

Supplementary material

Mapping forest types

The forest was classified into two forest types from aerial photographs: predominantly sugar maple or a mixture including a significant coniferous component. Fig. S.1 presents a map of the spatial distribution of these two forest types.



Fig. S.1 Forest type map of a portion of Haliburton Forest and Wildlife Reserve. The green regions are classified as predominantly sugar maple whilst the red regions are classified as a mixture of conifers and broadleaves. The blue regions are not forested and include lakes and major roads.

Parameterising the multivariate normal prior

The model was constrained using two multivariate normal priors, the parameters of which were obtained from the calibration plots. The parameter values are given in Table S.1.

Table S.1 The multivariate normal distributions (MVN) used as model priors require a vector of the means (μ) and a covariance matrix (Σ) of the variables extracted from the 114 calibration plots. The table presents general notation (G) as well as individual parameters for each forest type (Mh = sugar maple, Mix = mixture). The variables in the SDD parameter MVN (P_S) are: $\ln k$ = logged Weibull shape, λ = Weibull scale, $\varepsilon_N = N - \hat{N}$ = stem number residuals from equation (5), p_B = proportion of broadleaves. The variables in the stand metric MVN (P_M) are: BA_C/BA_B = basal area of conifers/broadleaves, QMD_C/QMD_B = quadratic mean diameter of conifers/broadleaves and TCH = top canopy height.

MVN	Means vector (μ)	Covariance matrix (Σ)
<i>Parameters (P_S)</i>		
G:	$\{\ln k \quad \lambda \quad \varepsilon_N \quad p_B\}$	$\begin{bmatrix} \text{var}(\ln k) & \text{cov}(\ln k, \lambda) & \dots & \text{cov}(\ln k, p_B) \\ \text{cov}(\lambda, \ln k) & \text{var}(\lambda) & \dots & \text{cov}(\lambda, p_B) \\ \vdots & \vdots & \ddots & \vdots \\ \text{cov}(p_B, \ln k) & \text{cov}(p_B, \lambda) & \dots & \text{var}(p_B) \end{bmatrix}$
Mh:	$\{0.03 \quad 12.4 \quad 0.00 \quad 0.89\}$	$\begin{bmatrix} 0.08 & 1.65 & -1.27 & 0.00 \\ 1.65 & 41.7 & -61.7 & -0.19 \\ -1.27 & -61.7 & 888 & -0.25 \\ 0.00 & -0.19 & -0.25 & 0.02 \end{bmatrix}$
Mix:	$\{-0.03 \quad 10.8 \quad 0.00 \quad 0.52\}$	$\begin{bmatrix} 0.09 & 1.55 & -0.02 & -0.01 \\ 1.55 & 38.7 & -85.2 & -0.01 \\ -0.02 & -85.2 & 3176 & -3.14 \\ -0.01 & -0.01 & -3.14 & 0.04 \end{bmatrix}$
<i>Stand metrics (P_M)</i>		
G:	$\{BA_C \quad BA_B \quad QMD_C \quad QMD_B \quad TCH\}$	$\begin{bmatrix} \text{var}(BA_C) & \text{cov}(BA_C, BA_B) & \dots & \text{cov}(BA_C, TCH) \\ \text{cov}(BA_B, BA_C) & \text{var}(BA_B) & \dots & \text{cov}(BA_B, TCH) \\ \vdots & \vdots & \ddots & \vdots \\ \text{cov}(TCH, BA_C) & \text{cov}(TCH, BA_B) & \dots & \text{var}(TCH) \end{bmatrix}$
Mh:	$\{3.26 \quad 18.5 \quad 18.9 \quad 22.1 \quad 14.7\}$	$\begin{bmatrix} 30.1 & -11.9 & 39.4 & 1.78 & 3.06 \\ -11.9 & 21.4 & -19.9 & 8.53 & 9.02 \\ 39.4 & -19.9 & 215 & -4.37 & 1.23 \\ 1.78 & 8.53 & -4.37 & 14.6 & 8.14 \\ 3.06 & 9.02 & 1.23 & 8.14 & 10.0 \end{bmatrix}$
Mix:	$\{14.5 \quad 13.7 \quad 23.7 \quad 21.7 \quad 13.6\}$	$\begin{bmatrix} 62.1 & -22.5 & 38.9 & 5.64 & 6.11 \\ -22.5 & 23.6 & -10.1 & 2.46 & 3.34 \\ 38.9 & -10.1 & 102 & 9.08 & 16.8 \\ 5.64 & 2.46 & 9.08 & 11.3 & 3.09 \\ 6.11 & 3.34 & 16.8 & 3.09 & 5.64 \end{bmatrix}$

Fig. S.2 presents an example of a joint relationship that would be accounted for in the multivariate prior P_S .

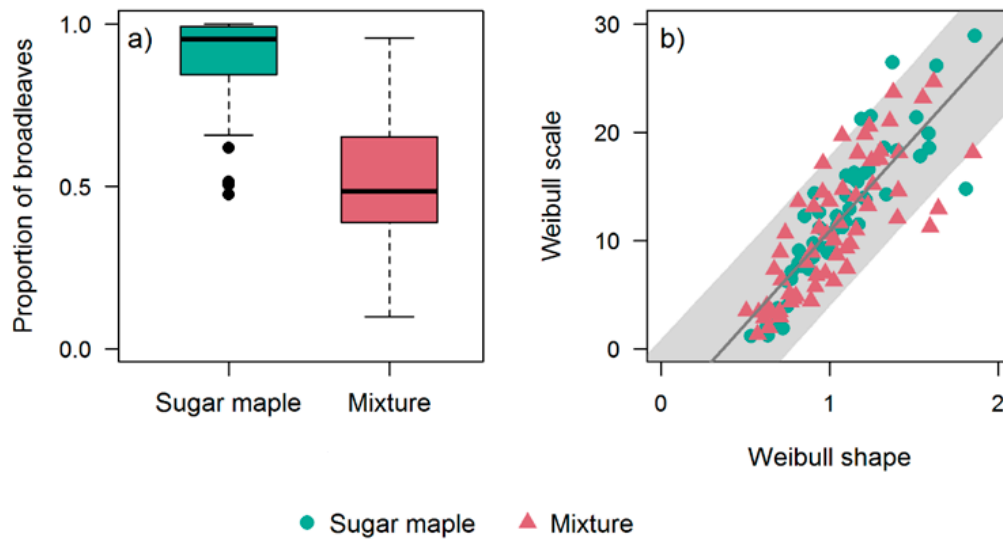


Fig. S.2 Example of joint relationships that are captured in the model priors. In a), the proportion of the total stems that were recorded as broadleaves in the calibration plots have been grouped according to the forest type classification. In b), the joint relationship between the Weibull scale and shape parameters (there was no significant difference between the two forest types) obtained by fitting a truncated Weibull distribution to the SDDs of the 114 calibration plots. The shaded area denotes the 95% prediction interval for the joint relationship.

Summarising stem diameter distributions by plot basal area

We can visualise consistent differences or errors in model predictions by grouping stem diameter distributions according to plot basal area (Fig. S.3). Although the typical SDD for each basal area quartile range was captured well by the mean predictions, small stems were consistently over-predicted in the lower basal area plots and under-predicted in the highest basal area plots. The SDDs of plots with low (0–25th percentiles) and somewhat above-average basal area (50–75th percentiles) were the most consistently well-predicted (Fig. S.3a,c).

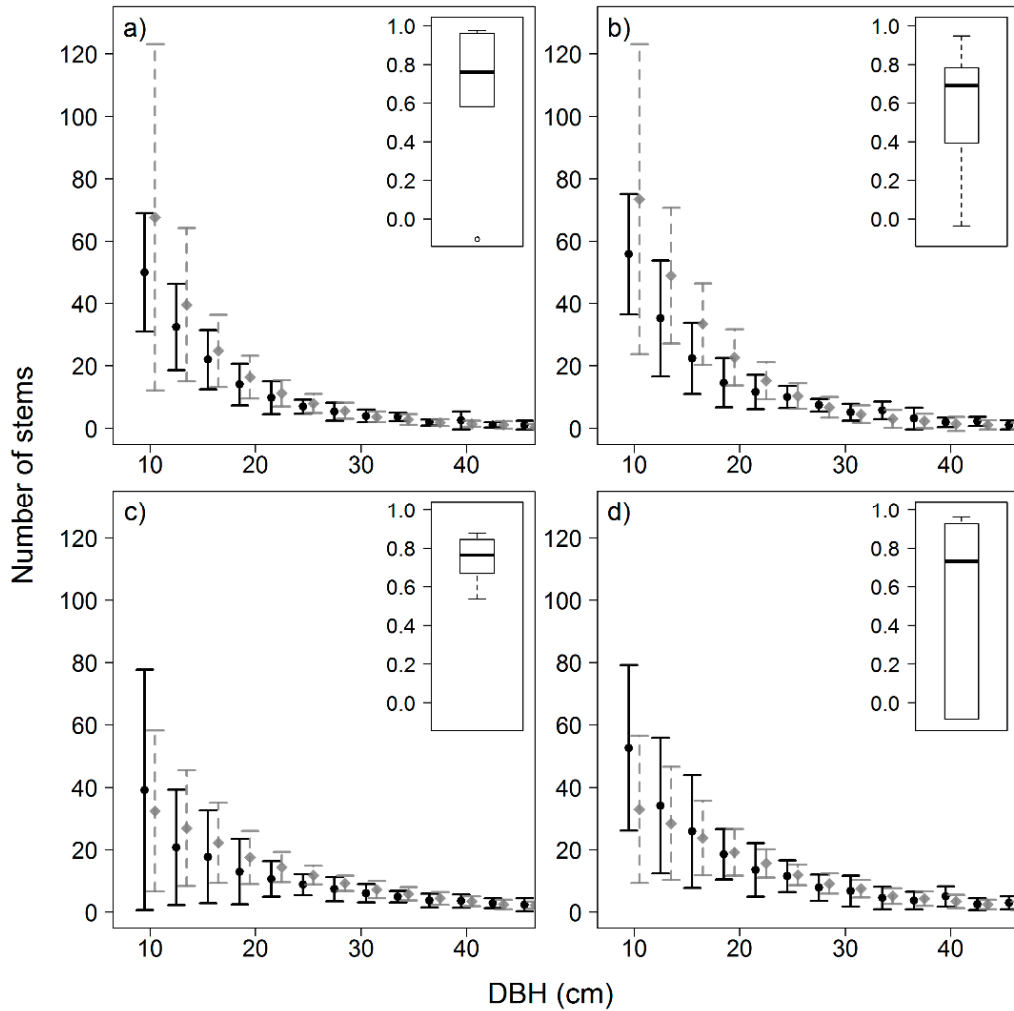


Fig. S.3 SDD predictions grouped according to plot basal area. Points and error bars (black = observed; grey = predicted) represent the mean number of stems within 3 cm diameter intervals, along with their associated standard deviation (grey points have been shifted to the right slightly for clarity). Each panel summarises SDDs for a different quartile range of plot basal areas (a) lowest 25% to d) highest 25%). Boxplots show the distribution of R^2 values for the plots included in each panel.