games

Article

Conflicts with Momentum

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Abstract: *Take the fort, then take the city*. In a two-stage, two-party contest, victory in the initial stage can provide an advantage in the final stage. We examine such momentum in conflict scenarios and investigate how valuable it must be to avoid a *Pyrrhic* victory. Our main finding is that although the elasticity of effort—which we allow to vary between the two stages—does impact the contestants' effort levels, it has no bearing on the endogenously determined value of momentum itself. Further, rent dissipation in the two-stage conflict is equal across party whether or not an individual obtains first-stage momentum. Thus, momentum helps a player solely by enhancing marginal ability for victory in the second-stage contest. It does not, however, change the player's net calculus of second-stage contest spending. Such contestable advantage is also found to be more rent-dissipative than innate/uncontestable advantage. Therefore, *Pyrrhic* victories should be more common for contests with an intermediate stage or stages in which advantages can be earned, *ceteris paribus*. While intermediate targets appear as useful conflict benchmarks, they dissipate additional expected contest rents. This additional rent-dissipative toll exists even for backward-inductive equilibrium behavior in a complete information setting. Whereas the quagmire theory suggests parties can become involved in problematic conflicts due to incomplete information, the present paper finds that the setting of conflict—namely, contestable intermediate advantage—can alternatively generate rent-dissipative tolls. Similarly, contestable advantage can lead parties to optimally forego contest participation (i.e., if conflict parameters do not meet the participation constraint). This is in contrast to a one-stage simultaneous contest with second-stage parametric values of the present contest.

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1. Introduction

The premise of our model is an intuitive one: *Take the fort, then take the city*. The concept is that many conflicts are not one-shot scenarios, but rather involve an initial stage in which one party can gain an advantage that improves its relative position in the ultimate stage.

More specifically, we set up a two-stage model of conflict in which the winner from the first-stage gains *momentum* in the sense that it has a reduced unit cost of effort in the second stage. One party gains an advantageous position or elevation through a first stage of conflict, so it is less costly in the second stage for that party to produce conflict inputs. One interpretation could be that one needs fewer soldiers to put forth an effective force when at an elevation gained earlier. The *Battle of the Alamo*, for example, had an ultimate 5-to-1 casualty ratio in favor of the advantaged side.

Our work is most similar to contest models that emphasize "head starts" in the sense of giving one party or another a cost advantage of some kind, as studied in [1–3]. These papers, however, focus in particular on the optimal setting of cost advantages (or disadvantages) in order to maximize effort expenditures by contestants, as if set by the contest designer. Furthermore, refs. [1,3] uses contest success function formats that differ importantly from ours and one-stage settings, while [2] focuses on a multi-stage setting in which some participants are disqualified from later stages. Ref. [2] also uses a classic [4] lottery-form success function. (Ref. [5] also study the role of cost asymmetries in conflict outcome). The two-stage nested conflict approach has been used to study aspects of conflict other than momentum (see, e.g., [6]).

Here, we assume that two parties in conflict exert efforts at a cost in a two-stage game with a more generalized contest success function of the Tullock variety, but more akin to the version generalized by works such as [7,8]. (The results obtained may in some ways depend upon the assumed functional form of the contest success function. We feel that a ratio-form success function—for which each party has a strictly positive probability of victory so long as a positive amount of effort is exerted—is appropriate for the types of conflict that we have in mind as the primary motivators for the analysis (e.g., multi-stage armed conflict between combatants).) Our model assumes that there is some cost advantage to be achieved by winning the first stage, but that the first stage's sensitivity to effort spending may differ from that of the second stage. Ultimately, we find that while the overall size of the (exogenous) cost advantage in the second stage—which we term *momentum*—does matter for the ultimate probability of victory and spending by parties in the second stage, the sensitivity to effort in either stage does not matter for how valuable that advantage is in a crucial sense. Although the elasticity of effort—which we allow to differ between the two stages of conflict—does impact the contestants' effort levels, it has no bearing on the endogenously determined value of momentum itself. Further, rent dissipation in the two-stage conflict is equal across party whether or not an individual obtains momentum in the first stage. Thus, momentum helps a player solely by enhancing the player's marginal ability for victory in the second-stage contest. It does not, however, change the player's net calculus of second-stage contest spending. Contestable advantage in conflict is also found to be more rent-dissipative than innate or otherwise incontestable advantage. Similarly, contestable advantage can lead parties to optimally forego contest participation (i.e., if conflict parameters do not meet the participation constraint). This is in contrast to a one-stage simultaneous contest that takes on the second-stage parametric values of the present contest.

Therefore, we expect *Pyrrhic* victories to be more common, *ceteris paribus*, for contests that feature an intermediate stage or stages in which subsequent advantages can be earned. While intermediate targets may appear as useful benchmarks in conflict, they in fact dissipate additional expected contest rents to each party. This additional rent-dissipative toll exists even given a backward-inductive (equilibrium) behavior in a setting of complete information rather than one characterized by "fog of war" effects. The quagmire theory suggests that countries can become involved in problematic (i.e., rent-dissipative) conflicts due to incomplete information. The present paper finds that the setting of conflict—namely, the contestibility of intermediate, momentous advantage in a conflict—can effectively substitute for incomplete information in generating rent-dissipative tolls.

2. A Model of Conflicts with Momentum

2.1. Model Setup

Consider two parties, $i = \{1, 2\}$, in a two-stage conflict. The ultimate winner of the final conflict in the second stage of the game is awarded a prize commonly valued at V by both parties. But winning the first stage of the game provides a cost advantage of $0 < \alpha < 1$ to the first-stage winner in terms of competing in the second stage.

Solving backwards, in the second stage of the game, one party has already won the first stage, making their objective function for that stage

$$\frac{s_w^{r_2}}{s_w^{r_2} + s_\ell^{r_2}} V - \alpha s_w - s_1$$

where s_w is the expenditure by the first-stage winner in the second stage, s_ℓ is the expenditure by the first-stage loser in the second stage, and s_1 is the expenditure of each party in the first stage. The expenditure by both parties is equal in the first stage since we assume symmetric valuations of V.

The r_2 parameter represents the sensitivity of the second-stage contest success function (CSF) to the relative expenditures chosen by the conflicting parties in that stage (see [7,9] for axiomitizations of CSFs). We initially impose the restriction $0 < r_2 \leq 2$ as is standard in the literature (e.g., [10]), but will both verify this assumption and explore the more detailed joint restrictions on this parameter and α necessary for the participation of both parties in the next subsection.

The party that loses the first stage has a similar objective function in the second stage to that of the first-stage winner but without the cost advantage of α,

$$\frac{s_\ell^{r_2}}{s_w^{r_2} + s_\ell^{r_2}} V - s_\ell - s_1.$$

These objective functions lead to the first order conditions

$$\frac{r_2 s_w^{r_2-1} s_\ell^{r_2}}{(s_w^{r_2} + s_\ell^{r_2})^2} V = \alpha$$

and

$$\frac{r_2 s_\ell^{r_2-1} s_w^{r_2}}{(s_w^{r_2} + s_\ell^{r_2})^2} V = 1,$$

which imply $s_\ell = \alpha s_w$, allowing the conditions to be solved to determine the equilibrium efforts of

$$s_w^* = \frac{r_2 \alpha^{r_2-1} V}{(1+\alpha^{r_2})^2}$$

and

$$s_\ell^* = \frac{r_2 \alpha^{r_2} V}{(1+\alpha^{r_2})^2}.$$

These then lead to probabilities of victory (which are the same as they would be as in the case of the classic Tullock CSF)

$$P_w^* = \frac{1}{(1+\alpha^{r_2})}$$

and

$$P_\ell^* = \frac{\alpha^{r_2}}{(1+\alpha^{r_2})}$$

and the corresponding expected payoffs

$$\pi_w^* = \frac{V}{(1+\alpha^{r_2})} - \alpha s_w^* - s_1$$

and

$$\pi_\ell^* = \frac{\alpha^{r_2} V}{(1+\alpha^{r_2})} - s_\ell^* - s_1.$$

We then refer to the *value of the momentum* as the difference between the winner's and loser's expected payoffs,

$$\pi_w^* - \pi_\ell^* = \frac{(1-\alpha^{r_2})}{(1+\alpha^{r_2})} V.$$

The first stage of the contest is then a battle for this value of momentum.

Assuming a different degree of elasticity for the contest success function in this initial stage, denoted r_1, the objective functions in this first-stage contest between players I and II are

$$u_I = \frac{s_I^{r_1}}{s_I^{r_1} + s_{II}^{r_1}} \frac{(1-\alpha^{r_2})}{(1+\alpha^{r_2})} V - s_I$$

and

$$u_{II} = \frac{s_{II}^{r_1}}{s_I^{r_1} + s_{II}^{r_1}} \frac{(1-\alpha^{r_2})}{(1+\alpha^{r_2})} V - s_{II}$$

which lead to the standard equilibrium efforts of

$$s_I^* = s_{II}^* = s_1^* = \frac{r_1 V}{4} \frac{(1-\alpha^{r_2})}{(1+\alpha^{r_2})}$$

The equilibrium (second-stage) expected payoffs are then

$$\pi_w^* = \frac{1}{(1+\alpha^{r_2})} V - \frac{r_2 \alpha^{r_2} V}{(1+\alpha^{r_2})^2} - \frac{r_1 V}{4} \frac{(1-\alpha^{r_2})}{(1+\alpha^{r_2})}$$

(because of the α-reduction in cost in the winner's payoff), and

$$\pi_\ell^* = \frac{\alpha^{r_2}}{1+\alpha^{r_2}} V - \frac{r_2 \alpha^{r_2} V}{(1+\alpha^{r_2})^2} - \frac{r_1 V}{4} \frac{(1-\alpha^{r_2})}{(1+\alpha^{r_2})}.$$

Before moving on to considerations of rent dissipation, we first must consider whether or not the parties will find it in their own best interests to participate in the conflict at each stage, which is the topic of the next subsection.

2.2. Participation, Parameter Restrictions, and Pyrrhic Victories

Though we have seemingly solved for the model's equilibrium, we must still verify that the positive resource expenditure by each party is better than the option of sitting out the conflict and not spending at all. (We restrict our attention to pure-strategy Nash equilibria, since the mixed strategy equilibria that would result from one (or both) players not spending would simply involve parties mixing between the original game's equilibrium spending levels and zero, as per [11,12]).

To show why participation may be an issue for some parameter configurations, we present three-dimensional graphs of π_w^* and π_ℓ^* as r_1 and r_2 range from just over zero to two (our previously assumed ranges, and the ranges for unique interior solutions to exist in standard contest models). We provide three graphs for the equilibrium expected payoff of each party, one with $\alpha = 0.75$, one with $\alpha = 0.5$, and one with $\alpha = 0.25$, to show how the relationship with the CSFs' parameters changes with a lower reward to the first-stage winner (higher α) vs. a higher reward (lower α).

Figures 1–3 illustrate the $\partial \pi_w^* / \partial r_1 \leq 0$ relationship, and how the negative relationship gets stronger as r_2 increases for given α. They also show that overall, for given (r_1, r_2), a lower $alpha$ (a bigger second-stage advantage to the first-stage winner) means a higher expected payoff: $\partial \pi_w^* / \partial \alpha < 0$.

The $\partial \pi_w^* / \partial r_2$ relationship is more nuanced. For larger α (e.g., $\alpha = 0.75$), $\partial \pi_w^* / \partial r_2 < 0 \; \forall r_1$. For mid-range α (e.g., $\alpha = 0.5$), $\partial \pi_w^* / \partial r_2 < 0$ for large enough r_1, since the increased effort cost effect of r_2 dominates. But for small r_1, $\partial \pi_w^* / \partial r_2$ begins negative but eventually becomes positive for larger r_2 as the improved probability of victory from increased r_2 dominates. This goes to the extreme for small α (e.g., $\alpha = 0.25$), when the $\partial \pi_w^* / \partial r_2 > 0 \; \forall r_2$ at low levels of r_1 and is still "U-shaped" (negative at low r_2, then becoming positive as r_2 gets closer to 2) when r_1 is closer to 2.

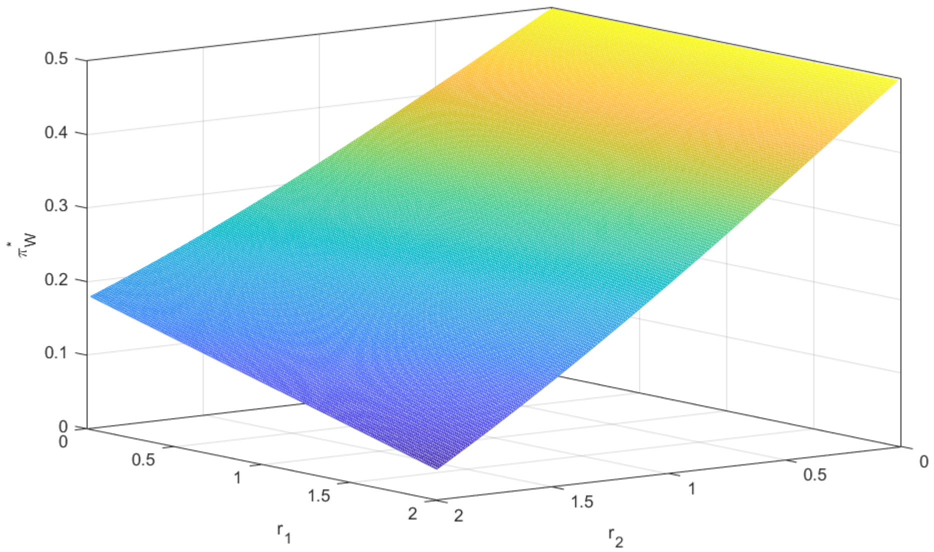

Figure 1. Equilibrium Payoff to the First−Stage Winner with $\alpha = 0.75$, $V = 1$, as r_1 and r_2 Vary.

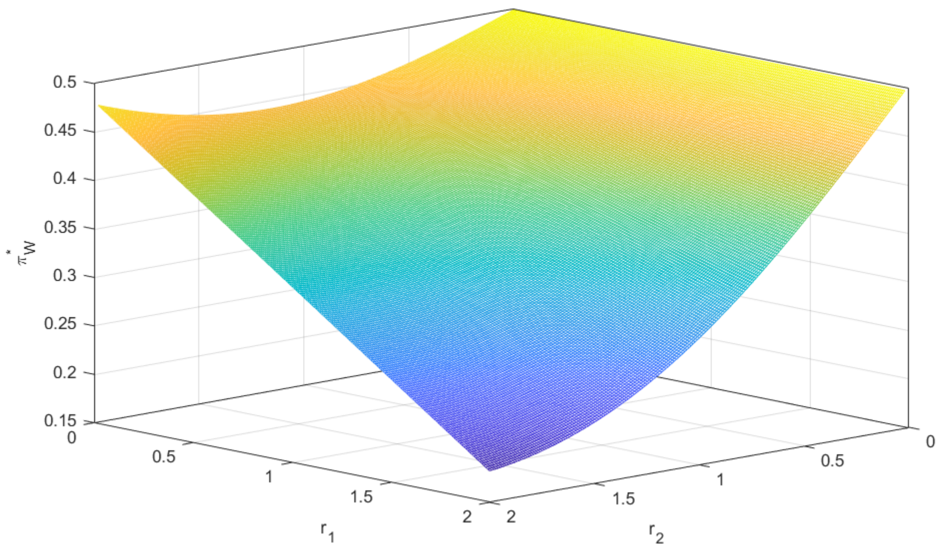

Figure 2. Equilibrium Payoff to the First−Stage Winner with $\alpha = 0.5$, $V = 1$, as r_1 and r_2 Vary.

Figures 4–6 illustrate the first-stage loser's equilibrium expected payoffs for the three example values of α ranging over r_1 and r_2. These relationships are similar to those for π^*_w but are easier to visualize, as $\partial\pi^*_\ell/\partial r_2 < 0$ for all $\alpha \in (0,1)$ and $r_1 \in (0.2]$, and $\partial\pi^*_\ell/\partial r_1 < 0$ for all $\alpha \in (0,1)$ and $r_2 \in (0,2]$. The bigger issue revealed by these graphs is that π_ℓ can be negative for a variety of parameter combinations, which brings into question whether or not the parties will necessarily want to participate in the conflict.

We begin by considering second stage and assume that a party will not spend at all if their equilibrium expected payoff from that stage is lower than simply dropping out of the conflict and spending zero in the second stage. Since $\pi^*_w > \pi^*_\ell$, we know that if the losing party from the first stage is willing to expend effort, the winning party will be willing to as well, so we only need to check the necessary condition for the first-stage losing party.

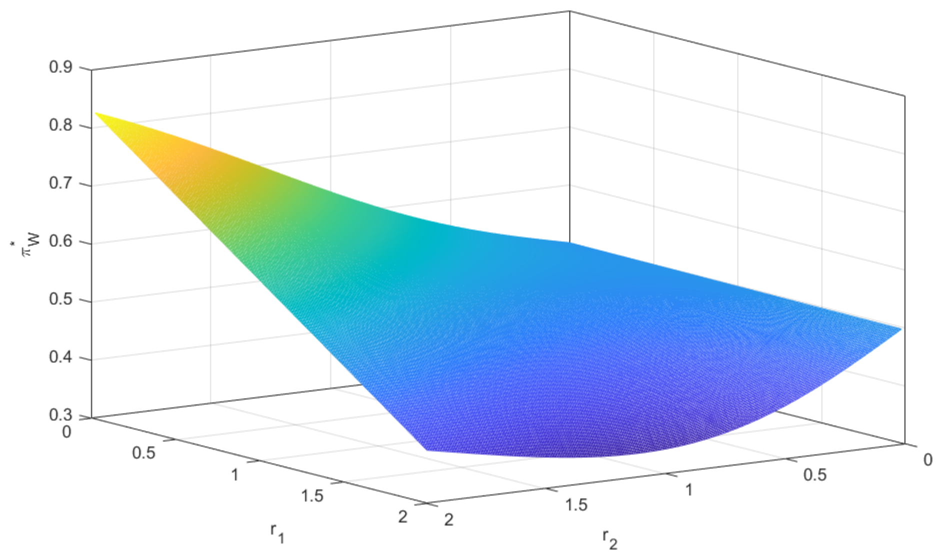

Figure 3. Equilibrium Payoff to the First−Stage Winner with $\alpha = 0.25$, $V = 1$, as r_1 and r_2 Vary.

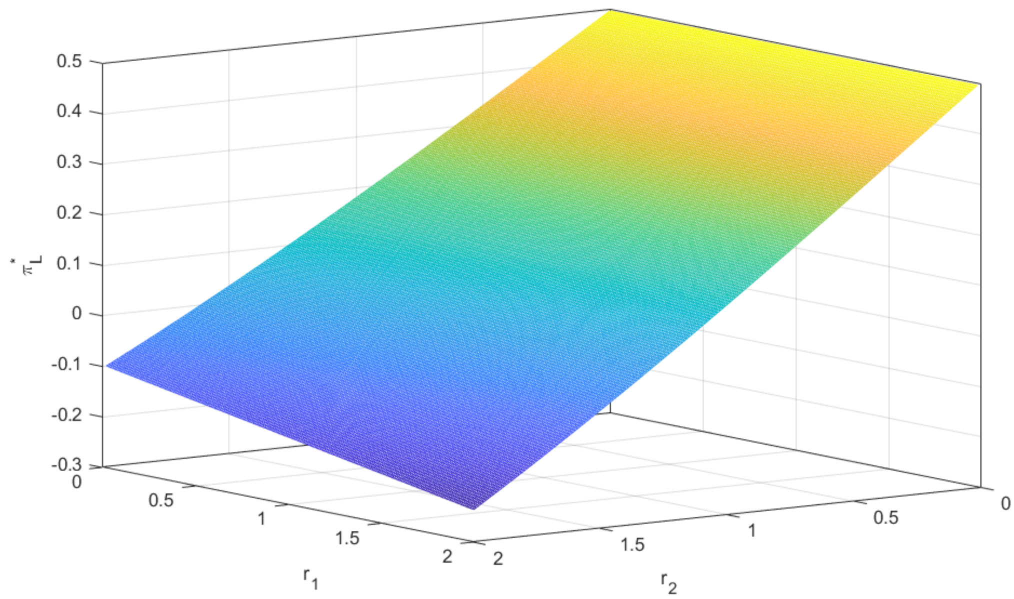

Figure 4. Equilibrium Payoff to the First−Stage Loser with $\alpha = 0.75$, $V = 1$, as r_1 and r_2 Vary.

Since spending from the first stage is sunk, the necessary participation constraint for the second stage is $\pi_\ell + s_1^* \geq 0$. That is, if a party spends nothing in the second stage, they simply lose the battle with certainty and the sunk effort cost with it, so only the portion of the party's expected payoff that is relevant to the second stage must be positive.

$$\pi_\ell^* + s_1^* = \frac{\alpha^{r_2}}{1+\alpha^{r_2}}V - \frac{r_2\alpha^{r_2}V}{(1+\alpha^{r_2})^2} \geq 0$$

which simplifies to the following condition.

Participation Constraint (i) (*PC (i)*): $\alpha^{r_2} \geq (r_2 - 1)$.

This restriction could of course be simplified a step further to isolate α in terms of r_2, but we keep it as in $PC(i)$ for the purposes of illustration since the relationship is nonlinear.

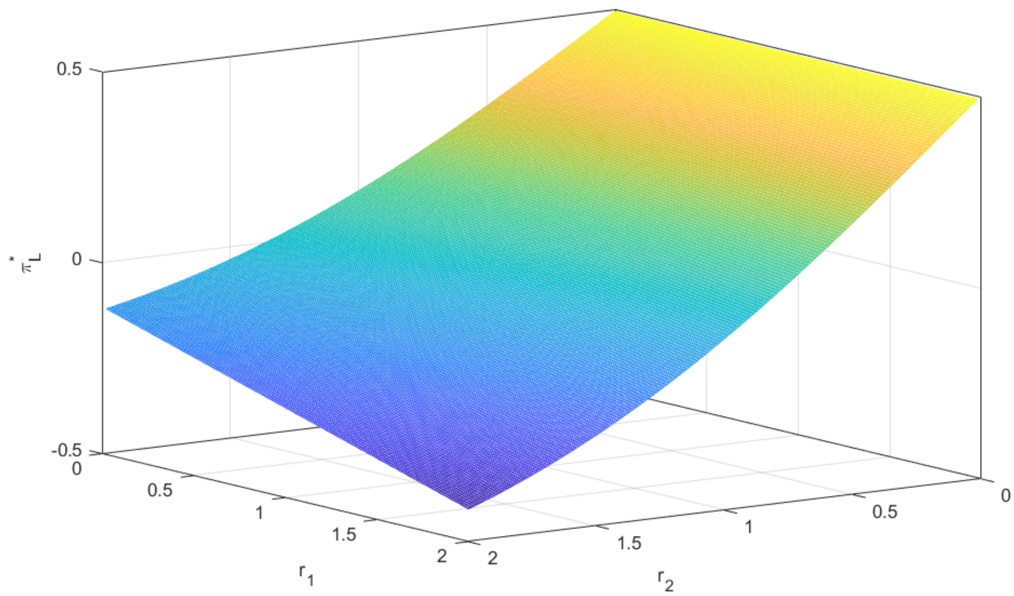

Figure 5. Equilibrium Payoff to the First−Stage Loser with $\alpha = 0.5$, $V = 1$, as r_1 and r_2 Vary.

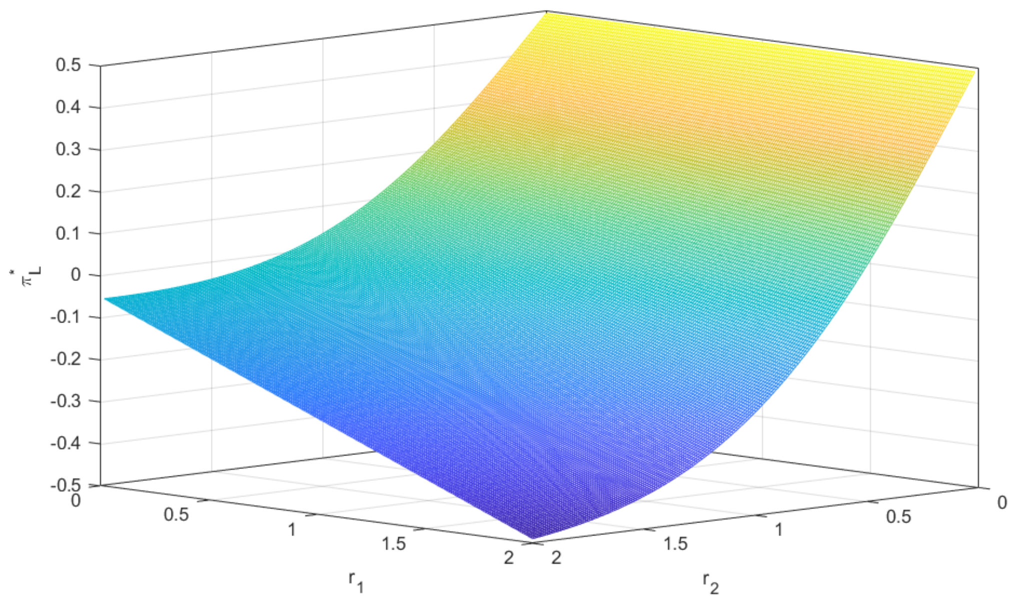

Figure 6. Equilibrium Payoff to the First−Stage Loser with $\alpha = 0.25$, $V = 1$, as r_1 and r_2 Vary.

Figure 7 graphs the left- and right-hand sides of $PC(i)$ in terms of α and r_2. All areas where $\alpha^{r_2} > (r_2 - 1)$ represent combinations of the two relevant parameters that result in second-stage conflict, with positive equilibrium expenditure by both parties as described in the previous section. All areas where $\alpha^{r_2} < (r_2 - 1)$ represent those combinations of α and r_2 that lead the losing party of the first stage to choose zero expenditure and abstain from conflict in the second stage. Intuitively, the participation constraint becomes more constrictive in terms of the allowable range of r_2 as the advantage to the first-stage winner increases (i.e., as α decreases) and vice versa. The larger the reward for winning the first stage, the less sensitive the second stage can be to effort without making it so much of an advantage that it completely deters the first-stage loser from continuing. The boundary of the maximum-allowable r_2 for given α can be traced along the curve of the intersection in Figure 7.

Our second participation constraint concerns the first stage of the conflict, when both parties have the option to either: expend effort seeking the advantage gained by the value of momentum or spend nothing and proceed to the second stage without that advantage with certainty (assuming the other party spends positively). In the latter case, of course, they also have zero sunk costs from the first stage. Thus, we compare the equilibrium expected payoff to a party—making positive equilibrium expenditures in each stage, since we assume $PC(i)$ is satisfied—to the expected payoff they would receive if they spend nothing in the first stage and competed only in the second stage at a disadvantage.

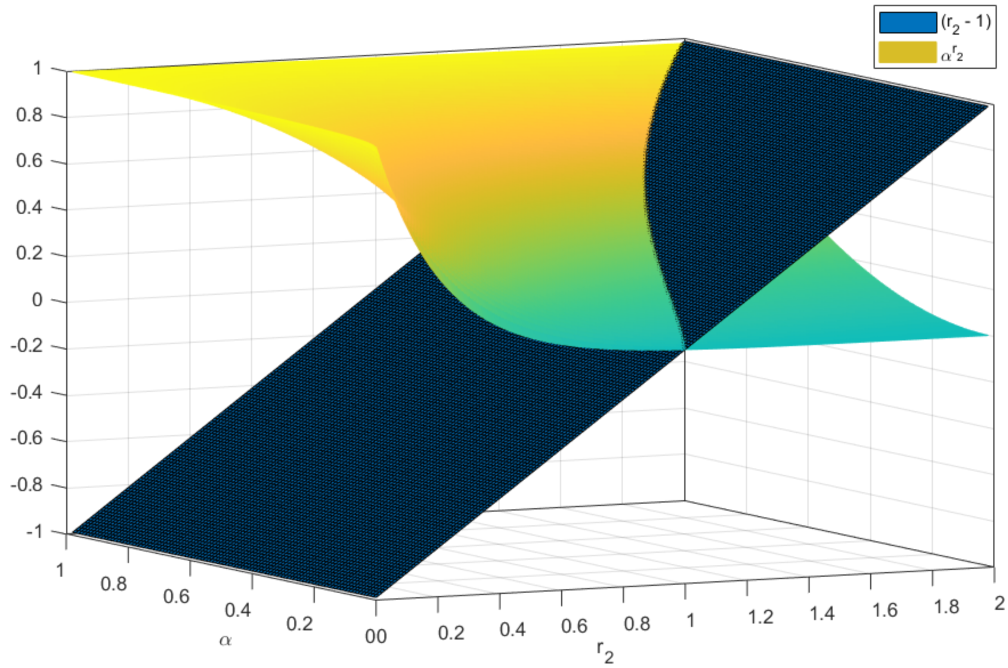

Figure 7. Participation Constraint (i): α^{r_2} vs. $(r_2 - 1)$.

Since the parties are equal in the first stage, the positive-effort equilibrium is symmetric, and each has an equal probability of victory or loss. Their expected payoff if they participate in the first-stage (again, assuming participation in the second stage) is $\frac{1}{2}\pi_w^* + \frac{1}{2}\pi_\ell^*$. If they choose to opt out of the first stage with certainty and sacrifice their chance at momentum, their equilibrium expected payoff is $P_\ell^* V - s_\ell^*$. The participation constraint is therefore

$$\frac{1}{2}\pi_w^* + \frac{1}{2}\pi_\ell^* \geq P_\ell^* V - s_\ell^*. \tag{1}$$

But since

$$\pi_w^* = P_w^* V - \alpha s_w^* - s_1^* \quad = \quad P_w^* V - s_\ell^* - s_1^*,$$

we have

$$\frac{1}{2}\pi_w^* + \frac{1}{2}\pi_\ell^* = \frac{1}{2}P_w^* V + \frac{1}{2}P_\ell^* V - s_\ell^* - s_1^*,$$

so (1) becomes

$$(P_w^* - P_\ell^*)V \geq 2s_1^*$$

or

$$\frac{(1-\alpha^{r_2})}{(1+\alpha^{r_2})}V \geq \frac{r_1 V}{2}\frac{(1-\alpha^{r_2})}{(1+\alpha^{r_2})},$$

which simplifies to the following condition.

Participation Constraint (ii) (*PC (ii)*): $2 \geq r_1$.

This makes sense given that the first stage is essentially a standard contest with the value of momentum as its prize, leading to the usual restriction for r_1 as our participation constraint for the first stage (assuming $PC(i)$ is satisfied).

The sensitivity of the first stage (or the initial "battle for momentum") to the expenditures of the conflicting parties is thus relatively unrestricted (so long as we are interested in pure-strategy equilibria), while the second stage must be permissible enough to vie for should a party lose the first stage. A second stage that is more sensitive to effort only enhances the advantage gained by momentum.

Even with these parameter constraints satisfied and both parties acting rationally according to backward induction, what is interesting is that Pyrrhic victories are still possible. In particular, note that in the game's first stage each party has an equal chance of victory and, so long as $PC(i)$ is satisfied, the loser of that stage will still compete in the second stage. But although $PC(i)$ ensures that positive spending is better than none at that point, it does not guarantee a net-positive overall expected payoff.

Consider $\pi_\ell^* = P_\ell^* V - s_\ell^* - s_1^*$. As long as ℓ competes (i.e., spends a positive amount) in the second-stage conflict, $P_\ell^* > 0$, meaning they have a positive probability of victory. But their expected payoff (including their effort expenditure from the first stage, s_1^*) may be negative. The condition $\pi_\ell^* < 0$ simplifies to $P_\ell^* V - s_\ell^* < s_1^*$ or

Pyrrhic Victory (PV): $\alpha^{r_2}(1+\alpha^{r_2}) - r_2\alpha^{r_2} < \frac{r_1}{4}(1-\alpha^{2r_2})$.

For example, consider $r_1 = r_2 = 1$, for which $PC(i)$ and $PC(ii)$ are each satisfied for all $0 < \alpha < 1$ (so that both parties compete in both stages of the conflict). Condition PV is satisfied, for all $\alpha < \sqrt{\frac{1}{5}}$. So, for $\alpha < \sqrt{\frac{1}{5}}$, the combatant who ends up losing the first-stage battle for momentum is set up to realize an expected Pyrrhic victory, in that his overall expected payoff is negative.

The condition under which such an expected Pyrrhic victory arises depends upon the values of all three parameters (i.e., r_1, r_2, and α). Figure 8 provides a plot of the left- and right-hand sides of condition PV as functions of r_1 and r_2 for $\alpha = \frac{1}{3}$. Condition PV is satisfied—so that the first-stage loser realizes an expected Pyrrhic victory—when the black surface lies above the multi-colored surface.

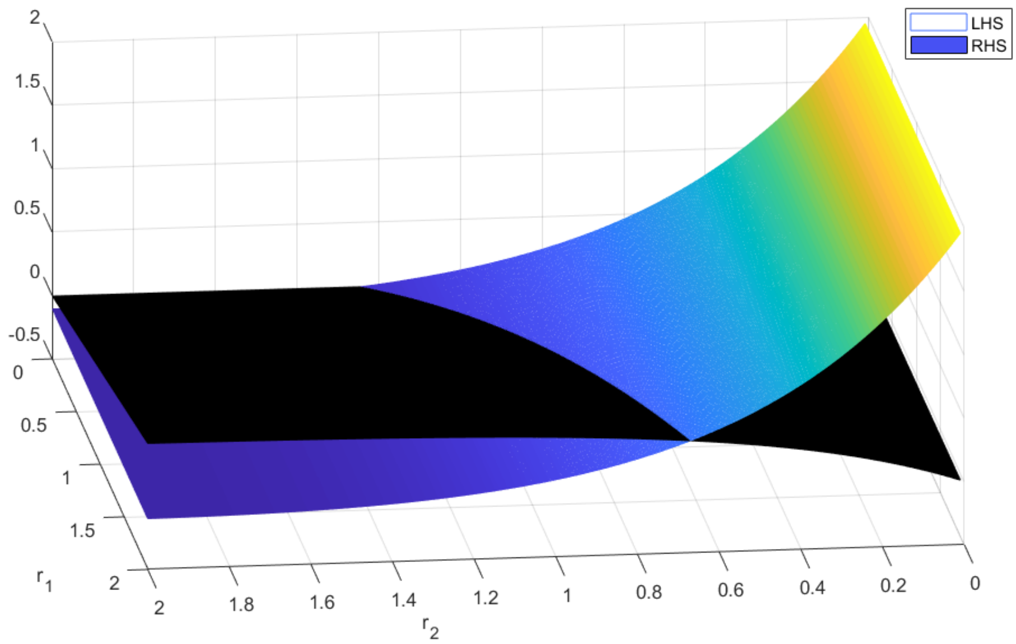

Figure 8. Pyrrhic Victory Conditions, $\alpha = 0.33$.

2.3. Rent Dissipation

Total rent dissipated across the two stages is

$$\frac{2r_2\alpha^{r_2}V}{(1+\alpha^{r_2})^2} + \frac{r_1 V}{2}\frac{1-\alpha^{r_2}}{1+\alpha^{r_2}}$$

such that total post-conflict rents are

$$V - \frac{2r_2\alpha^{r_2}V}{(1+\alpha^{r_2})^2} - \frac{r_1 V}{2}\frac{1-\alpha^{r_2}}{1+\alpha^{r_2}}.$$

Hence, the rent dissipation across the two players is equal, though the winner of the first stage does benefit from its win in terms of a higher likelihood of victory. What is perhaps interesting is that the elasticity of the contest success function does not determine that likelihood, but rather only the effort expenditure of the two contestants. Thus, momentum helps a player solely by enhancing the player's ability for victory in the second-stage contest. It does not, however, change the player's calculus of second-stage contest nor create any welfare changes therefrom.

From the second-stage objective functions, it is straightforward to see that the present contest is more rent-dissipative than a one-stage contest that features the same level of cost asymmetry, α, and noise parameter, r_2, as observed in the second round of this two-stage contest. Specifically, such an alternative contest would be less rent-dissipative by $2 \cdot s_1^*$ units of input expenditure. We know this because objective functions for each party in this alternative one-shot contest would be the same as the second-stage objective functions observed herein, but would exclude the $(-s_1)$ term from each function, where this term is not marginal to the decision calculus of that stage. Therefore, this term exactly measures additional rent-dissipation for each party in the two-stage game with contestable momentum. To check this reasoning, we can reconsider total rent-dissipation in the present contest:

$$\frac{2r_2\alpha^{r_2}V}{(1+\alpha^{r_2})^2} + \frac{r_1 V}{2}\frac{1-\alpha^{r_2}}{1+\alpha^{r_2}}$$

The second term in the sum above is simply $2 \cdot s_1^*$. Then, we expect the first term in the sum above to represent total rent dissipation for the alternative one-shot contest discussed previously. It is straightforward to verify that this is the case. That is, we find that $2 \cdot s_1^* = \frac{r_1 V}{2}\frac{1-\alpha^{r_2}}{1+\alpha^{r_2}}$. From this result, we conclude that, *ceteris paribus*, contestable advantage in conflict is more rent-dissipative than innate or otherwise incontestable advantage. Therefore, we expect *Pyrrhic* victories to be more common for contests that feature an intermediate stage or stages in which subsequent advantages can be earned, *ceteris paribus*. This additional rent-dissipative toll exists even given a backward-inductive (equilibrium) behavior in a setting of complete information rather than one characterized by "fog of war" effects. The quagmire theory suggests that countries can become involved in problematic (i.e., rent-dissipative) conflicts due to incomplete information. The present paper finds that the setting of conflict—namely, the contestibility of intermediate, momentous advantage in a conflict—can effectively substitute for incomplete information in generating rent-dissipative tolls.

3. Discussion and Conclusions

In this study, we have examined the role of momentum in conflict outcome. We model momentum as an "intermediate target". For example, one might take the fort before taking the city. By taking the fort, one then faces a lower unit input cost of contesting for the city. We model this as a two-stage contest in which the first stage is a conflict for the (value of) momentum, and the second stage is a battle for the ultimate conflict prize. The intermediate target is simply an instrument by which to gain an advantage toward the ultimate prize.

Our main finding is that although the elasticity of effort—which we allow to vary between the two stages of conflict—does impact the contestants' effort levels, it has no bearing on the endogenously determined value of momentum itself. Further, rent dissipation in the two-stage conflict is equal across party whether or not an individual obtains momentum in the first stage. Thus, momentum helps a player solely by enhancing the player's marginal ability for victory in the second-stage contest. It does not, however, change the player's net calculus of second-stage contest spending. Contestable advantage in conflict is also found to be more rent-dissipative than innate or otherwise incontestable advantage. Therefore, we expect *Pyrrhic* victories to be more common for contests that feature an intermediate stage or stages in which subsequent advantages can be earned, *ceteris paribus*.

An alternative version or extension of the model could incorporate additional parameters, for example giving the winner of the first stage a different elasticity of effort as compared to the first-stage loser in the second stage. In other words, an extra impact of momentum. Letting r_w and r_ℓ denote the second-stage elasticities, the results are qualitatively similar to the original model, but efforts would then be modified by those elasticities. Rather than $s_\ell = \alpha s_w$ as in the model analyzed, we would instead have $\frac{r_w}{r_\ell} s_\ell = \alpha s_w$. We chose to focus on the simpler model in this paper, with momentum as just a cost advantage, for clarity of presentation.

While intermediate targets may appear as useful benchmarks in conflict, they in fact dissipate additional expected contest rents to each party. This additional rent-dissipative toll exists even given a backward-inductive (equilibrium) behavior in a setting of complete information rather than one characterized by "fog of war" effects. That is, rather than countries becoming involved in problematic (i.e., rent-dissipative) conflicts due to incomplete information, the present paper finds that the setting of conflict—namely, the contestability of intermediate, momentous advantage in a conflict—can effectively substitute for incomplete information in generating rent-dissipative tolls. Similarly, we find that contestable advantage can lead parties to optimally forego contest participation (i.e., if conflict parameters do not meet the participation constraint). This is in contrast to a one-stage simultaneous contest that takes on the second-stage parametric values of the present contest.

An alternative application for our model could be a conflict between not military parties but business organizations—particularly a union organization versus management. Businesses may be very willing to invest early on to prevent workers' organizations from gaining any advantage going forward. And this may be true regardless of how hard the first-stage struggle is to prevent that advantage, or the degree of difficulty going forward. An additional example may be in the area of attack and defense of information networks, in which players engage in a first-stage battle over network access and alteration (e.g., undetected installation of a "backdoor" access point), sometimes followed by a second-stage battle over control of the network. In this case, undetected access can help an attacker gain knowledge about the architecture of the network. In turn, this knowledge will raise the attacker's effectiveness in stage 2 of the contest.

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