

Article

Informational Hold Up and Intermediaries

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Abstract: Why do some incomplete information markets feature intermediaries while others do not? I study the allocation of two goods in an incomplete information setting with a single principal, multiple agents with unit demand, and interdependent valuations. I construct a novel dynamic mechanism implemented by a principal who faces a set of intermediaries, each of whom represents an ex ante identical set of agents. This mechanism has a unique (up to permutation) weak perfect Bayesian equilibrium. The dynamic mechanism is inefficient with positive probability. Nevertheless, under mild conditions the agents are ex ante better off under the dynamic mechanism relative to a Vickrey-like auction because the intermediaries are more able to exploit information asymmetries in the dynamic mechanism than agents are able to exploit information asymmetries in the Vickrey-like auction. Finally, I show that in large markets the dynamic mechanism and Vickrey-like auction have the same expected total surplus. The comparison between the two mechanisms gives a stylized intuition for the hierarchical structure of larger markets and institutions.

Keywords: market design; matching; auctions; asymmetric information

JEL Classification: C73; C78; D44; D47; D82; D86



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1. Introduction

Consider a principal with multiple goods and agents with private single-dimensional information, unit demand, and interdependent valuations. In some such markets, the principal's market design problem is to elicit private information directly from the agents themselves. In other such markets, the principal's market design problem is to elicit information from the intermediaries who represent agents. Examples of possibly intermediated markets are hiring (in which a firm hires either by directly soliciting applications, or by contracting with one or more recruiters) and resource allocation within a firm (in which the CEO either allocates resources directly to individual contributors, or allocates them to middle managers who in turn allocate the resources among their respective direct reports). The principal's mechanism design problem facing the intermediaries differs substantially from the mechanism design problem facing the agents. Facing the agents themselves, the principal easily extracts all of the private information truthfully and implements the efficient allocation. Facing the intermediaries, the principal has few attractive mechanisms due to the intermediaries' multidimensional types. When the agents are represented by intermediaries, say that the principal faces an intermediated allocation problem; when the agents are not represented by intermediaries, say that the principal faces an immediate allocation problem.

This paper first presents a novel dynamic mechanism for an intermediated allocation problem and constructs its weak perfect Bayesian equilibrium. Second, I compare that equilibrium to an efficient Vickrey-like auction in the corresponding immediate allocation problem. I construct conditions under which agents ex ante prefer the dynamic mechanism to the Vickrey-like auction, as any reduction in an agent's expected valuation is more than offset by a reduction in expected payment. Finally, I show that in large markets (1) the agents ex ante strictly prefer the dynamic mechanism to the Vickrey-like auction and (2) the

dynamic mechanism and Vickrey-like auction have identical ex ante expected total surplus. This result provides intuition for the hierarchical structure of large markets and the flatter structure of small markets. Agents prefer the dynamic mechanism since intermediaries are better able to exploit the information asymmetries with the principal than individual agents. Conversely, the principal prefers the Vickrey-like auction, since the principal's mechanism design problem facing the intermediaries is harder than the problem when facing the agents themselves.

There are various reasons to consider a hybrid contract and auction allocation game. From a purely theoretical standpoint, it is well-known that auctions with externalities lead to demand reduction and inefficient outcomes [1]. The intermediated allocation mechanism of this paper allows explicit study of a specific, principal-selected form of demand reduction. Engelbrecht-Wiggans and Katok [2] consider a principal who has a preference for dealing with particular agents. They propose but do not model the reasons for the preference, such as existing relationships. A second interpretation views this paper's allocation game as a particular type of sequential allocation mechanism (see, for example, [3] for a discussion of sequential auctions for French timber lots). Finally, some objects are allocated in stages using different methods at each stage: Colorado [4] (among other states) allocates big game licenses via a series of draws with priorities based on type of license and residency status, followed by a first-come-first-served sale of left over licenses.

This paper contributes foremost to the (relatively small) literature on multidimensional mechanism design. Multidimensional allocation problems are notoriously difficult. Jehiel and Moldovanu [5] present an impossibility result which says that efficient and incentive compatible static mechanisms (usually) do not exist when agents have multidimensional types and interdependent valuations. In essence, a single dimensional payment typically cannot elicit multidimensional information. Their impossibility result includes the principal's mechanism design problem facing the intermediaries presented in this paper; the dynamic mechanism therefore contracts away one of the goods in order to reduce the principal's dimensional problem. Jehiel, Moldovanu, and Stacchetti [6] provide incentive compatibility constraints in an auction for a single item with externalities. Manelli and Vincent [7] provide a revenue-maximizing mechanism for a multi-good monopolist facing a single buyer with private valuations.

A second closely related literature concerns monopoly information brokers; this literature does not consider a principal facing the brokers themselves. Damiano and Li [8] consider a two-sided matching market in which a single price discriminating monopolist arranges a schedule of meeting locations and entry fees; men and women sort themselves assortatively (and possibly coarsely) to the locations, after which men (women) are randomly matched to women (men) at the same location. Each location serves as an intermediary to assure that agents at a given location fall within a narrow band of types; however, the monopolist controls the menu of prices and meeting locations. Johnson [9] considers a single profit maximizing principal as information broker. Admati and Pfleiderer [10] consider a monopolist who sells information by choosing a menu of noisiness.

Yet another class of models considers the intermediary as an agent with some particular capability that makes trading through intermediaries attractive to agents. Biglaiser [11] considers an intermediary who invests in quality detection. Diamond [12] considers intermediaries who are capable of costly monitoring on behalf of lenders. Rubinstein and Wolinsky [13] consider intermediaries who reduce search frictions between buyers and sellers. Their paper endogenously determines the activity of the intermediaries and their effects on the distribution of gains from trade. The current paper features intermediaries who can observe their own clients' private information and therefore extract larger rents from the principal on behalf of their clients than the agents could extract for themselves in a non-intermediated market.

Finally, a small trade literature explores the role of the intermediary as an access broker, an idea closely related to the reduction of search frictions in [13]. Notably, Antràs and Costinot [14] develop a general equilibrium using trade intermediation in a two island

model. They show that integration between traders and firms within an island always increases gains from trade, while the welfare effects of integration between traders from both islands are ambiguous. Another interpretation of intermediation is as a specific link in a larger trade network: Rauch [15] notes “the important role of intermediaries who can connect foreign agents to domestic networks.”

The rest of this paper proceeds as follows. Section 2 presents the model. Section 3 presents the dynamic mechanism. Section 4 presents the weak perfect Bayesian equilibrium of the dynamic mechanism. Section 5 describes the Vickrey-like auction, and constructs conditions under which agents ex ante prefer the dynamic mechanism to the Vickrey-like auction. Section 6 concludes.

2. The Model

For simplicity, consider the principal a seller who has two goods and the agents as buyers with unit demand. The principal s has two objects to allocate among NM unit-demand agents. Let $\mathcal{T} = \{(n, m) \mid n \in \{1, \dots, N\} \text{ and } m \in \{1, \dots, M\}\}$ denote the set of agents. Agent (n, m) privately observes her type t_{nm} . The t_{nm} are drawn independently across n and m according to a common, known distribution $F(t)$, with associated density $f(t)$ and support $\Theta = [0, \bar{t}] \subseteq [0, \infty)$. Let $t_{j,NM}$ denote the j th highest type among agents in \mathcal{T} , so that $t_{1,NM} \geq t_{2,NM} \geq \dots \geq t_{NM,NM}$. An allocation α is a $N \times M$ matrix with each entry $\alpha_{nm} \in \{0, 1\}$. Agent (n, m) receives an object under α exactly when $\alpha_{nm} = 1$. Let $|\alpha| = \sum_{n=1}^N \sum_{m=1}^M \alpha_{nm}$. An allocation is feasible if it respects the principal’s supply constraint.

Definition 1. A feasible allocation is an allocation α such that $|\alpha| \leq 2$.

Denote by \mathcal{A} the set of feasible allocations. Say agents $(n, m) \neq (n', m')$ are partners under α when $\alpha_{nm} = \alpha_{n'm'} = 1$.

2.1. Intermediaries

There are N intermediaries, $\{1, \dots, N\}$. Intermediary n represents agents $\mathcal{T}_n = \{(n, 1), \dots, (n, M)\}$ to the principal. An element of \mathcal{T}_n is a client of n . Let $\mathbf{t}_n = (t_{n1}, \dots, t_{nM})$ denote the vector of types of n ’s clients. Intermediary n privately observes \mathbf{t}_n .¹

Let $\mathbf{t}_{n,-m} = (t_{n1}, \dots, t_{n,m-1}, t_{n,m+1}, \dots, t_{nM})$. Let $t_{j,nM}$ denote the j th highest type among agents in \mathcal{T}_n (or alternatively, n ’s j th best client) so that $t_{1,nM} \geq t_{2,nM} \geq \dots \geq t_{M,nM}$. Define

$$\begin{aligned} \mathbf{T} &= (\mathbf{t}_1, \mathbf{t}_2, \dots, \mathbf{t}_N) \\ \mathbf{T}_{-n} &= (\mathbf{t}_1, \dots, \mathbf{t}_{n-1}, \mathbf{t}_{n+1}, \dots, \mathbf{t}_N) \\ \mathbf{T}_{-nm} &= (\mathbf{t}_1, \dots, \mathbf{t}_{n-1}, \mathbf{t}_{n,-m}, \mathbf{t}_{n+1}, \dots, \mathbf{t}_N). \end{aligned}$$

2.2. Valuations

Suppose (n, m) is assigned one object and the other object is assigned to an agent with type t . Agent (n, m) ’s valuation is $v(t_{nm}, t) = \lambda t_{nm} + (1 - \lambda)t$ with $\lambda \in [0, 1]$. An agent not assigned an object receives the reservation valuation 0. Agent (n, m) ’s valuation of allocation α is

$$v_{nm}(t_{nm}, \mathbf{T}_{-nm} \mid \alpha) = \alpha_{nm} v \left(t_{nm}, \sum_{n', m' \neq n, m} \alpha_{n'm'} t_{n'm'} \right). \tag{1}$$

The function $v_{nm}(\cdot, \cdot \mid \alpha)$ implies the following valuation function for intermediary n :

$$v_n(\mathbf{t}_n, \mathbf{T}_{-n} \mid \alpha) = \sum_{(n,m) \in \mathcal{T}_n} v_{nm}(t_{nm}, \mathbf{T}_{-nm} \mid \alpha). \tag{2}$$

Observe that $v(\cdot, \cdot)$ is weakly increasing in both arguments and weakly supermodular, which immediately implies that in the full information analog of this model, the unique efficient α allocates one object to each of the two highest type agents overall.²

3. The Dynamic Mechanism

The principal allocates the two objects via the dynamic mechanism Γ described below. The principal first allocates one object by contract (the “contracting round”) and subsequently allocates the remaining object by constrained Vickrey auction (the “auction round”). A contract is an ordered tuple $\mathbf{k} = (k_1, \dots, k_N)$ such that $k_n \in \{0, 1\}$ and $\sum_{n=1}^N k_n = 1$. Let $\mathbf{k}^n = (k_n, \mathbf{k}_{-n}) = (1, 0, \dots, 0)$. Say intermediary n is *contracted* if $k_n = 1$. The contract \mathbf{k}^n is a binding commitment by the principal to allocate one object to intermediary n , who in turn allocates it to the client of n ’s choice.

The timing of the dynamic mechanism Γ is:

1. Nature chooses \mathbf{t}_n ($n \in \mathcal{N}$); n privately observes \mathbf{t}_n .
2. The principal chooses the contract \mathbf{k} ; \mathbf{k} becomes common knowledge. Without loss of generality, assume $\mathbf{k} = \mathbf{k}^i$.
3. Intermediary i reports to the principal the name m_i^c of the agent to whom i allocates the first object; $t_{im_i^c}$ becomes common knowledge to the principal and i .
4. Intermediaries $n \in \mathcal{N}$ simultaneously and independently report to the principal the name and type m_n^a, \hat{t}_n^a of n ’s bidder for the second object.
5. The principal chooses assignment $\alpha(\mathbf{k}^i, m_i^c, \{m_n^a, \hat{t}_n^a\}_{n=1}^N)$ given by

$$\alpha_{nm} = \begin{cases} 1 & \text{if } (n, m) = (i, m_i^c) \text{ or } (j, m_j^a) \text{ such that } j = \operatorname{argmax}_n \hat{t}_n^a \\ 0 & \text{otherwise.} \end{cases}$$

6. The types $t_{im_i^c}, t_{jm_j^a}$ become common knowledge to the principal, i , and j .
7. The game ends; payoffs are
 - (a) $V_s = v(\max_{n \neq j} \hat{t}_n^a, t_{im_i^c})$
 - (b) If $i = j$, $V_i = V_j = v(t_{im_i^c}, t_{im_i^a}) + v(t_{im_i^c}, t_{im_i^a}) - v(\max_{n \neq i} \hat{t}_n^a, t_{im_i^c})$.
 - (c) If $i \neq j$, $V_i = v(t_{im_i^c}, t_{jm_j^a})$ and $V_j = v(t_{jm_j^a}, t_{im_i^c}) - v(\max_{n \neq j} \hat{t}_n^a, t_{im_i^c})$.
 - (d) If $n \neq i, j$, $V_n = 0$.

Per [5], there is no static, efficient direct mechanism which allows the principal to incentive compatibly elicit full information from the intermediaries, due to the multidimensional nature of each intermediary’s type and the interdependent valuations. Note that Γ is certainly not the only allocation mechanism available to the principal and I make no claims about whether Γ is second best. I merely posit that Γ is a plausible option in light of Theorem 4, and reasonable in the sense that the contracting round allocates one of the objects to an agent good enough in expectation, which simplifies the principal’s mechanism design problem selling the second object.

The principal strictly prefers that the contracted intermediary n allocate the object to $\operatorname{argmax}_m t_{nm}$. However, since types are private, the contracted intermediary can informationally hold up the principal, i.e., send (n, m) such that $t_{nm} = t_{2,nM}$. Sending (n, m) such that $t_{nm} = t_{2,nM}$ constitutes hold up because the intermediary n ’s power derives from the timing of Γ : intermediary n chooses m_n^c before the principal learns $t_{nm_n^c}$. The lack of payment in the contracting round creates a partial incentive for the contracted intermediary to informationally hold up. However, hold up creates ex post regret for the contracted intermediary if it subsequently loses the auction round. The payment rule in the auction round therefore partially incentivizes honesty in the contracting round as well as making the auction round truthful. The equilibrium analysis of Section 4 elaborates on this point.

The principal allocates the second object during the auction round using a constrained Vickrey auction. Let $\bar{t}_{-n} = \max_{j \neq n} \hat{t}_j^a$. Given $\alpha(\mathbf{k}^i, m_i^c, \{m_n^a, \hat{t}_n^a\}_{n=1}^N)$, intermediary n pays (and passes through to the agent who wins the auctioned object) the constrained Vickrey payment

$$\alpha_{nm_n^a} v(\bar{t}_{-n}, t_{im_i^c}). \tag{3}$$

Payoffs

Let

$$V_s(\mathbf{t}_n, \mathbf{T}_{-n} \mid \mathbf{k}^i, m_i^c, \{m_n^a, \hat{t}_n^a\}_{n=1}^N) = \sum_{n=1}^N \alpha_{nm_n^a} v(\bar{t}_{-n}, t_{im_i^c}) \tag{4}$$

$$V_n(\mathbf{t}_n, \mathbf{T}_{-n} \mid \mathbf{k}^i, m_i^c, \{m_n^a, \hat{t}_n^a\}_{n=1}^N) = v_n(\mathbf{t}_n, \mathbf{T}_{-nm} \mid \alpha(\mathbf{k}^i, m_i^c, \{m_n^a, \hat{t}_n^a\}_{n=1}^N)) - \alpha_{nm_n^a} v(\bar{t}_{-n}, t_{im_i^c}) \tag{5}$$

denote, respectively, the principal’s and intermediary n ’s payoffs given the sequence of actions $\mathbf{k}^i, m_i^c, \{m_n^a, \hat{t}_n^a\}_{n=1}^N$. Observe that since the intermediaries are ex ante identical, Equation (4) is independent of the principal’s strategy whenever the intermediaries adopt identical strategies. The expectation of Equation (5) at each information set determines the sequential rationality of an intermediary’s actions.

4. Perfect Bayesian Equilibrium

This section constructs the weak perfect Bayesian equilibria of Γ . For the remainder of the paper, I write “weak PBE.” Formally, a weak PBE is a strategy profile and a belief system such that (1) the strategy profile is sequentially rational at every information set given the belief system and (2) the belief system is derived using the strategy profile via Bayes’ rule.³ However, neither the principal’s nor the intermediaries’ beliefs change from their respective priors during the play of Γ . The principal moves just once in Γ , prior to actions by any of the intermediaries, so its beliefs do not change during the play of the game. Further, no intermediary acquires information about any other (beyond the common prior) during the play of Γ , so intermediary beliefs also do not change during the play of the game. Therefore, constructing the weak PBE of Γ merely requires sequential rationality at all information sets given the initial beliefs. Indeed, the proofs of Lemmas 2 and 3 do not rely on beliefs updated by Bayes’ rule whenever possible.

When $\mathbf{k} = \mathbf{k}^n$, say that n honors the contract if

$$m_n^c = \operatorname{argmax}_m t_{nm}$$

$$m_n^a = m \text{ such that } t_{nm} = t_{2,nM}$$

and say n dishonors the contract if

$$m_n^c = m \text{ such that } t_{nm} = t_{2,nM}$$

$$m_n^a = \operatorname{argmax}_m t_{nm}.$$

Consider the following strategy profile. Let

$$\sigma_s = \Pr(\mathbf{k}^n) = \frac{1}{N}. \tag{6}$$

For each $n \in \mathcal{N}$ let

$$\sigma_n = \begin{cases} m_n^a, \hat{t}_n^a = \operatorname{argmax}_m t_{nm}, t_{1,nM} & \text{if } \mathbf{k} \neq \mathbf{k}^n \\ m_n^c, m_n^a, \hat{t}_n^a = \text{honor}, t_{2,nM} & \text{if } \mathbf{k} = \mathbf{k}^n \text{ and } \\ & \lambda \geq \frac{\int_{t_{2,nM}}^{t_{1,nM}} [F(\bar{t}_{-n})]^{M(N-1)} d\bar{t}_{-n}}{t_{1,nM} - t_{2,nM}} \\ m_n^c, m_n^a, \hat{t}_n^a = \text{dishonor}, t_{1,nM} & \text{if } \mathbf{k} = \mathbf{k}^n \text{ and } \\ & \lambda < \frac{\int_{t_{2,nM}}^{t_{1,nM}} [F(\bar{t}_{-n})]^{M(N-1)} d\bar{t}_{-n}}{t_{1,nM} - t_{2,nM}}. \end{cases} \tag{7}$$

Lemmas 1–3 characterize sequentially rational actions in Γ and taken together show that the strategy profile given by Equations (6) and (7) describes the unique (up to permutation of the intermediaries) weak PBE of Γ .

Lemma 1. *Any lottery over \mathcal{K} is sequentially rational for the principal.*

Proof. See Appendix A. \square

Regardless of actions taken in the contracting round, truthful reporting is a weakly dominant action in the auction round.

Lemma 2. *During the auction round, it is sequentially rational for each intermediary to report truthfully the type of its best agent not allocated an object during the contracting round.*

Proof. See Appendix A. \square

Intermediary n learns its type \mathbf{t}_n and then (if contracted) chooses between honoring and dishonoring its contract. Lemma 3 characterizes the action of the contracted intermediary during the contracting round, while the uncontracted intermediaries necessarily take no action during the contracting round.

Lemma 3. *Suppose the principal announces $\mathbf{k} = \mathbf{k}^n$. Let*

$$\underline{\lambda} = \frac{\int_{t_{2,nM}}^{t_{1,nM}} [F(\bar{t}_{-n})]^{M(N-1)} d\bar{t}_{-n}}{t_{1,nM} - t_{2,nM}}. \tag{8}$$

It is sequentially rational for n to honor its contract whenever $\lambda \geq \underline{\lambda}$.

Proof. See Appendix A. \square

Equation (8) provides a lower bound on λ : it states that n weakly prefers to honor its contract whenever the weight placed on n 's client's type (λ) exceeds the average value of $[F(\bar{t}_{-n})]^{M(N-1)}$ on the interval $[t_{2,nM}, t_{1,nM}]$. Conversely, n strictly prefers to dishonor its contract when $\lambda < \underline{\lambda}$. When λ is small, the set of types such that the contracted intermediary prefers securing objects for each of its top two clients to securing an object for its best client while its second best client receives nothing is larger. However, the probability that a particular intermediary wins the auction round approaches zero when the number of intermediaries or number of clients per intermediary becomes large, so in the limit honoring the contract is sequentially rational for all λ .

Theorem 1. *The strategies in Equations (6) and (7) characterize the unique up to permutation of the intermediaries weak PBE of Γ , given N, M, λ , and the underlying distribution $F(\cdot)$. Furthermore, for absolutely continuous $F(\cdot)$*

$$\lim_{M \rightarrow \infty} \Theta^h = \lim_{N \rightarrow \infty} \Theta^h = \Theta^M. \tag{9}$$

Proof. See Appendix A. \square

4.1. Informational Hold Up

The constraint $\lambda \geq \underline{\lambda}$ is ambiguous; it may hold for some, all, or no \mathbf{t}_n , depending on $[F(\cdot)]^{M(N-1)}$. Generally, when the underlying distribution $F(\cdot)$ is continuous, then $\lambda \geq \underline{\lambda}$ holds for some but not all \mathbf{t}_n and Equation (8) describes a boundary between two sets

$$\Theta^h = \left\{ \mathbf{t}_n \in \Theta^M \mid \lambda \geq \frac{\int_{t_{2,nM}}^{t_{1,nM}} [F(\bar{t}_{-n})]^{M(N-1)} d\bar{t}_{-n}}{t_{1,nM} - t_{2,nM}} \right\}$$

$$\Theta^d = \left\{ \mathbf{t}_n \in \Theta^M \mid \lambda < \frac{\int_{t_{2,nM}}^{t_{1,nM}} [F(\bar{t}_{-n})]^{M(N-1)} d\bar{t}_{-n}}{t_{1,nM} - t_{2,nM}} \right\}$$

where Θ^h is the set of types for which the contracted intermediary honors its contract and Θ^d is the set of types for which the contracted intermediary dishonors its contract. The Bernoulli distribution illustrates that when $F(\cdot)$ is discontinuous, Equation (8) may hold for all or no \mathbf{t}_n . Let

$$[G(t)]^{M(N-1)} = \begin{cases} 0 & \text{if } t < 0 \\ p^{M(N-1)} & \text{if } 0 \leq t < 1 \\ 1 & \text{if } t \geq 1 \end{cases}.$$

If $\lambda \geq p^{M(N-1)}$, then $\lambda \geq \underline{\lambda}$ holds for all \mathbf{t}_n ; otherwise $\lambda \geq \underline{\lambda}$ does not hold for any \mathbf{t}_n . More generally, if $\lambda \geq [F(t)]^{M(N-1)}$ for all $t \in \Theta$ then $\Theta^h = \Theta^M$; if $\lambda < [F(t)]^{M(N-1)}$ for all $t \in \Theta$ then $\Theta^h = \emptyset$. Otherwise, Θ^h and Θ^d each have positive measure.

Loosely, a contracted intermediary rationally dishonors its contract when its top two clients' types are close and both types are reasonably high. Lemma 4 characterizes the boundary between Θ^h and Θ^d and makes this interpretation precise.

Lemma 4. *Suppose $F(\cdot)$ is strictly increasing. The boundary between Θ^h and Θ^d is*

$$\lambda = \frac{\int_{t_{2,nM}}^{t_{1,nM}} [F(\bar{t}_{-n})]^{M(N-1)} d\bar{t}_{-n}}{t_{1,nM} - t_{2,nM}} \tag{10}$$

and the boundary satisfies the following properties:

1. There exists a unique t_λ such that $[F(t_\lambda)]^{M(N-1)} = \lambda$, and $t_{1,nM} = t_{2,nM} = t_\lambda$ sits on the boundary.
2. Along the boundary, $t_{2,nM}$ decreases as $t_{1,nM}$ increases.
3. The boundary has slope -1 at (t_λ, t_λ) .

Proof. See Appendix A. \square

4.2. Expected Payoffs

Let t_{contract} denote the type of the contract winner, t_{auction} denote the type of the auction winner, and t_{loser} denote the type of the auction loser in the weak PBE of Γ . The agent who receives the contracted object receives valuation $\lambda t_{\text{contract}} + (1 - \lambda)t_{\text{auction}}$, pays 0, and receives payoff

$$\lambda t_{\text{contract}} + (1 - \lambda)t_{\text{auction}}. \tag{11}$$

The agent who wins the auctioned object receives valuation $\lambda t_{\text{auction}} + (1 - \lambda)t_{\text{contract}}$, pays $\lambda t_{\text{loser}} + (1 - \lambda)t_{\text{contract}}$, and receives payoff

$$\lambda(t_{\text{auction}} - t_{\text{loser}}). \tag{12}$$

Taking ex ante expectations over expressions (11) and (12), an agent's ex ante expected payoff under the weak PBE of Γ is

$$\begin{aligned} EV_{nm}^\Gamma &= \Pr((n, m) \text{ wins contract})E[\lambda t_{\text{contract}} + (1 - \lambda)t_{\text{auction}}] \\ &\quad + \Pr((n, m) \text{ wins auction})E[\lambda(t_{\text{auction}} - t_{\text{loser}})] \\ &= \frac{1}{NM}E[\lambda t_{\text{contract}} + t_{\text{auction}} - \lambda t_{\text{loser}}]. \end{aligned} \tag{13}$$

The ex ante expected total surplus is $EV^\Gamma = E[t_{\text{contract}} + t_{\text{auction}}]$.

5. Performance and Welfare Ranking

This section presents an agent’s ex ante expected payoff and the ex ante expected total surplus in a Vickrey-like auction run by the principal. I construct an upper bound on λ such that an agent ex ante prefers Γ to the Vickrey-like auction. Finally, I show that in large markets (1) each agent ex ante prefers Γ to the Vickrey-like auction and (2) the two mechanisms have the same expected total surplus.

5.1. Vickrey-like Auction and Expected Payoffs

The Vickrey-like auction elicits from each agent (n, m) a report \hat{t}_{nm}^V . Let $\hat{t}_{j,NM}^V$ denote the j th highest of the \hat{t}_{nm}^V . Truthful reporting is ex post incentive compatible (rather than weakly dominant) due to the interdependent valuations.⁴ If $\hat{t}_{nm}^V = \hat{t}_{1,NM}^V$ then (n, m) receives valuation $\lambda \hat{t}_{1,NM}^V + (1 - \lambda) \hat{t}_{2,NM}^V$, pays $\lambda \hat{t}_{3,NM}^V + (1 - \lambda) \hat{t}_{2,NM}^V$, and receives payoff

$$\lambda(\hat{t}_{1,NM}^V - \hat{t}_{3,NM}^V). \tag{14}$$

Similarly, if $\hat{t}_{nm}^V = \hat{t}_{2,NM}^V$ then (n, m) receives payoff

$$\lambda(\hat{t}_{2,NM}^V - \hat{t}_{3,NM}^V). \tag{15}$$

If $\hat{t}_{nm}^V < \hat{t}_{2,NM}^V$, agent (n, m) receives the reservation valuation 0. Under truthful reporting, $E[\hat{t}_{j,NM}^V] = E[t_{j,NM}]$; take ex ante expectations over Equations (14) and (15) to obtain an agent’s ex ante expected payoff under the Vickrey-like mechanism:

$$\begin{aligned} EV_{nm}^V &= \Pr(t_{nm} = t_{1,NM})E[\lambda(\hat{t}_{1,NM}^V - \hat{t}_{3,NM}^V)] \\ &\quad + \Pr(t_{nm} = t_{2,NM})E[\lambda(\hat{t}_{2,NM}^V - \hat{t}_{3,NM}^V)] \\ &= \frac{1}{NM}E[\lambda(t_{1,NM} + t_{2,NM} - 2t_{3,NM})]. \end{aligned} \tag{16}$$

The Vickrey-like auction is efficient in the usual sense that it allocates the objects to the two bidders with the highest types. The ex ante expected total surplus is $EV^V = E[t_{1,NM} + t_{2,NM}]$. While the Vickrey-like auction is not revenue-maximizing, the principal captures significant rent: the cost the top two bidders impose on the loser, which includes all of the interaction value in the agents’ valuations.

5.2. Welfare Ranking and Performance

Theorem 2. *Let*

$$\bar{\lambda} = \frac{E[t_{\text{auction}}]}{E[t_{1,NM} + t_{2,NM} - 2t_{3,NM} + t_{\text{loser}} - t_{\text{contract}}]}. \tag{17}$$

For all N, M , and $F(\cdot)$, $\bar{\lambda} > 0$. For all (n, m) , $EV_{nm}^\Gamma \geq EV_{nm}^V$ whenever $\lambda \leq \bar{\lambda}$.

Proof. See Appendix A. \square

Theorem 2 compares an agent’s ex ante expected payoff (ex ante expected surplus less ex ante expected transfer) between Γ and the Vickrey-like auction and provides an upper bound on λ : an agent weakly prefers Γ to the Vickrey-like auction whenever the weight $(1 - \lambda)$ placed on an agent’s partner’s type is high enough. Intuitively, an agent prefers Γ when she places sufficient weight on her partner’s type: the auction round guarantees a good partner to the agent who receives an object in the contracting round, and the agent who receives an object in the contracting round pays zero and thus receives all of the interaction value with that partner.

When the number of intermediaries is large and $F(\cdot)$ is absolutely continuous, agents always prefer Γ to the Vickrey-like auction.

Theorem 3. *Suppose $F(\cdot)$ is absolutely continuous. Then*

$$\lim_{N \rightarrow \infty} \bar{\lambda} = \lim_{N \rightarrow \infty} \frac{E[t_{\text{auction}}]}{E[t_{\text{loser}} - t_{\text{contract}}]} > 1. \quad (18)$$

Proof. See Appendix A. \square

Theorem 3 says that agents strictly benefit from the presence of intermediaries in large markets when the principal uses Γ to allocate the goods. Furthermore, as $N \rightarrow \infty$, the participation constraint on the agents from Theorem 2 and the “always honor” constraint on the intermediaries from Lemma 3 are both slack, so that $0 = \underline{\lambda} \leq \lambda \leq 1 < \bar{\lambda}$. Further, the slackness of these constraints implies that the asymptotic results hold even when the intermediaries extract positive rents from the agents. Finally, Γ is ex ante asymptotically efficient in the sense that it has the same ex ante expected total surplus as the Vickrey-like auction.

Theorem 4. *Suppose $F(\cdot)$ is absolutely continuous. Then as $M \rightarrow \infty$, Γ and the Vickrey-like auction have the same expected total surplus:*

$$\lim_{M \rightarrow \infty} E[t_{\text{contract}} + t_{\text{auction}}] = \lim_{M \rightarrow \infty} E[t_{1,NM} + t_{2,NM}]. \quad (19)$$

Proof. See Appendix A. \square

Theorem 4 says that the difference in total surplus between Γ and the Vickrey-like auction vanishes in probability. The dynamic mechanism Γ is thus a plausible alternative to the Vickrey-like auction in large markets; the presence of intermediaries provides an agent-preferred surplus division consistent with the claim that an intermediary is more able to exploit informational asymmetries than an individual agent.

6. Conclusions

This paper offers a novel dynamic mechanism for the allocation of multiple goods when agents have interdependent valuations. I construct the weak perfect Bayesian equilibrium of this mechanism, and provide conditions under which representation by intermediaries ex ante benefits agents while harming the principal. The agents’ preference for the dynamic mechanism stems from the fact that intermediaries are more able to exploit informational asymmetries than individual agents.

The dynamic mechanism uses a contracting round to reduce the dimensionality of the principal’s market design problem in the subsequent auction round. The existence of the contracting round itself is a form of informational rent in the sense that the principal gives away an object in exchange for information. However, an intermediary may informationally hold up the principal in the weak perfect Bayesian equilibrium, while the dynamic mechanism generally does not select the efficient allocation, in large markets the total surpluses of the dynamic mechanism and Vickrey-like auction converge in probability, offering intuition for the presence of intermediaries in large markets or organizations.

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Appendix A. Proofs

Proof of Lemma 1. Since the intermediaries are ex ante identical in expectation, all \mathbf{k} and therefore all lotteries over \mathcal{K} are sequentially rational. \square

Proof of Lemma 2. Equation (3) is the Vickrey payment rule, so truthful reporting of the name and type of n 's best agent not allocated an object during the contracting round follows from the standard weak dominance result in the Vickrey auction. \square

Proof of Lemma 3. Given $\mathbf{k} = \mathbf{k}^n$, the contracted intermediary n decides whether to honor its contract based on \mathbf{t}_n and its anticipation (following Lemma 2) that all intermediaries report truthfully during the auction round. When intermediary n honors its contract it receives

$$E_{\mathbf{T}_{-n}}[V_n(\mathbf{t}_n, \mathbf{T}_{-n} \mid \mathbf{k}^n, (\text{honor}, t_{2,nM}), \{m_i^a, \hat{t}_i^a\}_{i \neq n})] \tag{A1}$$

and when it dishonors its contract it receives

$$E_{\mathbf{T}_{-n}}[V_n(\mathbf{t}_n, \mathbf{T}_{-n} \mid \mathbf{k}^n, (\text{dishonor}, t_{1,nM}), \{m_i^a, \hat{t}_i^a\}_{i \neq n})]. \tag{A2}$$

It follows from Lemma 2 that the cumulative distribution function of $\bar{t}_{-n} = \text{argmax}_{j \neq n} \hat{t}_j^a$ is

$$[F(\bar{t}_{-n})]^{M(N-1)}. \tag{A3}$$

Substitute $v(t_{nm}, t) = \lambda t_{nm} + (1 - \lambda)t$, and Equations (1), (2) and (A3) into Equations (A1) and (A2) to obtain

$$\begin{aligned} & E_{\mathbf{T}_{-n}}[V_n(\mathbf{t}_n, \mathbf{T}_{-n} \mid \mathbf{k}^n, (\text{honor}, t_{2,nM}), \{m_i^a, \hat{t}_i^a\}_{i \neq n})] \\ &= [F(\bar{t}_{-n})]^{M(N-1)} E_{\mathbf{T}_{-n}}[\lambda t_{1,nM} + t_{2,nM} - \lambda \bar{t}_{-n} \mid \bar{t}_{-n} < t_{2,nM}] \\ & \quad + [1 - [F(\bar{t}_{-n})]^{M(N-1)}] E_{\mathbf{T}_{-n}}[\lambda t_{1,nM} + (1 - \lambda)\bar{t}_{-n} \mid \bar{t}_{-n} \geq t_{2,nM}] \end{aligned} \tag{A4}$$

$$\begin{aligned} & E_{\mathbf{T}_{-n}}[V_n(\mathbf{t}_n, \mathbf{T}_{-n} \mid \mathbf{k}^n, (\text{dishonor}, t_{1,nM}), \{m_i^a, \hat{t}_i^a\}_{i \neq n})] \\ &= [F(\bar{t}_{-n})]^{M(N-1)} E_{\mathbf{T}_{-n}}[t_{1,nM} + \lambda t_{2,nM} - \lambda \bar{t}_{-n} \mid \bar{t}_{-n} < t_{1,nM}] \\ & \quad + [1 - [F(\bar{t}_{-n})]^{M(N-1)}] E_{\mathbf{T}_{-n}}[\lambda t_{2,nM} + (1 - \lambda)\bar{t}_{-n} \mid \bar{t}_{-n} \geq t_{1,nM}]. \end{aligned} \tag{A5}$$

Intermediary n rationally honors its early admission contract in perfect Bayesian equilibrium for all \mathbf{t}_n such that Equation (A4) is greater than or equal to Equation (A5); expand the expectations in the resulting inequality to obtain

$$\begin{aligned} & \int_0^{\bar{t}} \lambda(t_{1,nM} - \bar{t}_{-n}) + \max\{t_{2,nM}, \bar{t}_{-n}\} d[F(\bar{t}_{-n})]^{M(N-1)} \\ & \geq \int_0^{\bar{t}} \lambda(t_{2,nM} - \bar{t}_{-n}) + \max\{t_{1,nM}, \bar{t}_{-n}\} d[F(\bar{t}_{-n})]^{M(N-1)}. \end{aligned}$$

Cancel common terms:

$$\begin{aligned} & \int_0^{\bar{t}} \lambda(t_{1,nM} - t_{2,nM}) d[F(\bar{t}_{-n})]^{M(N-1)} \\ & \geq \int_0^{\bar{t}} \max\{t_{1,nM}, \bar{t}_{-n}\} - \max\{t_{2,nM}, \bar{t}_{-n}\} d[F(\bar{t}_{-n})]^{M(N-1)}. \end{aligned}$$

Integrate the left hand side by parts and cancel common terms:

$$\lambda \geq \frac{\int_{t_{2,nM}}^{t_{1,nM}} [F(\bar{t}_{-n})]^{M(N-1)} d\bar{t}_{-n}}{t_{1,nM} - t_{2,nM}}. \tag{A6}$$

The right hand sides of inequality (A6) and Equation (8) are identical; the direction of inequality (A6) shows that it is sequentially rational for an intermediary to honor its contract whenever $\lambda \geq \underline{\lambda}$. \square

Proof of Theorem 1. Lemmas 1–3 characterize sequentially rational behavior at every information set given (trivially) updated beliefs. It follows that the strategies stated in Equations (6) and (7) characterize the unique up to permutation of the intermediaries weak perfect Bayesian equilibrium of Γ , given N, M, λ , and $F(\cdot)$.

When $F(\cdot)$ is absolutely continuous and at least one of $M, N \rightarrow \infty$, Equation (8) implies Equation (9):

$$\begin{aligned} \underline{\lambda} &= \lim_{M \rightarrow \infty} \frac{\int_{t_{2,nM}}^{t_{1,nM}} F(\bar{t}_{-n})^{M(N-1)} d\bar{t}_{-n}}{t_{1,nM} - t_{2,nM}} \\ &= \lim_{N \rightarrow \infty} \frac{\int_{t_{2,nM}}^{t_{1,nM}} F(\bar{t}_{-n})^{M(N-1)} d\bar{t}_{-n}}{t_{1,nM} - t_{2,nM}} \\ &= \frac{\int_{t_{2,nM}}^{t_{1,nM}} 0 d\bar{t}_{-n}}{t_{1,nM} - t_{2,nM}} \\ &= 0. \end{aligned}$$

\square

Proof of Lemma 4. Proof of the first property: It follows from Equation (10) that n is indifferent between honoring and holding up whenever the average value of $[F(\cdot)]^{M(N-1)}$ on the interval $[t_{2,nM}, t_{1,nM}]$ is λ :

$$\lambda = \frac{\int_{t_{2,nM}}^{t_{1,nM}} [F(\bar{t}_{-n})]^{M(N-1)} d\bar{t}_{-n}}{t_{1,nM} - t_{2,nM}}.$$

Since $[F(\cdot)]^{M(N-1)}$ is strictly increasing, there is a unique $t_\lambda \in \Theta$ such that $[F(t_\lambda)]^{M(N-1)} = \lambda$. Therefore, $t_{1,nM} = t_{2,nM} = t_\lambda$ sits on the boundary between Θ^h and Θ^d .

Proof of the second property: Implicitly differentiate Equation (10) with respect to $t_{1,nM}$ to obtain

$$\frac{dt_{2,nM}}{dt_{1,nM}} = \frac{[F(t_{1,nM})]^{M(N-1)} - \lambda}{[F(t_{2,nM})]^{M(N-1)} - \lambda}. \tag{A7}$$

It follows that $\frac{dt_{2,nM}}{dt_{1,nM}} < 0$ because either $t_{1,nM} > t_{2,nM}$, or else $t_{1,nM} = t_\lambda = t_{2,nM}$ and the third property holds.

Proof of the third property: Apply L'Hôpital's rule to Equation (A7) to obtain

$$\lim_{t_{1,nM} \rightarrow t_\lambda} \frac{dt_{2,nM}}{dt_{1,nM}} = \lim_{t_{1,nM} \rightarrow t_\lambda} \frac{f(t_{1,nM})[F(t_{1,nM})]^{M(N-1)-1}}{f(t_{2,nM})[F(t_{2,nM})]^{M(N-1)-1} \frac{dt_{2,nM}}{dt_{1,nM}}}.$$

The limit of the product is the product of the limits, ergo

$$\lim_{t_{1,nM} \rightarrow t_\lambda} \left(\frac{dt_{2,nM}}{dt_{1,nM}} \right)^2 = \lim_{t_{1,nM} \rightarrow t_\lambda} \frac{f(t_{1,nM})[F(t_{1,nM})]^{M(N-1)-1}}{f(t_{2,nM})[F(t_{2,nM})]^{M(N-1)-1}}.$$

Since $(t_{1,nM}, t_{2,nM})$ is on the boundary, $t_{2,nM} \rightarrow t_\lambda$ as $t_{1,nM} \rightarrow t_\lambda$

$$\lim_{t_{1,nM} \rightarrow t_\lambda} \left(\frac{dt_{2,nM}}{dt_{1,nM}} \right)^2 = 1.$$

In order to satisfy Equation (10), an increase in $t_{1,nM}$ above t_λ must be offset by a decrease in $t_{2,nM}$ below t_λ , ergo

$$\left. \frac{dt_{2,nM}}{dt_{1,nM}} \right|_{t_{1,nM}=t_\lambda} = -1.$$

□

Proof of Theorem 2. An agent weakly prefers Γ to the Vickrey-like auction whenever $EV_{nm}^\Gamma \geq EV_{nm}^V$; comparing Equations (13) and (16) obtain

$$E[\lambda t_{\text{contract}} + t_{\text{auction}} - \lambda t_{\text{loser}}] \geq E[\lambda(t_{1,NM} + t_{2,NM} - 2t_{3,NM})]$$

$$\lambda \leq \frac{E[t_{\text{auction}}]}{E[t_{1,NM} + t_{2,NM} - 2t_{3,NM} + t_{\text{loser}} - t_{\text{contract}}]}. \quad (\text{A8})$$

The right hand sides of inequality (A8) and Equation (17) are identical; the direction of inequality (A8) shows that an agent ex ante prefers Γ to the Vickrey-like auction whenever $\lambda \leq \bar{\lambda}$.

Now show that $\bar{\lambda} > 0$ for all N, M , and $F(\cdot)$. Observe the following:

1. If $t_{\text{contract}} = t_{1,NM}$, then $t_{\text{auction}} = t_{2,NM}$ and $t_{\text{loser}} = t_{3,NM}$.
2. If $t_{\text{contract}} = t_{2,NM}$, then $t_{\text{auction}} = t_{1,NM}$ and $t_{\text{loser}} = t_{3,NM}$.
3. If $j \geq 3$ and $t_{\text{contract}} = t_{j,NM}$, then $t_{\text{auction}} = t_{1,NM}$ and $t_{\text{loser}} = t_{2,NM}$.

Therefore, $E[t_{1,NM}] > E[t_{\text{contract}}]$, $E[t_{\text{loser}}] > E[t_{3,NM}]$ and $E[t_{\text{auction}}] \geq 0$, i.e., the numerator and denominator of the right hand side of inequality (A8) are positive. □

Proof of Theorem 3. When $F(\cdot)$ is absolutely continuous, $E[t_{1,NM} + t_{2,NM} - 2t_{3,NM}] \rightarrow 0$ as $N \rightarrow \infty$. Per Theorem 1, $\Theta^h \rightarrow \Theta^M$ as $N \rightarrow \infty$, so that

$$\lim_{N \rightarrow \infty} E[t_{\text{contract}}] = \lim_{N \rightarrow \infty} E[t_{1,nM}] = E[t_{1,nM}]. \quad (\text{A9})$$

Let

$$\beta_j = \Pr(t_{\text{contract}} = t_{j,NM} | \Theta^h = \Theta^M)$$

$$= \begin{cases} \frac{1}{N} \frac{\binom{NM-j}{M-1}}{\binom{NM-1}{M-1}} & \text{if } j \leq NM - M + 1 \\ 0 & \text{otherwise.} \end{cases} \quad (\text{A10})$$

It is easy to verify that $\lim_{N \rightarrow \infty} \beta_j = 0$, so that

$$\lim_{N \rightarrow \infty} E[t_{\text{auction}}] = \lim_{N \rightarrow \infty} E[\beta_1 t_{2,NM} + (1 - \beta_1) t_{1,NM}] = \lim_{N \rightarrow \infty} E[t_{1,NM}] \quad (\text{A11})$$

$$\lim_{N \rightarrow \infty} E[t_{\text{loser}}] = \lim_{N \rightarrow \infty} E[(\beta_1 + \beta_2) t_{3,NM} + (1 - \beta_1 - \beta_2) t_{2,NM}] = \lim_{N \rightarrow \infty} E[t_{2,NM}]. \quad (\text{A12})$$

Combine Equations (A9), (A11) and (A12) to show that Equation (18) holds:

$$\begin{aligned}
\lim_{N \rightarrow \infty} \bar{\lambda} &= \frac{E[t_{\text{auction}}]}{E[t_{\text{loser}} - t_{\text{contract}}]} \\
&= \lim_{N \rightarrow \infty} \frac{E[t_{1,NM}]}{E[t_{2,NM}] - E[t_{1,nM}]} \\
&= \lim_{N \rightarrow \infty} \frac{E[t_{1,NM}]}{E[t_{1,NM}] - E[t_{1,nM}]} \\
&> 1.
\end{aligned}$$

□

Proof of Theorem 4. From the proof of Theorem 2 and the definition of the β_j in Equation (A10), construct Equation (19)

$$\begin{aligned}
\lim_{M \rightarrow \infty} E[t_{\text{contract}} + t_{\text{auction}}] &= \lim_{M \rightarrow \infty} E[t_{1,nM} + (1 - \beta_1)t_{1,NM} + \beta_1 t_{2,NM}] \\
&= \bar{t} + (1 - \beta_1)\bar{t} + \beta_1 \bar{t} \\
&= 2\bar{t} \\
&= \lim_{M \rightarrow \infty} E[t_{1,NM} + t_{2,NM}]
\end{aligned}$$

□

Notes

- ¹ Consider an intermediary a pass through entity: an intermediary observes the types of its clients at zero cost, transmits information (possibly untruthfully) to the principal, and passes on to each client the object allocated by and payment owed to the principal. In large markets, the constraints of Theorems 1 and 3 are slack, so the main results of the paper hold in large markets even when the intermediary incurs some cost to observe t_n and charges its clients a representation fee, so long as the cost and fee are small.
- ² Shapley and Shubik [16] were the first to show that a match value function weakly increasing in both arguments and weakly supermodular implies the unique efficiency of positive assortative matching. In this context, the match is the assignment of objects to agents. The linear interdependence of $v(\cdot, \cdot)$ closely follows [5]. Another common valuation function with these properties is $v(a, b) = ab$ as in [17]. Other papers, including [8,9,18], exploit these assumptions in their incomplete information environments.
- ³ Mas-Colell, Whinston, and Green [19] provide a standard definition of weak PBE (p. 285) and discussion of the concept (pp. 283–285).
- ⁴ For a full discussion, see [18].

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