Supplementary Materials for

Enhanced Impact of Land Reclamation on the Tide in the Guangxi Beibu Gulf

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Supplementary Material includes the following:
Evaluation index system of the driving force factors of the reclamation (**Table S1**); Related index data of Guangxi Beibu Gulf (**Table S2**); And the numerical scheme of forward equations and adjoint equations

Table S1. Evaluation index system of the driving force factors of the reclamation

First-grade indicators	Second-grade indicators	Definition
1.Economic development	1. GDP of primary industry	the end result of agricultural economic activities (including farming, forestry, animal husbandry and fishery).
	2. GDP of secondary industry	the end result of industrial economic activities (including mining and quarrying, manufacturing, electricity, gas and water production and supply).
2.Population growth	3. Population growth rate	to the rate of population growth caused by natural and migration changes over a given time period (usually within a year).
	4. Urbanization degree	the urban population as a percentage of total population.
3.Marine industry development	5. Mariculture production	the output of aquatic products whose young are artificially released or naturally collected, and raised and managed artificially, and which are caught form the waters of mariculture.
	6. Cargo throughput	the weight of the cargo being loaded and unloaded.

Table S2. Related index data of Guangxi Beibu Gulf

year	GDP of primary industry(100 million yuan)	GDP of secondary industry(100 million yuan)	urbanization degree(%)	population growth rate(‰)	mariculture production(1000 tons)	cargo throughput of major port(10000 tons)	coastal reclamation area (km^2)	reclamation area in each year (km^2)	Natural coastline (km)	Artificial coastline (km)	Totel coastline length (km)

1997	53.5	37.3	21.8	5.1	498	938	0	0	765.7	195.1	960.8
1998	67.2	47.8	22.6	4.94	573	1071	1.4	1.4	741.3	225	966.3
1999	59.3	36	24.1	6.21	665	1163	5.4	4	713.4	255.3	968.7
2000	61	39	28.1	6.3	706	1288	5.6	0.2	692	279.4	971.4
2001	105.7	63.5	28.2	7.05	751	1442	9.7	4.1	655.8	320	975.8
2002	109	73.7	28.3	7.2	801	2049	11.1	1.4	653.5	334.3	987.8
2003	128	104	29.1	7.29	799	2016	13.6	2.5	636.7	351.9	988.6
2004	136.7	128.5	31.7	7.7	858	2400	13.4	0	614.2	390	1004.2
2005	149.1	157.3	33.6	8.16	929	2678	15.1	1.7	583	414.7	997.7
2006	160	220.3	34.6	8.32	801	4949	24.4	9.3	580.6	427.1	1007.7
2007	189.3	280.9	36.2	8.2	763	7192	32.6	8.2	555.1	434.8	989.9
2008	215.7	391.2	38.1	8.7	871	9408	36.5	3.9	523.1	461.8	984.9
2009	231	384.7	39.2	8.65	877	11923	56.6	20.1	509.9	483.7	993.6
2010	266.8	546.2	38.1	7.9	723	15331	69.2	12.6	507.4	550.6	1058
2011	329.2	715.7	39.8	7.93	977	17438	72.6	3.4	499.3	553.8	1053.1
2012	355.3	826.5	41.9	8.02	1056	18675	75.3	2.7	474.2	571.2	1045.4
2013	393	986.7	44.1	8.2	1090	20189	80.7	5.4	467.5	596.9	1064.4
2014	416	1133.8	45.1	8.37	1189	20482	83.8	3.1	467.3	610.1	1077.4
2015	448.8	1230	45.0	8.15	1185	22031	88	4.2	465.9	617.6	1083.5
2016	439.3	1384	45.9	7.87	1214	20392	94.5	6.5	462.4	631	1093.4
2017	514.7	1714.9	47.2	8.9	1308	23554	98.6	4.1	461.1	638.4	1099.5

Numerical scheme of forward equations (S1)

$$\frac{\zeta_{m,n}^{2j+1}-\zeta_{m,n}^{2j}}{\Delta t}+\frac{(h_{m+\frac{1}{2},n}+\zeta_{m+\frac{1}{2},n}^{2j})u_{m,n}^{2j}-(h_{m-\frac{1}{2},n}+\zeta_{m-\frac{1}{2},n}^{2j})u_{m-1,n}^{2j}}{a_n\Delta\lambda}$$

$$+\frac{(h_{m,n+\frac{1}{2}}+\zeta_{m,n+\frac{1}{2}}^{2j})v_{m,n}^{2j}\cos(\varphi_{n+\frac{1}{2}})-(h_{m,n-\frac{1}{2}}+\zeta_{m,n-\frac{1}{2}}^{2j})v_{m,n-1}^{2j}\cos(\varphi_{n-\frac{1}{2}})}{a_n\Delta\varphi}=0, \qquad (S1.1)$$

$$\frac{v_{m,n}^{2j+1}-v_{m,n}^{2j}}{\Delta t}+\frac{\overline{u}_{m,n}^{2j}(v_{m+1,n}^{2j}-v_{m-1,n}^{2j})}{2a_n\Delta\lambda}+\frac{v_{m,n}^{2j}(v_{m,n+1}^{2j}-v_{m,n-1}^{2j})}{2R\Delta\varphi}+f_n\overline{u}_{m,n}^{2j}$$

$$+\frac{k_{m,n+\frac{1}{2}}s_{m,n}^{2j}[\alpha v_{m,n}^{2j+1}+(1-\alpha)v_{m,n}^{2j}]}{h_{m,n+\frac{1}{2}}+\zeta_{m,n+\frac{1}{2}}^{2j}}+\frac{g(\zeta_{m,n+1}^{2j+1}-\zeta_{m,n}^{2j+1})}{R\Delta\varphi}-\frac{g(\overline{\zeta}_{m,n+1}^{2j+1}-\overline{\zeta}_{m,n}^{2j+1})}{R\Delta\varphi} \qquad (S1.2)$$

$$-A\Delta v_{m,n}^{2j}+\frac{\overline{u}_{m,n}^{2j^2}\tan(\varphi_n)}{R}=0,$$

$$\frac{u_{m,n}^{2j+1}-u_{m,n}^{2j}}{\Delta t}+\frac{u_{m,n}^{2j}(u_{m+1,n}^{2j}-u_{m-1,n}^{2j})}{2a_n\Delta\lambda}+\frac{\overline{v}_{m,n}^{2j+1}(u_{m,n+1}^{2j}-u_{m,n-1}^{2j})}{2R\Delta\varphi}-f_n\overline{v}_{m,n}^{2j+1}$$

$$+\frac{k_{m+\frac{1}{2},n}r_{m,n}^{2j}[\alpha u_{m,n}^{2j+1}+(1-\alpha)u_{m,n}^{2j}]}{h_{m+\frac{1}{2},n}+\zeta_{m+\frac{1}{2},n}^{2j}}+\frac{g(\zeta_{m+1,n}^{2j+1}-\zeta_{m,n}^{2j+1})}{a_n\Delta\lambda}-\frac{g(\overline{\zeta}_{m+1,n}^{2j+1}-\overline{\zeta}_{m,n}^{2j+1})}{a_n\Delta\lambda} \qquad (S1.3)$$

$$-A\Delta u_{m,n}^{2j}-\frac{u_{m,n}^{2j}\overline{v}_{m,n}^{2j+1}\tan(\varphi_n)}{R}=0,$$

$$\frac{\zeta_{m,n}^{2j+2}-\zeta_{m,n}^{2j+1}}{\Delta t}+\frac{(h_{m+\frac{1}{2},n}+\zeta_{m+\frac{1}{2},n}^{2j+1})u_{m,n}^{2j+1}-(h_{m-\frac{1}{2},n}+\zeta_{m-\frac{1}{2},n}^{2j+1})u_{m-1,n}^{2j+1}}{a_n\Delta\lambda}$$

$$+\frac{(h_{m,n+\frac{1}{2}}+\zeta_{m,n+\frac{1}{2}}^{2j+1})v_{m,n}^{2j+1}\cos(\varphi_{n+\frac{1}{2}})-(h_{m,n-\frac{1}{2}}+\zeta_{m,n-\frac{1}{2}}^{2j+1})v_{m,n-1}^{2j+1}\cos(\varphi_{n-\frac{1}{2}})}{R\Delta\varphi}=0, \qquad (S1.4)$$

$$\frac{u_{m,n}^{2j+2}-u_{m,n}^{2j+1}}{\Delta t}+\frac{u_{m,n}^{2j+1}(u_{m+1,n}^{2j+1}-u_{m-1,n}^{2j+1})}{2a_n\Delta\lambda}+\frac{\overline{v}_{m,n}^{2j+1}(u_{m,n+1}^{2j+1}-u_{m,n-1}^{2j+1})}{2R\Delta\varphi}-f_n\overline{v}_{m,n}^{2j+1}$$

$$+\frac{k_{m+\frac{1}{2},n}r_{m,n}^{2j+1}[\alpha u_{m,n}^{2j+2}+(1-\alpha)u_{m,n}^{2j+1}]}{h_{m+\frac{1}{2},n}+\zeta_{m+\frac{1}{2},n}^{2j+1}}+\frac{g(\zeta_{m+1,n}^{2j+2}-\zeta_{m,n}^{2j+2})}{a_n\Delta\lambda}-\frac{g(\overline{\zeta}_{m+1,n}^{2j+2}-\overline{\zeta}_{m,n}^{2j+2})}{a_n\Delta\lambda} \qquad (S1.5)$$

$$-A\Delta u_{m,n}^{2j+1}-\frac{u_{m,n}^{2j+1}\overline{v}_{m,n}^{2j+1}\tan(\varphi_n)}{R}=0.$$

$$\frac{v_{m,n}^{2j+2}-v_{m,n}^{2j+1}}{\Delta t}+\frac{\overline{u}_{m,n}^{2j+2}(v_{m+1,n}^{2j+1}-v_{m-1,n}^{2j+1})}{2a_n\Delta\lambda}+\frac{v_{m,n}^{2j+1}(v_{m,n+1}^{2j+1}-v_{m,n-1}^{2j+1})}{2R\Delta\varphi}+f_n\overline{u}_{m,n}^{2j+2}$$
$$+\frac{k_{m,n+\frac{1}{2}}s_{m,n}^{2j+1}[\alpha v_{m,n}^{2j+2}+(1-\alpha)v_{m,n}^{2j+1}]}{h_{m,n+\frac{1}{2}}+\zeta_{m,n+\frac{1}{2}}^{2j+1}}+\frac{g(\zeta_{m,n+1}^{2j+2}-\zeta_{m,n}^{2j+2})}{R\Delta\varphi}-\frac{g(\overline{\zeta}_{m,n+1}^{2j+2}-\overline{\zeta}_{m,n}^{2j+2})}{R\Delta\varphi} \qquad (S1.6)$$
$$-A\Delta v_{m,n}^{2j+1}+\frac{\overline{u}_{m,n}^{2j+2^2}\tan(\varphi_n)}{R}=0.$$

where $\Delta\lambda,\Delta\phi,\Delta t$ represent the longitude, the latitude and the time step respectively, $\phi_n=\phi_0+(n-1)\Delta\phi$, ϕ_0 is the initial latitude, $a_n=R\cos(\phi_n)$, $\overline{u}_{m,n}^{j}$, $r_{m,n}^{j}$, $\overline{v}_{m,n}^{j}$, $s_{m,n}^{j}$ are the same with Lardner (1993a).

Adjoint equations: In order to construct the adjoint equations, the cost function is defined as:

$$J(\zeta)=\frac{1}{2}K_\zeta\int_\Sigma(\zeta-\hat{\zeta})^2d\sigma,$$

and the Lagrangian function is defined as

$$L=\int\Bigg[\mu\left(\frac{\partial u}{\partial t}+\frac{u}{a}\frac{\partial u}{\partial\lambda}+\frac{v}{R}\frac{\partial u}{\partial\phi}-\frac{uv\tan\phi}{R}-fv+\frac{ku\sqrt{u^2+v^2}}{h+\zeta}-A\Delta u+\frac{g}{a}\frac{\partial(\zeta-\overline{\zeta})}{\partial\lambda}\right)+$$
$$\nu\left(\frac{\partial v}{\partial t}+\frac{u}{a}\frac{\partial v}{\partial\lambda}+\frac{v}{R}\frac{\partial v}{\partial\phi}+\frac{u^2\tan\phi}{R}+fu+\frac{kv\sqrt{u^2+v^2}}{h+\zeta}-A\Delta v+\frac{g}{R}\frac{\partial(\zeta-\overline{\zeta})}{\partial\phi}\right)+$$
$$\tau\left(\frac{\partial\zeta}{\partial t}+\frac{1}{a}\frac{\partial[(h+\zeta)u]}{\partial\lambda}+\frac{1}{a}\frac{\partial[(h+\zeta)v\cos\phi]}{\partial\phi}\right)+\frac{1}{2}W(\xi-\hat{\xi})^2\Bigg]dS+J(\zeta),$$

Similar with the method of He et al. (2004), the adjoint equations can be obtained as

$$
\frac{\partial \tau}{\partial t}+\frac{u}{a}\frac{\partial \tau}{\partial \lambda}+\frac{v}{a}\frac{\partial(\tau\cos\varphi)}{\partial \varphi}+\frac{k\mu u\sqrt{u^2+v^2}}{(h+\zeta)^2}+\frac{k\nu v\sqrt{u^2+v^2}}{(h+\zeta)^2}+\frac{g}{a}\frac{\partial \mu}{\partial \lambda}+\frac{g}{a}\frac{\partial \nu}{\partial \varphi}=K_\zeta(\zeta-\hat{\zeta}),
$$

$$
\begin{aligned}
&\frac{\partial \mu}{\partial t}-(f+\frac{kuv}{(h+\zeta)\sqrt{u^2+v^2}})\nu-\frac{\mu}{a}\frac{\partial u}{\partial \lambda}-\frac{\nu}{a}\frac{\partial v}{\partial \lambda}+\frac{1}{a}\frac{\partial}{\partial \lambda}(\mu u)+\frac{1}{R}\frac{\partial}{\partial \varphi}(\mu v)\\
&\qquad+\frac{h+\zeta}{a}\frac{\partial \tau}{\partial \lambda}+A\Delta\mu-\frac{k(2u^2+v^2)}{(h+\zeta)\sqrt{u^2+v^2}}\mu+\frac{\mu v\tan\varphi}{R}-\frac{2\nu u\tan\varphi}{R}=0, \qquad \text{(S2)}\\
&\frac{\partial \nu}{\partial t}+(f-\frac{kuv}{(h+\zeta)\sqrt{u^2+v^2}})\mu-\frac{\mu}{R}\frac{\partial u}{\partial \varphi}-\frac{\nu}{R}\frac{\partial v}{\partial \varphi}+\frac{1}{a}\frac{\partial}{\partial \lambda}(\nu u)+\frac{1}{R}\frac{\partial}{\partial \varphi}(\nu v)\\
&\qquad+\frac{h+\zeta}{a}\frac{\partial(\tau\cos\varphi)}{\partial \varphi}+A\Delta\nu-\frac{k(u^2+2v^2)}{(h+\zeta)\sqrt{u^2+v^2}}\nu+\frac{\mu u\tan\varphi}{R}=0.
\end{aligned}
$$

where τ,μ,ν denote the adjoint variables of ζ,u,v, respectively. For the adjoint equations (S2), the numerical scheme is given as S2.

Numerical scheme of adjoint equations (S2)

$$
\begin{aligned}
&\frac{\tau_{m,n}^{2j+2}-\tau_{m,n}^{2j+1}}{\Delta t}+\frac{(u_{m,n}^{2j+1}+u_{m-1,n}^{2j+1})(\tau_{m+1,n}^{2j+2}-\tau_{m-1,n}^{2j+2})}{2a_n\Delta\lambda}\\
&\quad+\frac{(v_{m,n}^{2j+1}+v_{m,n-1}^{2j+1})[\tau_{m,n+1}^{2j+2}\cos(\varphi_{n+\frac{1}{2}})-\tau_{m,n-1}^{2j+2}\cos(\varphi_{n-\frac{1}{2}})]}{2a_n\Delta\varphi}\\
&\quad+\frac{k_{m+\frac{1}{2},n}\mu_{m,n}^{2j+2}u_{m,n}^{2j+1}\sqrt{u_{m,n}^{2j+12}+\overline{v}_{m,n}^{2j+12}}}{(h_{m+\frac{1}{2},n}+\zeta_{m+\frac{1}{2},n}^{2j+1})^2}+\frac{k_{m,n+\frac{1}{2}}\nu_{m,n}^{2j+2}v_{m,n}^{2j+1}\sqrt{\overline{u}_{m,n}^{2j+12}+v_{m,n}^{2j+12}}}{(h_{m,n+\frac{1}{2}}+\zeta_{m,n+\frac{1}{2}}^{2j+1})^2} \qquad (S2.1)\\
&\quad+\frac{g(\mu_{m,n}^{2j+2}-\mu_{m-1,n}^{2j+2})}{a_n\Delta\lambda}+\frac{g(\nu_{m,n}^{2j+2}-\nu_{m,n-1}^{2j+2})}{a_n\Delta\varphi}-K_\zeta D_{m,n}(\zeta_{m,n}^{2j+1}-\hat{\zeta}_{m,n}^{2j+1})=0,
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\nu_{m,n}^{2j+2}-\nu_{m,n}^{2j+1}}{\Delta t}+(f_n-c_{m,n}^{2j+1})\overline{\mu}_{m,n}^{2j+2}-e_{m,n}^{2j+1}[(1-\alpha)\nu_{m,n}^{2j+1}+\alpha\nu_{m,n}^{2j+2}]\\
&\quad+\frac{(h_{m,n-\frac{1}{2}}+\zeta_{m,n-\frac{1}{2}}^{2j+1})[\tau_{m,n}^{2j+1}\cos(\varphi_n)-\tau_{m,n-1}^{2j+1}\cos(\varphi_{n-1})]}{a_n\Delta\varphi}\\
&\quad-\frac{\mu_{m,n}^{2j+2}(u_{m,n+1}^{2j+1}-u_{m,n-1}^{2j+1})}{2R\Delta\varphi}-\frac{\nu_{m,n}^{2j+2}(v_{m,n+1}^{2j+1}-v_{m,n-1}^{2j+1})}{2R\Delta\varphi} \qquad (S2.2)\\
&\quad+\frac{\nu_{m+1,n}^{2j+2}u_{m+1,n}^{2j+1}-\nu_{m-1,n}^{2j+2}u_{m-1,n}^{2j+1}}{2a_n\Delta\lambda}+\frac{\nu_{m,n+1}^{2j+2}v_{m,n+1}^{2j+1}-\nu_{m,n-1}^{2j+2}v_{m,n-1}^{2j+1}}{2R\Delta\varphi}\\
&\quad+A\Delta\nu_{m,n}^{2j+2}+\frac{u_{m,n}^{2j+1}\mu_{m,n}^{2j+2}\tan(\varphi_n)}{R}=0,
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\mu_{m,n}^{2j+2}-\mu_{m,n}^{2j+1}}{\Delta t}-(f_n+b_{m,n}^{2j+1})\bar{v}_{m,n}^{2j+1}-d_{m,n}^{2j+1}[(1-\alpha)\mu_{m,n}^{2j+1}+\alpha\mu_{m,n}^{2j+2}]\\
&\quad+\frac{(h_{m-\frac{1}{2},n}+\zeta_{m-\frac{1}{2},n}^{2j+1})(\tau_{m,n}^{2j+1}-\tau_{m-1,n}^{2j+1})}{a_n\Delta\lambda}\\
&\quad-\frac{\mu_{m,n}^{2j+2}(u_{m+1,n}^{2j+1}-u_{m-1,n}^{2j+1})}{2a_n\Delta\lambda}-\frac{v_{m,n}^{2j+1}(v_{m+1,n}^{2j+1}-v_{m-1,n}^{2j+1})}{2a_n\Delta\lambda}\\
&\quad+\frac{\mu_{m+1,n}^{2j+2}u_{m+1,n}^{2j+1}-\mu_{m-1,n}^{2j+2}u_{m-1,n}^{2j+1}}{2a_n\Delta\lambda}+\frac{\mu_{m,n+1}^{2j+2}v_{m,n+1}^{2j+1}-\mu_{m,n-1}^{2j+2}v_{m,n-1}^{2j+1}}{2R\Delta\varphi}\\
&\quad+A\Delta\mu_{m,n}^{2j+2}+\frac{\mu_{m,n}^{2j+2}v_{m,n}^{2j+1}\tan(\varphi_n)-2v_{m,n}^{2j+1}u_{m,n}^{2j+1}\tan(\varphi_n)}{R}=0,
\end{aligned}
\tag{S2.3}
$$

$$
\begin{aligned}
&\frac{\tau_{m,n}^{2j+1}-\tau_{m,n}^{2j}}{\Delta t}+\frac{(u_{m,n}^{2j}+u_{m-1,n}^{2j})(\tau_{m,n}^{2j+1}-\tau_{m-1,n}^{2j+1})}{2a_n\Delta\lambda}\\
&\quad+\frac{(v_{m,n}^{2j}+v_{m,n-1}^{2j})[\tau_{m,n+1}^{2j+1}\cos(\varphi_{n+\frac{1}{2}})-\tau_{m,n-1}^{2j+1}\cos(\varphi_{n-\frac{1}{2}})]}{2a_n\Delta\varphi}\\
&\quad+\frac{k_{m+\frac{1}{2},n}\mu_{m,n}^{2j+1}u_{m,n}^{2j}\sqrt{u_{m,n}^{2j\,2}+\bar{v}_{m,n}^{2j\,2}}}{(h_{m+\frac{1}{2},n}+\zeta_{m+\frac{1}{2},n}^{2j})^2}+\frac{k_{m,n+\frac{1}{2}}v_{m,n}^{2j+1}v_{m,n}^{2j}\sqrt{\bar{u}_{m,n}^{2j\,2}+v_{m,n}^{2j\,2}}}{(h_{m,n+\frac{1}{2}}+\zeta_{m,n+\frac{1}{2}}^{2j})^2}\\
&\quad+\frac{g(\mu_{m,n}^{2j+1}-\mu_{m-1,n}^{2j+1})}{a_n\Delta\lambda}+\frac{g(v_{m,n}^{2j+1}-v_{m,n-1}^{2j+1})}{a_n\Delta\varphi}-K_\zeta D_{m,n}(\zeta_{m,n}^{2j}-\hat{\zeta}_{m,n}^{2j})=0,
\end{aligned}
\tag{S2.4}
$$

$$
\begin{aligned}
&\frac{\mu_{m,n}^{2j+1}-\mu_{m,n}^{2j}}{\Delta t}-(f_n+b_{m,n}^{2j})\bar{v}_{m,n}^{2j+1}-d_{m,n}^{2j}[(1-\alpha)\mu_{m,n}^{2j}+\alpha\mu_{m,n}^{2j+1}]\\
&\quad+\frac{(h_{m-\frac{1}{2},n}+\zeta_{m-\frac{1}{2},n}^{2j})(\tau_{m,n}^{2j}-\tau_{m-1,n}^{2j})}{a_n\Delta\lambda}\\
&\quad-\frac{\mu_{m,n}^{2j+1}(u_{m+1,n}^{2j}-u_{m-1,n}^{2j})}{2a_n\Delta\lambda}-\frac{v_{m,n}^{2j+1}(v_{m+1,n}^{2j}-v_{m-1,n}^{2j})}{2a_n\Delta\lambda}\\
&\quad+\frac{\mu_{m+1,n}^{2j+1}u_{m+1,n}^{2j}-\mu_{m-1,n}^{2j+1}u_{m-1,n}^{2j}}{2a_n\Delta\lambda}+\frac{v_{m,n+1}^{2j+1}v_{m,n+1}^{2j}-v_{m,n-1}^{2j+1}v_{m,n-1}^{2j}}{2R\Delta\varphi}\\
&\quad+A\Delta\mu_{m,n}^{2j+1}+\frac{\mu_{m,n}^{2j+1}v_{m,n}^{2j}\tan(\varphi_n)-2v_{m,n}^{2j+1}u_{m,n}^{2j}\tan(\varphi_n)}{R}=0,
\end{aligned}
\tag{S2.5}
$$

$$
\begin{aligned}
&\frac{v_{m,n}^{2j+1}-v_{m,n}^{2j}}{\Delta t}+(f_n-c_{m,n}^{2j})\overline{\mu}_{m,n}^{2j}-e_{m,n}^{2j}[(1-\alpha)v_{m,n}^{2j}+\alpha v_{m,n}^{2j+1}] \\
&\quad+\frac{(h_{m,n-\frac{1}{2}}+\zeta_{m,n-\frac{1}{2}}^{2j})[\tau_{m,n}^{2j}\cos(\varphi_n)-\tau_{m,n-1}^{2j}\cos(\varphi_{n-1})]}{a_n\Delta\varphi} \\
&\quad-\frac{\mu_{m,n}^{2j}(u_{m,n+1}^{2j}-u_{m,n-1}^{2j})}{2R\Delta\varphi}-\frac{v_{m,n}^{2j+1}(v_{m,n+1}^{2j}-v_{m,n-1}^{2j})}{2R\Delta\varphi} \\
&\quad+\frac{v_{m+1,n}^{2j+1}u_{m+1,n}^{2j}-v_{m-1,n}^{2j+1}u_{m-1,n}^{2j}}{2a_n\Delta\lambda}+\frac{v_{m,n+1}^{2j+1}v_{m,n+1}^{2j}-v_{m,n-1}^{2j+1}v_{m,n-1}^{2j}}{2R\Delta\varphi} \\
&\quad+A\Delta v_{m,n}^{2j+1}+\frac{u_{m,n}^{2j}\mu_{m,n}^{2j}\tan(\varphi_n)}{R}=0.
\end{aligned}
\qquad (S2.6)
$$

where $\overline{\mu}_{m,n}^{j},\overline{v}_{m,n}^{j}$ are similar to $\overline{u}_{m,n}^{j},\overline{v}_{m,n}^{j}$, and

$$
b_{m,n}^{j}=\frac{k_{m+\frac{1}{2},n}}{h_{m+\frac{1}{2},n}+\zeta_{m+\frac{1}{2},n}^{j}}\frac{u_{m,n}^{j}\overline{v}_{m,n}^{j}}{\sqrt{{u_{m,n}^{j}}^2+{\overline{v}_{m,n}^{j}}^2}},\quad c_{m,n}^{j}=\frac{k_{m,n+\frac{1}{2}}}{h_{m,n+\frac{1}{2}}+\zeta_{m,n+\frac{1}{2}}^{j}}\frac{\overline{u}_{m,n}^{j}v_{m,n}^{j}}{\sqrt{{\overline{u}_{m,n}^{j}}^2+{v_{m,n}^{j}}^2}}
$$