

Table S1. Equations used for modeling sorption isotherms.

Model	Equation	Parameters	Ref.
Langmuir	$q_{eq} = \frac{q_{m,L} C_{eq}}{1 + b_L C_{eq}}$	$q_{m,L}$ (mmol g ⁻¹): Sorption capacity at saturation of monolayer b_L (L mmol ⁻¹): Affinity coefficient	[1]
Freundlich	$q_{eq} = k_F C_{eq}^{1/n_F}$	k_F and n_F : empirical parameters of Freundlich equation	[2]
Sips	$q_{eq} = \frac{q_{m,S} b_S C_{eq}^{1/n_S}}{1 + b_S C_{eq}^{1/n_S}}$	$q_{s,L}$, b_S and n_S : empirical parameters of Sips equation Sorption capacity at saturation, Affinity coefficient and Sips constant respectively	[3]

Table S2. Equations used for modeling uptake kinetics.

Model	Equation	Parameters	Ref.
PFORE	$q(t) = q_{eq,1}(1 - e^{-k_1 t})$	$q_{eq,1}$ (mmol g ⁻¹): sorption capacity at equilibrium k_1 (min ⁻¹): apparent rate constant of PFORE	[4]
PSORE	$q(t) = \frac{q_{eq,2}^2 k_2 t}{1 + k_2 q_{eq,2} t}$	$q_{eq,2}$ (mmol g ⁻¹): sorption capacity at equilibrium k_2 (g mmol ⁻¹ min ⁻¹): apparent rate constant of PSORE	[4]
RIDE	$\frac{q(t)}{q_{eq}} = 1 - \sum_{n=1}^{\infty} \frac{6\alpha(\alpha+1)\exp\left(\frac{-D_e q_n^2}{r^2} t\right)}{9 + 9\alpha + q_n^2 \alpha^2}$ With q_n being the non-zero roots of $\tan q_n = \frac{3 q_n}{3 + \alpha q_n^2}$ and $\frac{m q}{V C_0} = \frac{1}{1 + \alpha}$	D_e (m ² min ⁻¹): Effective diffusivity coefficient	[5]

(m (g): mass of sorbent; V (L): volume of solution; C_0 (mmol L⁻¹): initial concentration of the solution); r² radius of polymer particle.

Akaike Information Criterion, AIC:

$$AIC = N \ln \left(\frac{\sum_{i=0}^N (y_{i,exp.} - y_{i,model})^2}{N} \right) + 2N_p + \frac{2N_p(N_p + 1)}{N - N_p - 1}$$

Where N is the number of experimental points, N_p the number of model parameters, $y_{i,exp.}$ and $y_{i,model}$ the experimental and calculated values of the tested variable.

Table S3. Constituents of leaching solution before used for U removal at (pH=0.3)

Constituent	Conc. (mg/L)	Constituent	Conc. (mg/L)
U	530	Al ₂ O ₃	2510
REE	410	Pb	20.8
Fe	2300	Zr	29.5

Table S4. FTIR assignments of peaks for PGEG, TBP-PGEG, after loading with La and after 5 cycles of sorption desorption.

Vibration	Ref.	PGEG	PGEG+La a	PGEG+5 cycles	TBP- PGEG	TBP- PGEG+La	TBP- PGEG+5 cycles
N-H & O-H str.(overlapped)	[6-12]	3439	3464	3452	3398	3439	3442
C-H str. (aliphatic)	[13-16]	2931	2922	2929	2926, 2858	2922, 2850	2939
C(=O) O str.	[13,17,18]				1708		1745
C=O (amide)/(-C=N-) str. & 1°/2°	[19-23]	1631	1631	1631	1517	1681	1558
NH bend. (overlap)							
P=O (asym. Str.), C-H bend. /CH ₂ -N ⁺	[24-28]				1404	1465	1413
1°/2° OH bend. (in-plane)	[19,29]	1103	1105	1097	1172	1074	1168
C-O-C, C-O & C-C str.	[19,30,31]					1032	1071,1035
P-O-C str.	[26]				898		898
C-N str.	[32-34]	805	794	800	842	887(overlap)	(overlap)
P-O-C str.	[35]				611		
Fe-O	[36,37]	470	470	470	broad		615

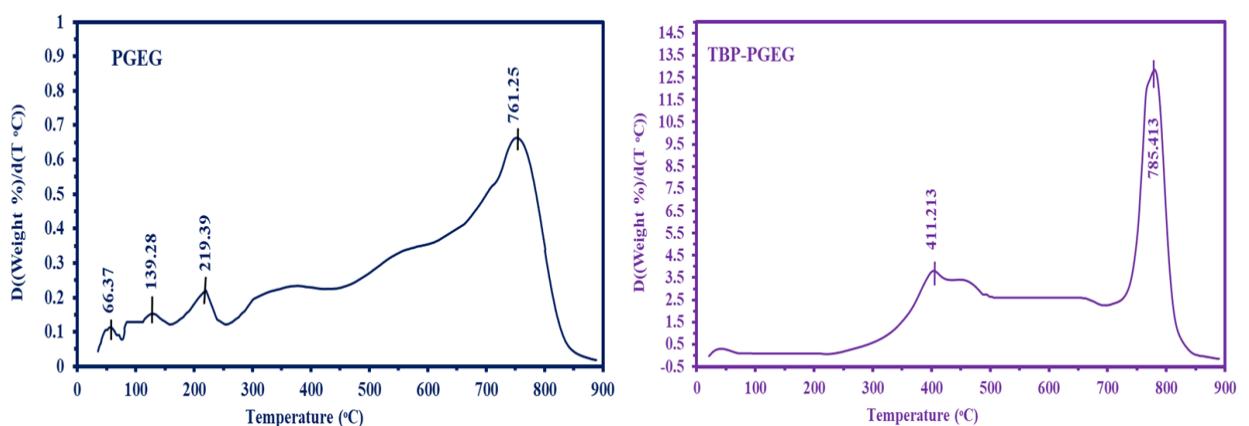


Figure S1. Dr-TG analysis of the PGEG and TBP-PGEG sorbents

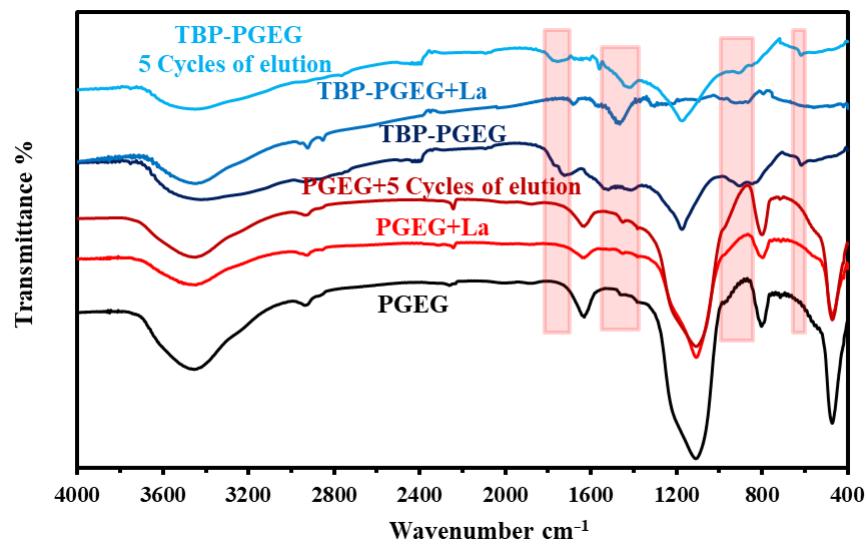


Figure S2. FTIR of collected spectra focusing on the main peaks.

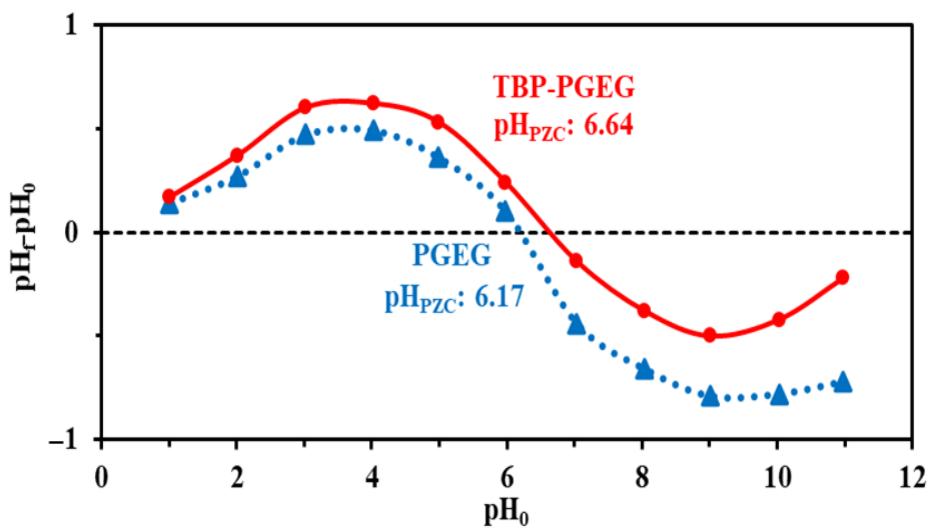


Figure S3. pH_{pzc} values of the PEG and TBP-PEG sorbents

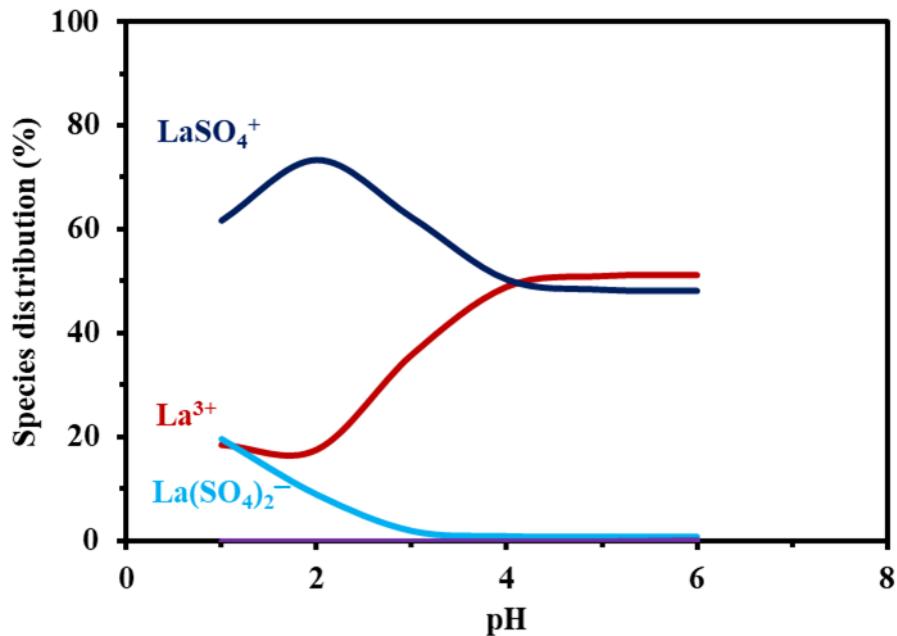


Figure S4. Speciation of La at different pH values

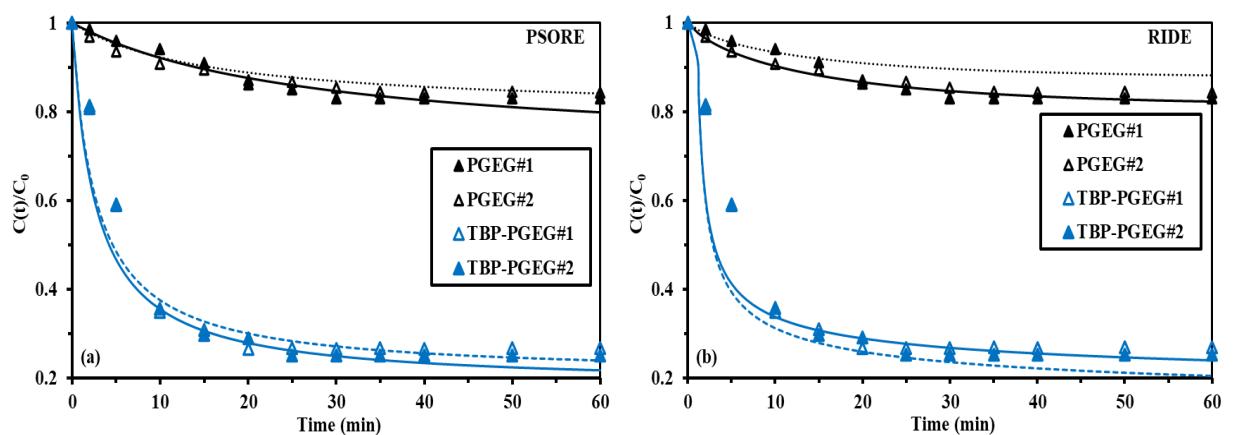


Figure S5. PSORE (a) and RIDE (b) fitting curves of PGEG and TBP-PGEG sorbents

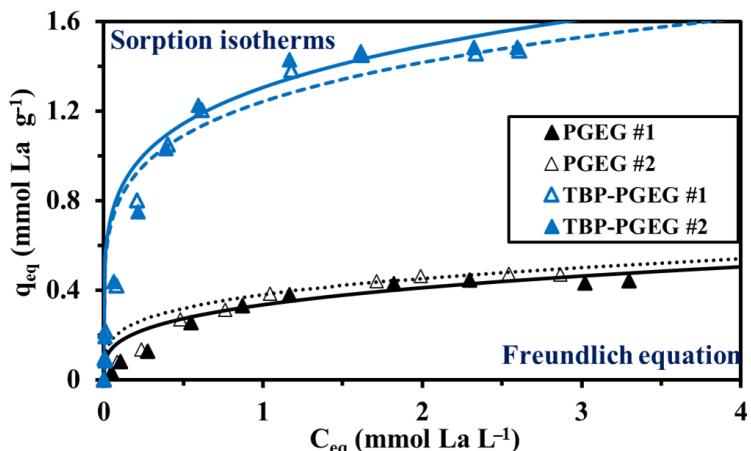


Figure S6. Freundlich equation fitting curves for the sorption isotherms of La(III) ions on PGEG and TBP-PGEG sorbents

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