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Game Theoretic Analysis of Pricing and Cooperative Advertising in a Reverse Supply Chain for Unwanted Medications in Households

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Abstract: Improper disposal of household unwanted medications (UMs) is an emergency problem around the world that adversely affects the sustainability of the environment and human's health. However, the current disposal practices, mainly based on advertising and collecting status, are unsatisfactory in most countries and regions. Thus, some scholars proposed an alternative disposal practice that is to provide incentives to customers. This study aims to compare a Single Model (advertising only) with a Joint Model (advertising with take-back pricing) in a two-echelon reverse supply chain (RSC) that is composed of one disposer and one collector. In each model, four games (non-cooperative, collector as the Stackelberg leader, disposer as the Stackelberg leader, and cooperative) were established in order to identify the optimal pricing and advertising strategies for both members. The results of the study indicate that there is a Pareto dominant range for Joint Model compared to Single Model, whereas Single Model has no Pareto improvement in any games. Furthermore, in non-cooperative games of Joint Model, it is better to implement the leader-follower structure rather than simultaneous movement structure. Additionally, it is verified that the cooperative game is feasible, which leads to the cooperation between the disposer and the collector, and the extra profit from the cooperation can be shared based on the Nash bargaining game. However, in Single Model, it is better for the disposer to act as a channel leader and the collector figures the follower.

Keywords: Game models; Unwanted medication; Reverse supply chain; Pricing; Vertical cooperative advertising; Pareto dominant

1. Introduction

Over the past decade, pharmaceutical expenditure and consumption have increased continuously as a result of population aging, the rising prevalence of chronic diseases, new treatment opportunities, and so forth [1]. In addition, more spare medications are stored up for any contingency [2]. However, these medications may not be consumed entirely, owing to expiry, condition resolved or symptoms improved, side effects or other reasons [3], resulting in a universal issue of massively unwanted medications (UMs) accumulating in households around the world.

UMs include expired, unused, spilled and contaminated pharmaceutical products, drugs, vaccines, and sera that are no longer required and need to be disposed of appropriately [4]. Improper disposal of UMs can cause damage to human health (deterioration due to expiry or taken by anyone other than the prescribed patient) and the environment [5–8], especially to the aquatic ecosystems [9–11]. Moreover, UMs may be repackaged and resold to the market illegally [7], which constitutes a grave threat to public safety. Accelerative concern has been given to UMs and their proper disposal.

Although the European Union (EU) has a directive statement that member states are obligated to ensure appropriate collection systems are in place for unused or expired medicinal products [12,13], this code is applied inconsistently and seems to be unenforced [2]. Numerous studies suggested that the sustainable environmentally appropriate method to dispose of UMs is to return them to the pharmacy [2] or take-back locations [14]. Nonetheless, households' most common method for disposal of pharmaceuticals is still by garbage and sewage in many countries including the United Kingdom [5], the United States [3,15], Kuwait [16], Lebanon [7], Saudi Arabia [17], and Serbia [18]. Furthermore, in some regions where pharmacies offer the service of collecting UMs, the evaluated effectiveness has a number of weaknesses [19].

There exists some literature on the performance of reverse supply chains (RSCs). For instance, Yoon and Jeong [20] focused on implementing coordinative contracts between manufacturers and retailers in the RSC so as to maximize the total performance. With respect to general RSCs, the RSC for UMs is rather challenging and tough to implement. Xie and Breen [21] explained that the reasons were lack of commercial motivation and legislative enforcement for actors to return or collect UMs and requirement for more investment, which is not cost-effective. Thus, researchers paid less attention to RSCs for UMs.

Commercially, advertising can largely promote sales. Different cooperative advertising models have been studied more sufficiently in the forward supply chain (FSC) [22–32] including two excellent reviews [22,23]. On the other hand, advertising occupies a crucial position in promoting the quantity of collected used products [33]. It is because better collection efficiency required end-users' consciousness [34] and advertising can raise public awareness about the environment by highlighting the environmental risk associated with inappropriately disposed of Ums [35]. However, few articles have investigated the role of advertising in the RSCs for UMs. Although Hong et al. [33] have investigated the optimal decisions of cooperative advertising in closed-loop supply chains that considered the retailer's local advertising expenditure, it is not suitable for studying the collection of UMs. Thus, different from previous research, this paper firstly supposes a model where the quantity of collected UMs is influenced by channel parties' advertising expenses in the RSCs (referred to as Single Model).

To improve the situation of UMs holders' deficiency to return, Huang et al. [36] and Weraikat et al. [37] suggested providing incentives to customers. Huang et al. [36] established a tri-level programming model for recycling logistics networks in which the government undertook responsibility and related cost for recycling and getting rid of expired medications. In this research, UMs are eliminated as garbage and have no salvage value. Weraikat et al. [37] proposed a two-echelon pharmaceutical RSC with two coordination models, which are Producer-customer scheme and Producer-customer-3PL scheme. They provided incentives to stimulate customers to return UMs before their shelf life was up. Different from Huang et al. [36], this study classified UMs according to their shelf life and recycled reusable ones for more profit. However, they both seem to leave out of consideration of a crucial marketing tool, advertising. In view of this point, this research considers providing the customers with a take-back price incentive that is similar to the works of Huang et al. [36] and Weraikat et al. [37]. Furthermore, advertising is taken into account from collectors who are facing customers and can take UMs back conveniently and easily. Hence, this study refers to cooperative advertising to jointly promote the UMS take-back quantity that is rarely considered in existing research on RSCs. Thus, this paper secondly supposes a model where the quantity of collected UMs is influenced by both take-back price and channel parties' advertising expenses in the RSCs (referred to as Joint Model).

To this end, this paper proposes a two-echelon RSC composed of one disposer and one collector. It is assumed that advertising is indispensable while the RSC can choose whether to deploy a take-back price incentive. There is a threefold consideration for this assumption. First, if there does not exist publicity and advertising, the information of collection activities cannot spread out and nobody would join in. Second, advertising is a kind of education investment to improve the awareness of the environment and medication safety. Third, in real cases, the vast majority of collecting activities do not provide a take-back price incentive. Therefore, two models as mentioned before are developed: (i) take-back pricing and advertising jointly impact to the quantity of collecting UMs (Joint Model); (ii) the quantity is influenced only by advertising (Single Model).

Within the research of cooperative advertising composed of a manufacturer and a retailer, Yue et al. [24] and Wei and Xie [28] investigated two game models including Stackelberg-manufacturer and cooperative game. Meanwhile, considering retailers' increasing power in supply chains, another two games, Stackelberg-retailer and equal power as in Nash game, were taken into account [26,27,29,31]. As for the issue of UMs collection, the deficiency or ambiguity of related laws and regulations in most countries and regions results in the uncertainty surrounding who should take the responsibility to collect UMs. Thus, there exist different collection parties from organizers to executors with diverse channel power. Hence, in each model, four types of disposer-collector relationship are considered: non-cooperative, disposer as Stackelberg leader, collector as Stackelberg leader, and cooperative game models. The reason why we propose a RSC instead of a closed-loop supply chain is that UMs are useless and cannot be remanufactured, resulting in no close relationship with the FSC. This study focuses on the following decision-making problems:

(1) On which condition the RSC could and would like to use a take-back price incentive;
(2) how to choose best RSC structure under different situations;
(3) how to effectively allocate the fund between pricing and national and local advertising;
(4) and wether one party would like to share another's advertising expense?

This paper is structured as follows. In Section 2, the basic problems and assumptions in the two models are described, followed by analyzing the models in Section 3. Then, comparisons and discussions of the two models and numerical examples are made in Section 4. Finally, in Section 5, the main results are concluded with suggestions for future research.

2. Problem Description and Assumptions

This paper focuses on a two-echelon RSC composed of a single collector and a single disposer, since UMs collection activity lacks competition for less monetary benefit from collecting expired medications (except the illegal business which collects and repackages expired medications then sells them to suburbs or villages) and are generally formed by the driving force of related law and regulation, social responsibility, and resell reusable unexpired medications. From practical perspectives, the original price and categories of medications are ignored for their uselessness to patients. The responsibility of the collector is to collect and make a preliminary classification of UMs and the disposer takes all the collected UMs from the collector and disposes of them, such as incinerating medicine waste, recycling glass and packages, repacking reused medications, and so on. The collector can be a pharmacy, community, third-party logistics provider, etc., who faces UMs holders and can take them back conveniently and easily. The disposer can be played by the pharmaceutical company, government, and specialized agency. The common disposer-collector combinations are pharmaceutical company-pharmacy [37,38], government-pharmacy (most European countries) and government-community (some states in America).

In addition, two models are designed with and without an incentive by offering customers take-back price. In Section 4, this paper will compare and analyze the results of the two models and the suitable conditions for potential application.

The symbols are explained in Table 1 and proposed models are based on the following assumptions. Disposer's unit revenue s_d and collector's unit revenue s_c are set as constants that are in connection with the parties' own characteristics. More specifically, if the disposer is a government who regards improper disposal of medications as of special importance that will cause a huge reduction of its utility for the damage to resident and environment, the disposer will make more policies (e.g., toll tax for pharmaceutical industry and more collection subsidy) to support collection activities, then s_d increases. When a manufacturer acts as the disposer, s_d can be derived from a corporate sense of responsibility, actual monetary profit from resale or donation of reusable medications or the benefit from avoiding being punished in an existing related law. Likewise, when the collector is a pharmacy, s_c may be generated from the benefit of growing sales by collecting activity or sense of responsibility. Here, it needs to be explained that s_d and s_c are NET values, in which the related cost has been deducted. This treatment does not affect models. t denotes the fraction of collector's advertising expenditure, which is the percentage of the disposer agreeing to share with the collector. Therefore, $0 \leq t \leq 1$.

Table 1. Definition of symbols.

Parameters	
β	Customers' sensitivity coefficient to the collector's take-back price
α	Potential take-back scale
s_d	Disposer's unit revenue incurred by disposal activity
s_c	Collector's unit revenue incurred by collection activity
K_c	Effectiveness of collector's local advertising
K_d	Effectiveness of disposer's national advertising
k	Advertising ratio (i.e., K_d/K_c)
Variables	
p_c	Collector's take-back price
p_d	Disposer's price claimed for collector
m	Collector margin (i.e., $m = p_d + s_c - p_c$)
n	Disposer margin (i.e., $n = s_d - p_d$)
a	Collector's local advertising expenditure
A	Disposer's national advertising investment
t	Advertising participation rate
π	Profit

2.1. Disposer-Collector RSC with an Incentive of Take-Back Price

Model with a take-back price incentive (Joint Model) is illustrated in Figure 1. For more quantity of collected UMs and profit, the collector offers a take-back price to customers and both disposer and collector make advertising expenses. Throughout this study, the mass of collected UMs is stated as quantity, however, the model can be adjusted according to the actual situation. For instance, the UMs can be collected according to weight. Accordingly, all the corresponding parameters should be tied to weight.

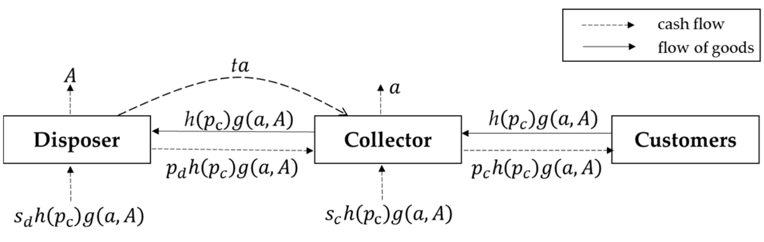

Figure 1. Disposer-collector RSC with a price incentive to customer.

There are vertical cooperative advertising strategies that the disposer shares part of the collector's advertising expenditure. Decision variables for the disposer are national advertising investment, the price given to the collector and the cooperative advertising compensation. The collector's decision variables are advertising expenditure and the take-back price.

In view of the situation that the quantity of collected UMs is influenced by the take-back price and advertising, the take-back quantity $Q(p_c, a, A)$ depends on the take-back price p_c and the advertising levels a and A as $Q(p_c, a, A) = h(p_c)g(a, A)$, which means that the quantity is jointly affected by price-related function $h(p_c)$ and advertising-related function $g(a, A)$. A simple price model is assumed to express the positive effect of the take-back price on the collection quantity,

$$h(p_c) = \alpha + \beta p_c \tag{1}$$

$h(p_c)$ increases with respect to p_c. The two parameters α and β are positive and can be interpreted as a potential take-back scale without any incentive from price and advertising and price sensitivity of customers. $Q(p_c, a, A) = h(p_c)g(a, A) = (\alpha + \beta p_c)g(a, A) = \alpha g(a, A) + \beta g(a, A)\ p_c$. It can be seen that advertising has two effects in promoting customers' environmental awareness and sense of responsibility, where the front part of the formula expresses that advertising could increase the voluntary quantity and the latter partial indicates that advertising makes the customers more sensitive to the take-back price. $g(a, A)$ is the function proposed by Xie and Wei [28]. That is $h(a, A) = K_c\sqrt{a} + K_d\sqrt{A}$. The square root advertising demand function indicates a diminishing marginal demand for increasing advertising expenditure and is further used by SeyedEsfahani and Biazaran [29] and Aust and Buscher [31]. a and A denote local and advertising expenses and the two parameters K_c and K_d are positive constants and can be interpreted as the effectiveness of the collector's and the disposer's advertising. According to Aust and Buscher [31], the collector's net profit margin m is brought into the equation as a new decision variable,

$$m = p_d + s_c - p_c \tag{2}$$

Hence, the following modified price-related and advertising-related functions are derived.

$$h(p_d, m) = \alpha + \beta(p_d + s_c - m) \tag{3}$$

$$g(a, A) = K_c\sqrt{a} + K_d\sqrt{A} \tag{4}$$

For the sake of simplicity, denote

$$\overline{\alpha} = \alpha + \beta s_c \tag{5}$$

Thus,

$$h(p_d, m) = \alpha + \beta(p_d + s_c - m) = \overline{\alpha} + \beta(p_d - m) \tag{6}$$

The subscripts "d", "c", and "$d + c$" represent parameters related to the disposer, the collector, and the whole system. The profits of disposer π_d, the collector π_c, and the entire system π_{d+c} can be written as follows respectively.

$$\pi_d = (s_d - p_d)[\overline{\alpha} + \beta(p_d - m)]\left(K_c\sqrt{a} + K_d\sqrt{A}\right) - A - ta \tag{7}$$

$$\pi_c = m[\overline{\alpha} + \beta(p_d - m)]\left(K_c\sqrt{a} + K_d\sqrt{A}\right) - (1 - t)a \tag{8}$$

$$\pi_{d+c} = (s_d + s_c - p_c)(\overline{\alpha} + \beta p_c)\left(K_c\sqrt{a} + K_d\sqrt{A}\right) - a - A \tag{9}$$

Note that, in this model, s_c and s_d can be less than 0 when the cost caused by the activity exceeds the revenue. However, they cannot be less than 0 simultaneously for the non-negativity of the system's profit, which should guarantee $s_d + s_c - p_c > 0$. Profits of the disposer and the collector should be nonnegative, which implies $n = s_d - p_d > 0$ and $m = p_d + s_c - p_c > 0$.

2.2. Disposer-Collector RSC without an Incentive of Take-Back Price

Another model without regard to customer incentive (Single Model) is presented in Figure 2. The collector takes no account of take-back price to promote quantity of collected Ums, which is solely affected by two members' advertising expenses.

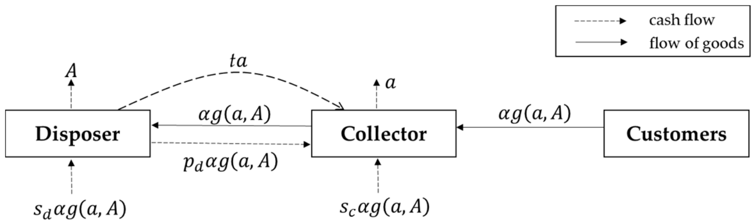

Figure 2. Disposer-collector RSC without a price incentive to customer.

Regardless of the take-back price paid to the customers, the quantity of collected UMs is just a function of the collector's local advertising expenditure a and disposer's national advertising investment A, which is

$$Q(a, A) = \alpha\left(K_c\sqrt{a} + K_d\sqrt{A}\right) \tag{10}$$

Then, the functions of the disposer's profit π_d, the collector's profit π_c and the profit of whole supply chain π_{d+c} are as follows.

$$\pi_d = (s_d - p_d)\alpha\left(K_c\sqrt{a} + K_d\sqrt{A}\right) - A - ta \tag{11}$$

$$\pi_c = (p_d + s_c)\alpha\left(K_c\sqrt{a} + K_d\sqrt{A}\right) - (1 - t)a \tag{12}$$

$$\pi_{d+c} = (s_d + s_c)\alpha\left(K_c\sqrt{a} + K_d\sqrt{A}\right) - A - a \tag{13}$$

3. Models

3.1. Four Games of Disposer-Collector Relationship with Customer Incentive

3.1.1. The Non-Cooperative Nash Game

This model is based on the situation that the disposer and the collector have a symmetrical distribution of power. The situation can be modeled through the Nash equilibrium, where both members make decisions concurrently and non-cooperatively to maximize their own profit. Hence, the decision problems of the disposer can be formulated as follows.

$$\begin{aligned}&\max_{p_d, A, t} \pi_d = (s_d - p_d)[\overline{\alpha} + \beta(p_d - m)]\left(K_c\sqrt{a} + K_d\sqrt{A}\right) - A - ta \\ &s.t.\ A > 0,\ 0 \leq t \leq 1 \text{ and } p_d > m - s_c.\end{aligned} \tag{14}$$

The decision problem of the collector is

$$\begin{aligned}&\max_{m, a} \pi_c = m[\overline{\alpha} + \beta(p_d - m)]\left(K_c\sqrt{a} + K_d\sqrt{A}\right) - (1 - t)a \\ &s.t.\ a > 0 \text{ and } m < p_d + s_c.\end{aligned} \tag{15}$$

The constraints $p_d > m - s_c$ in Equation (14) and $m < p_d + s_c$ in Equation (15) are to ensure a positive take-back price. These decision problems can be solved by setting the first-order conditions $\partial\pi_d/\partial p_d$, $\partial\pi_d/\partial A, \partial\pi_c/\partial m$ and $\partial\pi_c/\partial a$ to zero (see Appendix B for the details). The disposer will choose $t = 0$, because the cooperative advertising fraction has a negative influence on his profit function Equation (14). After some simplifications, the results of the Nash equilibrium are obtained.

For simplicity of expression, define

$$s = s_d + s_c \tag{16}$$

where s stands for the total unit revenue of the whole RSC. In Section 3.1, denote superscripts N, S_d, S_c and C as the optimal solutions in non-cooperative, disposer as the Stackelberg leader, collector as the Stackelberg leader, and cooperative games.

Theorem 1. *If the RSC channel is characterized by a symmetrical distribution of power and there is no cooperation between the disposer and the collector, the model can be represented by a Nash equilibrium with:*

1. $m^N = n^N = \frac{\beta s+\alpha}{3\beta}$, $p_d^N = \frac{2\beta s_d - \beta s_c - \alpha}{3\beta}$ and $p_c^N = \frac{\beta s - 2\alpha}{3\beta}$;
2. $A^N = \left(\frac{K_d(\beta s+\alpha)^2}{18\beta}\right)^2$ and $a^N = \left(\frac{K_c(\beta s+\alpha)^2}{18\beta}\right)^2$;
3. $t^N = 0$.

Part 1 of Theorem 1 suggests that in a game with symmetric distribution of power, the net profit margins for both players are the same in the RSC. This result is the same as that in FSC [29,31]. Furthermore, m and n increase as the parameter α increases and decrease as β increases, while p_d and p_c are just on the contrary. The four optimal solutions are all irrelevant to advertising effectiveness parameters. Comparing the optimal expressions for national and local advertising expenses in part 2 of Theorem 1, it is clear that both players only consider their own advertising effectiveness. From part 3 of Theorem 1, the disposer does not intend to share the collector's local advertising expenditure in the Nash equilibrium.

3.1.2. Asymmetric Relationship with Disposer-Leadership

If the disposer possesses more power to dominate the collector, we consider an asymmetrical relationship. Based upon Xie and Neyret [26], this model situation can be solved by the Stackelberg equilibrium. Specifically, as a leader, the disposer can learn about the collector's reaction to his decision and embrace it into his decision making on pricing and advertising. The collector's decision problem in the Stackelberg equilibrium is the same as Equation (15) in the above section.

$$m = \frac{\beta p_d + \overline{\alpha}}{2\beta} \tag{17}$$

$$a = \frac{K_c^2 m^2 [\overline{\alpha} + \beta(p_d - m)]^2}{4(1-t)^2} \tag{18}$$

Substituting Equation (17) into Equation (18) to eliminate the variable m, the disposer's decision problem is formulated below.

$$\begin{gathered} \max_{p_d,A,t} \pi_d = (s_d - p_d)[\overline{\alpha} + \beta(p_d - m)]\left(K_c\sqrt{a} + K_d\sqrt{A}\right) - A - ta \\ s.t.\ m = \frac{\beta p_d + \overline{\alpha}}{2\beta},\ a = \frac{K_c^2(\beta p_d + \overline{\alpha})^4}{64\beta^2(1-t)^2},\ p_d > m - s_c, A > 0 \text{ and } 0 < t < 1. \end{gathered} \tag{19}$$

The results stated in Theorem 2 can be derived after substituting the constraints into the objective function to eliminate the collector's decision variables and then setting the first order derivatives of the disposer's variables to zero. For simplicity, introduce a parameter r in the optimal expressions:

$$r = \frac{12k^2 + 9 - \sqrt{16k^4 + 16k^2 + 9}}{16k^2 + 9} \tag{20}$$

where $k = K_d/K_c$.

Theorem 2. *The RSC with an asymmetrical distribution of power, where the disposer acts as a leader, has the following Stackelberg disposer equilibrium.*

1. $m^{S_d} = \frac{(\beta s+\alpha)r}{2\beta}$, $n^{S_d} = \frac{(\beta s+\alpha)(1-r)}{\beta}$, $p_d^{S_d} = \frac{r\beta s_d-(1-r)\beta s_c-(1-r)\alpha}{\beta}$ and $p_c^{S_d} = \frac{r\beta s-(2-r)\alpha}{2\beta}$;
2. $A^{S_d} = \left(\frac{K_d(\beta s+\alpha)^2 r(1-r)}{4\beta}\right)^2$ and $a^{S_d} = \left(\frac{K_c(\beta s+\alpha)^2 r(4-3r)}{16\beta}\right)^2$;
3. $t^{S_d} = \frac{4-5r}{4-3r}$.

Considering that r is decreasing with respect to k in Equation (20), the range of r is $1/2 < r < 2/3$. Part 1 of Theorem 2 shows that in the range of r, the collector's margin m is always less than disposer's margin profit n, which illustrates the dominant of Stackelberg leader. In addition, $p_d^{S_d}$ and $p_c^{S_d}$ increase as r increases (k decreases). In Part 2 of Theorem 2, when making decisions on advertising expenses, the two players will take their relative advertising effectiveness into account. Part 3 of Theorem 2 illustrates that as a Stackelberg leader, the disposer will share collector's investment in local advertising from $1/3$ to $3/5$. Furthermore, the degree of participation increases with respect to k because t decreases as r increases and r decreases as k increases, implying that the more effective of the disposer's own advertising investment compared to that of the collector's, the more the disposer will share the collector's advertising expenditure.

It concludes that in a Stackelberg game that the disposer acts as a leader, if the disposer's national advertising is more effective than the collector's, it is better for the disposer to decrease the price to the collector and share more collector's advertising expenditure. Accordingly, the collector would reduce the take-back price.

3.1.3. Asymmetric Relationship with Collector-Leadership

In this section, the situation is considered where the collector occupies more power than the disposer, which can be modeled as collector acting as the Stackelberg leader. The disposer's decision problem in the Stackelberg equilibrium is the same as Equation (14) in Section 3.1.1.

$$p_d = \frac{\beta(s_d+m)-\overline{\alpha}}{2\beta} \tag{21}$$

$$A = \frac{(s_d-p_d)^2[\overline{\alpha}+\beta(p_d-m)]^2 K_d^2}{4} \tag{22}$$

$$t = 0 \tag{23}$$

Variable p_d in Equation (22) can be eliminated by substituting Equation (21) into it, then the collector's decision problem can be described below:

$$\begin{aligned} &\max_{m,a} \pi_c = m[\overline{\alpha}+\beta(p_d-m)]\left(K_c\sqrt{a}+K_d\sqrt{A}\right)-(1-t)a \\ &s.t.\ p_d = \tfrac{\beta(s_d+m)-\overline{\alpha}}{2\beta},\ A = \tfrac{K_d^2[\beta(s_d-m)+\overline{\alpha}]^4}{64\beta^2},\ t=0,\ m < p_d+s_c \text{ and } a > 0. \end{aligned} \tag{24}$$

Similar to the method in Section 3.1.2, plug the constraints for p_d, A and t into the objective function and then set the first order derivatives of collector's variables to zero. The following parameter h in the optimal expressions of the collector-led game is obtained. When $k=1$, $h=1/3$; When $k \neq 1$,

$$h = \frac{5k^2-2-\sqrt{9k^4-4k^2+4}}{8(k^2-1)} \tag{25}$$

Theorem 3. *The RSC with an asymmetrical distribution of power, where the collector acts as a leader, has the following Stackelberg disposer equilibrium:*

1. $m^{Sc} = \frac{(\beta s+\alpha)h}{\beta}, n^{Sc} = \frac{(\beta s+\alpha)(1-h)}{2\beta},\ p_d^{Sc} = \frac{\beta s_d(1+h)-(1-h)(\beta s_c+\alpha)}{2\beta},\ p_c^{Sc} = \frac{(1-h)\beta s-(1+h)\alpha}{2\beta};$
2. $A^{Sc} = \left(\frac{K_d(\beta s\ +\alpha)^2(1-h)^2}{8\beta}\right)^2$ and $a^{Sc} = \left(\frac{K_c(\beta s+\alpha)^2 h(1-h)}{4\beta}\right)^2;$
3. $t^{Sc} = 0.$

h decreases with respect to k. Thus, the range of h is $1/4 < h < 1/2$. From 1 of Theorem 3, m and n are equal when $h = 1/3$. Here, $K_c = K_d$. Also, if h changes from $1/3$ to $1/2$ with k varying from 0 to 1 (i.e., $K_c > K_d$), m is larger than n. On the contrary, if $K_c < K_d$, m is less than n. It is obvious that the collector cannot guarantee her larger net profit margin than the disposer. In addition, $p_d^{S_c}$ increases and $p_c^{S_c}$ decrease as h increases (k decreases). Moreover, as a consequence of follower status, the disposer has no motivation to share the collector's advertising expenditure for the negative effect on his own profit which stated in part 3 that t always equals zero.

In summary, in a Stackelberg game where the collector acts as a leader, if the collector's local advertising is relatively effective, it is better for the collector to implement a lower take-back price to UMs holders and not share any disposer's advertising investment. Accordingly, the disposer would increase the price offered to the collector.

3.1.4. Cooperative Game

In this section, a cooperative game approach is applied between the disposer and the collector in the RSC, in which the disposer and the collector jointly determine p_c, a and A. The decision problem is to maximize the total profit,

$$\begin{aligned} &\max_{p_c,a,A} \pi_{d+c} = (s-p_c)(\alpha+\beta p_c)\left(K_c\sqrt{a}+K_d\sqrt{A}\right) - A - a \\ &s.t.\ a,\ A > 0. \end{aligned} \tag{26}$$

When the disposer and the collector cooperate, there exists only three decision variables p_c, A, and a, while m, p_d and t do not affect the total profit anymore. The problem in Equation (26) is solved by derivation and set the first order equations to zero. The results obtained can be described as follows.

Theorem 4. *A cooperation RSC between the disposer and the collector with an objective of maximum total profit has the following equilibrium:*

1. $p_c^C = \frac{\beta s-\alpha}{2\beta};$
2. $A^C = \left(\frac{K_d(\beta s+\alpha)^2}{8\beta}\right)^2$ and $a^C = \left(\frac{K_c(\beta s+\alpha)^2}{8\beta}\right)^2.$

We suppose that both players will approve cooperation only if they gain a higher profit than in any other non-cooperative games described above [26,29,31]. That is

$$\Delta\pi_d = \pi_d^C - \pi_d^{\max} \geq 0 \tag{27}$$

$$\Delta\pi_c = \pi_c^C - \pi_c^{\max} \geq 0 \tag{28}$$

where π_d^C and π_c^C respectively denote the disposer's and the collector's profits in a cooperative game; $\pi_d^{\max}$ and $\pi_c^{\max}$ stand for the players' maximum profit in any non-cooperative games. A cooperation satisfying these inequalities simultaneously is called feasible. Note that this assumption is restrictive, because the members may be willing to obtain less profit in reality if there is no opportunity to gain the desired market structure.

Hence, the total extra profit can be derived,

$$\Delta\pi_{d+c} = \Delta\pi_d + \Delta\pi_c = \pi_{d+c}^C - \pi_d^{\max} - \pi_c^{\max} \geq 0 \quad (29)$$

From Theorem 4, the total profit in the cooperative game (π_{d+c}^C) can be easily obtained. Then, it is necessary to compare the results of the other equilibriums from three non-cooperative games in order to ascertain $\pi_d^{\max}$ and $\pi_c^{\max}$. Once the shareable extra profit $\Delta\pi_{d+c}$ is determined, the cooperating players have to agree on its division.

If p_c, a and A are, respectively, equal to p_c^C, a^C and A^C, then the channel's profit is maximized. However, the determination of the remaining variables t *and* p_d cannot be affected unambiguously, as there is an infinite amount of sets of t and p_d which can yield the particular division of profits which will be discussed in Section 4.

3.2. Four Games of Disposer-Collector Relationship without Customer Incentive

For the similar solving method and procedures, the game solutions in Single Model are listed in Table 2. In the first three non-cooperative models, the optimal p_d are all zero and disposers' advertising investment are identical, which is positively correlated to changes in his own advertising effectiveness and bear no relation to the collector's. Moreover, the disposer has no intention to share collector's advertising expenditure in games of the Nash and the Stackelberg collector. Nevertheless, in the disposer-led game, the disposer will afford advertising allowance for the collector, which is positively and negatively correlated to changes in the disposer's marginal profit and the collector's marginal profit, respectively. Note that the disposer's advertising participation comes into existence on condition that his unit revenue is more than half of the collector's ($s_d > s_c/2$).

Table 2. Optimal solutions of four games in Single Model.

Variables	Nash	Stackelberg Disposer	Stackelberg Collector	Cooperation
p_d	0	0	0	N/A
$\sqrt{a}$	$\frac{\alpha K_c s_c}{2}$	$\frac{\alpha K_c (2s_d+s_c)}{4}$	$\frac{\alpha K_c s_c}{2}$	$\frac{\alpha K_c s}{2}$
$\sqrt{A}$	$\frac{\alpha K_d s_d}{2}$	$\frac{\alpha K_d s_d}{2}$	$\frac{\alpha K_d s_d}{2}$	$\frac{\alpha K_d s}{2}$
t	0	$\frac{2s_d - s_c}{2s_d + s_c}$	0	N/A

Note: the optimal solutions of Stackelberg disposer game are in the situation that $s_d > s_c/2$. When $s_d \leq s_c/2$, the results are the same as Nash equilibrium.

4. Discussion of the Results and Numerical Examples

In the previous sections, the optimal solutions of the four games in two models are identified, respectively. In this section, the solutions are compared and analyzed. The existence and discussion of Single Model are reasonable and necessary. One reason is that in Joint Model, the take-back price should be no less than zero, thereby allowing, in a certain condition, for the formation of optimal decision-making of Joint Model, which will be discussed in Section 4.1. Another reason is that even if the RSC does not offer any price incentive, there is still a portion of UMs holders willingly returning UMs to the collector due to environmental protection considerations. In some situations, it is better for the RSC members to only use an advertising approach that is diverse from FSC and the RSC remanufacturing productions with residual value. These situations will be discussed in Section 4.2. Then, we compare the optimal solutions of different game scenarios in Joint Model and Single Model in Sections 4.3 and 4.4, separately. Also, the feasibility of the cooperation and a bargaining game of Joint Model are explored in Section 4.3.

Throughout this section, superscripts P and N in the first position denote the results in Joint Model and Single Model, respectively. The superscripts N, S_d, S_c and C in the second position respectively represent the games of Nash, Stackelberg disposer, Stackelberg collector, and Cooperation.

For example, $p_c^{PS_d}$ is the optimal solution of the collector's take-back price in Stackelberg disposer game of Joint Model.

4.1. Conditions for Formation of Optimal Decision-Making of RSC in Two Models

In Joint Model, considering that the take-back price cannot be less than zero (the collector cannot charge fees on returning behavior), the thresholds of s in different games are obtained (Table 3). Note that the implementations of price incentives only simply require the sum of two member's unit revenue (s) to meet certain conditions, because these models assume that the games are played in complete information scenarios, where both participants' unit revenue are common knowledge. As a consequence, the unit revenue of one player (s_d or s_c) can be less than 0 and p_d plays the role in adjusting the profit margin (m and n) between the members.

Table 3. The condition for the formation of optimal decision-making of the RSC in two models.

Models	Nash	Stackelberg Disposer	Stackelberg Collector	Cooperation
Joint Model	$s > \frac{2\alpha}{\beta}$	$s > \frac{\alpha}{\beta}\frac{2-r}{r}$	$s > \frac{\alpha}{\beta}\frac{1+h}{1-h}$	$s > \frac{\alpha}{\beta}$
Single Model	$s_d > 0$ $s_c > 0$	$s_d > 0$ $s_c > 0$	$s_d > 0$ $s_c > 0$	$s > 0$

While in Single Model, since the RSC does not consider a take-back price incentive, the optimal solutions of p_d in Nash, Stackelberg disposer, and Stackelberg collector are all equal to zero. Consequently, in these three games, the unit revenue of each player should be positive, or their total profits will be less than 0. In addition, the condition for cooperation game in Single Model is the lowest, where s needs to be greater than zero.

Next, we further analyze the thresholds of s to implement take-back price incentive in Joint Model. In the Nash game and the cooperation game, the threshold of s is only related to α/β, while in games of Stackelberg disposer and Stackelberg collector, besides the ratio, the thresholds are also affected by k, which is the relative strength of their advertising effectiveness. In the Stackelberg disposer game, the threshold of s is increasing with k, which means it is more difficult to offer a price incentive to customers. In addition, this value is twice to three times that of the cooperation game. Meanwhile, it is opposite in the Stackelberg collector game, in which the threshold of s is negatively correlated to changes in k. The better the disposer's advertising effectiveness, the easier the collector provides the price incentives. Moreover, cooperation of the RSC leads to the lowest level of threshold s. Figure 3 depicts thresholds of s respect to k in different games of Joint Model ($\alpha/\beta = 1$).

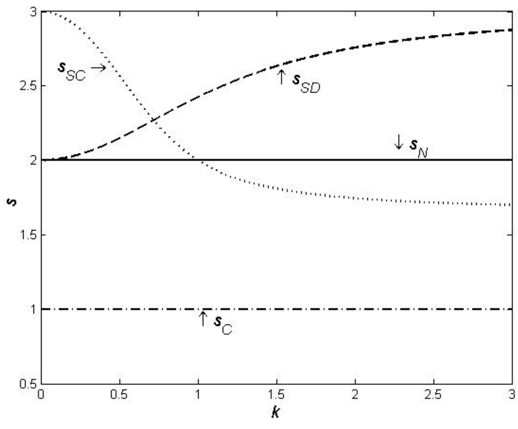

Figure 3. Thresholds of s to implement take-back price incentive in k in different games of Joint Model.

4.2. Comparison of the Two Models

In Section 4.1, we solved the questions if the games in the two models could be played. However, to a certain game, for instance, the Nash game, when both models can be played (i.e. conditions in Table 3 are satisfied under each game), does it always gain large quantity of collected UMs and more players' profits to offer UMs holders price incentive (Joint Model)? This issue is settled by comparison of the two models in this section. (Optimal expressions in each game scenario of two models are shown in Appendix A).

Supposing $s_d, s_c > 0$ and denoting $t_1 = s\beta/\alpha$, $ss = s_d/s_c$ and $k = K_d/K_c$, the following propositions are provided. The details of proofs of Proposition 1–4 can be derived in Appendix B.

Proposition 1. *In the Nash game, in the range of $t_1 > 2$, both players' profits in Joint Model are always higher than those in Single Model when the following constraints are satisfied.*

$$\begin{cases} \frac{(1+t_1)^4}{81t_1^2}(1+ss)^2(2+k^2) - (2ss + k^2ss^2) > 0 \\ \frac{(1+t_1)^4}{81t_1^2}(1+ss)^2(1+2k^2) - (1+2k^2ss) > 0 \end{cases} \tag{30}$$

The first constraint is derived from $\pi_d^{PN} > \pi_d^{NN}$ and the second results from $\pi_c^{PN} > \pi_c^{NN}$.

Proposition 2. *In the Stackelberg disposer game, suppose $t_1 > (2-r)/r$, then both players' profits in Joint Model are always higher than those in Single Model when following constraints are satisfied.*

When $s_d > s_c/2$,

$$\begin{cases} \frac{(1+t_1)^4(1+ss)^2r^2}{16t_1^2}\left[\frac{(4-3r)^2}{4} + 4k^2(1-r)^2\right] - \left(\frac{1}{2}+ss\right)^2 - k^2ss^2 > 0 \\ \frac{(1+t_1)^4(1+ss)^2r^3}{16t_1^2}\left[\frac{(4-3r)}{4} + 4k^2(1-r)\right] - s - 2kss - \frac{1}{2} > 0 \end{cases} \tag{31}$$

When $s_d \le s_c/2$,

$$\begin{cases} \frac{(1+t_1)^4(1+ss)^2r^2}{16t_1^2}\left[\frac{(4-3r)^2}{4} + 4k^2(1-r)^2\right] - (2ss + k^2ss^2) > 0 \\ \frac{(1+t_1)^4(1+ss)^2r^3}{16t_1^2}\left[\frac{(4-3r)}{4} + 4k^2(1-r)\right] - (1+2k^2ss) > 0 \end{cases} \tag{32}$$

Similarly, the first inequalities in both cases are derived from $\pi_d^{PS_d} > \pi_d^{NS_d}$ and the second come from $\pi_c^{PS_d} > \pi_c^{NS_d}$. The two cases is caused by Single Model in which the disposer would like to share part of the collector's advertising expenditure only when $s_d > s_c/2$. $t_1 > (2-r)/r$ ensures that Joint Model can be played. In the range of $2 < t_1 < 3$, assume that k_0 makes $t_1 = (2-r)/r$. Since $(2-r)/r$ is increasing with respect to k, $t_1 > (2-r)/r$ holds when $k < k_0$. When $t_1 \ge 3$, k can be arbitrary.

Proposition 3. *In the Stackelberg collector game, suppose $t_1 > (1+h)/(1-h)$, then both players' profits in Joint Model are always higher than those in Single Model when following constraints are satisfied:*

$$\begin{cases} \frac{(1+t_1)^4(1+ss)^2(1-h)^3}{16t_1^2}\left[4h + k^2(1-h)\right] - 2ss - k^2ss^2 > 0 \\ \frac{(1+t_1)^4(1+ss)^2h(1-h)^2}{4t_1^2}\left[h + k^2(1-h)\right] - 1 - 2k^2ss > 0 \end{cases} \tag{33}$$

The first inequality is introduced from $\pi_d^{PS_c} > \pi_d^{NS_c}$ and the second is derived from $\pi_c^{PS_c} > \pi_c^{NS_c}$. Similar to the Stackelberg disposer game, $t_1 > (1+h)/(1-h)$ is to ensure that Joint Model can be played. In the range of $5/3 < t_1 < 3$, it is assumed that k_0 makes $t_1 = (1+h)/(1-h)$, since $(1+h)/(1-h)$ is decreasing with respect to k, $t_1 > (1+h)/(1-h)$ holds when $k > k_0$. When $t_1 \geq 3$, k can be arbitrary.

Since the implicit function forms of the conditional expression are inconvenient for analysis, some numerical examples are given. The expressions in Proposition 1–3 are with respect to three parameters, t_1, ss, and k. t_1 reflects the whole unit revenue of RSC, while ss describes the allocation of this whole unit revenue in two players. k represents the relative effectiveness of disposer's national advertising to collector's local advertising. Set t_1 as a constant value and make the sensitivity analysis on ss and k. Figure 4 depicts the comparisons of two players' profits of two models in different games.

The lines denote relations of equality and divide the considered region into several areas. For example, the first figure describes the situations in the Nash game and there exist two lines. The top right one is $\pi_d^{PN} = \pi_d^{NN}$ and it splits the whole region into two areas with $\pi_d^{PN} > \pi_d^{NN}$ on the left and $\pi_d^{PN} < \pi_d^{NN}$ on the right. As a consequence, the central area, where both two members' profits in Joint Model are larger than those in Single Model, is the Pareto improvement area of Joint Model compared to Single Model. Interestingly, there is no area where both members' profits in Single Model are larger than those in Joint Model simultaneously which means Single Model has no Pareto improving range to Joint Model in any game.

In each game, we choose three different values of t_1(2.9, 3 and 3.1). Regardless of the game, raising the overall unit revenue (the growth of t_1 indicates an increasing in s) can increase application range of Joint Model (more extensive s and k), which suggests that the larger the s, the more the RSC of UMS should consider choosing Joint Model to offer UMs holders a take-back price incentive, which can increase both players' profit.

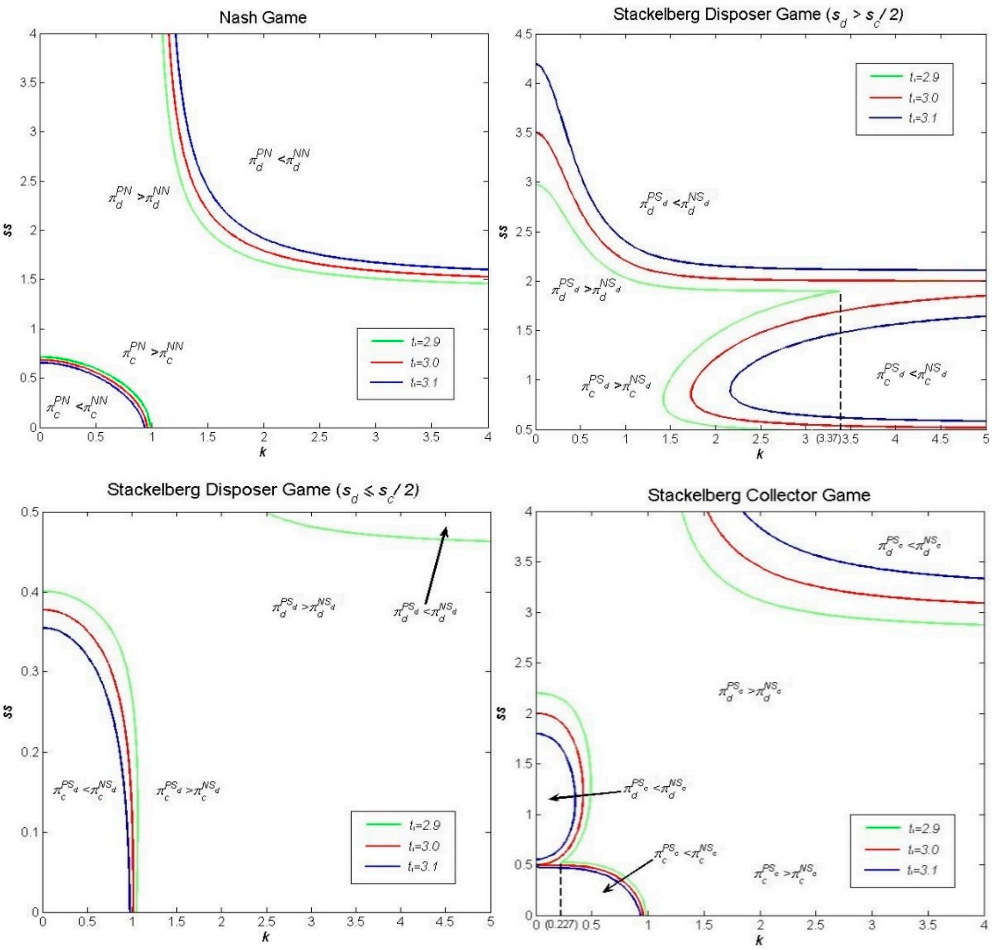

Figure 4. Comparisons of two players' profit of two models in different games.

Note that in the Stackelberg disposer game ($s_d > s_c/2$), when $t_1 < 3$, k is not arbitrary for the constraint $t_1 \geq (2-r)/r$. In this computation of $t_1 = 2.9$, k should be less than 3.37. Similarly, in the Stackelberg collector game of $t_1 = 2.9$, k should be more than 0.227.

Furthermore, in Pareto improvement area of Joint Model, not only both members' profits are predominant, but also the quantity of collected UMs is more than that in Single Model (See Figure 5).

The green lines in the Stackelberg disposer game ($s_d > s_c/2$) and the Stackelberg collector game in Figure 5 are shaped by the equalities of total RSC profit in two models. There exists a region in each of the two games that the total RSC profit of Single Model is larger than that in Joint Model (Region I and II). However, in these regions, the total RSC profit increases, the leader's profit increases, but the follower's profit decreases. Then, the follower may refuse to join in the RSC of UMs in Single Model.

The Pareto dominant region combination of ss and k can be a reference for the government to make a subsidy policy for the collection of UMs. For instance, a RSC composed of a pharmaceutical manufacturer and a pharmacy and the manufacturer leads the game. The total unit revenue (s) only comes from the subsidies of the government. If the government considers the two parties' advertising effectiveness is similar ($K_d = K_c$), then it is better for him to provide subsidies to not only the manufacturer but also the pharmacy (when $k = 1$, the Pareto dominant region requires ss be less than about 2.25). In turn, the government can choose the best RSC structure by the Pareto dominant region based on given ss and k practical situation.

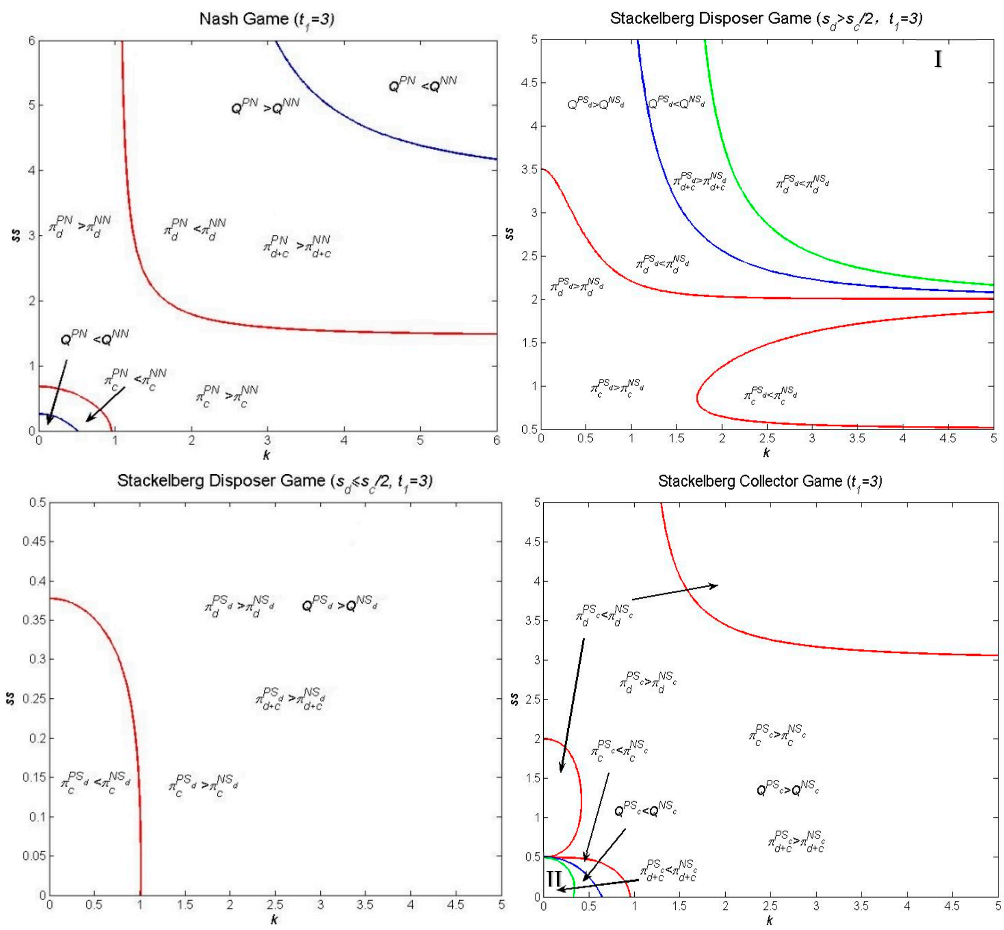

Figure 5. Comparisons of two players' profits and quantity of collected UMs of two models in different games ($t_1 = 3$).

Through the analysis of numerical examples above, it turns out that in decentralized games, it does not always gain more profit for each player and quantity of collected UMs to implement price incentive. However, comparing the results of the two models in the centralized game, we have the following observation.

Proposition 4. *When* $s > \alpha/\beta$, $Q^{PC} > Q^{NC}$ *and* $\pi_{d+c}^{PC} > \pi_{d+c}^{NC}$.

Through the analysis of the numerical examples above, it turns out that in decentralized games, it does not always gain more profit for each player and quantity of collected UMs to implement a price incentive. However, comparing the results of the two models in the centralized game, we have the following observation.

Proposition 4 shows that in the cooperation game, when the condition to implement price incentive ($s > \alpha/\beta$) is achieved, the RSC should choose Joint Model and consider offering UMs take-back price incentive which may reach a higher quantity of collected UMs and total supply chain profit.

4.3. Comparisos within Joint Model

From the analysis of Section 4.1, it has determined that the whole unit revenue of the RSC should achieve a specific threshold to perform a take-back price incentive model (Joint Model). This section compares the optimal solutions including prices, advertising expenditures, quantity of collected UMs, and members' profits in different game scenarios of Joint Model. Then, the feasibility of the cooperative games is explored in Section 4.3.4. The comparisons are on the base of an assumption that the thresholds in Section 4.1 of Joint Model have been achieved with all k, namely, $s > 3\alpha/\beta$. The optimal solutions are summarized and presented in Table A1 in Appendix A. and the proofs of proposition 5–11 are shown in Appendix B.

4.3.1. Comparisons on Prices

Proposition 5. *The take-back prices in four game scenarios have the following ordinal relationship. (1)* $p_c^{PC} > p_c^{PN} > p_c^{PS_d} \geq p_c^{PS_c}$, *if* $0 < k \leq k_1$ *; (2)* $p_c^{PC} > p_c^{PN} \geq p_c^{PS_c} > p_c^{PS_d}$, *if* $k_1 < k \leq 1$ *; (3)* $p_c^{PC} > p_c^{PS_c} > p_c^{PN} > p_c^{PS_d}$, *if* $k > 1$. *(* k_1 *satisfies that* $r + h = 1$ *(*$k_1 \approx 0.7164$*)).*

Proposition 5 reveals that the ordinal relationships of the take-back prices simply depend on k and are irrelevant to s, s_d, s_c, α and β. From the following Propositions 6 to 11, which are about p_d, a, A, Q, π_c and π_d, their ordinal relationships merely rely on k and are independent of other parameters. It shows that the highest take-back price is always in the cooperative game whereas the lowest prices occur in the Stackelberg disposer game when k is lower ($k_1 < 0.7164$), and in the Stackelberg collector game when k is higher ($k_1 > 0.7164$). The take-back price offered to the UMs holders can be regarded as a kind of welfare and the increasing of the price would add the surplus of UMs holders.

Proposition 6. *The prices given to the collector by the disposer in four game scenarios have the following ordinal relationship. (1)* $p_d^{PS_c} \geq p_d^{PN} > p_d^{PS_d}$, *if* $0 < k \leq 1$*; (2)* $p_d^{PN} \geq p_d^{PS_c} > p_d^{PS_d}$, *if* $1 < k$.

From Proposition 6, regardless of k, the price in Stackelberg disposer game is the lowest indicating the leader position. When the effectiveness of the disposer's national advertising is lower than that of the collector's local advertising, the price is the highest in Stackelberg collector game, and followed by the Nash game.

4.3.2. Comparisons on Advertising Expenditures

Proposition 7. *The collector's advertising expenditures in four game scenarios have the following ordinal relationship. (1)* $a^{PC} > a^{PS_d} > a^{PS_c} \geq a^{PN}$, *if* $0 < k \leq 1$*; (2)* $a^{PC} > a^{PS_d} > a^{PN} > a^{PS_c}$, *if* $1 < k$.

Proposition 7 concludes that in cooperative game, the collector's local advertising expenditure is the highest and followed by Stackelberg disposer game. From Section 4.2, the disposer does not provide any collector's advertising expenditure share in the Nash and the Stackelberg collector games. However, in the Stackelberg disposer game, he prefers sharing local advertising expenditure with the collector according to their relative advertising effectiveness. This expenditure sharing may result in the collector's willingness to increase investment in advertising compared with the other two games.

Proposition 8. *The disposer's advertising expenditures in four game scenarios have the following ordinal relationship. (1)* $A^{PC} > A^{PS_d} > A^{PN} \geq A^{PS_c}$, *if* $0 < k \leq 1$; *(2)* $A^{PC} > A^{PS_d} > A^{PS_c} > A^{PN}$, *if* $1 < k \leq k_2$; *(3)* $A^{PC} > A^{PS_c} > A^{PS_d} > A^{PN}$, *if* $k_2 < k$. *(*k_2 *satisfies* $2r(1-r) = (1-h)^2$ *(*$k_2 \approx 1.338$ *)).*

Similarly, based on Proposition 8, the disposer's national advertising investment is the highest in the cooperative game and followed by the Stackelberg disposer game or the Stackelberg collector game. The disposer always invests more in the Stackelberg game as a leader than in the Nash game.

4.3.3. Comparisons on Quantity of Collected UMs and Profits

Proposition 9. *The quantity of collected UMs in four game scenarios have the following ordinal relationship. (1)* $Q^{PC} > Q^{PS_d} > Q^{PN} \geq Q^{PS_c}$, *if* $0 < k \leq 1$; *(2)* $Q^{PC} > Q^{PS_d} \geq Q^{PS_c} > Q^{PN}$, *if* $1 < k \leq k_3$; *(3)* $Q^{PC} > Q^{PS_c} > Q^{PS_d} \geq Q^{PN}$, *if* $k_3 < k \leq k_4$; *(4)* $Q^{PC} > Q^{PS_c} > Q^{PN} > Q^{PS_d}$, *if* $k_4 < k$. *(*k_3 *satisfies* $r^2\left[(4-3r)/2 + k^2 2(1-r)\right] = (1-h)^2\left[2h + k^2(1-h)\right]$ *(*$k_3 \approx 1.2060$*);* k_4 satisfies $8\left(1+k^2\right) = 27r^2\left[(4-3r)/2 + k^2 2(1-r)\right]$ ($k_4 \approx 1.5385$)).

The largest amount of collected UMs is also from the cooperative game, followed by the Stackelberg disposer or the Stackelberg collector, instead of Nash game. It is evident from Proposition 9 that when $k < 1.21$, the quantity of collected UMs in the Stackelberg disposer game is more than other two games and when $k > 1.21$, it is in the Stackelberg collector game that the quantity is the most.

Proposition 10. *The disposers' profits in four game scenarios have the following ordinal relationship. (1)* $\pi_d^{PS_d} > \pi_d^{PN} \geq \pi_d^{PS_c}$, *if* $0 < k \leq 1$; *(2)* $\pi_d^{PS_d} \geq \pi_d^{PS_c} > \pi_d^{PN}$, *if* $1 < k \leq k_5$; *(3)* $\pi_d^{PS_c} > \pi_d^{PS_d} > \pi_d^{PN}$, *if* $k_5 < k$. *(*k_5 *satisfies* $r^3\left[(4-3r)/2 + k^2 4(1-r)\right] = 4h(1-h)^2\left[h + k^2(1-h)\right]$ *(*$k_5 \approx 1.3835$*)).*

It is obvious from Proposition 10 that the disposer prefers the leader-follower structure (the Stackelberg disposer game or the Stackelberg collector game) rather than the simultaneous move structure (the Nash game). If the disposer's advertising effectiveness is relatively weak ($k < 1.3835$), he tends to lead the game to obtain more channel profit, while if his advertising effectiveness is comparatively high, he would like to be a follower, which makes him gain more profit.

Proposition 11. *The collectors' profits in four game scenarios have the following ordinal relationship. (1)* $\pi_c^{PS_d} \geq \pi_d^{PS_c} > \pi_d^{PN}$, *if* $0 < k \leq k_6$; *(2)* $\pi_d^{PS_c} \geq \pi_d^{PS_d} > \pi_d^{PN}$, *if* $k_6 < k \leq k_7$; *(3)* $\pi_d^{PS_c} > \pi_d^{PN} > \pi_d^{PS_d}$, *if* $k_7 < k$. (k_6 satisfies $r^3\left[(4-3r)/2 + k^2 4(1-r)\right] = 4h(1-h)^2\left[h + k^2(1-h)\right]$ ($k_6 \approx 0.8457$); k_7 satisfies $16\left(1+2k^2\right) = 81r^3\left[(4-3r)/2 + 4k^2(1-r)\right]$ ($k_7 \approx 0.8552$)).

Proposition 11 reveals that the collector has an aptitude for being a leader when $k > 0.85$ and being a follower when $k < 0.85$, which can bring her more profit. Thus, combining Proposition 10 and Proposition 11, it demonstrates that there exist accordance and conflict in the leader-follower structure (Figure 6). More specifically, when $k < 0.85$, the disposer prefers being a leader in the RSC and precisely the collector is willing to be a follower. In this structure, both players can gain their maximum profit in non-cooperative games. Similarly, if $k > 1.38$, a game structure with the collector as the leader and the disposer as the follower is both members' preference. Therefore, these two ranges are their accordance regions. However, if $0.85 < k < 1.38$, the two players' optimal profits are all in their own leading game, which resulted in a conflict with each other rather than being a Stackelberg follower. It is worth mentioning that in this conflict region, if $k < 1.12$, the quantity of collected UMs is largest in the Stackelberg disposer game, which is better for the collecting activity.

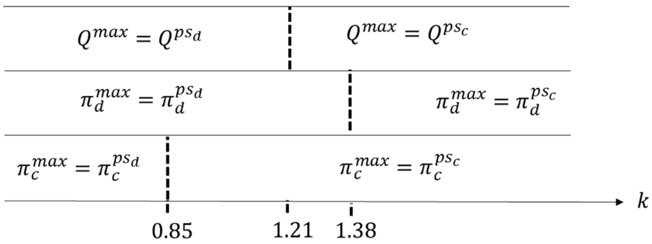

Figure 6. The accordance and conflict of the leader-follower structure.

Besides, the collection efficiency and channel efficiency are analyzed, and calculated as the ratio of non-cooperative games to the cooperative game and the sum of two players' profits in any of the non-cooperative games to the total profit of RSC in the cooperative game solution (see Figure 7). From Figure 7, the following proposition is presented.

Proposition 12. *The coordination of the Stackelberg disposer game and the Stackelberg collector game decreases and increases with k, respectively, while it is independent with k in the Nash game.*

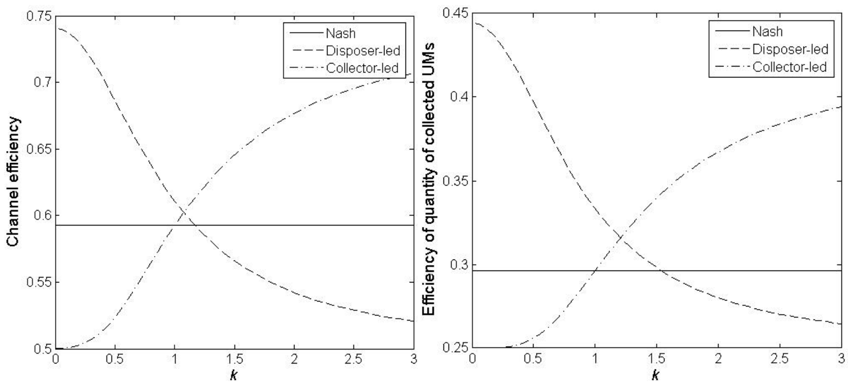

Figure 7. Collection efficiency and channel efficiency with respect to k.

The cooperative game is a more coordinated model than the non-cooperative games. When the collector's advertising is more efficient, a more coordinated model will return a higher profit to the RSC in the Stackelberg disposer game. By contrast, when the disposer's advertising is more efficient, the Stackelberg collector game is more coordinated.

In summary, the analytical results favor that cooperative game which is the best at offering the highest take-back price and invest the most in both national and local advertising, leading to the most preferable collected quantity and total profit. Furthermore, in non-cooperative games, it is better to implement the leader-follower structure rather than simultaneous movement structure. In details, when the disposer's national advertising effect is relatively lower than the collector's, the Stackelberg disposer game would be more satisfactory. However, if the disposer's advertising is efficient, a Stackelberg collector game is favorable.

4.3.4. Feasibility of the Cooperation

With the analytical solutions of the cooperative game in Section 3.1.4, it is observed that the disposer and the collector would only agree to make joint decisions if their individual profits are higher in the cooperative game than those in the non-cooperative structures. In this section, the comparative results in Section 4.3 are applied to verify the feasibility of the problem. To achieve this, it is necessary to prove the existence of $\left(p_c^{PC}, p_d^{PC}, a^{PC}, A^{PC},\ t^{PC}\right)$. Based on Equation (29), it derives

$$\pi_{d+c}^{PC} \geq \pi_d^{\max} + \pi_c^{\max} \tag{34}$$

Based on Proposition 10 and Proposition 11, the maximum profits of the disposer and collector are obtained in three regions. Table 4 determines the maximum profits of each player in each region.

Table 4. The maximum profits of the disposer and the collector.

k	π_d^{max}	π_c^{max}
$k \leq 0.85$	$\pi_d^{PS_d}$	$\pi_c^{PS_d}$
$0.85 < k < 1.38$	$\pi_d^{PS_d}$	$\pi_c^{PS_c}$
$1.38 \leq k$	$\pi_d^{PS_c}$	$\pi_c^{PS_c}$

It is evident that when $k \leq 0.85$ and $k \geq 1.38$, the inequality Equation (34) is obtained, because $\pi_d^{\max} + \pi_c^{\max}$ equals to $\pi_{d+c}^{PS_d}$ and $\pi_{d+c}^{PS_c}$ respectively and they are all less than π_{d+c}^{PC}. Similarly, the ratio $(\pi_d^{PS_d} + \pi_c^{PS_c})/\pi_{d+c}^{PC}$ can be calculated as being less than 1 in the range of $0.85 < k < 1.38$. Consequently, it shows that the cooperative game is feasible, which leads to the disposer and the collector's willingness to cooperate. Therefore, next section will make a trial to settle the problem that how to share the extra profit from cooperation. The profit-sharing issue is discussed in Section 4.3.5.

4.3.5. Bargaining Model

In this section, a feasible region is proposed for the two internal variables p_d and t. Then, the Nash bargaining model will be conducted to resolve the profit-sharing problem in this region. The feasible region for this problem can be clarified based on the method used by Xie and Wei [28]. The extra profit of the disposer and the collector are as follows:

$$\begin{aligned} \Delta\pi_d &= \pi_d^{PC} - \pi_d^{\max} \\ &= (s_d - p_d)\left(\alpha + \beta p_c^{PC}\right)\left(K_c\sqrt{a^{PC}} + K_d\sqrt{A^{PC}}\right) - A^{PC} - ta^{PC} - \pi_d^{\max} \\ &= -p_d B - ta^{PC} + C > 0 \end{aligned} \tag{35}$$

$$\begin{aligned} \Delta\pi_c &= \pi_c^{PC} - \pi_c^{\max} \\ &= \left(p_d + s_c - p_c^{PC}\right)\left(\alpha + \beta p_c^{PC}\right)\left(K_c\sqrt{a^{PC}} + K_d\sqrt{A^{PC}}\right) - (1-t)a^{PC} - \pi_c^{\max} \\ &= p_d B + ta^{PC} - D > 0 \end{aligned} \tag{36}$$

where B, C, and D are $B = \left(\alpha + \beta p_c^{PC}\right)\left(K_c\sqrt{a^{PC}} + K_d\sqrt{A^{PC}}\right) > 0$, $C = s_d B - A^{PC} - \pi_d^{\max} > 0$ and $D = -(s_c - p_c)B + a^{PC} + \pi_c^{\max} > 0$.

Thus, it shows that inequalities (35) and (36) specify an area between two parallel lines. Every combination (p_d, t) in this region presents a feasible solution to the bargaining problem. According to Nash [39], the bargaining outcome (p_d^*, t^*) can be obtained by maximizing the product of player's utilities over the feasible solution. λ_d and λ_c can be regarded as the bargaining power of the disposer and collector, respectively. Thus, the utility functions for the disposer and the collector are $u_d(p_d, t) = \Delta\pi_d(p_d, t)^{\lambda_d}, u_c(p_d, t) = \Delta\pi_c(p_d, t)^{\lambda_c}$, respectively.

Then Nash's solution can be derived from the following optimization,

$$\max u_d(p_d, t) \cdot u_c(p_d, t) = \Delta\pi_d(p_d, t)^{\lambda_d} \cdot \Delta\pi_c(p_d, t)^{\lambda_c} \tag{37}$$

The solution of Equation (37) is achieved:

$$\begin{aligned}\Delta\pi_d\left(p_d^*, t^*\right) &= \tfrac{\lambda_d}{\lambda_d+\lambda_c}\Delta\pi_{d+c} = \tfrac{\lambda_d}{\lambda_d+\lambda_c}(C-D),\\ \Delta\pi_c\left(p_d^*, t^*\right) &= \tfrac{\lambda_c}{\lambda_d+\lambda_c}\Delta\pi_{d+c} = \tfrac{\lambda_c}{\lambda_d+\lambda_c}(C-D).\end{aligned} \tag{38}$$

Therefore, if $\lambda_d > \lambda_c$, the disposer shares more extra profit than the collector's and the same λ makes the two players equally divide profits. From Equation (38), it is easy to obtain

$$p_d^* B + t^* a^{PC} = \frac{D\lambda_d + C\lambda_c}{\lambda_d + \lambda_c} \tag{39}$$

Equation (39) describes the relationship between the optimal solutions of p_d and t in the cooperative game, which is a line parallel to $\pi_d^{PC} = \pi_d^{max}$.

4.4. Comparisons within Single Model

From Table A2 in Appendix A, it turns out that the optimal solutions are identical in the Nash game, the Stackelberg collector game, and the Stackelberg disposer game without cooperative advertising ($s_d/s_c \leq 1/2$). Consequently, these optimal solutions are labeled as a superscript S+ and make comparisons on these same solutions with the Stackelberg disposer game with $s_d/s_c > 1/2$.

Proposition 13. *In the model without take-back price incentive, the following ordinal relationships are always satisfied. (1)* $a^{NC} > a^{NS_d} > a^{NS_+}$*; (2)* $A^{NC} > A^{NS_d} = A^{NS_+}$*; (3)* $Q^{NC} > Q^{NS_d} > Q^{NS_+}$*; (4)* $\pi_c^{NS_d} > \pi_c^{NS_+}$*; (5)* $\pi_d^{NS_d} > \pi_d^{NS_+}$*; (6)* $\pi_{d+c}^{NC} > \pi_{d+c}^{NS_d} > \pi_{d+c}^{NS_+}$.

From Proposition 13, it reveals that only when the unit revenue of the disposer is larger than half of the collector's, the disposer prefers sharing the responsibility of the collector's advertising expenditure, which leads to a better collective effect including more quantity of collected UMs and more profit for both members. Figure 8 presents the collection efficiency and channel efficiency of the Stackelberg disposer with cooperative advertising ($s_d/s_c > 1/2$). It shows that both efficiencies are high under the circumstance where *ss* is large and *k* is small. Especially, the channel efficiency is extremely close to 1, which means the coordination of RSC.

In summary, when the RSC does not consider offering a take-back price incentive, it is better for the disposer to undertake a channel leader position and the collector acts as the follower. The disposer should choose a collector with better advertising effectiveness and share a proportion of the collector's advertising expenditure. Besides, if the government would like to make a subsidy or penalty policy (the penalty can be treated as an opportunity cost to increase the unit revenue in turn) to the UMs RSC, it is favorable for him to aim at the disposer, for instance, the pharmaceutical manufacture, since it can increase the disposer's unit revenue compared with the collector's (an increase in *ss*).

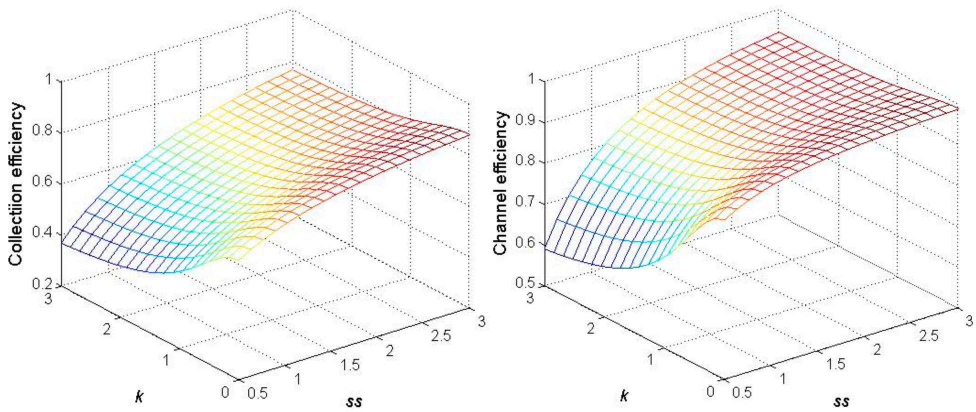

Figure 8. Collection efficiency and channel efficiency of Stackelberg disposer game when $s_d/s_c > 1/2$ with respect to *t* and *ss* in Single Model.

5. Conclusions

For collecting more UMs from households, this study develops and tests two models in a RSC composed of a single disposer and a collector. In Joint Model, the quantity of collected UMs is affected by take-back price and advertising, while in Single Model, the quantity of collected UMs is influenced only by advertising. In each model, four games are played: non-cooperative, disposer as the Stackelberg leader, disposer as the Stackelberg leader, and cooperative. The conditions for the formation of optimal decision-making of the RSC in two models are explored.

In Single Model, both players' unit revenue should be more than 0. Whereas the total unit revenue of the two players should be more than a specific threshold in Joint Model, and the price paid by the disposer to the collector adjusts their corresponding margin profit. In real life, most countries are currently adopting Single Model and do not offer take-back price. Therefore, it is hard for the disposers to get any revenue. If so, the disposers are not willing to pay any to the collectors to adjust collectors' profit margin. How both players can receive benefits becomes a practical issue. Among all UMs, some UMs are unused but near expired. The RSC can offer a channel for disposers to pay extra money for replacing these kinds of UMs. After verifying the packages of these UMs, the collectors can sell these UMs in a good discount rate to the consumers who will use them immediately. This arrangement can satisfy the win-win requirement in Single Model. Of course, this suggestion is based on an assumption that regulation allows reselling reusable unexpired medications.

Comparing the equilibrium results in the four games of the two models, we conclude that, for a certain game, for instance the Nash game, when both models can be played, there always exists a Pareto dominance range with respect to the total unit revenue (t_1) and its allocation (ss) between two members and their relative effectiveness of advertising expenses (k) for Joint Model compared with Single Model, where more players' profits and larger quantity of collected UMs can be obtained. Under these conditions, RSC prefers choosing Joint Model in which a take-back price should be considered offered to UMs holders. Therefore, offering take-back price can reduce the cost of advertising because less advertising is needed due to its effectiveness. If so, the RSC or the government can relocate its budget from doing advertising to giving a take-back price. So the RSC or the government does not really pay a big money for executing "take-back price" policy. On the other hand, the take-back price may not be instant "cash". It can be in a form of a cash coupon for environmentally friendly grocery products. Therefore, it can promote a sustainable concept to the UMs holders. This arrangement can also be interpreted by the public as a way for the RSC or the government to support sustainable development by supporting the producers of environmentally friendly grocery products. Furthermore, the RSC or the government can set a one-to-one sustainability fund to subsidize poor people to purchase medications. When a UMs holder returns the UMs, the government will donate the same amount of money to the fund as the amount of "take-back price" to the UMs holder. This arrangement provides a good excuse for the UMs holders to return the UMs (not for money) and creates a good citizen image for these UMs holders.

However, there exists no area where both members' profits in Single Model are larger than those in Joint Model simultaneously, which means Single Model has no Pareto improving range to Joint Model in any games. The cooperative game has the lowest implement threshold compared with other three games in Joint Model. Furthermore, when the condition to implement price incentive is achieved, regardless of t_1, ss, and k, the RSC should choose Joint Model to reach the higher quantity of collected UMs and total supply chain profit. As discussed above, in the real life, it is hard to design a practical strategy that can facilitate both parties to obtain a desirable amount of benefits in Single Model. However, for Joint Model, not limited to above recommendations, there are a lot of areas for the RSC and the government to implement price incentive strategy. However, in considering the effectiveness of Joint Model (in term of the quantity of collected UMs and total supply chain profit), the real cooperation of the RSC for UMs becomes a matter of cardinal significance.

Furthermore, the total unit revenue is vital to the RSC. The larger the total unit revenue, the more likely the two members are to implement a take-back price incentive where more quantity of collected

UMs is achieved. In this research, it is assumed that it is positive which may form the driving force of the growth of sales in FSC and social responsibility [36]. Through the health promotion and education for disease prevention, more people will care about their health and know how to consume medications for improving their quality of life. Then, the sales of medications will grow. Social responsibility is a fundamental aspect of citizenship, so citizens are educated to participate in creating a safer, more humane, sustainable world. The education for social responsibility can take place in different structures and processes in communities. Another positive driving force is related law and regulation, such as the permission to resell reusable unexpired medications [37], so the government is taking a major role in implementing the "take-back price" in real life through revising the law and regulation.

In most cases, it is undeniable that this revenue is difficult to measure in reality and the immediate and obvious financial benefit is lacking compared with WEEE RSC. This situation calls for regulations and subsidies from the government who could levy a disposing tax on the pharmaceutical industry or medication users, and then assist the RSC for UMs. Kotchen et al. [40] made a survey to study the residents' willingness to pay for a disposal program, and the results show that the conservative estimate of mean willingness to pay is $1.53 per prescription and this assumption implies $320 million for the United States as a whole, which easily outweighs the costs of establishing disposal programs.

This research has multiple limitations. When considering the pricing model, it chooses a simplest linear model. It may not be highly appropriate for the practice and needs further discussion. Besides, the total available quantity of collected UMs is not unlimited in reality, whereas the quantitative restriction is not taken into consideration and simply treated as unrestricted. However, it is usually insufficient to consider the fund for the collection of UMs and this assumption is of rationality. Furthermore, for the convenience of analysis, it is supposed that the collected UMs is homogeneous or can be explained as a mean value. Nevertheless, it does not conform to the reality and needs further research.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Table A1. Optimal expressions in each game scenario in models with customers' take-back price incentive (Joint Model).

Variables	Nash	Stackelberg Disposer	Stackelberg Collector	Cooperation
p_d	$\frac{2\beta s_d-\beta s_c-\alpha}{3\beta}$	$\frac{r\beta s_d-(1-r)\beta s_c-(1-r)\alpha}{\beta}$	$\frac{\beta s_d(1+h)-(1-h)\beta s_c-(1-h)\alpha}{2\beta}$	*N/A*
p_c	$\frac{\beta s\ -2\alpha}{3\beta}$	$\frac{r\beta s-(2-r)\alpha}{2\beta}$	$\frac{(1-h)\beta s-(1+h)\alpha}{2\beta}$	$\frac{\beta s\ -\alpha}{2\beta}$
$\sqrt{a}$	$\frac{K_c(\beta s+\alpha)^2}{18\beta}$	$\frac{K_c(\beta s+\alpha)^2 r(4-3r)}{16\beta}$	$\frac{K_c(\beta s+\alpha)^2 h(1-h)}{4\beta}$	$\frac{K_c(\beta s+\alpha)^2}{8\beta}$
$\sqrt{A}$	$\frac{K_d(\beta s+\alpha)^2}{18\beta}$	$\frac{K_d(\beta s+\alpha)^2 r(1-r)}{4\beta}$	$\frac{K_d(\beta s+\alpha)^2(1-h)^2}{8\beta}$	$\frac{K_d(\beta s+\alpha)^2}{8\beta}$
t	0	$\frac{4-5r}{4-3r}$	0	*N/A*
Q	$\frac{(\beta s+\alpha)^3\left(K_c^2+K_d^2\right)}{54\beta}$	$\frac{(\beta s+\alpha)^3 r^2}{16\beta}\left[K_c^2\frac{(4-3r)}{2}+K_d^2 2(1-r)\right]$	$\frac{(\beta s+\alpha)^3(1-h)^2}{16\beta}\left[K_c^2 2h+K_d^2(1-h)\right]$	$\frac{(\beta s+\alpha)^3\left(K_c^2+K_d^2\right)}{16\beta}$
π_d	$\frac{(\beta s+\alpha)^4\left(2K_c^2+K_d^2\right)}{324\beta^2}$	$\frac{(\beta s+\alpha)^4 r^2}{64\beta^2}\left[K_c^2(4-3r)^2/4+K_d^2 4(1-r)^2\right]$	$\frac{(\beta s+\alpha)^4(1-h)^3}{64\beta^2}\left[K_c^2 4h+K_d^2(1-h)\right]$	*N/A*
π_c	$\frac{(\beta s+\alpha)^4\left(K_c^2+2K_d^2\right)}{324\beta^2}$	$\frac{(\beta s+\alpha)^4 r^3}{64\beta^2}\left[K_c^2\frac{(4-3r)}{2}+K_d^2 4(1-r)\right]$	$\frac{(\beta s+\alpha)^4 h(1-h)^2}{16\beta^2}\left[K_c^2 h+K_d^2(1-h)\right]$	*N/A*
π_{d+c}	$\frac{(\beta s+\alpha)^4\left(K_c^2+K_d^2\right)}{108\beta^2}$	$\frac{(\beta s+\alpha)^4 r^2}{64\beta^2}\left[K_c^2\frac{(4-3r)(4-r)}{4}+K_d^2 4(1-r)\right]$	$\frac{(\beta s+\alpha)^4(1-h)^2}{64\beta^2}\left[K_c^2 4h+K_d^2(1-h)(1+3h)\right]$	$\frac{(\beta s+\alpha)^4\left(K_c^2+K_d^2\right)}{64\beta^2}$
Collection efficiency	$\frac{8}{27}$	$\frac{r^2\left[(4-3r)/2+2k^2(1-r)\right]}{1+k^2}$	$\frac{(1-h)^2\left[2h+k^2(1-h)\right]}{1+k^2}$	*N/A*
Channel efficiency	$\frac{16}{27}$	$\frac{r^2\left[(4-3r)(4-r)/4+k^2 4(1-r)\right]}{1+k^2}$	$\frac{(1-h)^2\left[4h+k^2(1-h)(1+3h)\right]}{1+k^2}$	*N/A*
π_d/π_c	1	$\frac{(4-3r)^2/4+k^2 4(1-r)^2}{r[(4-3r)/2+k^2 4(1-r)]}$	$\frac{(1-h)\left[4h+k^2(1-h)\right]}{4h[h+k^2(1-h)]}$	*N/A*

Note: 1. Collection efficiency = Q^{+}/Q^{c}. 2. Channel efficiency = $\pi_{d+c}^{+}/\pi_{d+c}^{C}$. + stands for a game of Nash, Stackelberg disposer or Stackelberg collector.

Table A2. Optimal expressions in each game scenario in models without customers' take-back price incentive (Single Model).

Variables	Nash	Stackelberg Disposer	Stackelberg Collector	Cooperation
p_d	0	0	0	*N/A*
p_c	0	0	0	0
$\sqrt{a}$	$\frac{\alpha K_c s_c}{2}$	$\frac{\alpha K_c(2s_d+s_c)}{4}$	$\frac{\alpha K_c s_c}{2}$	$\frac{\alpha K_c s}{2}$
$\sqrt{A}$	$\frac{\alpha K_d s_d}{2}$	$\frac{\alpha K_d s_d}{2}$	$\frac{\alpha K_d s_d}{2}$	$\frac{\alpha K_d s}{2}$
t	0	$\frac{2s_d-s_c}{2s_d+s_c}$	0	*N/A*
Q	$\frac{\alpha^2}{2}\left(K_c^2 s_c + K_d^2 s_d\right)$	$\frac{\alpha^2}{2}\left[K_c^2\left(s_d + \frac{s_c}{2}\right) + K_d^2 s_d\right]$	$\frac{\alpha^2}{2}\left(K_c^2 s_c + K_d^2 s_d\right)$	$\frac{\alpha^2 s}{2}\left(K_c^2 + K_d^2\right)$
π_d	$\frac{\alpha^2}{4}\left(K_c^2 2s_c s_d + K_d^2 s_d^2\right)$	$\frac{\alpha^2}{4}\left[K_c^2\left(s_d + \frac{s_c}{2}\right)^2 + K_d^2 s_d^2\right]$	$\frac{\alpha^2}{4}\left(K_c^2 2s_c s_d + K_d^2 s_d^2\right)$	*N/A*
π_c	$\frac{\alpha^2}{4}\left(K_c^2 s_c^2 + K_d^2 2s_c s_d\right)$	$\frac{\alpha^2 s_c}{4}\left[K_c^2\left(s_d + \frac{s_c}{2}\right) + 2K_d^2 s_d\right]$	$\frac{\alpha^2}{4}\left(K_c^2 s_c^2 + K_d^2 2s_c s_d\right)$	*N/A*
π_{d+c}	$\frac{\alpha^2}{4}\left[K_d^2 s_d^2 + K_c^2 s_c^2 + 2s_c s_d\left(K_c^2 + K_d^2\right)\right]$	$\frac{\alpha^2}{4}\left[\frac{K_c^2(2s_d+s_c)(2s_d+3s_c)}{4} + K_d^2 s_d(2s_c + s_d)\right]$	$\frac{\alpha^2}{4}\left[K_d^2 s_d^2 + K_c^2 s_c^2 + 2s_c s_d\left(K_c^2 + K_d^2\right)\right]$	$\frac{\alpha^2 s^2}{4}\left(K_c^2 + K_d^2\right)$
Collection efficiency	$\frac{\left(1+k^2 ss\right)}{(1+ss)(1+k^2)}$	$\frac{ss+1/2+k^2 ss}{(1+ss)(1+k^2)}$	$\frac{\left(1+k^2 ss\right)}{(1+ss)(1+k^2)}$	*N/A*
Channel efficiency	$\frac{k^2 ss^2+1+2ss\left(1+k^2\right)}{(1+ss)^2(1+k^2)}$	$\frac{\left[(2ss+1)(2ss+3)+k^2\left(8ss+ss^2\right)\right]}{4(1+ss)^2(1+k^2)}$	$\frac{k^2 ss^2+1+2ss\left(1+k^2\right)}{(1+ss)^2(1+k^2)}$	*N/A*
π_d/π_c	$\frac{2ss+k^2 ss^2}{1+2k^2 ss}$	$\frac{(ss+1/2)^2+k^2 ss^2}{2k^2 ss+ss+1/2}$	$\frac{2ss+k^2 ss^2}{1+2k^2 ss}$	*N/A*

Note: The optimal solutions of Stackelberg disposer game are in the situation $s_d > s_c/2$. When $s_d \leq s_c/2$, the results are the same as those in Nash equilibrium.

Appendix B

Proof of Theorem 1. The disposer's problem Equation (14) can be solved by taking the first order equations $\partial\pi_d/\partial p_d$ and $\partial\pi_d/\partial A$ to zero, $\partial\pi_d/\partial p_d = \left(K_c\sqrt{a}+K_d\sqrt{A}\right)(-\overline{\alpha}-2\beta p_d+\beta m+\beta s_d)=0$ and $\partial\pi_d/\partial A = \frac{K_d}{2\sqrt{A}}(s_d-p_d)[\overline{\alpha}+\beta(p_d-m)]-1=0$. t has a negative influence on the disposer's profit function, so $t=0$. After algebraic simplification, the two decision variables are

$$p_d = \frac{\beta(s_d+m)-\overline{\alpha}}{2\beta} \tag{A1}$$

$$A = \frac{K_d^2(s_d-p_d)^2[\overline{\alpha}+\beta(p_d-m)]^2}{4} \tag{A2}$$

To prove its optimality, the Hessian matrix $H^{N_d} = \begin{pmatrix} \frac{\partial^2\pi_d}{\partial p_d^2} & \frac{\partial^2\pi_d}{\partial p_d\partial A} \\ \frac{\partial^2\pi_d}{\partial A\partial p_d} & \frac{\partial^2\pi_d}{\partial A^2} \end{pmatrix}$ should be calculated. The second order partial derivatives are $\partial^2\pi_d/\partial p_d^2 = -2\beta\left(K_c\sqrt{a}+K_d\sqrt{A}\right)$, $\partial^2\pi_d/\partial p_d\partial A = \partial^2\pi_d/\partial A\partial p_d = K_d/2\sqrt{A}(-\overline{\alpha}-2\beta p_d+\beta m+\beta s_d)$ and $\partial^2\pi_d/\partial A^2 = -K_d(s_d-p_d)[\overline{\alpha}+\beta(p_d-m)]/4A^2\sqrt{A}$.

Thus, the first principle minor $H_1^{N_d}$ and second principle minor $H_2^{N_d}$ of H^{N_d} at the solution (A3) and (A4) are as follows.

$$\begin{aligned} H_1^{N_d} &= \frac{\partial^2\pi_d}{\partial p_d^2} = -2\beta\left\{K_c\sqrt{a}+\frac{K_d^2[\overline{\alpha}+\beta(s_d-m)]^2}{8\beta}\right\}, \\ H_2^{N_d} &= \frac{\partial^2\pi_d}{\partial p_d^2}\frac{\partial^2\pi_d}{\partial A^2}-\frac{\partial^2\pi_d}{\partial p_d\partial A}\frac{\partial^2\pi_d}{\partial A\partial p_d} = \frac{8\beta^2\left\{8\beta K_c\sqrt{a}+K_d^2[\overline{\alpha}+\beta(p_d-m)]^2\right\}}{K_d^2[\overline{\alpha}+\beta(s_d-m)]^4}. \end{aligned}$$

Considering that $H_1^{N_d}$ is always negative and $H_2^{N_d}$ is always positive, H^{N_d} is negative definite, which can guarantee π_d is concave at this point and π_d is a local maximum. Because the solution is the only stationary point in the domain of definition, π_d is a global maximum at (A1) and (A2).

Similarly, the collector's problem Equation (15) can be solved by setting the first order equations $\partial\pi_c/\partial m$ and $\partial\pi_c/\partial a$ to zero. From $\partial\pi_c/\partial m = \left(K_c\sqrt{a}+K_d\sqrt{A}\right)(\overline{\alpha}+\beta p_d-2\beta m)=0$ and $\partial\pi_c/\partial a = \frac{K_c m}{2\sqrt{a}}[\overline{\alpha}+\beta(p_d-m)]-(1-t)=0$, the decision variables can be obtained as below.

$$m = \frac{\overline{\alpha}+\beta p_d}{2\beta} \tag{A3}$$

$$a = \frac{m^2K_c^2[\overline{\alpha}+\beta(p_d-m)]^2}{4(1-t)^2} \tag{A4}$$

The Hessian matrix of the collector is $H^{N_c} = \begin{pmatrix} \frac{\partial^2\pi_c}{\partial m^2} & \frac{\partial^2\pi_c}{\partial m\partial a} \\ \frac{\partial^2\pi_c}{\partial a\partial m} & \frac{\partial^2\pi_c}{\partial a^2} \end{pmatrix}$. The second order partial derivatives are $\partial^2\pi_c/\partial m^2 = -2\beta\left(K_c\sqrt{a}+K_d\sqrt{A}\right)$, $\partial^2\pi_c/\partial m\partial a = \partial^2\pi_c/\partial a\partial m = K_c(\overline{\alpha}+\beta p_d-2\beta m)/2\sqrt{a}$ and $\partial^2\pi_c/\partial a^2 = -mK_c[\overline{\alpha}+\beta(p_d-m)]/4a\sqrt{a}$.

Hence, the first principle minor $H_1^{N_c}$ and second principle minor $H_1^{N_c}$ of H^{N_c} are

$$\begin{aligned} H_1^{N_c} &= \frac{\partial^2\pi_c}{\partial m^2} = -2\beta\left[\frac{K_c^2(\overline{\alpha}+\beta p_d)^2}{8\beta(1-t)}+K_d\sqrt{A}\right], \\ H_2^{N_c} &= \frac{\partial^2\pi_c}{\partial m^2}\frac{\partial^2\pi_c}{\partial a^2}-\frac{\partial^2\pi_c}{\partial m\partial a}\frac{\partial^2\pi_c}{\partial a\partial m} = \frac{8\beta^2(1-t)^2\left[K_c^2(\overline{\alpha}+\beta p_d)^2+8\beta K_d\sqrt{A}(1-t)\right]}{K_c^2(\overline{\alpha}+\beta p_d)^4}. \end{aligned}$$

By the same token, $H_1^{N_C}$ is always negative and $H_2^{N_C}$ is always positive, therefore, H^{N_C} is negative definite, which can guarantee π_c is concave at this specific point and the point is a local maximum. Because the solution is the only stationary point in the domain of definition, π_c is a global maximum. Thus, the Equations (A1)–(A4) and $t = 0$ can be solved together, then replace $\overline{\alpha}$ by $\alpha + \beta s_c$ to get the expressions stated in Theorem 1.

Proof of Theorem 2. In Stackelberg disposer game, as the follower, the collector's decision problem is the same as that in the Nash equilibrium. Thus, Equations (A3) and (A4) can also be applied in Stackelberg disposer equilibrium. Therefore, by substituting these equations into the disposer's objective function Equation (14) and through some simplification, the disposer's profit is

$$\pi_d = \frac{1}{2}(s_d - p_d)(\overline{\alpha} + \beta p_d)\left[\frac{K_c^2(\overline{\alpha} + \beta p_d)^2}{8\beta(1-t)} + K_d\sqrt{A}\right] - \frac{K_c^2(\overline{\alpha} + \beta p_d)^4 t}{64\beta^2(1-t)^2} - A.$$

Then, the first order derivatives $\partial\pi_d/\partial p_d, \partial\pi_d/\partial A$ and $\partial\pi_d/\partial t$ can be acquired and then take them to zero.

$$\begin{aligned}\frac{\partial\pi_d}{\partial p_d} = \tfrac{1}{2}(\beta s_d - \overline{\alpha} - 2\beta p_d)&\left[\tfrac{K_c^2(\overline{\alpha}+\beta p_d)^2}{8\beta(1-t)} + K_d\sqrt{A}\right] \\ &+ \tfrac{K_c^2(s_d-p_d)(\overline{\alpha}+\beta p_d)^2}{8(1-t)} - \tfrac{tK_c^2(\overline{\alpha}+\beta p_d)^3}{16\beta(1-t)^2} = 0\end{aligned} \tag{A5}$$

$$\frac{\partial\pi_d}{\partial A} = \frac{K_d(s_d - p_d)(\overline{\alpha} + \beta p_d)}{4\sqrt{A}} - 1 = 0 \tag{A6}$$

$$\frac{\partial\pi_d}{\partial t} = \frac{K_c^2(\overline{\alpha} + \beta p_d)^3}{16\beta}\left[\frac{s_d - p_d}{(1-t)^2} - \frac{(\overline{\alpha} + \beta p_d)(1+t)}{4\beta(1-t)^3}\right] = 0 \tag{A7}$$

After simplifying Equations (A6) and (A7), the expressions of A and t can be derived.

$$A = \frac{1}{16}K_d^2(s_d - p_d)^2(\overline{\alpha} + \beta p_d)^2 \tag{A8}$$

$$t = \frac{4\beta(s_d - p_d) - (\overline{\alpha} + \beta p_d)}{4\beta(s_d - p_d) + (\overline{\alpha} + \beta p_d)} \tag{A9}$$

Note that t should be non-negative in Equation (A9), so here two situations are discussed, $p_d < \frac{4\beta s_d - \overline{\alpha}}{5\beta}$ and $0 < t \le 1$, or $p_d \ge \frac{4\beta s_d - \overline{\alpha}}{5\beta}$ and $t = 0$.

In the first situation, solve Equations (A5), (A8), and (A9) together. By substituting (A8) and (A9) into (A5) and after algebraic calculations and replacing K_d/K_c with k, an equation expressed as functions of p_d can be obtained,

$$\left(16k^2 + 9\right)(\overline{\alpha} + \beta p_d)^2 - \left(18 + 24k^2\right)(\overline{\alpha} + \beta s_d)(\overline{\alpha} + \beta p_b) + 8\left(k^2 + 1\right)(\overline{\alpha} + \beta s_d)^2 = 0 \tag{A10}$$

Treat the operator $\overline{\alpha} + \beta p_b$ as a variable and use the formula of equation extracting roots to derive the solution, $\overline{\alpha} + \beta p_d = (\overline{\alpha} + \beta s_d)\frac{12k^2+9\pm\sqrt{16k^4+16k^2+9}}{16k^2+9}$. Here, for $\frac{\overline{\alpha}+\beta p_d}{\overline{\alpha}+\beta s_d} < 1(p_d < s_d)$, $\frac{12k^2+9\pm\sqrt{16k^4+16k^2+9}}{16k^2+9}$ should be less than 1. Therefore, the only feasible solution of this equation is $\overline{\alpha} + \beta p_d = (\overline{\alpha} + \beta s_d)\frac{12k^2+9-\sqrt{16k^4+16k^2+9}}{16k^2+9}$. Denote $r = \frac{12k^2+9-\sqrt{16k^4+16k^2+9}}{16k^2+9}$, then p_d can be obtained, which is

$$p_d = \frac{\beta s_d r - (1-r)\overline{\alpha}}{\beta} \tag{A11}$$

Note that r is increasing with respect to k, so the range of r is $1/2 < \mathrm{r} < 2/3$, where $\frac{\beta s_d r-(1-r)\overline{\alpha}}{\beta} < \frac{4\beta s_d - \overline{\alpha}}{5\beta}$. Therefore, (A11) is the only feasible solution of Equation (A10).

In the second case where $p_d \geq \frac{4\beta s_d - \overline{\alpha}}{5\beta}$ and *t* = *0*, solve Equation (A5) by substituting (A8) and *t* = *0*.

$$4\left(k^2-1\right)(\overline{\alpha}+\beta p_d)^2+3\left(1-2k^2\right)(\overline{\alpha}+\beta s_d)(\overline{\alpha}+\beta p_d)+2k^2(\overline{\alpha}+\beta s_d)^2=0 \tag{A12}$$

When $k = 1$,

$$p_d=\frac{2\beta s_d-\overline{\alpha}}{3\beta} \tag{A13}$$

In other cases that $k \neq 1$, p_d is obtained by the formula of equation extracting roots:

$$p_d=\frac{(\alpha+\beta s_d)v-\overline{\alpha}}{\beta} \tag{A14}$$

where $v=\frac{6k^2-3-\sqrt{4k^4-4k^2+9}}{8(k^2-1)}$.

However, the value of p_d in Equations (A13) and (A14) is not in the feasible region (i.e., more than $\frac{4\beta s_d-\overline{\alpha}}{5\beta}$), resulting in that (A12) has no feasible solution with $p_d \geq \frac{4\beta s_d-\overline{\alpha}}{5\beta}$. Therefore, Equation (A11) is the only feasible solution of p_d in Stackelberg disposer equilibrium.

Similarly, to prove that these solutions are optimal, the Hessian matrix should be calculated,

$$H^{S_d}=\begin{pmatrix} \frac{\partial^2\pi_d}{\partial p_d^2} & \frac{\partial^2\pi_d}{\partial p_d\partial A} & \frac{\partial^2\pi_d}{\partial p_d\partial t} \\ \frac{\partial^2\pi_d}{\partial A\partial p_d} & \frac{\partial^2\pi_d}{\partial A^2} & \frac{\partial^2\pi_d}{\partial A\partial t} \\ \frac{\partial^2\pi_d}{\partial t\partial p_d} & \frac{\partial^2\pi_d}{\partial t\partial A} & \frac{\partial^2\pi_d}{\partial t^2} \end{pmatrix}.$$

The second order partial derivatives are as follows.

$$\frac{\partial^2\pi_d}{\partial p_d^2}=\frac{K_c^2(\overline{\alpha}+\beta p_d)}{8(1-t)}\left[-3\overline{\alpha}-\beta p_d+3\beta s_d-\frac{3t(\overline{\alpha}+\beta p_d)}{2(1-t)}\right]-\beta K_d\sqrt{A}\,,$$
$$\frac{\partial^2\pi_d}{\partial A^2}=-\frac{K_d(s_d-p_d)(\overline{\alpha}+\beta p_d)}{8A\sqrt{A}}\,,\ \frac{\partial^2\pi_d}{\partial t^2}=\frac{K_c^2(\overline{\alpha}+\beta p_d)^2}{16(1-t)^2}\left[2(s_d-p_d)-\frac{(\overline{\alpha}+\beta p_d)(2+t)}{2\beta(1-t)}\right],$$
$$\frac{\partial^2\pi_d}{\partial A\partial p_d}=\frac{\partial^2\pi_d}{\partial p_d\partial A}=\frac{K_d}{4\sqrt{A}}(-\overline{\alpha}+\beta p_d-2\beta s_d),$$
$$\frac{\partial^2\pi_d}{\partial p_d\partial t}=\frac{\partial^2\pi_d}{\partial t\partial p_d}=\frac{K_c^2(\overline{\alpha}+\beta p_d)^2}{16\beta(1-t)^2}\left[3\beta(s_d-p_d)-\frac{2(\overline{\alpha}+\beta p_d)}{1-t}\right],\ \frac{\partial^2\pi_d}{\partial A\partial t}=\frac{\partial^2\pi_d}{\partial t\partial A}=0.$$

For the complexty of the expressions and difficulty of analytical solution, we adopt the method of Aust and Buscher [31], which produced 150,000 sets of parameters and computed numerical studies. In our study, parameters were set that $0.1 \leq \alpha, \beta, s_d, K_d, K_c \leq 5$. Hence we could prove numerically that H^{S_d} is negative definite at the solution given by Equations (A8), (A9), and (A11), which means that π_d is concave in p_d, A, and t; and the specific point is a local maximum of the disposer's decision problem. As Equations (A5)–(A7) have only one root expressed in Equations (A8), (A9), and (A11) within the considered domain of definition, the local optimum is the global optimum of π_d. Thus, the expression presented in Theorem 2 can be derived by substituting Equation (A11) into (A8), (A9), (A3) for t and m, respectively, and (A3), (A11) into (A4) for a then replace $\overline{\alpha}$ by $\alpha+\beta s_c$.

Proof of Theorem 3. In the Stackelberg collector game, as a follower, the disposer's decision problem is the same as that in the Nash equilibrium. Thus, Equations (A1) and (A2), and *t* = *0* can also be applied in the Stackelberg collector equilibrium. Therefore, by substituting these equations into the collector's objective function Equation (24) and through some simplification, the collector's profit can be obtained as below.

$$\pi_c=\frac{m}{2}[\overline{\alpha}+\beta(s_d-m)]\left[K_c\sqrt{a}+\frac{K_d^2[\overline{\alpha}+\beta(s_d-m)]^2}{8\beta}\right]-a \tag{A15}$$

Then, the first order derivatives $\partial\pi_c/\partial m$ and $\partial\pi_c/\partial a$ can be acquired, then set them as zero.

$$\frac{\partial\pi_c}{\partial m}=\frac{1}{2}(\overline{\alpha}+\beta s_d-2\beta m)\left[K_c\sqrt{a}+\frac{K_d^2[\overline{\alpha}+\beta(s_d-m)]^2}{8\beta}\right]-\frac{mK_d^2[\overline{\alpha}+\beta(s_d-m)]^2}{8}=0 \tag{A16}$$

$$\frac{\partial\pi_c}{\partial a}=\frac{K_c m[\overline{\alpha}+\beta(s_d-m)]}{4\sqrt{a}}-1=0 \tag{A17}$$

From Equation (A17), a can be expressed as

$$a=\frac{1}{16}K_c^2m^2[\overline{\alpha}+\beta(s_d-m)]^2 \tag{A18}$$

Substituting Equation (A18) into (A16) and replacing K_d/K_c with k, the following equality can be derived.

$$4\beta^2\left(k^2-1\right)m^2+\beta(\overline{\alpha}+\beta s_d)\left(2-5k^2\right)m+k^2(\overline{\alpha}+\beta s_d)^2=0$$

When $k = 1$, $m=\frac{\overline{\alpha}+\beta s_d}{3\beta}$. In other cases that $k\neq 1$, m can be obtained by the formula of equation extracting roots,

$$m=\frac{(\overline{\alpha}+\beta s_d)h}{\beta} \tag{A19}$$

where $h=\frac{5k^2-2-\sqrt{9k^4-4k^2+4}}{8(k^2-1)}$. Thus, h can be set as $1/3$ when $k=1$.

To prove these solutions are optimal, the Hessian matrix should be calculated, $H^{S_c}=\begin{pmatrix}\frac{\partial^2\pi_c}{\partial a^2} & \frac{\partial^2\pi_c}{\partial a\partial m}\\ \frac{\partial^2\pi_c}{\partial m\partial a} & \frac{\partial^2\pi_c}{\partial m^2}\end{pmatrix}$.

The second order partial derivatives are $\frac{\partial^2\pi_c}{\partial a^2}=\frac{mK_c[\overline{\alpha}+\beta(s_d-m)]}{8a\sqrt{a}}$, $\frac{\partial^2\pi_c}{\partial a\partial m}=\frac{\partial^2\pi_c}{\partial m\partial a}=\frac{K_c}{4\sqrt{a}}(\overline{\alpha}+\beta s_d-2\beta m)$ and $\frac{\partial^2\pi_c}{\partial m^2}=-\beta K_c\sqrt{a}-\frac{3K_d^2[\overline{\alpha}+\beta(s_d-m)](\overline{\alpha}+\beta s_d-2\beta m)}{8}$.

Similar to Theorem 3. Parameters were set that $0.1\leq\alpha,\beta,s_d,K_d,K_c\leq 5$. Hence H^{S_c} can be proved numerically that it is negative definite at the solution given by Equations (A18) and (A19), which means that π_c is concave in m and a; and the specific point is a local maximum of the collector's decision problem. As Equations (A16) and (A17) have only one root expressed in Equations (A18) and (A19) within the considered domain of definition, the local optimum is the global optimum of π_c. Thus, by substituting Equation (A19) into (A18), (A1) for a and p_d, respectively, and (A19),(A1) into (A2) for A, then replace $\overline{\alpha}$ by $\alpha+\beta s_c$, the expression presented in Theorem 3 can be obtained.

Proof of Theorem 4. The total profit function is expressed in Equation (9). With the variables p, a and A, make $s=s_d+s_c$ and then set the first order derivatives $\partial\,\pi_{d+c}/\partial p_c$, $\partial\,\pi_{d+c}/\partial a$ and $\partial\,\pi_{d+c}/\partial A$ to zero. These are $\frac{\partial\pi_{d+c}}{\partial p_c}=\left(K_c\sqrt{a}+K_d\sqrt{A}\right)(s\beta-\beta p_c-\alpha)=0$, $\frac{\partial\pi_{d+c}}{\partial a}=\frac{K_c(s-p_c)(\alpha+\beta p_c)}{2\sqrt{a}}-1=0$ and $\frac{\partial\pi_{d+c}}{\partial A}=\frac{K_d(s-p_c)(\alpha+\beta p_c)}{2\sqrt{A}}-1=0$.

After some simplification, the decision variables can be derived as below.

$$p_c=\frac{s\beta-\alpha}{2\beta} \tag{A20}$$

$$a=\left[\frac{K_c(s-p_c)(\alpha+\beta p_c)}{2}\right]^2 \tag{A21}$$

$$A=\left[\frac{K_d(s-p_c)(\alpha+\beta p_c)}{2}\right]^2 \tag{A22}$$

To prove these solutions are optimal, the Hessian matrix should be calculated, $H^C = \begin{pmatrix} \frac{\partial^2 \pi_{d+c}}{\partial p_c{}^2} & \frac{\partial^2 \pi_{d+c}}{\partial p_c \partial a} & \frac{\partial^2 \pi_{d+c}}{\partial p_c \partial A} \\ \frac{\partial^2 \pi_{d+c}}{\partial a \partial p_c} & \frac{\partial^2 \pi_{d+c}}{\partial a^2} & \frac{\partial^2 \pi_{d+c}}{\partial a \partial A} \\ \frac{\partial^2 \pi_{d+c}}{\partial A \partial p_c} & \frac{\partial^2 \pi_{d+c}}{\partial A \partial a} & \frac{\partial^2 \pi_{d+c}}{\partial A^2} \end{pmatrix}$. The second order partial derivatives are $\frac{\partial^2 \pi_{d+c}}{\partial p_c^2} = -2\beta\left(K_c\sqrt{a} + K_d\sqrt{A}\right)$, $\frac{\partial^2 \pi_{d+c}}{\partial a^2} = -\frac{K_c(s-p_c)(\alpha+\beta p_c)}{4a\sqrt{a}}$, $\frac{\partial^2 \pi_{d+c}}{\partial A^2} = \frac{K_d(s-p_c)(\alpha+\beta p_c)}{4A\sqrt{A}}$, $\frac{\partial^2 \pi_{d+c}}{\partial p_c \partial a} = \frac{\partial^2 \pi_{d+c}}{\partial a \partial p_c} = \frac{K_c(s\beta-\beta p_c-\alpha)}{2\sqrt{a}}$, $\frac{\partial^2 \pi_{d+c}}{\partial p_c \partial A} = \frac{\partial^2 \pi_{d+c}}{\partial A \partial p_c} = \frac{K_d(s\beta-\beta p_c-\alpha)}{2\sqrt{A}}$ and $\frac{\partial^2 \pi_{d+c}}{\partial a \partial A} = \frac{\partial^2 \pi_{d+c}}{\partial A \partial a} = 0$.

Then, at the solution (A10)–(A22), the Hessian matrix can be expressed as
$$H^C = \begin{pmatrix} -\frac{(\alpha+\beta s)^2\left(K_c^2+K_d^2\right)}{4} & 0 & 0 \\ 0 & -\frac{32\beta^2}{K_c^2(\alpha+\beta s)^2} & 0 \\ 0 & 0 & -\frac{32\beta^2}{K_d^2(\alpha+\beta s)^2} \end{pmatrix}.$$

Therefore, H^C is negative definite and π_{d+c} is concave at this specific point, which is local maximum. As it is the only maximum candidate, the point is the globally profit maximizing solution. Then, by inserting Equation (A20) into (A21) and (A22), the optimal expressions shown in Theorem 4 can be obtained.

Proof of Proposition 1. Setting $s = t_1\alpha/\beta(t_1 \geq 2)$, then $\beta = t_1\alpha/s(t_1 \geq 2)$.

$$\pi_d^{PN} = \frac{(\beta s+\alpha)^4\left(2K_c^2+K_d^2\right)}{324\beta^2} = \frac{\left(\frac{t_1\alpha}{s}s+\alpha\right)^4\left(2K_c^2+K_d^2\right)}{324\left(\frac{t_1\alpha}{s}\right)^2} = \frac{(t_1+1)^4\alpha^2 s^2\left(2K_c^2+K_d^2\right)}{324t_1^2}$$

Thus,
$$\pi_d^{PN} - \pi_d^{NN} = \frac{(t_1+1)^4\alpha^2 s^2\left(2K_c^2+K_d^2\right)}{324t_1^2} - \frac{\alpha^2\left(K_c^2 2s_c s_d + K_d^2 s_d^2\right)}{4} \tag{A23}$$

After substituting $s = s_c + s_d$ and setting $K_d/K_c = k$ and $s_d/s_c = ss$, equation (A23) can be simplified to
$$\frac{\alpha^2 K_c^2 s_c^2}{4}\left[\frac{(1+t_1)^4}{81t_1^2}(1+ss)^2\left(2+k^2\right) - \left(2ss+k^2 ss^2\right)\right] \tag{A24}$$

Similarly, $\pi_c^{PN} = \frac{(\beta s+\alpha)^4\left(K_c^2+2K_d^2\right)}{324\beta^2} = \frac{\left(\frac{t_1\alpha}{s}s+\alpha\right)^4\left(K_c^2+2K_d^2\right)}{324\left(\frac{t_1\alpha}{s}\right)^2} = \frac{(t_1+1)^4\alpha^2 s^2\left(K_c^2+2K_d^2\right)}{324t_1^2}$, then
$$\begin{aligned} \pi_c^{PN} - \pi_c^{NN} &= \frac{(t_1+1)^4\alpha^2 s^2\left(K_c^2+2K_d^2\right)}{324t_1^2} - \frac{\alpha^2\left(K_c^2 s_c^2 + K_d^2 2s_c s_d\right)}{4} \\ &= \frac{\alpha^2 K_c^2 s_c^2}{4}\left[\frac{(1+t_1)^4}{81t_1^2}(1+ss)^2\left(1+2k^2\right) - \left(1+2k^2 ss\right)\right] \end{aligned} \tag{A25}$$

When the two players' profits in Joint Model are all larger than those in Single Model respectively, Joint Model should be chosen for Pareto improvement. Therefore, Equations (A24) and (A25) are more than zero simultaneously which is the condition stated in Proposition 1.

Proof of Proposition 2 and 3. The proofs of Proposition 2 and 3 are similar to those of Proposition 1, except that the expressions of optimal solutions are diffident when $s_d > s_c/2$ and $s_d \leq s_c/2$. Hence, the results should be discussed in two cases.

Proof of Proposition 4.

$$\frac{\pi_{d+c}^{PC}}{\pi_{d+c}^{NC}} = \frac{\frac{(\beta s+\alpha)^4\left(K_c^2+K_d^2\right)}{64\beta^2}}{\frac{\alpha^2 s^2\left(K_c^2+K_d^2\right)}{4}} = \left\{\frac{1}{4}\left[\left(\frac{\beta s}{\alpha}\right) + \frac{1}{\left(\frac{\beta s}{\alpha}\right)} + 2\right]\right\}^2 > \left\{\frac{1}{4}\left[2\sqrt{\left(\frac{\beta s}{\alpha}\right)\frac{1}{\left(\frac{\beta s}{\alpha}\right)}} + 2\right]\right\}^2 = 1$$

Therefore, $\pi_{d+c}^{PC} > \pi_{d+c}^{NC}$.

$$\frac{Q^{PC}}{Q^{NC}} = \frac{\frac{(\beta s+\alpha)^3\left(K_c^2+K_d^2\right)}{16\beta}}{\frac{\alpha^2 s\left(K_c^2+K_d^2\right)}{2}} = \frac{1}{8}\left[\left(\frac{\beta s}{\alpha}\right)^2 + \frac{1}{\left(\frac{\beta s}{\alpha}\right)} + 3\left(\frac{\beta s}{\alpha}\right) + 3\right] \tag{A26}$$

When $s > \alpha/\beta$, $\beta s/\alpha > 1$. Regarding $\beta s/\alpha$ as a whole x and setting $f(x) = \frac{1}{8}\left(x^2 + \frac{1}{x} + 3x + 3\right)(x > 1)$, $f(x)$ can be testified as increasing in x for $f''(x) = \frac{1}{4}\left(1 + \frac{1}{x^3}\right) > 0$ and then $f'(x) = \frac{1}{8}\left(2x - \frac{1}{x^2} + 3\right) > f'(1) = \frac{1}{2} > 0$. Therefore, $f(x) > f(1) = 1$ which is the same result in Equation (A26) and means $Q^{PC} > Q^{NC}$.

Proof of Proposition 5. To compare p_c in four game scenarios, their difference should be calculated. $p_c^{PC} - p_c^{PN} = \frac{1}{6}\left(s + \frac{\alpha}{\beta}\right) > 0$ apparently for s and α/β are all positive, so $p_c^{PC} > p_c^{PN}$. $p_c^{PC} - p_c^{PS_d} = \frac{1-r}{2}\left(s + \frac{\alpha}{\beta}\right) > 0$ and $p_c^{PN} - p_c^{PS_d} = \frac{2-3r}{6}\left(s + \frac{\alpha}{\beta}\right) > 0$, because $1/2 < r < 2/3$. Similarly, $1/4 < h < 1/2$, then $p_c^{PC} - p_c^{PS_c} = \frac{h}{2}\left(s + \frac{\alpha}{\beta}\right) > 0$. $p_c^{PN} - p_c^{PS_c} = \frac{3h-1}{6}\left(s + \frac{\alpha}{\beta}\right)$. When $1/3 < h < 1/2$ $(k < 1)$, $p_c^{PN} > p_c^{PS_c}$; when $1/4 < h < 1/3$ $(k > 1)$, $p_c^{PN} < p_c^{PS_c}$; $h = 1/3$ $(k = 1)$ is the condition for equality. $p_c^{PS_d} - p_c^{PS_c} = \frac{r+h-1}{2}\left(s + \frac{\alpha}{\beta}\right)$ equals to 0 when $r + h - 1 = 0$ which is a complicated function of k, so we solve it by software and obtain the solution $k_1 \approx 0.7164$. Moreover, $r + h - 1 = 0$ is decreasing with k because r and h are all decreasing with k. Therefore, when $k < k_1$, $r + h - 1 > 0$ which leads to $p_c^{PS_d} > p_c^{PS_c}$ and vice-versa. Based on the above relationships, the results listed in Proposition 5 can be gained.

Proof of Proposition 6 to 11. These proofs are exactly similar to the proof of Proposition 5 except that in proofs of Propositions 7–11, their ordinal relationships can be cleared by comparing their quotients with 1.

Proof of Proposition 13. It is evident from Table A2 in Appendix A that $a^{NC} > ^{NS_d} > a^{NS_+}$ and $A^{NC} > ^{NS_d} = A^{NS_+}$. Because $ss = s_d/s_c > 1/2$, the following relationships can be proved.

$$\frac{Q^{NS_d}}{Q^{NS_+}} = \frac{ss+\frac{1}{2}+k^2 ss}{1+k^2 ss} > \frac{1+k^2 ss}{1+k^2 ss} = 1$$

$$\frac{\pi_d^{NS_d}}{\pi_d^{NS_+}} = \frac{\left(ss+\frac{1}{2}\right)^2+k^2 ss^2}{2ss+k^2 ss^2} = \frac{\left(ss-\frac{1}{2}\right)^2+2ss+k^2 ss^2}{2ss+k^2 ss^2} = \frac{\left(ss-\frac{1}{2}\right)^2}{2ss+k^2 ss^2} + 1 > 1$$

$$\frac{\pi_c^{NS_d}}{\pi_c^{NS_+}} = \frac{ss+\frac{1}{2}+2k^2 ss}{1+2k^2 ss} = \frac{ss-\frac{1}{2}}{1+2k^2 ss} + 1 > 1$$

$$\frac{\pi_{d+c}^{NS_d}}{\pi_{d+c}^{NC}} = \frac{\frac{(2ss+1)(2ss+3)}{4}+k^2\left(2ss+ss^2\right)}{(1+ss)^2\left(1+k^2\right)} = \frac{\left(ss^2+2ss+\frac{3}{4}\right)+k^2\left(ss^2+2ss\right)}{(1+ss)^2+k^2(1+ss)^2} < 1$$

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