Supplementary Material

S.1. Derivations of *ye* and *ys*

The Transfer Matrix of each layer in the stack is as follows:

$$M_{BE}=\begin{pmatrix} \cos(\sqrt{\frac{\rho_{Mo}}{c_{44,Mo}}}\cdot t_{BE}\cdot\omega) & 0 & 0 & \frac{\sin(\sqrt{\frac{\rho_{Mo}}{c_{44,Mo}}}\cdot t_{BE}\cdot\omega)}{\omega\cdot\sqrt{\rho_{Mo}\cdot c_{44,Mo}}} \\ 0 & \cos(\sqrt{\frac{\rho_{Mo}}{c_{11,Mo}}}\cdot t_{BE}\cdot\omega) & \frac{\sin(\sqrt{\frac{\rho_{Mo}}{c_{11,Mo}}}\cdot t_{BE}\cdot\omega)}{\omega\cdot\sqrt{\rho_{Mo}\cdot c_{11,Mo}}} & 0 \\ 0 & -\omega\cdot\sqrt{\rho_{Mo}c_{11,Mo}}\cdot\sin(\sqrt{\frac{\rho_{Mo}}{c_{11,Mo}}}\cdot t_{BE}\cdot\omega) & \cos(\sqrt{\frac{\rho_{Mo}}{c_{11,Mo}}}\cdot t_{BE}\cdot\omega) & 0 \\ -\omega\cdot\sqrt{\rho_{Mo}c_{44,Mo}}\cdot\sin(\sqrt{\frac{\rho_{Mo}}{c_{44,Mo}}}\cdot t_{BE}\cdot\omega) & 0 & 0 & \cos(\sqrt{\frac{\rho_{Mo}}{c_{44,Mo}}}\cdot t_{BE}\cdot\omega) \end{pmatrix} \tag{1}$$

$$M_{Pz}=\begin{pmatrix} \cos(\sqrt{\frac{\rho_{AlN}}{c_{44,AlN}}}\cdot t_{Pz}\cdot\omega) & 0 & 0 & \frac{\sin(\sqrt{\frac{\rho_{AlN}}{c_{44,Mo}}}\cdot t_{Pz}\cdot\omega)}{\omega\cdot\sqrt{\rho_{AlN}\cdot c_{44,AlN}}} \\ 0 & \cos(\frac{t_{Pz}\cdot\omega}{C}) & \frac{\sin(\frac{t_{Pz}\cdot\omega}{C})}{\omega\cdot\rho_{AlN}\cdot C} & 0 \\ 0 & -\frac{\omega\cdot c_{33,AlN}\cdot\sin(\frac{t_{Pz}\cdot\omega}{C})}{C}-\frac{\omega\cdot e_{33}{}^{2}\cdot\sin(\frac{t_{Pz}\cdot\omega}{C})}{\varepsilon_{33}\cdot C} & \cos(\frac{t_{Pz}\cdot\omega}{C}) & 0 \\ -\omega\cdot\sqrt{\rho_{AlN}c_{44,AlN}}\cdot\sin(\sqrt{\frac{\rho_{AlN}}{c_{44,AlN}}}\cdot t_{Pz}\cdot\omega) & 0 & 0 & \cos(\sqrt{\frac{\rho_{AlN}}{c_{44,AlN}}}\cdot t_{Pz}\cdot\omega) \end{pmatrix} \tag{2}$$

$$M_{TE}=\begin{pmatrix} \cos(\sqrt{\frac{\rho_{Mo}}{c_{44,Mo}}}\cdot t_{TE}\cdot\omega) & 0 & 0 & \frac{\sin(\sqrt{\frac{\rho_{Mo}}{c_{44,Mo}}}\cdot t_{TE}\cdot\omega)}{\omega\cdot\sqrt{\rho_{Mo}\cdot c_{44,Mo}}} \\ 0 & \cos(\sqrt{\frac{\rho_{Mo}}{c_{11,Mo}}}\cdot t_{TE}\cdot\omega) & \frac{\sin(\sqrt{\frac{\rho_{Mo}}{c_{11,Mo}}}\cdot t_{TE}\cdot\omega)}{\omega\cdot\sqrt{\rho_{Mo}\cdot c_{11,Mo}}} & 0 \\ 0 & -\omega\cdot\sqrt{\rho_{Mo}c_{11,Mo}}\cdot\sin(\sqrt{\frac{\rho_{Mo}}{c_{11,Mo}}}\cdot t_{TE}\cdot\omega) & \cos(\sqrt{\frac{\rho_{Mo}}{c_{11,Mo}}}\cdot t_{TE}\cdot\omega) & 0 \\ -\omega\cdot\sqrt{\rho_{Mo}c_{44,Mo}}\cdot\sin(\sqrt{\frac{\rho_{Mo}}{c_{44,Mo}}}\cdot t_{TE}\cdot\omega) & 0 & 0 & \cos(\sqrt{\frac{\rho_{Mo}}{c_{44,Mo}}}\cdot t_{TE}\cdot\omega) \end{pmatrix} \tag{3}$$

$$M_{Ox}=\begin{pmatrix} \cos(\sqrt{\frac{\rho_{Ox}}{c_{44,Ox}}}\cdot t_{Ox}\cdot\omega) & 0 & 0 & \frac{\sin(\sqrt{\frac{\rho_{Ox}}{c_{44,Ox}}}\cdot t_{Ox}\cdot\omega)}{\omega\cdot\sqrt{\rho_{Ox}\cdot c_{44,Ox}}} \\ 0 & \cos(\sqrt{\frac{\rho_{Ox}}{c_{11,Ox}}}\cdot t_{Ox}\cdot\omega) & \frac{\sin(\sqrt{\frac{\rho_{Ox}}{c_{11,Ox}}}\cdot t_{Ox}\cdot\omega)}{\omega\cdot\sqrt{\rho_{Ox}\cdot c_{11,Ox}}} & 0 \\ 0 & -\omega\cdot\sqrt{\rho_{Ox}c_{11,Ox}}\cdot\sin(\sqrt{\frac{\rho_{Ox}}{c_{11,Ox}}}\cdot t_{Ox}\cdot\omega) & \cos(\sqrt{\frac{\rho_{Ox}}{c_{11,Ox}}}\cdot t_{Ox}\cdot\omega) & 0 \\ -\omega\cdot\sqrt{\rho_{Ox}c_{44,Ox}}\cdot\sin(\sqrt{\frac{\rho_{Ox}}{c_{44,Ox}}}\cdot t_{Ox}\cdot\omega) & 0 & 0 & \cos(\sqrt{\frac{\rho_{Ox}}{c_{44,Ox}}}\cdot t_{Ox}\cdot\omega) \end{pmatrix} \tag{4}$$

$$M_{TM}=\begin{pmatrix} \cos(\sqrt{\frac{\rho_{Mo}}{c_{44,Mo}}}\cdot t_{TM}\cdot\omega) & 0 & 0 & \frac{\sin(\sqrt{\frac{\rho_{Mo}}{c_{44,Mo}}}\cdot t_{TM}\cdot\omega)}{\omega\cdot\sqrt{\rho_{Mo}\cdot c_{44,Mo}}} \\ 0 & \cos(\sqrt{\frac{\rho_{Mo}}{c_{11,Mo}}}\cdot t_{TM}\cdot\omega) & \frac{\sin(\sqrt{\frac{\rho_{Mo}}{c_{11,Mo}}}\cdot t_{TM}\cdot\omega)}{\omega\cdot\sqrt{\rho_{Mo}\cdot c_{11,Mo}}} & 0 \\ 0 & -\omega\cdot\sqrt{\rho_{Mo}c_{11,Mo}}\cdot\sin(\sqrt{\frac{\rho_{Mo}}{c_{11,Mo}}}\cdot t_{TM}\cdot\omega) & \cos(\sqrt{\frac{\rho_{Mo}}{c_{11,Mo}}}\cdot t_{TM}\cdot\omega) & 0 \\ -\omega\cdot\sqrt{\rho_{Mo}c_{44,Mo}}\cdot\sin(\sqrt{\frac{\rho_{Mo}}{c_{44,Mo}}}\cdot t_{TM}\cdot\omega) & 0 & 0 & \cos(\sqrt{\frac{\rho_{Mo}}{c_{44,Mo}}}\cdot t_{TM}\cdot\omega) \end{pmatrix} \quad (5)$$

where t_{BE}, t_{Pz}, t_{TE}, t_{Ox}, and t_{TM} are the thicknesses of the bottom electrode layer, the piezoelectric layer, the top electrode layer, the silicon oxide (SiO_2) reflection layer, and the top molybdenum reflection layer, respectively; ω is the angular frequency; ρ_{Mo} and c_{Mo} are the density and the stiffness of the molybdenum; ρ_{Ox} and c_{Ox} are the density and the stiffness of the SiO_2; ρ_{AlN} and c_{AlN} are the density and the stiffness of the aluminum nitride (AlN). In equation (2), C is expressed in the form:

$$C=\sqrt{\frac{c_{33,AlN}}{\rho_{AlN}}+\frac{{e_{33}}^2}{\rho_{AlN}\cdot\varepsilon_{33}}} \quad (6)$$

where e and ε are the piezoelectric coupling coefficient and the relative dielectric constant of AlN, respectively.

The 4 × 4 Transfer Matrix of the whole stack can be obtained:

$$M=M_{BE}\cdot M_{Pz}\cdot M_{TE}\cdot M_{Ox}\cdot M_{TM} \quad (7)$$

where M is a 4 × 4 Transfer Matrix, which condenses a multilayered system into a set of four equations correlating the boundary conditions at the first interface to that at the last interface.

$$\begin{pmatrix} u \\ v \\ \sigma \\ \tau \end{pmatrix}_{last}=M\cdot\begin{pmatrix} u \\ v \\ \sigma \\ \tau \end{pmatrix}_{first} \quad (8)$$

where u, v, σ, and τ are the normal displacement, the tangential displacement, the normal stress, and the tangential stress, respectively; The subscripts *top* and *bot* represent the top and the bottom surface. Since both the upper and the lower surfaces of the device are exposed to air, the stresses on them are zero. Hence, the determination of the 2 × 2 bottom left sub-matrix of M is zero:

$$M_{31}\cdot M_{42}-M_{41}\cdot M_{32}=0 \quad (9)$$

The determination can be rewritten in the form of a product of two factors, which are named as *ye* and *ys*, respectively:

$$ye\cdot ys=0 \quad (10)$$

The expressions of *ys* and *ye* are as follows:

$$\begin{aligned}
ys = {} & a_1Sin[(\frac{t_{BE}}{m_1}-\frac{t_{Pz}}{m_2}-\frac{t_{TE}}{m_1}-\frac{t_{Ox}}{m_3}-\frac{t_{TM}}{m_1})\omega]+a_2Sin[(\frac{t_{BE}}{m_1}-\frac{t_{Pz}}{m_2}+\frac{t_{TE}}{m_1}+\frac{t_{Ox}}{m_3}+\frac{t_{TM}}{m_1})\omega]+a_3Sin[(\frac{t_{BE}}{m_1}+\frac{t_{Pz}}{m_2}-\frac{t_{TE}}{m_1}+\frac{t_{Ox}}{m_3}+\frac{t_{TM}}{m_1})\omega] \\
& +a_4Sin[(\frac{t_{BE}}{m_1}+\frac{t_{Pz}}{m_2}+\frac{t_{TE}}{m_1}-\frac{t_{Ox}}{m_3}+\frac{t_{TM}}{m_1})\omega]+a_5Sin[(\frac{t_{BE}}{m_1}+\frac{t_{Pz}}{m_2}+\frac{t_{TE}}{m_1}+\frac{t_{Ox}}{m_3}-\frac{t_{TM}}{m_1})\omega]+a_6Sin[(\frac{t_{BE}}{m_1}+\frac{t_{Pz}}{m_2}-\frac{t_{TE}}{m_1}-\frac{t_{Ox}}{m_3}-\frac{t_{TM}}{m_1})\omega] \\
& +a_7Sin[(\frac{t_{BE}}{m_1}-\frac{t_{Pz}}{m_2}+\frac{t_{TE}}{m_1}-\frac{t_{Ox}}{m_3}-\frac{t_{TM}}{m_1})\omega]+a_8Sin[(\frac{t_{BE}}{m_1}-\frac{t_{Pz}}{m_2}-\frac{t_{TE}}{m_1}+\frac{t_{Ox}}{m_3}-\frac{t_{TM}}{m_1})\omega]+a_9Sin[(\frac{t_{BE}}{m_1}-\frac{t_{Pz}}{m_2}-\frac{t_{TE}}{m_1}-\frac{t_{Ox}}{m_3}+\frac{t_{TM}}{m_1})\omega] \\
& +a_{10}Sin[(\frac{t_{BE}}{m_1}-\frac{t_{Pz}}{m_2}-\frac{t_{TE}}{m_1}+\frac{t_{Ox}}{m_3}+\frac{t_{TM}}{m_1})\omega]+a_{11}Sin[(\frac{t_{BE}}{m_1}-\frac{t_{Pz}}{m_2}+\frac{t_{TE}}{m_1}-\frac{t_{Ox}}{m_3}+\frac{t_{TM}}{m_1})\omega]+a_{12}Sin[(\frac{t_{BE}}{m_1}-\frac{t_{Pz}}{m_2}+\frac{t_{TE}}{m_1}+\frac{t_{Ox}}{m_3}-\frac{t_{TM}}{m_1})\omega] \\
& +a_{13}Sin[(\frac{t_{BE}}{m_1}+\frac{t_{Pz}}{m_2}-\frac{t_{TE}}{m_1}-\frac{t_{Ox}}{m_3}+\frac{t_{TM}}{m_1})\omega]+a_{14}Sin[(\frac{t_{BE}}{m_1}+\frac{t_{Pz}}{m_2}-\frac{t_{TE}}{m_1}+\frac{t_{Ox}}{m_3}-\frac{t_{TM}}{m_1})\omega]+a_{15}Sin[(\frac{t_{BE}}{m_1}+\frac{t_{Pz}}{m_2}+\frac{t_{TE}}{m_1}-\frac{t_{Ox}}{m_3}-\frac{t_{TM}}{m_1})\omega] \\
& +a_{16}Sin[(\frac{t_{BE}}{m_1}+\frac{t_{Pz}}{m_2}+\frac{t_{TE}}{m_1}+\frac{t_{Ox}}{m_3}+\frac{t_{TM}}{m_1})\omega]
\end{aligned} \tag{11}$$

$$\begin{aligned}
ye = {} & b_1Sin[(\frac{t_{BE}}{n_1}-\frac{t_{Pz}}{n_2}-\frac{t_{TE}}{n_1}-\frac{t_{Ox}}{n_3}-\frac{t_{TM}}{n_1})\omega]+b_2Sin[(\frac{t_{BE}}{n_1}-\frac{t_{Pz}}{n_2}+\frac{t_{TE}}{n_1}+\frac{t_{Ox}}{n_3}+\frac{t_{TM}}{n_1})\omega]+b_3Sin[(\frac{t_{BE}}{n_1}+\frac{t_{Pz}}{n_2}-\frac{t_{TE}}{n_1}+\frac{t_{Ox}}{n_3}+\frac{t_{TM}}{n_1})\omega] \\
& +b_4Sin[(\frac{t_{BE}}{n_1}+\frac{t_{Pz}}{n_2}+\frac{t_{TE}}{n_1}-\frac{t_{Ox}}{n_3}+\frac{t_{TM}}{n_1})\omega]+b_5Sin[(\frac{t_{BE}}{n_1}+\frac{t_{Pz}}{n_2}+\frac{t_{TE}}{n_1}+\frac{t_{Ox}}{n_3}-\frac{t_{TM}}{n_1})\omega]+b_6Sin[(\frac{t_{BE}}{n_1}+\frac{t_{Pz}}{n_2}-\frac{t_{TE}}{n_1}-\frac{t_{Ox}}{n_3}-\frac{t_{TM}}{n_1})\omega] \\
& +b_7Sin[(\frac{t_{BE}}{n_1}-\frac{t_{Pz}}{n_2}+\frac{t_{TE}}{n_1}-\frac{t_{Ox}}{n_3}-\frac{t_{TM}}{n_1})\omega]+b_8Sin[(\frac{t_{BE}}{n_1}-\frac{t_{Pz}}{n_2}-\frac{t_{TE}}{n_1}+\frac{t_{Ox}}{n_3}-\frac{t_{TM}}{n_1})\omega]+b_9Sin[(\frac{t_{BE}}{n_1}-\frac{t_{Pz}}{n_2}-\frac{t_{TE}}{n_1}-\frac{t_{Ox}}{n_3}+\frac{t_{TM}}{n_1})\omega] \\
& +b_{10}Sin[(\frac{t_{BE}}{n_1}-\frac{t_{Pz}}{n_2}-\frac{t_{TE}}{n_1}+\frac{t_{Ox}}{n_3}+\frac{t_{TM}}{n_1})\omega]+b_{11}Sin[(\frac{t_{BE}}{n_1}-\frac{t_{Pz}}{n_2}+\frac{t_{TE}}{n_1}-\frac{t_{Ox}}{n_3}+\frac{t_{TM}}{n_1})\omega]+b_{12}Sin[(\frac{t_{BE}}{n_1}-\frac{t_{Pz}}{n_2}+\frac{t_{TE}}{n_1}+\frac{t_{Ox}}{n_3}-\frac{t_{TM}}{n_1})\omega] \\
& +b_{13}Sin[(\frac{t_{BE}}{n_1}+\frac{t_{Pz}}{n_2}-\frac{t_{TE}}{n_1}-\frac{t_{Ox}}{n_3}+\frac{t_{TM}}{n_1})\omega]+b_{14}Sin[(\frac{t_{BE}}{n_1}+\frac{t_{Pz}}{n_2}-\frac{t_{TE}}{n_1}+\frac{t_{Ox}}{n_3}-\frac{t_{TM}}{n_1})\omega]+b_{15}Sin[(\frac{t_{BE}}{n_1}+\frac{t_{Pz}}{n_2}+\frac{t_{TE}}{n_1}-\frac{t_{Ox}}{n_3}-\frac{t_{TM}}{n_1})\omega] \\
& +b_{16}Sin[(\frac{t_{BE}}{n}+\frac{t_{Pz}}{n_2}+\frac{t_{TE}}{n_1}+\frac{t_{Ox}}{n_3}+\frac{t_{TM}}{n_1})\omega]
\end{aligned} \tag{12}$$

where m_1, m_2, m_3, n_1, n_2, n_3 are the expressions of the material parameters with the following elements:

$$\begin{cases}
m_1 = \sqrt{c_{44,Mo} / \rho_{Mo}} \\
m_2 = \sqrt{c_{44,AlN} / \rho_{AlN}} \\
m_3 = \sqrt{c_{44,Ox} / \rho_{Ox}} \\
n_1 = \sqrt{c_{11,Mo} / \rho_{Mo}} \\
n_2 = \sqrt{\dfrac{c_{33,AlN}}{\rho_{AlN}} + \dfrac{{e_{33}}^2}{\rho_{AlN} \times \varepsilon_{33}}} \\
n_3 = \sqrt{c_{11,Ox} / \rho_{Ox}}
\end{cases} \tag{13}$$

Here, a_1–a_{16} and b_1–b_{16} are expressions of material parameters.