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Adaptive Least-Squares Collocation Algorithm Considering Distance Scale Factor for GPS Crustal Velocity Field Fitting and Estimation

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Abstract: High-precision, high-reliability, and high-density GPS crustal velocity are extremely important requirements for geodynamic analysis. The least-squares collocation algorithm (LSC) has unique advantages over crustal movement models to overcome observation errors in GPS data and the sparseness and poor geometric distribution in GPS observations. However, traditional LSC algorithms often encounter negative covariance statistics, and thus, calculating statistical Gaussian covariance function based on the selected distance interval leads to inaccurate estimation of the correlation between the random signals. An unreliable Gaussian statistical covariance function also leads to inconsistency in observation noise and signal variance. In this study, we present an improved LSC algorithm that takes into account the combination of distance scale factor and adaptive adjustment to overcome these problems. The rationality and practicability of the new algorithm was verified by using GPS observations. Results show that the new algorithm introduces the distance scale factor, which effectively weakens the influence of systematic errors by improving the function model. The new algorithm can better reflect the characteristics of GPS crustal movement, which can provide valuable basic data for use in the analysis of regional tectonic dynamics using GPS observations.

Keywords: GPS velocity field; Least-squares collocation algorithm (LSC); Distance scale factor; Adaptive adjustment; fitting and estimation

1. Introduction

GPS velocity field provides the most intuitive representation of regional crustal movement and deformation, and is also the basis of geodynamic studies. However, due to the complexity of regional terrain and economic factors of GPS station construction, the GPS monitoring data may be affected by errors [1], which hinders obtaining a GPS velocity field with a large range, high density and high precision. Therefore, many crustal movement models were established to improve the abovementioned problems, such as the multi-face function method [2], the finite element interpolation method [3], the polynomial fitting method [4], and the least-squares collocation (LSC) algorithm [5].

Compared to crustal motion models, the LSC has unique advantages in that it considers both function model parameters and random signals, and can be applied to establish regional crustal movement models [6]. The least-squares collocation (LSC) algorithm has been developed as a general least-squares theory for estimating any element of the earth's gravity field based on studies conducted by Moritz [7] and Krarup [8] on least-squares estimation of gravity anomalies. The LSC is indispensable in the study of crustal movement and deformation. The LSC can weaken the influence of points position distribution on a deformation field by establishing the covariance matrix between the estimated and

observation points and can accurately describe the regional crustal movement trend [6]. The LSC algorithm has been used to estimate crustal movement signals from GPS velocity fields [9–15]. It has also been used in various earth science fields to control systematic and anomaly errors in GIS spatial data [16], detect outliers in multibeam bathymetric data [17], solve common point coordinate errors in 3D coordinate transformation [18], improve the accuracy of mobile light detection and ranging (LiDAR) systems in hostile environments [19], improve the results of downward continuation values of airborne gravity data [20], and refine the local covariance model of gravity anomalies [21].

The reliability and accuracy of the LSC algorithm is based on the exact covariance function; however, at present, it is difficult to establish an ideal empirical covariance function. Unreliable covariance function can cause inconsistencies in observation noise and signal variance. Furthermore, in view of the complexity of the measured data, during the process of using Gaussian functions (the most commonly used empirical covariance function) as an empirical covariance function to establish the mapping of distance and covariance, the covariance will appear negative, which violates the positive definiteness of the Gaussian function. In this case, the commonly used approach is to avoid the negative value, which affects the data utilization and leads to the reduction in the reliability of the fitting model.

In this study we propose an adaptive least-squares collocation algorithm considering the distance scale factor and show that the new algorithm has great advantages in actual GPS observation studies. The improvements of the new algorithm over previous ones are as follows: the distance scale factor is first introduced to establish a comparative relationship between the covariance of observations and the cross-covariance of two observations with the farthest distance. This step ensures the positive definiteness of the data and allows a reasonable Gaussian model parameter to be obtained. It also weakens the systematic error of the model. Then, on the basis of the first step, the variance component estimation method (VCEM) is created to construct an adaptive factor. This step is further adjusted to ensure a coordinated weight ratio between the signals and the observation so as to improve the accuracy of the estimated points.

2. Methods

2.1. Traditional LSC Algorithm

The LSC algorithm is an optimal estimation algorithm that simultaneously determines the tendency parameters and the signals with randomness [22]:

$$L_{q \times 1} = A_{q \times p} X_{p \times 1} + U_{q \times m} S_{m \times 1} + n_{q \times 1} \quad (1)$$

where L is the observation vector; X is the non-random tendency parameter; A is coefficient matrix, reflecting the contribution of X to L ; S is random signal, which includes the signal of observation points and unobserved points (estimated points) in the study area; U is the rectangular matrix, the left q columns of the U matrix is the unit matrix, corresponding to the observation points signal, the right $m - q$ columns of the U matrix is null matrix, corresponding to the estimated points signal; n represents observation noise.

The observed values L include the tendency part AX determined by fixed parameters, the random signal part S , and the observation noise part n . When the traditional LSC algorithm (TLSC) is applied to establish the GPS crustal horizontal velocity model, the Equation (1) can be rewritten as follows [13]:

$$V_o_{q \times 1} = A_{q \times 33} \Omega_{33 \times 1} + U_{q \times m} V_s_{m \times 1} + n_{q \times 1} \quad (2)$$

where V_o represents GPS horizontal velocity field; Ω is a parameter of the Euler rigid motion model, and A is the coefficient matrix that is calculated from the coordinates of the GPS stations; $A\Omega$ represents the overall motion tendency of the study area; V_s is the effective random signal after deducting the

overall motion tendency from V_o , and U is the coefficient matrix. The adjustment solution of the tendency parameter Ω and the signal V_s in Equation (2) can be calculated as follows [6]:

$$\hat{\Omega} = (A^T \bar{C}^{-1} A)^{-1} A^T \bar{C}^{-1} V_o \quad (3)$$

$$\hat{V}_s = C_{uo} \bar{C}^{-1} (V_o - A \hat{\Omega}) \quad (4)$$

$$\bar{C} = (C_{oo} + C_{nn}) \quad (5)$$

where C_{oo} is the covariance of the V_o , which is used to describe the spatial distribution correlation and dispersion among GPS stations; C_{nn} is the self-covariance of the observation error of V_o , which is used to describe the dispersion of the GPS velocity itself and reflect the observation precision; C_{uo} is the covariance matrix between the estimated points signal and the observation points signal. The C_{nn} can be obtained from the GPS velocity field. C_{oo} and C_{uo} are unknown quantities, but can be determined from the functional relationship between signal covariance and the distance between different observation points. The covariance of signals between observation points will decrease with the increase of the distance between observation points. This feature is consistent with the Gaussian function, which is usually used as the signal empirical covariance function [13–15]:

$$F(d) = a \exp(-k^2 d^2) \quad (6)$$

where a represents the first undetermined parameter, and also represents the self-covariance of the observation signal; d is the distance between the observation points; k is the second undetermined parameter, denoting the smoothness of the continuous velocity field distribution. Normally, a and k can be obtained by least squares fitting [13]. The value required to fit the Gaussian function $F(d)$ can be composed of the following two parts: [6]:

$$C(0) = \frac{1}{q} \sum_{i=1}^q V s_i^2 \quad (7)$$

$$C(d_i) = \frac{1}{q_{di}} \sum V s_i V s_j \quad (8)$$

where $C(0)$ is the self-covariance of the observation points; q is the number of observation points that participate in the modelling construction; $C(d_i)$ is the cross-covariance of the observation points, corresponding to d_i after dividing the distance segment.

2.2. Considering Distance Scale Factor

In the TLSC algorithm, according to the least square adjustment principle, the Gaussian function is fitted mainly by establishing the mapping relation between distance and covariance. However, negative values often appear in covariance calculation results, which violates the positive definiteness of the Gaussian function. If the negative value is removed and the positive value is retained, a large amount of data will be lost, resulting in poor fitting and estimation. Therefore, Gaussian function considering distance scale factor is constructed (DLSC). The prior Gaussian function model can be established as follows:

$$F(d) = C(0) \exp(-k^2 d^2) \quad (9)$$

where $C(0)$ is the self-covariance of the observation signal, which can be solved based on Equation (7). According to the regularity of the stationary random function (Gaussian function) that $C(d) \leq C(0)$, if the distance d goes to infinity, the covariance $C(d)$ will be close to zero. Therefore, when the maximum

distance between the two observation points is defined (d_{max}), the covariance $C(d_{max})$ is much less than $C(0)$, namely:

$$C(d_{max}) \ll C(0) \quad (10)$$

Defining $C(d_{max}) = RC(0)$, $R \ll 1$, where R is the distance scale factor of signal covariance. Taking into account the Equation (9), there is:

$$F(d_{max}) = RC(0) \quad (11)$$

$$RC(0) = C(0) \exp(-k^2 d_{max}^2) \quad (12)$$

$$R = \exp(-k^2 d_{max}^2) \quad (13)$$

$$-k^2 = \ln R / d_{max}^2 \quad (14)$$

The corresponding value of k can be determined by taking different values of R . Then the Gaussian function can be established to determine the covariance matrix of signals between observation points. From the root mean square error (RMS) of the checking residuals, the k value with the smallest RMS value will be selected appropriately adjusting the R value.

2.3. Adaptive Collocation

On the other hand, in the TLSC algorithm, the coordination between the variance-covariance of the signal and the observation may not be unbalanced. To solve this problem, an adaptive factor can be introduced to adjust contribution of the observation error and signal in the process of solving model parameters. The function model of the adaptive collocation algorithm (ALSC) based on Helmert VCEM is [23,24]:

$$V^T P_e V + \alpha S^T P_s S = \min \quad (15)$$

where α is the adaptive factor; V is the correction value of the observation; S is the correction value of the signal; P_e and P_s are weight matrices of the observation and signal vectors, respectively. Equation (15) takes the signal as the virtual observation value and combines the observations to form two kinds of observation values [25]:

$$\left. \begin{aligned} V &= AX + BS - L \\ V_s &= S - 0 \end{aligned} \right\} \quad (16)$$

The normal equation can be obtained [25]

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix} \begin{bmatrix} \sigma_e^2 \\ \sigma_s^2 \end{bmatrix} = \begin{bmatrix} V^T P_e V \\ S^T P_s S \end{bmatrix} \quad (17)$$

where $S_{ii} = n_i - 2\text{tr}(N^{-1}N_i) + \text{tr}(N^{-1}N_iN^{-1}N_i)$; $S_{ij} = \text{tr}(N^{-1}N_iN^{-1}N_j)$; n_1 is the number of observations; n_2 is the number of signals; $N = \begin{bmatrix} A^T P_e A & A^T P_e B \\ B^T P_e A & B^T P_e B + P_s \end{bmatrix}$; $N_1 = \begin{bmatrix} A^T P_e A & A^T P_e B \\ B^T P_e A & B^T P_e B \end{bmatrix}$; $N_2 = \begin{bmatrix} 0 & 0 \\ 0 & P_s \end{bmatrix}$; σ_s^2 represents the variance of the signal, σ_e^2 represents the variance of the noise.

Then σ_e^2 and σ_s^2 can be solved based on the Equation (17), and the adaptive factor can be constructed:

$$\alpha = \sigma_e^2 / \sigma_s^2 \quad (18)$$

Using the adaptive factor to calculate the iterative adjustment makes the signal weight matrix increase or decrease, which ensures coordination between the observation and the signal weight matrix:

$$P_s = \alpha P_s \quad (19)$$

Combined with the above adjusted observed weight matrix, the posterior Euler parameters and posterior signal correction values can be recalculated, as follows:

$$\hat{\Omega} = (A^T \bar{C}^{-1} A)^{-1} A^T \bar{C}^{-1} V_o \quad (20)$$

$$\hat{V}_s = C_{uo} \bar{C}^{-1} (V_o - A \hat{\Omega}) \quad (21)$$

where $\bar{C} = (C_{oo} + C_{mm})$; $\hat{\Omega}$ is the posterior Euler parameter; $\hat{V}_s$ is the posterior signal; C_{oo} is the covariance matrix of observation noise; C_{mm} is the covariance matrix of signal.

2.4. Fusion Algorithm

For the DLSC algorithm, the weight ratio of the observation noise and the effective signal cannot be guaranteed to be consistent, although the problem of negative signal covariance is overcome. For the ALSC algorithm, the a priori weight ratio between the signal and the observation vector is consistent; however, only the positive signal covariance value will be intercepted to fit the empirical Gaussian function. This will affect the accuracy of the fitting parameters of the Gaussian function.

In order to fully guarantee the reliability of the fitting and estimation results, and effectively solve the limitations of the above two problems existing in the DLSC and ALSC algorithms, we propose an adaptive least-squares collocation algorithm considering the distance scale factor (ADLSC). The new algorithm will overcome the problem of negative signal covariance, and ensure that the signal is consistent with the observed noise. The approximate frame diagram of the ADLSC is shown in Figure 1:

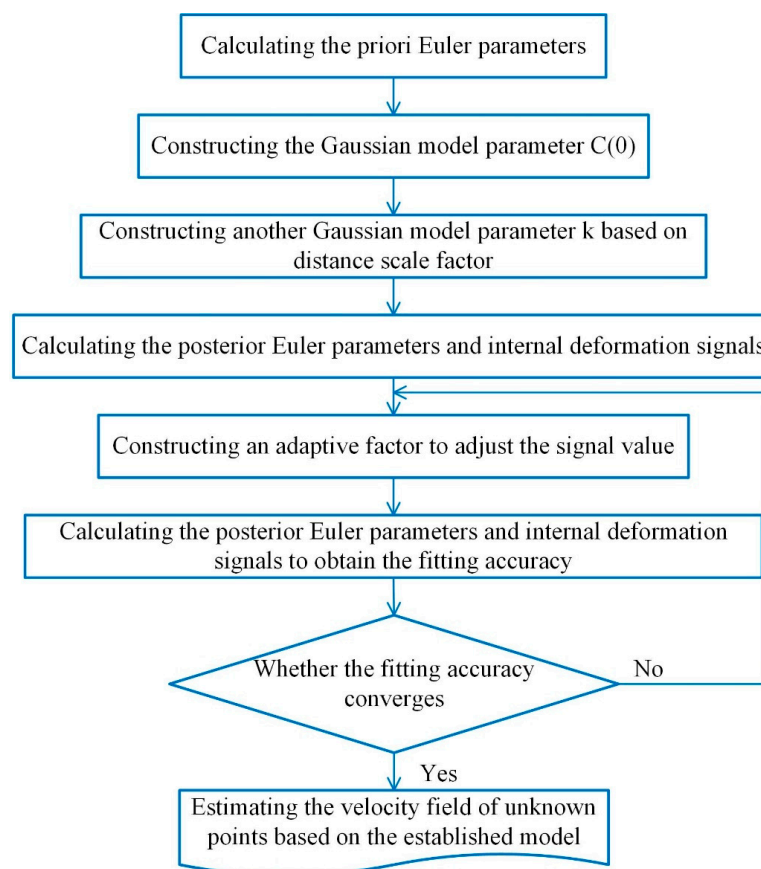


Figure 1. The frame diagram of the an adaptive least-squares algorithm considering the distance scale factor (ADLSC algorithm).

The specific implementation steps are as follows:

(1) Solving the overall motion trend of the study area. A priori Euler rigid parameter Ω (the random signal is not considered, or it is attributed to the observation noise) can be calculated according to the principle of least squares based on the GPS velocity. The model is as follows:
$$V_o = A \Omega + n$$
 where V_o is the GPS horizontal velocity; n is observation noise. Then the residual components can be calculated based on the trend term $A\Omega$.

where V_o is the GPS horizontal velocity; n is observation noise. Then the residual components can be calculated based on the trend term $A\Omega$.

(2) Constructing parameters of the priori Gauss function. According to Equations (7) and (8), the parameters $C(0)$ and $C(d_i)$ in the priori Gaussian function can be calculated based on the residual component solved in step (1).

(3) Introducing the distance scale factor. According to Equation (14), by analyzing the RMS of the checking residual, the k value with the smallest RMS value will be selected by appropriately adjusting the R value. Then, the specific mathematical expression of the empirical Gaussian function can be determined.

In this step, the introduced distance scale factor takes into account the scale of the study area, and will significantly improve data utilization. Because this strategy avoids negative cross-covariance variance, their guarantees a more accurate and reliable empirical Gaussian function.

(4) Constructing the covariance matrix of observation signal based on the Gaussian function of the prior signal determined in step (3). The initial signal weight matrix $P_s^{(0)}$ can be obtained by inverting the covariance. In order to analyze the influence of the noise variance on fitting and estimation results, it is usually assumed that the signal variance $\sigma_s^{2(0)} = 1$ and the different noise variance $\sigma_e^{2(0)}$ will be given. Then the posterior Euler vector parameter $\hat{\Omega}^{(0)}$ and the posterior signal $\hat{V}_s^{(0)}$ can be further solved according to Equations (3) and (4).

(5) Performing the adaptive iterative adjustments. The adaptive factor $\alpha^{(0)}$ can be solved by using the VCEM according to Equations (16)–(18), which is used to adjust the signal weight matrix to obtain a new signal weight matrix $P_s^{(1)}$. Then the adjustment process is carried out again, and a new posterior Euler vector parameter $\hat{\Omega}^{(1)}$ and the posterior signal $\hat{V}_s^{(1)}$ can be calculated. During K iterations, until the RMS in the checking residual is the smallest, the observed noise is consistent with the signal variance component, and the optimal Euler vector parameter $\hat{\Omega}^{(K)}$ and the signal $\hat{V}_s^{(K)}$ are obtained.

(6) Solving the signal of the estimated points. The signal covariance matrix between the estimated point and the observed point can be constructed based on the priori Gaussian function, which is determined in step (3). Finally, the signal of estimated point can be solved according to the Equation (4) based on the optimal signal $\hat{V}_s^{(K)}$ that is determined in step (5).

3. Results and Analysis

To verify the rationality of the new algorithm in real cases, we selected the Sichuan–Yunnan block located in southern China [15] and used long-term GPS velocities to test the ADLSC algorithm. The GPS measurements were taken from the Crustal Movement Observation Network of China (CMONOC I) [26] and the Tectonic and Environmental Observation Network of Mainland China (CMONOC II) [27]. The horizontal GPS velocities used in this study were with respect to stable Eurasia based on the ITRF2008 reference frame and included a total of 81 stations from 1999 to 2013 (Figure 2) [28]. These stations contain 22 continuously operating GPS stations and 59 campaign-mode GPS stations. The latter have reinforced concrete monuments with forced-centering apparatus for GPS antennas. In each survey, dual-frequency GPS receivers and choke ring antennas were used, and each station was occupied for at least three days, to ensure the quality of the data. As the stations ran at least five observation campaigns over a 10-year span, highly accurate crustal horizontal velocity fields can be obtained.

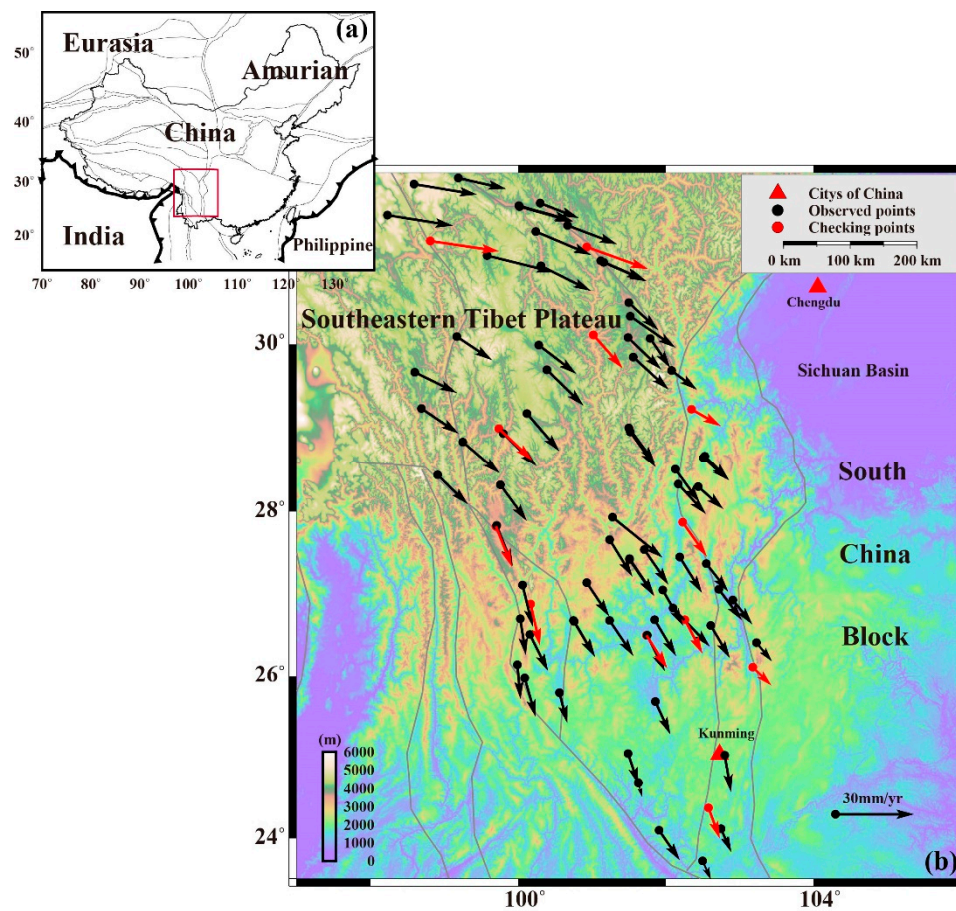


Figure 2. The GPS velocities for 1999–2013 (mm/a) and the location of the Sichuan–Yunnan block. Red rectangle outlines the study area in mainland China (a). Gray solid lines represent the major faults [29] (b). Red triangles represent major cities (capital cities) (b). Black arrows are the observed GPS velocities, and the red color at the GPS station represents the checking points (b).

Among the 81 GPS monitoring data values, 12 GPS points are randomly selected as checking points and the remaining 69 GPS points are fitting points. The checking points are not included in the calculation of unknown parameters. Then, the velocity at each fitting and checking point can be estimated using different collocation algorithms, and the residuals between the estimated and observed velocity can be also calculated. The RMS is used to assess the quality of the results, as follows:

$$RMS = \left[\frac{1}{m} \sum_{i=1}^m (d_i)^2 \right]^{1/2} \quad (22)$$

where m represents the numbers of fitting and checking points; d_i is the residual between the estimated and observed velocity at each fitting and checking point. We use four collocation algorithms to calculate the RMS of the fitting points and checking points, namely, the TLSC, DLSC, ALSC, and ADLSC algorithms (Table 1).

Table 1. Root mean square error (RMS) of the fitting points and checking points using four algorithms.

Algorithms	Fitting Points RMS (mm/a)		Checking Points RMS (mm/a)		Running Time (s)
	(E)	(N)	(E)	(N)	
TLSC	2.43	1.32	3.21	1.86	1.29
DLSC	2.00	1.19	2.61 (18.7%)	1.73 (6.9%)	0.81
ALSC	2.15	0.96	2.82 (12.1%)	1.24 (33.3%)	1.42
ADLSC	1.96	0.54	2.57 (19.9%)	1.03 (44.6%)	1.05

Compared to the other three algorithms, the ADLSC has the smallest RMS both in the fitting points and checking points (Table 1). It indicates that the ADLSC has the optimal fitting effect and estimation effect. Figure 3 shows the residual between the estimated and observed velocity of each checking point. The overall amplitude of the residuals derived from ADLSC is relatively smaller and closer to zero (Figure 3). For the DLSC and ALSC, the RMS are about 18.7% and 12.1% better than the TLSC in the east direction, and are about 6.9% and 33.3% better than the TLSC in the north direction, respectively. However, for the ADLSC, the RMS is about 19.9% and 44.6% better than the TLSC in the east and north directions, respectively (Table 1). In addition, we can see that there are no significant differences in program running time among the four algorithms, which are within two seconds (Table 1). The subtle differences may be the adaptive computing is an iterative process, which will take a relatively long time.

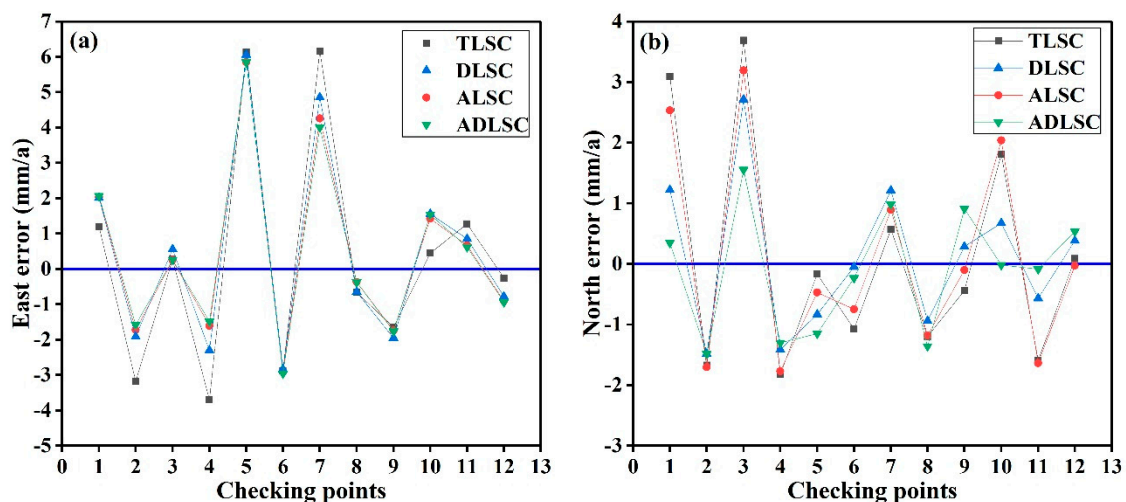
**Figure 3.** Residual between the estimated and observed velocity of the checking points: (a) east direction; (b) north direction.

Figure 4 shows the residuals between the estimated and observed velocity of the fitting points. The overall amplitude of the residuals derived from ADLSC is also relatively smaller, closer to zero (Figure 4), and has the smallest RMS value (only has 1.96 mm/a to the east and 0.54 mm/a to the north).

In addition, we further conduct correlation analysis on the fitting and observed velocity in the east and north directions to comprehensively test the fitting effect of each algorithm (Figure 5). Compared with the other three algorithms, the correlation coefficient R of the TLSC is relative weak, and the values are 0.9077 and 0.9233 in the east and north directions, respectively (Figure 5a1,b1)). For the DLSC and ALSC, the value of R are 0.9383 and 0.9281 in the east direction, and are 0.9418 and 0.9548 in the north direction (Figure 5a2,a3,b2,b3); however, for ADLSC, the value of R are 0.9404 and 0.9863 in the east and north directions, respectively (Figure 5a4,b4). The above analysis shows that the positive correlation between the observed and fitting velocity calculated by the ADLSC is very strong, which is further proves that the ADLSC algorithm has the best fitting effect.

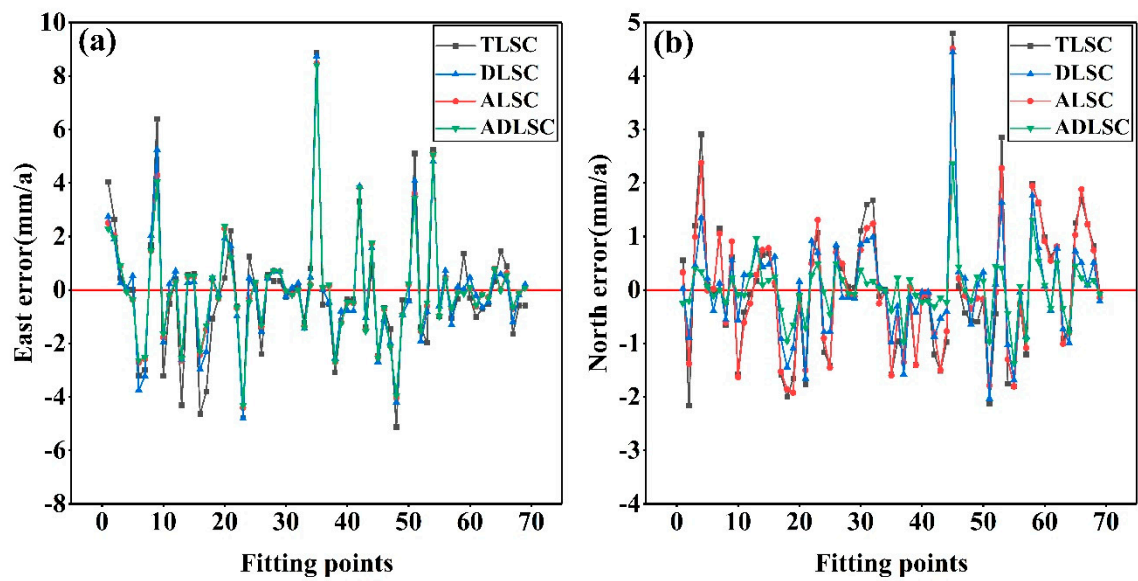


Figure 4. Residual between the estimated and observed velocity of the fitting points in the (a) east and (b) north.

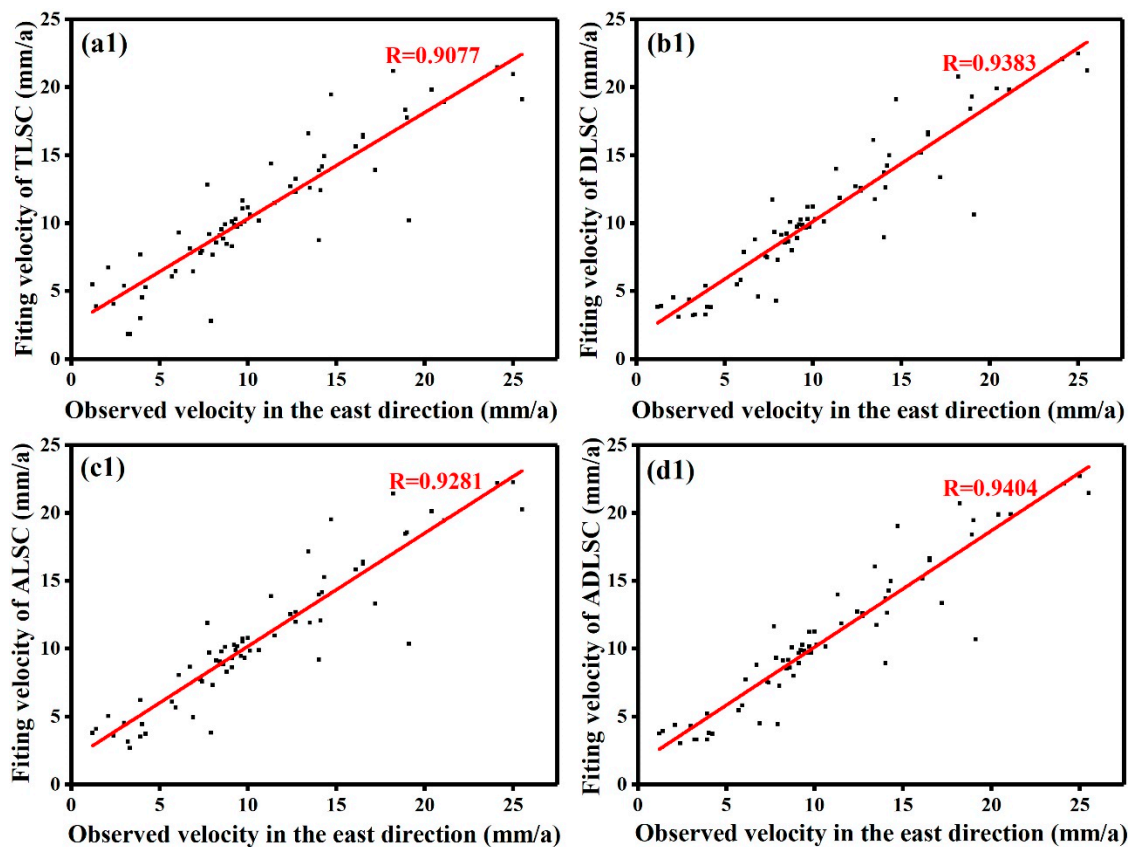


Figure 5. Cont.

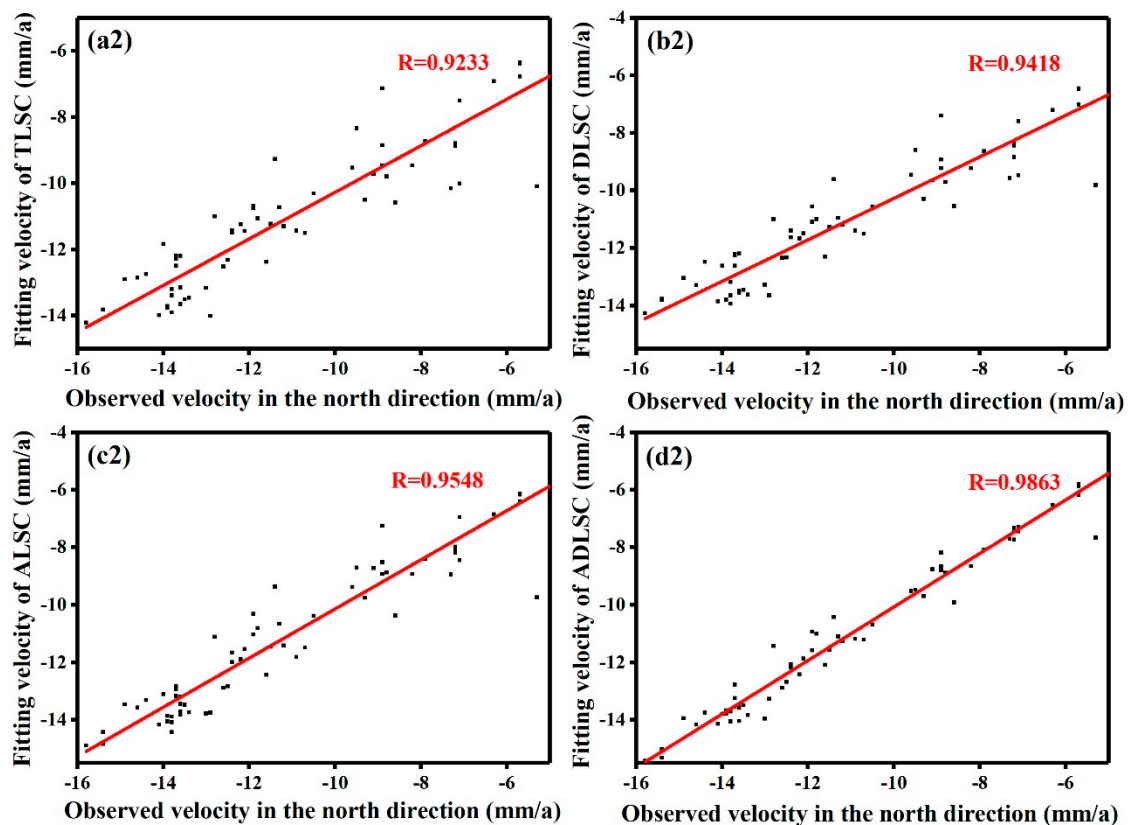


Figure 5. Correlation analysis between the fitting and observed velocity in the east (a1, b1, c1, and d1) and north directions (a2, b2, c2, and d2).

4. Discussion

4.1. Influence of Noise Levels on the New Algorithm

In practical applications, the prior noise level of the observed data is unknown, while the random signal is partially dependent on the prior noise level. Thus, we set three noise levels, $\sigma_0^2 = 0.1$, $\sigma_0^2 = 1$, and $\sigma_0^2 = 10$, to analyze the influence of different noise levels on the fitting and estimation results of the TLSC and ALSC algorithms (Table 2).

Table 2. Influence of different noise levels on fitting and estimation results of the traditional LSC algorithm (TLSC) and ALSC algorithms in the actual case.

Algorithms	Noise Levels (σ_0^2)	RMS (mm/a) (Fitting Points)		RMS (mm/a) (Check Points)	
		(E)	(N)	(E)	(N)
TLSC	0.1	1.82	0.64	2.42	1.04
	1	2.08	0.91	2.73	1.18
	10	2.43	1.32	3.21	1.86
ALSC	0.1	2.15	0.96	2.82	1.24
	1	2.15	0.96	2.82	1.24
	10	2.15	0.96	2.82	1.24

Table 2 shows that the TLSC are affected significantly by the observation noise levels. When higher observation accuracy is gained artificially (small noise level, such as $\sigma_0^2 = 0.1$), most of the residuals will be absorbed as signals. Therefore, the observation corrections will be smaller, resulting in the illusion that the fitting and estimation results have higher accuracy. However, for the ALSC, the fitting

and estimation results are not affected by the noise levels, and the results are more stable. Because the ADLSC algorithm incorporates the ALSC algorithm, it will also not be affected by the noise levels.

4.2. Influence of Randomly Selected Checking Points on Algorithms

In this study we set checking points that accounted for 15% of the total number of observations. In theory, the more checking points involved in the calculation, the more effective the algorithm will prove to be. Nevertheless, although the total number of observations is usually fixed, a good checking result should be based on a well-fitting model. However, a certain sufficient number of fitting points are needed to get a well-fitting model. In general situation, the checking points selected can account for approximately 10% of the total number of observations [25].

Analyzing the selection of different random checking points may lead to different solutions for the four algorithms. We set up two more experiments (Figure 6), and the results are shown in Tables 3 and 4. Comparing Tables 3 and 4 with Table 1, we can see that there are no significant differences in the fitting and estimation results of the four algorithms. The ADLSC always has the smallest RMS, for both the fitting points and the checking points. This further proves the strength of the new algorithm to densify the description of a regional crustal velocity field.

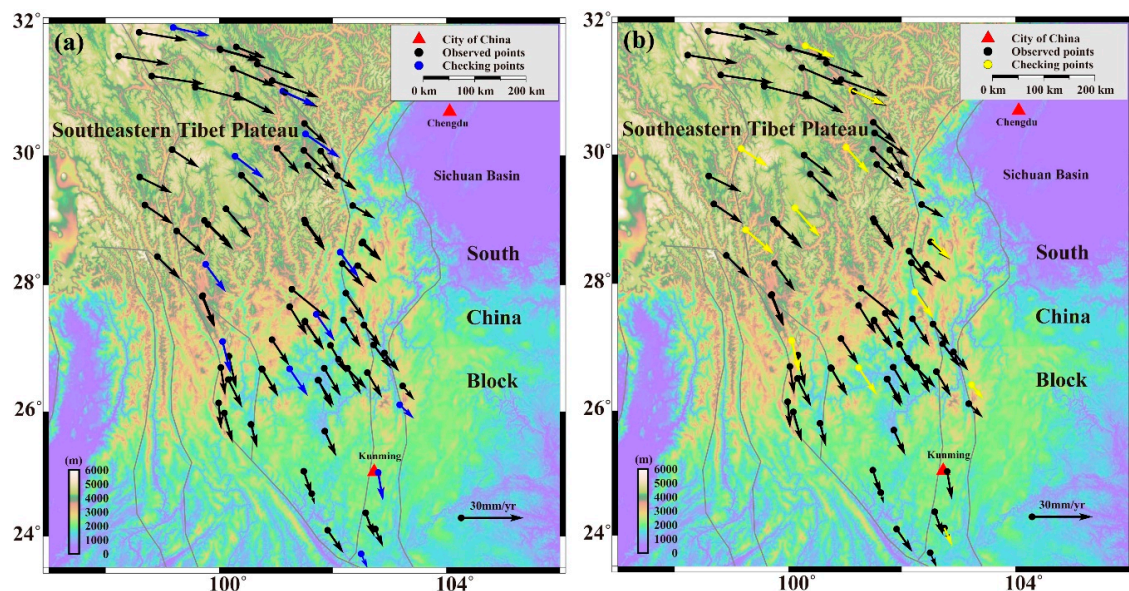


Figure 6. The different and random selection of checking points in (a) experiment 2 and (b) experiment.

Table 3. RMS of the fitting points and checking points using four algorithms (Experiment 2).

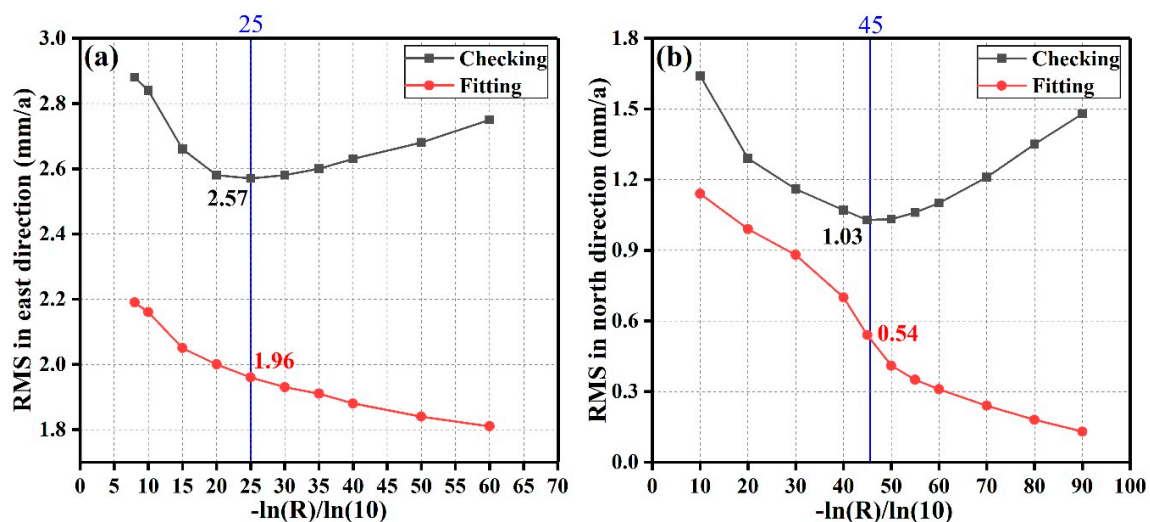
Algorithms	Fitting Points RMS (mm/a)		Checking Points RMS (mm/a)		Running Time (s)
	(E)	(N)	(E)	(N)	
TLSC	2.32	1.35	2.81	2.00	1.29
DLSC	1.98	1.09	2.37 (15.7%)	1.79 (10.5%)	0.81
ALSC	2.12	0.99	2.55 (9.3%)	1.72 (14.0%)	1.42
ADLSC	1.94	0.68	2.34 (16.7%)	1.36 (32.0%)	1.05

Table 4. RMS of the fitting points and checking points using four algorithms (Experiment 3).

Algorithms	Fitting Points RMS (mm/a)		Checking Points RMS (mm/a)		Running Time (s)
	(E)	(N)	(E)	(N)	
TLSC	2.38	1.59	2.84	1.88	1.29
DLSC	1.89	1.42	2.32 (18.3%)	1.56 (17.0%)	0.81
ALSC	2.01	1.29	2.71 (4.6%)	1.35 (28.2%)	1.42
ADLSC	1.87	1.14	2.30 (19.0%)	1.11 (41.0%)	1.05

4.3. Determination of the Optimal Distance Scale Factor of the New Algorithm

In the ADLSC algorithm, a key issue is determining the optimal distance scale factor according to the monitoring data. Based on the principle of $R \ll 1$ (Section 2.2), we constantly try to give different R values and calculate the corresponding RMS of the fitting and checking results. Detailed statistical results are shown in Figure 7, which shows that the fitting RMS decreases as the magnitude of the distance scale factor decreases. This may be caused by a high signal weight due to the decrease of the distance scale factor value, which will lead excessive noise absorption as the signal and the fitting residual are gradually reduced. However, the checking RMS presents a decreasing trend, reaches the lowest value, and then increases again. Therefore, for the ADLSC algorithm, the distance scale factor corresponding to the minimum checking RMS point in each direction is selected as the optimal value. Finally, the optimal scale factor of the eastward distance is 10^{-25} and that of the northward distance is 10^{-45} .

**Figure 7.** Optimal distance scale factor of the ADLSC in the (a) east direction and (b) north direction.

4.4. Advantage of the New Algorithm for Treating Functional Model Error

From the perspective of functional model error, the problem of unreliable covariance function can be attributed to systematic model errors that can be solved using the residual re-fitting method. From the perspective random model error, the unreliability of the covariance function can be partially adjusted by the variance factor, which can re-adjust the priori weight ratio between the signal and the observation vector based on the VCEM.

In this study, the ADLSC algorithm is used to study the unreliable covariance function in the fitting and estimation of GPS crustal horizontal velocity. In practice, it is hard to say whether the unreliable covariance will result in systematic errors or random errors, but both of them should exist. That is to say, the internal correlation of the variance-covariance of signal is inaccurate, or the variance-covariance of observation is inconsistent with the signal. In order to solve the above-mentioned problem, the ADLSC algorithm first introduces the distance scale factor to weaken the influence of systematic errors by

improving the function model, which plays an important role in adjusting the inter-correlation of signals. Secondly, the ADLSC algorithm combines the ALSC algorithm to weaken the influence of random error and adjust the overall coordination relationship between the variance-covariance of the signal and the observation.

We showcase the advantage of the ADLSC in dealing with functional model errors using the signal variance results estimated by the first Helmert VCEM calculation because this depends entirely on the signal covariance function model established earlier. The results are shown in Table 5, Figure 8, and Figure 9.

Table 5. The signal variance estimated results by the first Helmert VCEM calculation of the two algorithms under different noise levels.

Noise Levels (σ_0^2)	E (mm/a)		N (mm/a)	
	ALSC	ADLSC	ALSC	ADLSC
0.1	18.4682	2.3495 (87.3%)	3.6667	2.0740 (43.4%)
1	5.4206	0.8802 (83.8%)	1.2534	1.1009 (12.2%)
10	2.7854	0.8020 (71.2%)	1.0777	0.8921 (17.2%)

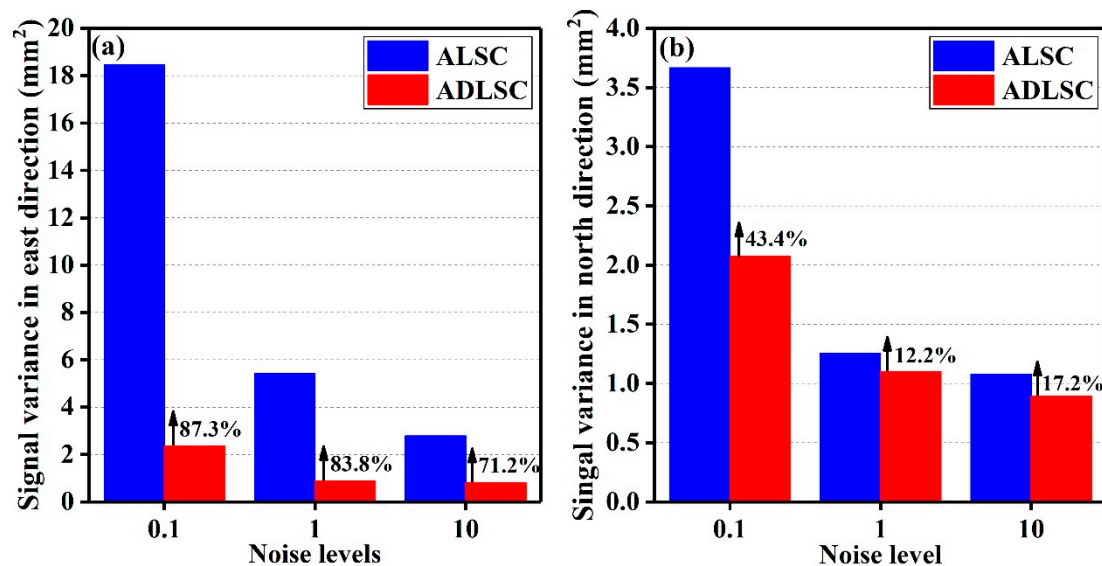


Figure 8. Comparison of the signal variance estimated results by the first Helmert VCEM calculation of the two algorithms under different noise levels: (a) east direction; (b) north direction.

It can be seen from Table 5 and Figure 8 that the estimated signal variance results by the first Helmert VCEM calculation of the ADLSC are all less than that of the ALSC under different noise levels. When the noise levels are $\sigma_0^2 = 0.1$, $\sigma_0^2 = 1$, and $\sigma_0^2 = 10$, the signal variance of the ADLSC are about 87.3%, 83.8%, and 71.2% better than the ALSC in the east direction (Figure 8a), and are about 43.4%, 12.2%, 17.2% better than the ALSC in the north direction, respectively, (Figure 8b). The above results clearly show that the strategy of introducing distance scale factor can significantly reduce the signal variance estimate value and obtain more reliable signal fitting and estimation results.

Figure 9 shows that the improvement of the east and north fitting accuracy is significantly reduced with the increase of iteration times. Because the contribution of the first iteration is the largest in the whole adaptive iteration calculation, the first iteration calculation is directly related to the parameters of the covariance function model. Figure 9 also shows that the distance scale factor can effectively weaken the systematic error of the model (Figure 9a,b), and plays an important role in the subsequent adaptive fitting estimation. It further indicates the rationality of the ADLSC algorithm that considered the unreliable covariance function as a combination of systematic error of the model and the random

error of the model, and is better suited to the treatment of the functional model error. It also should be noted that in this study, we mainly focus on the stochastic signals with isotropic characteristics, even if two directions are considered: east and north. More complex anisotropic situations will be further studied in the future.

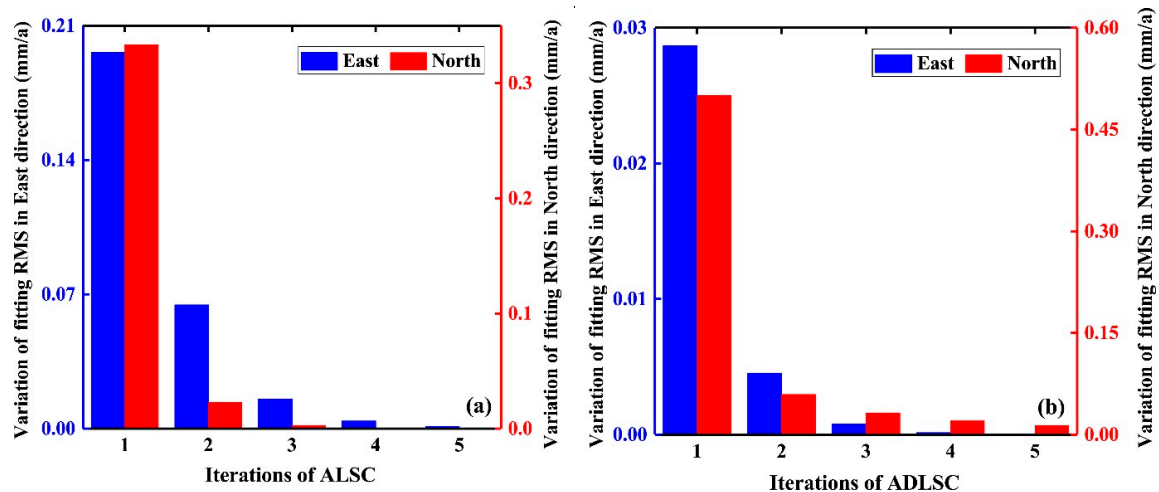


Figure 9. Comparison of the iteration times and fitting RMS in the iterative process of the VCEM calculation: (a) east direction; (b) north direction.

5. Conclusions

In this study, we propose an adaptive least-squares algorithm considering the distance scale factor (ADLSC), and successfully apply it to the fitting and estimation of the GPS crustal horizontal velocity field. The new algorithm considers distance scale factor to establish the Gaussian covariance function, the data utilization rate is improved, and the influence of the negative value of the cross-covariance on the construction of the covariance model is also avoided. Furthermore, the new algorithm introduces adaptive factors to adjust the balance between the observation and prior information during the solving process of the model parameters. In particular, it should be noted that the coordination of the signal covariance matrix and the observed covariance matrix established is further improved after the distance scale factor is taken into account.

The new fusion algorithm can more provide more accurate fitting and estimation of regional GPS crustal movement characteristics and provide basic data of a higher spatial distribution density to solve the crustal strain field. We only studied the GPS crustal velocity field fitting and estimation, but the global navigation satellite system (GNSS) is becoming increasingly diverse due to the technology and algorithms now available. However, we believe that our new algorithm is also applicable to the crustal velocity field measured from other satellite constellations (such as Beidou, GLONASS, or Galileo). This is because the crustal velocity calculated from these satellites can be also used as the basic data for the new algorithm. More importantly, the latter is not limited to using GPS data; it can also use other observation data such as level and gravity data. Our future work will focus on fusing robust estimation and anisotropic covariance functions into the ADLSC algorithm.

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