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Article

Deriving an Improved Dynamic EROI to Provide Better Information for Energy Planners

Ioannis N. Kessides [1,⋆] **and David C. Wade** [2]

[1] The World Bank, 1818 H Street, NW, Washington, DC 20433, USA

[2] Argonne National Laboratory, 9700 S. Cass Avenue, Argonne, IL 60439, USA

⋆ Author to whom correspondence should be addressed; E-Mail: ikessides@worldbank.org; Tel.: +1-202-473-9345.

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Abstract: The two most frequently quantified metrics of net energy analysis–the energy return on (energy) investment and the energy payback period–do not capture the growth rate potential of an energy supply infrastructure. This is because the analysis underlying these metrics is essentially static–all energy inputs and outputs are treated the same, regardless of where they occur in the life cycle of the infrastructure. We develop a dynamic energy analysis framework to model the growth potential of alternative electricity supply infrastructures. An additional figure of merit, the infrastructure doubling time, is introduced. This metric highlights the critical importance of the time phasing of the initial energy investment for emplacing a given infrastructure, as opposed to the ongoing O&M energy expenditures, for the infrastructure's growth potential. The doubling time metric also captures the influence of capacity factor, licensing and construction time lags.

Keywords: doubling time; dynamic EROI; sustainability; net energy

1. Introduction

The world is facing an enormous energy challenge. Concerns about the rate of depletion of the more readily accessible fossil fuel resources, energy security, and climate change, are giving rise to a raging policy debate at the global level. A number of energy sources and technological options are being

examined and actively pursued. However, there are highly divergent views on the constraints and opportunities associated with all of these options. Consequently, the energy environment remains opaque and uncertain. There have been persistent questions, for example, as to whether some of the energy options (e.g., corn-based ethanol) produce less energy than they consume (directly plus indirectly). These concerns in turn have generated renewed interest in net energy analysis. In particular, recent work has produced new perspectives, deeper insights, and more careful calculations of the energy return on (energy) investment ($EROI$), and the energy pay back period (τ_1)—two of the most frequently quantified metrics of net energy and life cycle analysis. Research efforts during the past few years have focussed on: evaluating the potential impacts of a declining $EROI$ on economic activity [1]; calculating the minimum $EROI$ for a sustainable society [2]; providing a systematic review of what is known about the $EROI$ and τ_1 of major fossil fuels and renewable resources [3-7]; and analyzing the measurement errors associated with current estimates of the $EROI$ [8].

There is also an ongoing debate about the ability of renewable generating technologies for scaling to materiality—*i.e.*, scaling to the terawatt level [9-11]. This is an important consideration, because global electricity demand is projected to almost double from around 16,000 TWh in 2007 to just under 29,000 TWh in 2030. Over 80% of that growth is projected to come from developing countries. The compound average growth rate of demand between 2007 and 2030 is estimated to be around 2.5% per annum for the world as a whole (1% for OECD and 3.9% for non-OECD countries). Such a growth rate might, at first glance, appear to be modest. However, the base is substantial—so the implied absolute increase in demand is huge. To put things into perspective: In 2007, global electricity generation capacity was around 4,500 GW. By 2030 it is projected to increase to just under 8,000 GW. This would be equivalent to adding 3.5 countries like the US (1039 GW) or 5 continents like OECD-Europe (847 GW) to the electricity supply pool [12].

The life cycle parameters derived from net energy analysis are helpful in assessing energy systems on the basis of energetics—*i.e.*, in terms of energy input and output over their lifetime. As such, they are useful in comparing alternative energy systems in terms of their use of society's productive resources for delivering a given amount of energy, and ultimately, in terms of their efficiency. However, the conventional energy analysis is essentially static. All energy inputs and outputs are treated the same, regardless of where they occur temporally in the life cycle of the energy technology [13]. The underlying equations of such analysis do not have a transient term. This limits the potential role of net energy analysis in energy planning where human preferences in energy use across time should properly be taken into account.

This paper develops a model describing the dynamic behavior of an energy facility (or a technology) under a plowback constraint—*i.e.*, a certain fraction of the facility's (or technology's) power output is plowed back into the self-replicating construction of new facilities and their associated resource supply and delivery infrastructures, while the rest of its output is made available to meet society's active energy demand. The requirement that each energy technology makes a contribution towards the national energy demand besides taking care of its own expansion (and thus avoids being a net energy sink) [14] is motivated by the tight demand and supply balance facing most countries around the world. Our dynamic energy analysis indicates that the single numerical values of life cycle energy metrics, $EROI$ and τ_1, are not sufficient for assessing the capacity of a given infrastructure to support rapid growth rates.

It is important to understand the "structure" and time dependencies of the energy investments required for emplacing and maintaining such an infrastructural facility. For this we propose and derive a third metric, the Doubling Time, τ_2, which clarifies the way in which certain physical characteristics (e.g. the intermittency and low conversion efficiency and power density of energy flows) limit a given energy infrastructure's capacity for expansion and self-replication. The doubling time metric τ_2 measures the amount of time required for a given energy facility to produce and accumulate enough excess energy, after making a contribution to national energy demand, to replicate itself by constructing another facility of similar capacity–*i.e.*, it measures the capability of a given energy infrastructure to sustain and reproduce itself from its own output while making sufficient residual energy available for societal use. The doubling time metric for a given energy facility depends on several fundamental characteristics of its underlying technology, including: the capacity factor; amount of energy required for constructing and emplacing a unit of nameplate capacity; fraction of the facility's gross energy output used for its operation and maintenance; time required for constructing and emplacing a new facility; and effective lifetime of the facility.

Utilizing literature values of $EROI$, τ_1, and other physical parameters based on life cycle analyses of different electric power generation sources, we find significant differences between fossil fuel fired plants, nuclear power, and renewable technologies in terms of their ability to achieve high rates of indigenous capacity expansion. The low power density of renewable energy extraction and the intermittency of renewable flows impose deep physical limits to their growth trajectory.

2. Historical Evolution of Energy Supply Infrastructures

At a simplified level of representation, an energy supply infrastructure consists of resource collection and concentration channels feeding into a conversion node that transforms the energy resource into more "convenient" energy carriers. These in turn supply distribution channels delivering energy to final users of energy services. The energy carriers have the capacity to deliver either heat or work. The converters transform a resource energy carrier into a delivery energy carrier that is more suitable to user needs. The resource delivery and the distribution channels involve spatial transport of energy carriers. In combination, they ship the energy content of the resource to the end user.

In general, each link in the energy supply chain produces wastes during the process of resource harvesting, transport, conversion and end use. The wastes range from solid to gaseous to heat. They may be chemically or radio toxic. They may be persistent or transitory.

Prior to the late 1700's the underlying energy resource was derived strictly from the sun. The radiant energy fluxes from the sun were collected and concentrated principally in three forms: (i) harvesting of foodstuffs which were carted to towns where they supplied animals and men who in turn were capable of delivering work; (ii) harvesting wood and straw and putting it into a processed form suitable for conversion into fire to heat and light; (iii) rain water concentrated onto streams and rivers running downhill where a waterwheel energy converter transformed it to work. Thus, the pre-industrial societies relied mainly on biomass fuels and animate energy converters [15]. This multi-millennium-old energy infrastructure prevailed in Europe and the Americas until the beginning of the 19th century and in most of Asia and Africa until the middle of the 20th century [16]. It still comprises the principal energy supply for a large segment of the world population today.

2.1. The Role of Energy Density and the Power Density of Energy Conversion

Table 1 compares the gravimetric energy density (amount of energy per unit of weight) of pre-industrial biomass fuels with the fossil resources of the industrial era and the nuclear fuel resource of the atomic era. Air-dry wood has energy density around 16 MJ/kg and most other biomass fuels have energy densities below 20 MJ/kg, while good quality bituminous coal is over 24 MJ/kg. The energy density of crude oil is just below 42 MJ/kg and that of refined oil products is 43-46 MJ/kg. Moreover, the energy density of uranium is over $3 \cdot 10^6$ MJ/kg. Solid and liquid fuels have an even greater advantage in terms of volumetric density (amount of energy per unit of volume) in comparison to biomass and gaseous fluids: natural gas rates around 35 MJ/m^3 while crude oil has approximately 35 GJ/m^3, *i.e.* its volumetric density is one thousand times higher [17-19].

Table 1. Energy densities of energy carriers.

Energy Carrier	Energy Density (MJ/kg)
Wood	16
Coal	22-25
Oil	42
Nuclear fuel	$3 \cdot 10^6$

The historic transitions from biomass to coal and then from coal to petroleum entailed a movement towards more concentrated sources of energy. Higher energy density carriers present significant advantages in terms of their extraction, portability, shipping and storage costs, and conversion options [17]. The greater the energy density (gravimetric and volumetric) the more energy transported or stored for the same amount of weight or volume. The changeover to a high energy density supply infrastructure took place not only at the resource harvest links in the supply chain but also at the conversion nodes and delivery links.

Table 2 compares the power densities (energy flux per unit of horizontal surface) of alternative electricity supply infrastructures. All renewable generation technologies have power densities that are substantially lower (2-3 orders of magnitude) than the fossil-fuelled modes. The modest energy density of renewable sources and the very low power density of renewable energy extraction imply that these new technologies will require much larger infrastructures, spread over significantly greater areas, relative to today's infrastructure of fossil fuel extraction, combustion and electricity generation, to produce an equivalent quantity of energy [17,20]. Renewable technologies will generally require larger energy expenditures for the initial emplacement of their facilities–*i.e.* they will entail higher emplacement energy costs per unit of nameplate capacity.

Table 2. Power densities of alternative electricity supply infrastructures.

Power Source	**Power Density (W/m^2)**	
	Low	High
Natural Gas	200	2000
Coal	100	1000
Solar (PV)Oil	4	9
Solar (CSP)	4	10
Wind	0.5	1.5
Biomass	0.5	0.6

2.2. Excess Energy Availability as a Driver of Social Change

Advances in most human endeavors—transportation, agriculture, commerce, science and technology, health care, household life—were driven directly or indirectly by the changes in society's underlying energy systems and the availability of surplus energy. Indeed, the extraordinary expansion of the human population, economic growth and rising standards of living were powered by high-$EROI$, high energy surplus fossil fuels [2,20-21].

In the pre-industrial age, the attainable energy density at the output of a converter node was constrained by practical capacities of the harvest and shipping channels which supplied it. This in turn constrained the population density that could be supported. Food collection and delivery constrained the concentration of population to what its hinterland could support–primarily small villages embedded in the hinterland itself, to reduce delivery distances to a day's travel or so. Waterwheels driving grain mills and sawmills of the 1800's delivered very modest amounts of power. As a result, the cities of the preindustrial age were relatively small and societies were predominantly rural and agricultural [22].

Throughout the pre-industrial era, not only population densities, but overall populations were constrained by the infrastructure for food (energy) delivery. Malthus stated the constraint on a nation's overall population as a function of arable land availability (*i.e.*, food/energy supply). Sustainability was maintained for many centuries preceding the late 1700's as a quasi-steady state balance of energy supply and population – but it was maintained at a small world population and at a medieval lifestyle of stagnant (and small) GDP per capita [23].

The transition to high energy density carriers and converters, where it has taken place in the Western industrialized nations, has dramatically changed the character of society. Population is now concentrated in cities, many of which are huge compared to the pre-industrial era. Population migrated to the factory towns of England and America during the 1800's to exploit the concentrated energy density from coal-fueled steam power. Factory production rapidly replaced the earlier cottage industry regime of societal organization.

There are several underlying causes for these historical changes in society that accompanied the evolution in energy supply infrastructure. They happened in part because the new energy supply infrastructure delivered an increased net surplus energy relative to that required to maintain the earlier medieval

steady state. Also, institutional arrangements were made which facilitated corresponding concentration of capital and labor to match the concentration of energy.

3. Transition Towards a Sustainable Global Energy Supply Infrastructure

In the 21st century, world society is attempting to achieve a transition to a new energy supply infrastructure that supports the tenets of sustainable development. The requirements of the enabling energy supply infrastructure include:

- Capacity to deliver net excess energy;
- Scalability;
- Longevity;
- Environmental friendliness;
- Capacity to achieve required growth rates.

The requirement for "generation of net excess energy" is the essential ingredient for the supply infrastructure to facilitate economic growth.

"Scalability" pertains to its practical capacity to supply the required vast amounts of energy to support rising global energy demand–the New Policies Scenario of the International Energy Agency (IEA) projects global energy consumption to increase by 36% from 2008 to 2035, rising from 12,300 Mtoe to 16,750 Mtoe [12].

"Longevity" pertains to the long term availability of the energy resource at current and projected levels of use into the distant future.

"Environmental friendliness" pertains to minimizing the waste burden generated by the infrastructure emplacement and operation and to reducing the carbon footprint.

"Capacity to achieve required growth rates" pertains to dynamic response capability of the infrastructure to grow under constraints of energy plowback required to support infrastructure growth and operation.

For the purposes of this paper, it is postulated that the energy carriers that deliver end use services will remain unchanged (electricity and liquid chemical fuels) because they are already optimized for high energy density, versatility and convenience. Rather, it is assumed that the transition to a sustainable energy supply infrastructure will occur in the resource harvesting and concentration and in the associated energy conversion nodes in the supply chain.

As evidence mounts on the threats of climate change, pressures are increasing for a major shift away from fossil fuels and towards renewable and other low-carbon energy sources. However, if history is of any guide, the transition to a low-carbon economy will be slower and more challenging than some optimists have claimed. Fossil fuels will be displaced but only gradually. In the New Policies Scenario of the IEA, for example, the share of fossil fuels in the primary energy mix will decline only modestly from 81% in 2008 to 74% in 2035 [12].

The impediments to a rapid energy transition derive from technological, economic, and social factors. First, technological innovations have to become available. Transitions require a specific sequence of

scientific advances, technical innovations, and organizational changes. If any one element of this sequence is missing or delayed, then the transition period becomes lengthier [24]. Second, since existing assets usually have long economic lifetimes, there is active resistance on the part of their owners to any change that would lead to their premature replacement. Finally, social reluctance to change and active resistance of stakeholders in the legacy infrastructure retard entry of new technologies. Significantly in that regard, market entry sometimes can result only because of changes in institutional arrangements (that are resisted by the stakeholders in the current regime).

Immutable physical upper bounds are imposed on the pace of entry by the energy balance of the proposed infrastructure itself. Energy must be expended to emplace and operate new infrastructure and this reduces the excess energy available for societal use. There are fears, for example, that a very rapid transition to a renewable-energy economy could lead to the cannibalization of energy from existing power plants and thus exacerbate the current global energy scarcity [25].

4. Figures of Merit for Energy Supply Infrastructures

Assume energy demand increases incrementally due to population growth and/or increase in annual energy use per capita. This demand increase will be accommodated by increasing the capacity of the supply infrastructure. When the number of deployed converter nodes, extent of area required to harvest the fuel resource, and the associated shipping needed for delivery to the converter are increased, then an incremental cost is incurred in the form of energy expended to emplace the new infrastructure and to operate it. This energy cost must be borne by the existing and new infrastructure. If this cost gets larger as a fraction of the capacity of the infrastructure to deliver energy, then the rate of delivery of net energy declines.

To assess the ability of alternative electricity generating technologies in facilitating the transition towards a sustainable global energy supply infrastructure we employ two existing figures of merit and propose a third one. Our analysis is guided by a simple and yet fundamental principle invoked by Hall et al [2]: that for any being or system to survive and grow, and thus make a contribution to sustainable development, it must gain substantially more energy than it uses in obtaining that energy. Moreover, as Cleveland [25] rightly notes, the size and rate of delivery of such surplus energy are important in assessing sustainability.

4.1. Energy Return in Energy Investment

Several figures of merit are in use to characterize the net (excess) energy output to be derived from emplacing or enlarging an energy supply infrastructure under an energy plowback constraint. One example is the energy return on (energy) investment ($EROI$). It is defined as [2, 26-28]:

$$EROI = \frac{\text{gross quantity of energy delivered over the infrastructure lifetime}}{\text{quantity of energy expended to emplace and operate the infrastructure over its lifetime}}$$

The numerator is given by:

(Numerator) = $P_{np} \cdot \psi \cdot T$
where

- P_{np} is the nameplate power capacity of the infrastructural facility;
- ψ is the facility's average load or capacity factor;
- T is the effective lifetime of the facility.

The denominator is equal to: the sum of the energy, E, expended to initially emplace (and ultimately decommission) the infrastructural facility, plus the energy expended for the operation and maintenance (O&M) of the facility over its lifetime:

(Denominator) = $E + P_{np}\psi hT$
where

- E is the energy expended to emplace and ultimately decommission the facility;
- h is the "hotel load fraction", *i.e.* the fraction of gross energy produced that is diverted for the operation and maintenance of the facility.

Thus,

$$EROI = \frac{P_{np}\psi T}{E + P_{np}\psi hT}. \quad (1)$$

Note that in this definition:

- the energy content of the fuel resource harvested and processed is not included in the denominator – *i.e.*, only the energy needed for the infrastructure per se is accounted for;
- the conversion efficiency from fuel energy carrier to product energy carrier is embedded in the numerator – and the energy unit is joules of heat for both numerator and denominator.

Inspection of the formula shows that $EROI$ will be increased by:

- reducing $\frac{E}{P_{np}}$, the emplacement energy/nameplate capacity;
- increasing ψ, the load factor;
- reducing h, the O&M energy fraction of production;
- increasing T, the facility's lifetime.

When determining the numerical value of $EROI$ for an energy supply infrastructure, it is necessary to consider the entire supply chain from harvesting the resource to delivering energy services to end users – summing up all the energy expended to emplace and operate that supply chain. This involves a life cycle analysis (LCA) to evaluate energy consumption elements in every stage of the supply chain—the resource collection, energy conversion, and the energy distribution and delivery links of the infrastructure.

The extent of upstream and downstream energy expenditures to emplace new infrastructure is not standardized, and as a result, ranges of values for $EROI$ are found in the literature. LCAs are extensive undertakings when performed individually for a specific infrastructure, and are even more complex when done to enable a comparison of alterative infrastructures under consistent assumptions. Still, there are several comparative LCAs in the literature that have taken care to report values for $EROI$ under consistent assumptions for alternative candidate infrastructures.

Given an $EROI$ value for a candidate infrastructure, the excess energy available for societal use is [3,29]

$$\begin{aligned}\text{(Lifetime Excess Energy)} &= \text{(Lifetime Gross Energy Output) - (Lifetime Energy Input)} \\ &= \text{(Lifetime Gross Energy Output)} \cdot \left[1 - \frac{1}{EROI}\right].\end{aligned}$$

As Hall [30] points out, while much of human progress can be attributed to technology, much of that technology has been a means for using more energy for human ends. Surplus energy is what facilitates economic growth, technological progress, and most human endeavors. If energy supply infrastructures with high $EROI$ are deployed then only a small portion of society's energy budget would be required by the energy sector itself. The rest could be utilized to support all economic, commercial, and social activities that are so critically dependent upon energy [20].

The $EROI$ as a figure of merit pertains to "efficiency" of an infrastructure–its ability to deliver excess energy to society–integrated over its lifetime (given an assumption about the availability of a resource input). However, it does not address:

- scalability (because it is a ratio);
- longevity (because it assumes the availability of a resource);
- capacity to achieve required growth rates.

An informative way to think about $EROI$ is in terms of the fraction of lifetime gross energy output that is expended for initial emplacement and that which is needed for operation and maintenance:

$$EROI = \frac{P_{np}\psi T}{E + P_{np}\psi h T} = \frac{1}{\dfrac{E}{P_{np}\psi T} + h} \tag{2}$$

i.e.,

$$EROI = \frac{1}{\left(\begin{array}{c}\text{fraction of gross production} \\ \text{expended for initial emplacement}\end{array}\right) + \left(\begin{array}{c}\text{fraction of gross production} \\ \text{expended for O\&M}\end{array}\right)}$$

The first term represents an initial "capitalization" expenditure of energy. And the second term represents an ongoing "variable" expenditure. However, unless the details of the LCA are available, this breakdown is not evident because when $EROI$ values are reported, the two components of the denominator become subsumed and indistinguishable within a single number.

4.2. Energy Payback Period

A second figure of merit, the "energy payback period", τ_1, is used to characterize the "capital" component. The energy payback period is a measure of the time interval required for the infrastructure – once it is installed – to deliver net energy sufficient to cover the initial energy investment [31,32]

$$\tau_1 P_{np} \psi (1 - h) = E \tag{3}$$

or

$$\tau_1 = \frac{E}{P_{np} \psi (1 - h)} \tag{4}$$

Writing

$$\frac{\tau_1}{T} = \frac{E}{P_{np} \psi (1 - h) T} \tag{5}$$

it can be seen that the ratio of energy payback period to infrastructure lifetime, τ_1/T, will be much less than 1.0 (which is obviously desirable) if:

- $\frac{E}{P_{np}}$ the energy expended for emplacing a unit of nameplate capacity is small;
- ψ the capacity factor is large;
- h the portion of gross production going to O&M is much less than 1;
- T the infrastructure's effective lifetime is long.

If the detailed reporting of a LCA includes values for both $EROI$ and τ_1, then it is possible to "back calculate" the two subcomponents of $EROI$.

4.3. Doubling Time

The doubling time metric, τ_2, is a measure of the time interval required to accumulate enough excess energy to deploy new infrastructure sufficient to double power output. It is necessary to develop equations for the infrastructure's dynamic response in order to find a mathematical expression for the doubling time figure of merit [33].

Consider the growth of an energy park where at time t:

$P(t)$ = power available for societal use
$P_{np}(t)$ = installed nameplate power capacity

The annual-average power available for societal use is related to installed nameplate power as:

$$P(t) = P_{np}(t)\psi(1-h)(1-\beta) \tag{6}$$

where β is the fraction of produced power that is plowed back into the construction of new plants and their associated resource supply and delivery infrastructures.

Based on the availability of energy diverted from societal use to build new infrastructure, the incremental capacity $dC(t)$ under construction during time period dt is:

$$dC(t) = C(t+dt) - C(t) = dt\left[\frac{(1-h)\psi\beta P_{np}(t)}{q} - \frac{C(t)}{\lambda}\right] \tag{7}$$

where:

- $C(t) =$ nameplate plant capacity under construction at time t;
- $q = \dfrac{E}{P_{np}}$ is the energy expended to construct a unit of nameplate power capacity and its supporting infrastructure;
- $\lambda =$ average facility licensing and construction time.

The net capacity of nameplate power coming on line during time interval dt is the net of new build and decommissioning:

$$dP_{np}(t) = P_{np}(t+dt) - P_{np}(t) = dt\left[-\frac{1}{T}P_{np}(t) + \frac{C(t)}{\lambda}\right] \tag{8}$$

Collecting the above results, the system equations that define the dynamics of growth for the energy supply under an energy plowback constraint are:

$$\frac{d}{dt}P_{np}(t) = -\frac{1}{T}P_{np}(t) + \frac{C(t)}{\lambda} \tag{9}$$

$$\frac{d}{dt}C(t) = \frac{(1-h)\psi\beta P_{np}(t)}{q} - \frac{C(t)}{\lambda}$$

or in matrix notation

$$\frac{d}{dt}\left\{\begin{array}{c} P_{np}(t) \\ C(t) \end{array}\right\} = \mathbf{A}\left\{\begin{array}{c} P_{np}(t) \\ C(t) \end{array}\right\} \tag{10}$$

where

$$\mathbf{A} = \left\{\begin{array}{cc} -\frac{1}{T} & \frac{1}{\lambda} \\ \frac{(1-h)\psi\beta}{q} & -\frac{1}{\lambda} \end{array}\right\} \tag{11}$$

The initial boundary conditions (assuming steady state) are

$$P_{np}(t_0) = P_0 \tag{12}$$

$$C(t_0) = \lambda \frac{(1-h)\psi\beta}{q} P_0 \, .$$

The most positive eigenvalue of the state transition matrix, $\mathbf{A}$, sets the upper bound on the rate of energy supply growth attainable for a given reinvestment factor, β. The eigenvalues, α, of A are solutions of the quadratic equation:

$$\det |\mathbf{A} - \alpha \mathbf{I}| = 0 \tag{13}$$

$$\det \begin{vmatrix} -\frac{1}{T} - \alpha & \frac{1}{\lambda} \\ \frac{(1-h)\psi\beta}{q} & -\frac{1}{\lambda} - \alpha \end{vmatrix} = 0$$

or

$$\alpha^2 + \alpha(\frac{1}{\lambda} + \frac{1}{T}) + \frac{1}{\lambda T} - \frac{1}{\lambda}\frac{(1-h)\psi\beta}{q} = 0 \tag{14}$$

This equation can be solved for the eigenvalues, α, using the quadratic formula:

$$\alpha = \frac{-(\frac{1}{\lambda} + \frac{1}{T}) \pm \sqrt{(\frac{1}{\lambda} + \frac{1}{T})^2 + \frac{4}{\lambda T}\left[\frac{(1-h)\psi T\beta}{q} - 1\right]}}{2} \tag{15}$$

There are two eigenvalues. By inspection, at least one is positive if $\frac{(1-h)\psi T\beta}{q} > 1$. Calling the most positive eigenvalue α^*, the persisting solution of the state equations is simple exponential growth at an annual rate of α^*, starting from the steady state initial condition:

$$P_{np}(t) = P_0 e^{\alpha^* t} \tag{16}$$

$$C(t) = \lambda \frac{(1-h)\psi\beta}{q} P_0 e^{\alpha^* t}$$

The doubling time figure of merit τ_2 – applicable to growing infrastructures under an energy plowback constrained is defined as

$$\tau_2 = \frac{\ln 2}{\alpha^*} \tag{17}$$

and is the time interval it takes to accumulate the energy needed to double the emplaced infrastructure–given a specified energy reinvestment fraction, β.

The upper bound on achievable growth rates, α^*, constrained by energy plowback, is determined by the following infrastructural characteristics:

- λ : licensing and construction time period;
- T : asset lifetime;
- ψ : capacity factor;
- β : energy plowback fraction;
- h : O&M plowback fraction;
- q : amount of energy expended to emplace a unit of nameplate capacity.

The sources for numerical values for these parameters are as follows: T, ψ, h,and q are usually documented in the life cycle analyses that produce values for $EROI$ and/or τ_1; licensing and construction time period, λ, is known from actual plant construction practice; and the energy plowback fraction, β, is a parameter to be assumed in parametric scoping studies.

5. The Structure of Net Excess Energy and the Growth Potential of Alternative Infrastructures

We have noted above the essential role of surplus energy availability ($EROI >> 1$) in enabling economic growth and the historical evolution toward higher energy density carriers and higher power density converters as an effective way to increase the value of $EROI$.

The current energy infrastructure of industrial and many developing countries is based on fossil resources. This infrastructure does not meet the tenet of sustainable development. But it is not enough merely to restructure it at its current overall level because energy demand will be growing in the 21st century in response to increasing per capita energy use and increasing world population.

After improvements in efficiency of energy use and conversion are exhausted, growth in energy supply will necessitate emplacement of additional energy infrastructure assets. These emplacements will consume energy. Indeed, to support energy infrastructure expansion, it will be necessary for some fraction of the energy from both legacy and newly emplaced assets to be diverted from societal use and reinvested in order to support the next increment of capacity expansion.

This section will examine the dynamics of growth of energy supply under the constraint of energy plowback for incremental infrastructure emplacement. Using the idealized model developed above, it is possible to: (i) identify the essential constraints on feasible rates of growth; and (ii) clarify why the single numerical values of the $EROI$ are not by themselves sufficient for assessing the growth potential of alternative energy infrastructures–*i.e.* that it is important to analyze and understand the structure and time dependencies of the energy investments that are required for emplacing and maintaining these infrastructures.

5.1. The Importance of Up-Front (Emplacement) Energy Investment and Load Factor

To meet rapid growth in energy demand a high value of α^* (a short doubling time) is desirable. Examination of (14) indicates that the infrastructure's indigenous growth α^* will be larger when

$$\frac{(1-h)\psi T\beta}{q} > 1 \text{ and } \lambda T < 1 \tag{18}$$

With the exception of the energy plowback fraction, β, all the other parameters determining the above inequalities reflect the infrastructure's underlying physical and technological characteristics. Substantial differences between fossil fuel-based and renewable infrastructures in terms of these underlying characteristics have very significant implications for their differential ability to sustain high rates of indigenous growth.

One of the most fundamental attributes of renewable technologies is intermittency, which refers to the fraction of time that a given energy source/facility is available to society [20]. An important consequence of the intermittency of these technologies (*i.e.*, the fact that wind does not blow all the time and the sun does not shine all the time) is their low capacity or load factor–*i.e.*, low ψ values. By contrast, because of the continuous nature of fossil fuel extraction, most conventional (fossil-fueled and nuclear) generating technologies have very high load factors (high ψ values) and are "dispatchable."

Fossil-fueled and renewable technologies also have substantially different energy and power densities. The lower energy density of renewable sources as compared to fossil fuels implies that the former require significantly larger infrastructures–labor, capital, materials and energy–to produce an equivalent amount of energy [20]. Similarly, the low power density of renewable energy extraction implies that for renewable infrastructures large quantities of energy must be expended to emplace a unit of nameplate power capacity–*i.e.*, for renewable conversion nodes, q is large.

The fact that renewable technologies have low ψ and high q values while fossil-fueled generating modes have high ψ and low q values (and hence the ratio $\frac{\psi}{q}$has much larger values for fossil-fueled as compared to renewable technologies), has important consequences for their respective abilities to achieve high rates of indigenous growth. What matters to doubling time is the time phasing of the initial capital vs the ongoing O&M components of $EROI$. Consider the case of a renewable technology whose $EROI$ is similar to that of a fossil-fueled generation mode. Given a value for $EROI$, it is easy to see from (3) that

$$\frac{(1-h)\psi T\beta}{q} = \left\{\frac{\psi T}{q}\left[1-\frac{1}{EROI}\right]+1\right\}\beta \tag{19}$$

Even with the same value of $EROI$, the renewable technology could still have a much smaller value $\frac{\psi}{q}$ (relative to the fossil fueled technology) because of its intermittency and low power density. Assuming that the two technologies have similar T values (and the energy plowback fraction β is the same), then (18) implies that the value of $\frac{(1-h)\psi T\beta}{q}$ for the renewable technology will be much smaller relative to that of the fossil fueled technology. This in turn implies, according to (14), that the renewable

technology will have a much lower achievable growth rate, α^* (and correspondingly longer doubling time τ_2) relative to the fossil fueled generating mode.

5.2. Illustrative Example: Growth Potential under an Energy Plowback Constraint

IEA [34] performs a life cycle analysis (LCA) of different electricity generation sources (coal, oil, LNG, nuclear, wind, PV, solar thermal, hydro, and geothermal) in Japan. And it applies a consistent set of net energy formulas across the different generation options. The study's analysis is based on power outputs and annual capacity factors for the most typical generation plants in Japan–for fossil fuels and nuclear the reported values for annual capacity take into account periodic inspections while for renewables they are the maximum obtained under normal operating conditions in Japan. The estimates of the net supplied energy by each power generation system are based on a standardized power plant with nameplate capacity of 1,000 MW and an assumed life expectancy of 30 years for each plant. From the net supplied energy data, the energy payback period of each generation option is being estimated.

The IEA study was published in 2002. Thus, the study's reported estimates of LCA parameters are considerably outdated–especially for wind which has been experiencing very rapid technical change. Moreover, capacity factors and consequently net energy returns for renewables are highly site-dependent. Clearly, the wind and solar resources of Japan are not necessarily comparable to those found in the best sites around the world. However, the objective of our illustrative analysis is not to obtain the most accurate point estimates or representative values of net energy parameters. Instead, whet we seek to show is that the single numerical values of $EROI$ are not by themselves sufficient to evaluate the potential of alternative energy supply infrastructures for indigenous growth.

The study provides estimates of $EROI$, ψ, τ_1 and assumes that $T = 30$. For both coal-fired generation and wind power, $EROI = 6$. Coal has a much larger capacity factor ($\psi = .75$) relative to wind ($\psi = .20$). Moreover, coal has a much shorter estimated energy payback period ($\tau_1 = 0.15$ years) in comparison to wind ($\tau_1 = 3.39$ years). From these values we can back-calculate $\dfrac{E}{P_{np}\psi T}$, q, and h. These estimates are presented in Table 3. For coal, $q = .094$ and thus $\dfrac{\psi}{q} = 7.98$. For wind, $q = .637$ and $\dfrac{\psi}{q} = 0.31$. With a 20% plowback (*i.e.* $\beta = .2$), coal-fired plants can attain 73% annual expansion growth rate while for wind power the computed annual growth rate is only 2%.

Thus, coal-fired generation shows potential to support rapid indigenous growth. Wind, on the other hand, seems quite constrained. This at first might appear to be surprising in light of wind's $EROI$ being as large as coal's and its O&M plowback fraction h being smaller than that of coal. To understand this outcome it is necessary to recognize the time phasing of the initial capital energy input vs the ongoing O&M energy inputs making up $EROI$ in (3). The initial capital component for wind, representing the fraction of gross production expended for initial emplacement, is over 25 times larger than that of coal's–or equivalently for coal the ratio $\dfrac{\psi}{q}$ is over 25 times larger than that of wind's. According to (18), this implies that the ratio $\dfrac{(1-h)\psi T\beta}{q}$ has a much larger value of 40.08 for coal relative to wind for which the corresponding value is just 1.75. The doubling time τ_2, again with a 20% plowback, is 1.3

years for coal and 28.5 years for wind. Thus, the ability of wind to rapidly scale up its production by bootstrapping its own energy appears to be limited relative to coal.

Table 3. Breakout of components of EROI and growth potential under energy plowback constraint.

	$EROI$	T	ψ	τ_1	$\frac{E}{\psi P_{np} T}$	h	$q = \frac{E}{P_{np}}$	λ	β	α^*	τ_2
Coal	6	30	0.75	0.15	0.004	0.163	0.094	4	0.2	0.55	1.3
Wind	6	30	0.20	3.39	0.106	0.061	0.637	1	0.2	0.02	28.5

6. Summary and Conclusions

Among the desirable features of an energy supply infrastructure are the ability to deliver large amounts of surplus energy and to grow at the rate required by societal need. The latter is becoming increasingly important in view of the expected substantial growth in global energy demand (mainly from developing countries) and the urgency to stabilize greenhouse gas emissions by transitioning as rapidly as possible to low-carbon energy systems. The two most frequently quantified metrics of net energy analysis, the energy return on (energy) investment and the energy payback period, do not capture the growth rate potential of an energy supply infrastructure. This is because in the analysis underlying these metrics, all energy inputs and outputs are treated the same, regardless of where they occur in the life cycle of a given infrastructure.

We develop a dynamic energy analysis framework to model the growth potential of alternative electricity supply infrastructures. A key feature of our model is the requirement that part of the energy output from a given infrastructure is reinvested for capacity expansion (*i.e.*, the construction of new plants) while the rest is made available to meet society's demand for energy. An additional figure of merit, the infrastructure doubling time, is introduced. This metric highlights the critical importance of the time phasing of the initial energy investment for emplacing a given infrastructure, as opposed to the ongoing O&M energy expenditures, for the infrastructure's growth potential. The doubling time metric also captures the influence of capacity factor, licensing and construction time lags.

The efficacy of the doubling time metric is illustrated by comparing the growth rate potential of fossil (coal) versus renewables (wind) technologies with similar EROIs and using the same energy plowback (reinvestment) fraction for each. The illustration shows that the lower capacity factor and front-loaded capital versus operating energy requirements of wind slow down its achievable growth rate, compared to that of coal.

When the growth rate for a specific supply option is specified by societal need or by policy, the necessary energy input for growth of the chosen supply option will be diverted from societal usage – either by increasing its indigenous energy plowback fraction or by subsidizing its energy requirement from another supply option. While an EROI value well in excess of unity is necessary for self-supplied infrastructure growth, it is not sufficient; capacity factor and energy necessary for emplacement and for operation and time lags for licensing and construction also play an important role.

This paper focusses on the growth potential of alternative electricity supply infrastructures as constrained by innate physical energy balance and dynamic response limits. It seeks to provide a deeper understanding of the powerful physical limits that are facing the alternative generating technologies–physical limits and constraints that cannot be relaxed through economic policy measures. However, the paper's emphasis on the technical headroom of alternative generating technologies does not seek to supplant the time-honored economic cost-benefit analysis. Nor does it question the power of the incentives provided by market pricing mechanisms for the efficient allocation of scarce energy resources. Instead, it seeks to facilitate a technical reality check on the potential of these technologies to have an impact on the scale required by the global energy problem. It can also furnish more accurate and timely signals of impending critical conditions [35]. Especially in the presence of significant market imperfections and externalities, the paper's net energy methodology could serve as an important complement to economic analysis for evaluating prospective energy supply architectures.

References

1. Hall, C.A.S.; Powers, R.; Schoenberg, W. In peak oil, EROI, investments and the economy in an uncertain future. In *Renewable Energy Systems: Environmental and Energetic Issues*; Pimentel, D., Ed.; Elsevier: London, UK, 2008.
2. Hall, C.A.S.; Balogh, S.; Murphy, D.J. What is the minimum EROI that a sustainable society must have? *Energies* **2009**, *2*, 25-47.
3. Murphy, D.J.; Hall, C.A.S. Year in review–EROI or energy return on (energy) invested. *Ann. N. Y. Acad. Sci.* **2010**, *1185*, 102-118.
4. Kubiszewski, I.; Cleveland, C.; Endres, P. Meta-analysis of net energy return for wind power systems. *Renewable Energy* **2010**, *35*, 218-225.
5. Bankier, C.; Gale, S. Energy payback of roof mounted photovoltaic cells. *The Env. Eng.* **2006,** *7*, 11-14.
6. Fthenakis, V.; Alsema, E. Photovoltaics energy payback times, greenhouse gas emissions and external costs: 2004-early 2005 status. *Prog. in Photov. : Res. and Appl.* **2006,** *14*, 275-280.
7. Fthenakis, V.; Kim, V.; Alsema, E. Emissions from photovoltaic life cycles. *Environ. Sci. Technol.* **2008**, *42*, 2168-2174.
8. Murphy, D.J.; Hall, C.A.S.; Powers, R. New perspectives on the energy return on (energy) investment (EROI) of corn ethanol. *Environ. Dev. Sustain.* **2011**, *13*, 179-202.
9. Jacobson, M.Z.; Delucchi, M.A. Providing all global energy with wind, water, and solar power, part I: technologies, energy resources, quantities and areas of infrastructure, and materials. *Energy Policy* **2011,** in press.
10. Jacobson, M.Z.; Delucchi, M.A. Providing all global energy with wind, water, and solar power, part II: reliability, system and transmission costs, and policies. *Energy Policy* **2011**, in press.
11. MacKay, D. *Sustainable Energy—Without the Hot Air*; Cambridge University Press: Cambridge, UK, 2008.
12. IEA (International Energy Agency). *World Energy Outlook 2010*; OECD/IEA: Paris, France, 2010.

13. Hagens, N. Applying time to energy analysis. *The Oil Drum* **2010**. Available online: http://www.energybulletin.net/ stories/2010-12-13/applying-time-energy-analysis (accessed on 4 December 2011).
14. Mathur, J.; Bansal, N.K.; Wagner, H-J. Dynamic energy analysis to assess maximum growth rates in developing power generation capacity: case study of India. *Energy Policy* **2004**, *32*, 281-287.
15. Cleveland, C. Energy quality, net energy, and the coming energy transition. In *Proceedings of Sixth Annual Energy Conference on the Future of Oil as an Energy Source,* Abu Dhabi, United Arab Emirates, 7–8 October 2000.
16. Smil, V. Moore's curse and the great energy delusion. *The American,* November 19, **2008**.
17. Smil, V. Power density primer, parts i-v. *Master Resource*, **2010**, http://www.vaclavsmil.com/wp-content/uploads/docs/smil-article-power-density-primer.pdf.
18. Maciejewska, A.; Veringa, H.; Sanders, J.; Peteves, S.D. Co-Firing of Biomass with Coal: Constraints and Role of Biomass Pre-treatment. DG JRC Institute for Energy, 2006.
19. What is nuclear? http://www.whatisnuclear.com/articles/nucenergy.html (accessed on 4 December 2011).
20. Cleveland, C. Energy Transitions Past and Future. *Encyclopedia of Earth*, 2008.
21. Hall, C.A.S.; Klitgaard, K. *Energy and the wealth of nations: understanding the biophysical economy*; Springer: New York, USA, forthcoming.
22. Munro, J.H. Industrial energy from water-mills in the European economy, 5th to 18th centuries: the limitations of power. MPRA Paper No. 11027, **2002**. Available online: http://mpra.ub.uni-muenchen.de/11027/ (accessed on 4 December 2011).
23. Braudel, F. *Civilization and Capitalism 15th–18th Century: Vol I The Structure of Everyday Life*; University of California Press: Berkeley, CA, USA, 1992.
24. Smil, V. Energy Transitions: History, Requirements, Prospects; Praeger: Oxford, UK, 2010.
25. Pearce, J. Optimizing greenhouse gas mitigation strategies to suppress energy cannibalism. 2nd Climate Change Technology Conference, Hamilton, Ontario, Canada, 12–15 May 2009.
26. Cleveland, C. Ten fundamental principles of net energy. In *The Encyclopedia of Earth*, 2008; Available online: http://www.eoearth.org/article/Ten_fundamental_principles_of_net_energy (accessed on 1 September 2011).
27. Hall, C.A.S.; Cleveland, C.; Kaufmann, R. *Energy and Resource Quality: The Ecology of the Economic Process*; Wiley Interscience: New York, USA, 1986.
28. Hall, C.A.S.; Cleveland, C. EROI: definition, history and future implications. Presented at ASPO-US Conference, Denver, 10 November 2005.
29. Mulder, K.; Hagens, N. Energy return on investment: towards a consistent framework. *Ambio*, 2008, 39, 74-79.
30. Hall, C.A.S. Provisional results from EROI assessments. *The Oil Drum,* **2008**. Available online: http://www.theoildrum.com/node/3810 (accessed on 4 December 2011).
31. Gagnon, L. Civilisation and energy payback. *Energy Policy* **2008**, *36*, 3317-3322.
32. Knapp, K.; Jester, T. Empirical investigation of the energy payback time for photovoltaic modules. *Solar Energy* **2001**, *71*,165-172.

33. Utamura, M. Analytical model of carbon dioxide emission with energy payback effect. *Energy* **2005**, *30*, 2073-2088.
34. IEA (International Energy Agency). Environmental and Health Impacts of Electricity Generation; The International Energy Agency–Implementing Agreement for Hydropower Technologies and Programmes, 2002. Available online: http://www.ieahydro.org/reports/ST3-020613b.pdf (accessed on 4 December 2011).
35. Berndt, E. From technocracy to net energy analysis: engineers, economists and recurring energy theories of value. MIT WP#1353-82, 1982.