## **Supplementary Materials**

To theoretically calculate the NA (numerical aperture) of  $\mu$ OIL chip, the maximum half angle of ( $\theta$ ) the cone of rays collected by the lens, the geometry and material properties of the 2 media and the ball mini-lens are needed.

The ball mini-lens of the  $\mu$ OIL chip can be modelled as a thick lens. According to the thick-lens equations [1–3]:

$$\Phi_{\rm IS} = \frac{n_{\rm air} - n_{\rm L}}{R_{\rm l}} \tag{S1}$$

$$\Phi_{\rm OS} = \frac{n_{\rm L} - n_{\rm M}}{R_2} \tag{S2}$$

$$\Phi = \Phi_{\rm IS} + \Phi_{\rm OS} + \Phi_{\rm IS} \Phi_{\rm OS} \left(\frac{CT}{n_{\rm L}}\right)$$
(S3)

$$P = \frac{\Phi_{\rm IS}}{\Phi} \left(\frac{n_{\rm M}}{n_{\rm L}}\right) {\rm CT}$$
(S4)

$$P' = -\frac{\Phi_{\rm OS}}{\Phi} \left(\frac{n_{\rm air}}{n_{\rm L}}\right) \rm CT \tag{S5}$$

$$EFL = \frac{1}{\Phi}$$
(S6)

$$f_{\rm F} = -n_{\rm M} {\rm EFL} \tag{S7}$$

$$f_{\rm R} = n_{\rm air} EFL \tag{S8}$$

$$BFL = f_R + P' \tag{S9}$$

$$FFL = f_R + P \tag{S10}$$

$$NPS = f_F + f_R \tag{S11}$$

where  $\Phi_{IS}$  is the power of surface 1,  $\Phi_{OS}$  is the power of surface 2 and  $\Phi$  is Lens Power,  $n_M$  is object space medium index,  $n_L$  is Lens index,  $n_{air}$  is image space index (air),  $R_1$  is Radius of surface 1 and  $R_2$ is radius of surface 2, CT is center thickness of the ball mini-lens, P is primary principle point, P' is secondary principle point, EFL is effective focal length, measured from principle point to the focal point. BFL is back focal length of the ball lens measured from the lens surface to the focal point, FFL is the front focal length, NPS is the dodal point shift,  $f_F$  is front focal point and  $f_R$  is rear focal point.

Substitute Equation (S1) and Equation (S2) into Equation (S3):

$$\Phi = \frac{n_{\text{air}} - n_{\text{L}}}{R_{\text{l}}} + \frac{n_{\text{L}} - n_{\text{M}}}{R_{2}} + \left(\frac{n_{\text{air}} - n_{\text{L}}}{R_{\text{l}}}\right) \left(\frac{n_{\text{L}} - n_{\text{M}}}{R_{2}}\right) \left(\frac{R_{\text{l}} + R_{2}}{n_{\text{L}}}\right)$$
(S12)

For the lens fully immersed in air, the focal point is expressed as follow:

$$\frac{1}{\text{EFL}} = \Phi = (n_{\text{L}} - 1)(\frac{1}{R_{\text{I}}} + \frac{1}{R_{2}} + (\frac{n_{\text{L}} - 1}{R_{1}R_{2}})(\frac{d}{n_{\text{L}}}))$$
(S13)

The inverse lens power ( $\Phi$ ) is the effective focal point (EFL) of the lens where d is lens diameter. When the ball mini-lens is immersed in two different media, as in the case of  $\mu$ OIL, half of the lens is exposed in air where  $n_{air} = 1$  and half immersed in oil medium,  $\Phi$  can be written as:

$$\Phi = \frac{1}{R} (n_{\rm M} + 1 - \frac{2n_{\rm M}}{n_{\rm L}})$$
(S14)

Finding NA:

At the maximum acceptance angle, the optical rays originating from the sample will be tangent to the ball-lens surface (Figure S1) and NA can be expressed as:

$$NA = n_{M} \sin\theta \text{ or } n_{M} \left(\frac{R}{BFL + R}\right)$$
(S15)

NA = 
$$n_{\rm M} \left( \frac{\Phi R n_{\rm L}}{\Phi R n_{\rm L} + n_{\rm M} n_{\rm L} - 2(n_{\rm L} - 1) n_{\rm M}} \right)$$
 (S16)

Sapphire ball-lenses have a refractive index of 1.768. With the ball mini-lenses half immersed in immersion oil, the back focal length (BFL) of  $\mu$ OIL was calculated to be 124.5  $\mu$ m away from ball-lens surface. The Lens power is 1.6 mm<sup>-1</sup>, while BFL is 124.5  $\mu$ m, the maximum acceptance angle ( $\theta_{max}$ ) is 53°. The theoretical NA of the Sapphire ball-lens can be as high as 1.2.

Figure S1. Schematic of a thick-lens immerse in two media.



**Figure S2.** Intensity profile of a 3  $\mu$ m grid pattern obtained from the combined system. The spacer thickness was 5  $\mu$ m. The field of view (FOV) is approximately 60  $\mu$ m (where the intensity decreases by 20%).



## References

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