the help of aggregate shock, uncertainty spontaneously triggers firms' precautionary saving behavior and breaks the liquidity balance in the market, which happens to be irrelevant to the fundamental change.

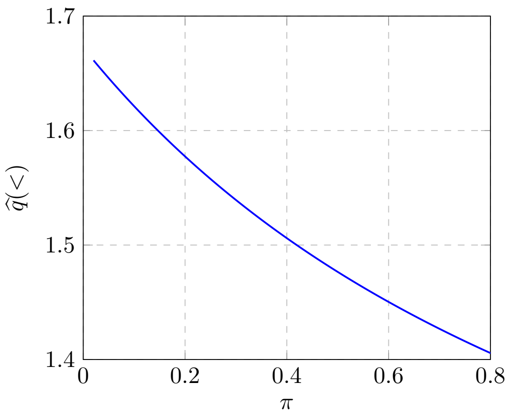


Figure 4. The upper bound of equilibrium price under uncertainty.

Given Propositions 1 and 2, we have the properties of reserve prices and equilibrium price under uncertainty; the combination of these two parts is worth commenting on. Geometrically, for any equilibrium price q, it should be equal or less than $\widehat{q}$ due to $\widehat{q}$ being a cutoff of the equilibrium price; it hence has $q \leq \widehat{q}$. Meanwhile, the reserve prices define the interval for equity prices trading between liquidity supply and demand, suggesting that a deal can be made only when $q \in [\underline{q}, \overline{q}]$. Therefore, the condition for equilibrium to be steady, unique, and existing is to satisfy $\{q : q \leq \widehat{q}\} \cap \{q : \underline{q} \leq q \leq \overline{q}\}$. However, there exists a specific region that any subset in that region could violate the equilibrium condition. To show this disequilibrium thoroughly, we simplify the Euler equations for two different types of firms as follows:

$$\Gamma(q) = (1-\pi)\beta\left[\frac{r+\lambda q}{q} - P\right]u'(c^s),$$

$$\Psi(q) = \pi\beta\left[P - \frac{r+\phi\lambda q + (1-\phi)\lambda q^R}{q}\right]u'(c^i).$$

We now know that $q > 1, m > 0$ if the equilibrium is IE, then the inflation at steady state is $P = 1$. In contrast, assume the equilibrium is still IE but that agents do not hold any money, it then has $q > 1, m = 0$ and the steady state inflation is $P = 0$. Accordingly, the expected utilities for both types are $u'(c^s) \equiv \Gamma(q) + \Psi(q)|_{P=0}, u'(c^i) \equiv \Gamma(q) + \Psi(q)|_{P=0}$. On the basic parameterization of Kiyotaki and Moore [1], it provides $\widehat{q} < \underline{q} < \overline{q}$ if the wedge is assumed to be $\Delta = 0.01$. On the one hand, this result clearly violates the equilibrium condition. On the other hand, if firms pick a trade price within interval $[\underline{q}, \overline{q}]$ and follows $q > \widehat{q}$, this would cause investing firms to experience a liquidity shortage but with zero money balance, which matches $q > 1, m = 0, P = 0$. Given the worst case scenario of $\underline{\pi}$, the two following Euler equations hold:

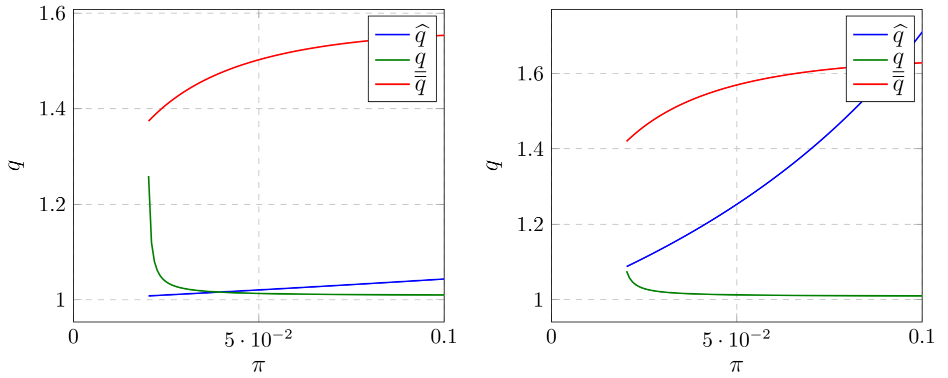
(1) For the saving firms,

$$(1-\underline{\pi})\left(\frac{1}{1-\underline{\pi}} - \frac{r+\lambda q}{q}\beta\right)u'(c^s) = \underline{\pi}\frac{r+\lambda\phi q + \lambda(1-\phi)q^R}{q}\beta u'(c^i);$$

(2) For the investing firms,

$$(1-\underline{\pi})\frac{r+\lambda q}{q}\beta u'(c^s) = \underline{\pi}\left(\frac{1}{\underline{\pi}} - \frac{r+\lambda\phi q + \lambda(1-\phi)q^R}{q}\beta\right)u'(c^i).$$

On the parameterization of $\theta = 0.19, \lambda = 0.975, \beta = 0.99$, and $\underline{\pi}$ are assumed temporally to be 0.05; a little computation shows that the solution of item (1) is $q^s \approx 1.003$, while the solution of (2) is $q^i \approx 1.212$. The buying equilibrium price and selling equilibrium price are not equal. The left panel of Figure 5 provides the disequilibrium with a straightforward example. To avoid such a disequilibrium, a supplementary constraint is needed, which is essential for the stability and sustainability of the aggregate liquidity.



(a) $\Delta \to 0.01$ (b) $\Delta \to 0.1$

Figure 5. The existence of equilibrium under uncertainty.

Proposition 3. *To guarantee the uniqueness and the existence of equilibrium under uncertainty, the tuple* $(\underline{\pi}, \Delta)$ *for the equity prices should satisfy that*

$$\underline{q}(\underline{\pi}, \Delta) < \widehat{q}(\underline{\pi}, \Delta) < \overline{q}(\underline{\pi}, \Delta).$$

To properly show the geometric intuition and the insight of Proposition 3, it is useful to use Figure 5 as the illustration, where wedge Δ is fixed at two different values and the equity prices are depicted by three lines, blue, green and red, respectively, varying against the expectation of π. On the left panel, there exists a certain region in which the blue line $\widehat{q}$ is below the green line $\underline{q}$ when wedge Δ is fixed at 0.01 and the expectation threshold is less than 0.05. Within this region, any equilibrium price candidate q will not surpass the upper bound $\widehat{q}$ ($q \leq \widehat{q}$) and it is hence not qualified for the market trading price ($q \notin [\underline{q}, \overline{q}]$). This geometric description is corresponding to the numerical result of $q^s \neq q^i$ as shown in the above. In contrast, the right panel plots the result that the constraint in Proposition 3 is implemented, where the blue line ($\widehat{q}$) lies in between the red ($\overline{q}$) and green ($\underline{q}$); it can also be verified that $q^i = q^s \approx 1.063$, meaning the existence of equilibrium under uncertainty is assured.

4. Conclusions

In this paper, we ask two pivotal questions: What worsens the aggregate liquidity in the absence of exogenous simulation? How does a firm act and cause this effect under uncertainty? By responding to these unresolved questions, the exogenous shock in Kiyotaki and Moore [1] is replaced with endogenous uncertainty to change the model's perturbation from fundamental changes/shocks to expectation shifting. Meanwhile, the conditions of the firm dynamics and individual financing decisions are solved for anatomizing why these factors can cause precautionary saving and liquidity shortages. With a detailed and thorough analysis, three main conclusions are found for the topic.

First, given that agents are uncertain about the investment chance π and making decisions under the worst case scenario, the model generates a unique expectation threshold that drags the market's (or firms') liquidity status from sufficiency to insufficiency. The main reason is that uncertainty motivates agents to begin precautionary saving, which irresistibly raises the liquidity demand, and, when the liquidity supply is fixed at a certain level in the short run, this mismatch occurs.

Second, by splitting firms into saving and investing types, the intertemporal return and reserve price of each type are obtained, as the uncertainty is interacting with them directly and explicitly. Use an investing firm as an intuitive example: to have enough external financing to offset the loss under the worst case scenario, it is reasonable for the investing firm to raise the selling price of equities, including the ask price $\underline{q}$. Meanwhile, the investing firm's return R^i is strictly decreasing in the selling price, resulting in a downfall of the firm's intertemporal return. For the saving firm, the reverse is also true.

Third, the uncertainty pushes up the upper bound of the equilibrium equity price and the value of fiat money as well, justifying that firms try to secure the external financing by raising the equity issuing price and hoarding liquid assets, such as fiat money, as the precautionary saving against the liquidity shortage. Moreover, an extra mathematical constraint is supplemented for the uniqueness and the existence of the equilibrium under uncertainty, which is also the essential element for the stability and sustainability of the liquidity.

Our theoretical results match the evidence of the agent's precautionary behavior and aggregate fluctuations before the financial crisis in Greece, the UK, and the US as Bachmann et al. [2], Acharya and Merrouche [16], and Acharya and Merrouche [16] documented. For the sake of application and further research, we would like to discuss one computational example to implement the empirical test on our theory. Specifically, note that the stochastic discounters in our model are:

$$\Lambda^{c^i}_{t,t+1} = \underbrace{\frac{\exp\left((1-\eta)\mathbb{E}U_{t+1}(c^i_{t+1})\right)}{\sum_j \pi(j)_t \exp\left((1-\eta)\mathbb{E}U_{t+1}(c^i_{t+1})\right) + \sum_j (1-\pi(j)_t) \exp\left((1-\eta)\mathbb{E}U_{t+1}(c^s_{t+1})\right)}}_{\chi_{c^i,t}} \cdot \beta \frac{u'(c^i_{t+1})}{u'(c_t)},$$

$$\Lambda^{c^s}_{t,t+1} = \underbrace{\frac{\exp\left((1-\eta)\mathbb{E}U_{t+1}(c^s_{t+1})\right)}{\sum_j \pi(j)_t \exp\left((1-\eta)\mathbb{E}U_{t+1}(c^i_{t+1})\right) + \sum_j (1-\pi(j)_t) \exp\left((1-\eta)\mathbb{E}U_{t+1}(c^s_{t+1})\right)}}_{\chi_{c^s,t}} \cdot \beta \frac{u'(c^s_{t+1})}{u'(c_t)},$$

and the ambiguous term for the investing and saving firm are $\Lambda^{c^i}_{t,t+1}$ and $\Lambda^{c^s}_{t,t+1}$, respectively. Mathematically, $\chi_{c^i,t} = \chi_{c^s,t} = 1$ if the ambiguous parameter $\eta = 1$, where the model's economy is free from uncertainty. Meanwhile, these two ambiguous terms $\chi_{c^i,t}, \chi_{c^s,t}$ are decreasing in the ambiguous parameter η, saying the bigger η is, the less resources there are for agent to have after discounting. For simplicity, use $\eta \in (1, \infty)$ as the basic measure for the ambiguity and, with additional data to calibrate the parameter array, $\{\beta, \gamma, \phi, \pi, 1-\lambda, 1-\theta, r/q+\lambda-1\}$ using the formal macroeconometric method (for example, the combination of the Kalman filter and MLE for unobservable parameters), the model can be simulated numerically. In contrast to the original equilibrium, one can assume that the agent's expectations start to shift after period 10 and the model converges to a new equilibrium at period 100 such that $X_{100} = X_{101} = X_{102} = \cdots$. Instead of using the perturbation method (because expectation shifting causes the steady state to not be fixed), projection and function approximation, such as Chebyshev polynomials, are essential for solving the model's transitional path.

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