Supplementary Materials

Theorem:

The aggregate GSV measure $aggrGSV_I = n \cdot \left(\bar{y} - f\left(\bar{\mathbf{x}}\right)\right)$ *indicates the sum of profit efficiencies of the firms in group I at the most favorable non-negative input prices. Specifically:*

$$aggrGSV_I = \max_{\mathbf{w} \geq \mathbf{0}} \sum_{i=1}^{n} \left((y_i - \mathbf{w}'\mathbf{x}_i) - \pi(\mathbf{w}) \right).$$

***Proof*:**

Starting from the average profit inefficiency stated on the right-hand side of the equation posited in the Theorem, we can reorganize the expression as:

$$\max_{\mathbf{w} \geq \mathbf{0}} \sum_{i=1}^{n} \left((y_i - \mathbf{w}'\mathbf{x}_i) - \pi(\mathbf{w}) \right) \tag{A1}$$

$$= n \cdot \max_{\mathbf{w} \geq \mathbf{0}} \left(\sum_{i=1}^{n} y_i / n - \sum_{i=1}^{n} \mathbf{w}'(\mathbf{x}_i / n) - \pi(\mathbf{w}) \right) \tag{A2}$$

$$= n \cdot \max_{\mathbf{w} \geq \mathbf{0}} \left(\bar{y} - \mathbf{w}'\bar{\mathbf{x}} - \pi(\mathbf{w}) \right) \tag{A3}$$

$$= n \cdot \left(\bar{y} - \min_{\mathbf{w} \geq \mathbf{0}} \left(\mathbf{w}'\bar{\mathbf{x}} + \pi(\mathbf{w}) \right) \right). \tag{A4}$$

Regarding the minimization problem, note that the cost of the average input vector (i.e., $\mathbf{w}'\bar{\mathbf{x}}$) increases as the input prices $\mathbf{w}$ increase, whereas the profit function $\pi(\mathbf{w})$ is a decreasing function of $\mathbf{w}$. Since f is concave, $\pi(\mathbf{w})$ is convex, and thus the minimization problem has a unique global optimum.

Differentiating $\left(\mathbf{w}'\bar{\mathbf{x}} + \pi(\mathbf{w})\right)$ with respect to input prices $\mathbf{w}$, we have the first-order conditions:

$$\bar{\mathbf{x}} + \nabla\pi(\mathbf{w}) = 0, \tag{A5}$$

where $\nabla\pi(\mathbf{w})$ is the subgradient of the profit function at $\mathbf{w}$. If f is differentiable, then the subgradient reduces to the gradient vector:

$$\nabla\pi(\mathbf{w}) = \left(\frac{\partial\pi(\mathbf{w})}{\partial w_1} \cdots \frac{\partial\pi(\mathbf{w})}{\partial w_R} \right)'.$$

By Hotelling's lemma (Hotelling 1932):

$$\nabla\pi(\mathbf{w}) = \begin{pmatrix} -x_1^*(\mathbf{w}) \\ \vdots \\ -x_R^*(\mathbf{w}) \end{pmatrix} \quad \mathbf{x}^*(\mathbf{w}), \tag{A6}$$

where $\mathbf{x}^*(\mathbf{w})$ is the optimal profit maximizing input vector at prices $\mathbf{w}$.

Hotelling's lemma can also be established for non-differentiable functions by using the sub-gradients (see e.g., Blume 2008 for details). In that case, $\mathbf{x}^*(\mathbf{w})$ is not unique, but it does not influence the optimal solution to the minimization problem in (A4).

Inserting the right-hand side of (A6) to equality (A5), we have the first-order condition:

$$\bar{\mathbf{x}} - \mathbf{x}^*(\mathbf{w}) = 0. \tag{A7}$$

Therefore, the optimal solution to the minimization problem of (A4) can be expressed as:

$$\min_{\mathbf{w}\geq\mathbf{0}}\left(\mathbf{w}'\overline{\mathbf{x}}+\pi(\mathbf{w})\right)=\mathbf{w}'\overline{\mathbf{x}}+(f(\overline{\mathbf{x}})-\mathbf{w}'\overline{\mathbf{x}})=f(\overline{\mathbf{x}})\,. \tag{A8}$$

Inserting the last expression of (A8) back to (A4), we have:

$$\max_{\mathbf{w}\geq\mathbf{0}}\left(\sum(y_i-\mathbf{w}'\mathbf{x}_i)-\pi(\mathbf{w})\right)=n\cdot\left(\overline{y}-f(\overline{\mathbf{x}})\right)=aggrGSV_I\,. \tag{A9}$$