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The Art of Sharing Resources: How to Distribute Water during a Drought Period

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Abstract: Water scarcity is a growing problem in many regions worldwide. According to the United Nations, around one-fifth of the world's population lives in areas where water is scarce. Another one-quarter of the world's population has to face water supply cuts, mainly because this proportion of the population lacks the necessary infrastructure to acquire water from rivers and aquifers (UN, 2005). Water is a resource that is essential to human survival and is also present in all productive processes in the economy. Therefore, we are challenged to adequately manage water to ensure the population's well-being and to achieve socioeconomic development. Specifically, this paper analyzes the situation present in the summer of 2022 at Riudecanyes (a village in Catalonia, Spain), where a drought problem exists. We propose applying the conflicting claims problem theory to give possible solutions to distribute the water. We propose to use this theory to describe the distribution of the available irrigation hours in 2022, considering the demand made by the farmers in the previous year, when there was regular irrigation.

Keywords: water scarcity; conflicting claims problems; drought

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1. Introduction

As is well known, water is a resource that is necessary for the functioning of the economy and is used by the population in everyday activities, food, culture, health, and education. Therefore, the main problem at present is to satisfy the water demands of the energy, agriculture, and water sector. According to the Director General of the United Nations Educational, Scientific, and Cultural Organization (UNESCO), "More and more water resources are being polluted, over-exploited, and dried up by humans, sometimes with irreversible consequences". Moreover, there are several risks related to water, such as shortages, droughts, floods, lack of access to this resource, and environmental degradation. Also, it is essential to emphasize other problems that contribute to scarcity, such as demographic changes and global warming.

Regarding demographic change, according to the United Nations, in November 2022, the world's population will have reached 8 billion people, which is considered to be significant in human development. However, by 2037 the population is expected to reach 9 billion people, which represents a decrease in the growth rate (see Figure 1). Therefore, although a reduction in birth may be observed, an improvement in the inhabitants' quality of life must be ensured. To achieve this, the population should have the necessary resources, such as food, energy, and water. With regards to the effects of global warming, severe weather events, such as droughts, environmental degradation, changes in aquatic ecosystems, and the loss of water quality, among others, should be considered. Therefore, a more intelligent use of sparse water resources is required, as well as their protection from pollution and over-exploitation, because these effects seriously affect human health, food security, and economic stability.

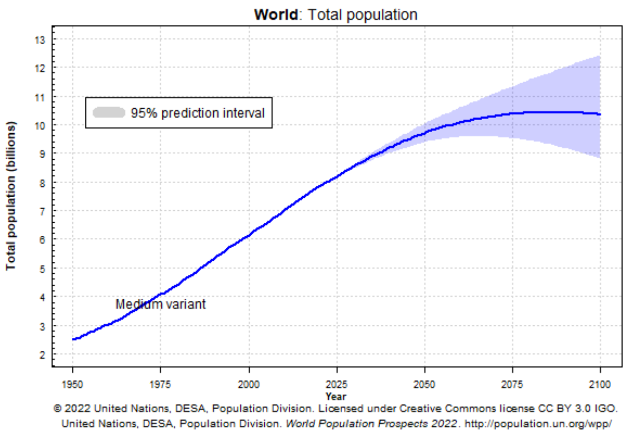

Figure 1. Total population. The 2022 Revision of World Population Prospects, UN.

Globally, groundwater provides about 50% of all drinking water and 43% of all agricultural irrigation (Food and Agriculture Organization of the United Nations, FAO, 2023). According to the UN, groundwater withdrawal for human use is very important. Still, it corresponds to only one category of the services offered by groundwater systems (including provisioning services, see Figure 2). If the groundwater level declines, there may be some problems such as increasing costs (because of the technical complexity and energy requirements of groundwater extraction), increasing water scarcity, the degradation of ecosystems, competition between groundwater user sectors (or between individual wells); and increasingly inequitable access to this resource. As already mentioned, water is essential for agricultural production.

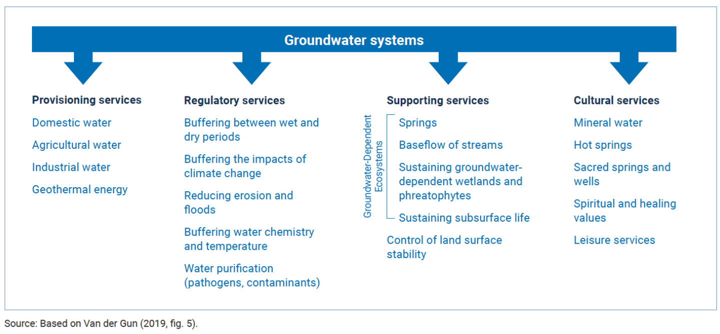

Figure 2. Groundwater functionality. The United Nations World Water Development Report 2022.

In this paper, we focus on water scarcity caused by a continued drought at the Riudecanyes reservoir in Catalonia, Spain. Specifically, we focus on how to distribute water for irrigation and private consumption. Ref. [1] defined water scarcity as an imbalance between the supply and demand or when excessive demand exists for the available supply. The Spanish ministry for ecological transition and the demographic challenge reported Spain's water reserves as 48.9 percent of its capacity[1].

During recent years in Spain, there have been several drought seasons in different regions of the country, thus significantly affecting the agriculture sector and the water supply. The most affected parts are the peninsula's interior and some Mediterranean coastal areas, mainly because rainfall is scarce, and the average temperatures are increasing. Figure 3 shows a drought monitoring map for Spain, which clearly shows the areas at risk of shortages.

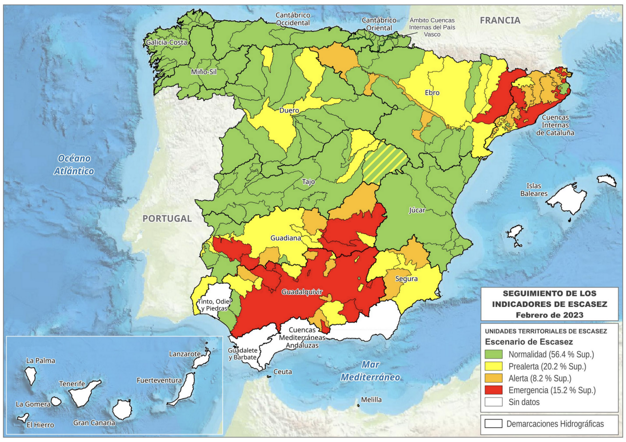

Figure 3. Drought monitoring map; different levels are indicated by the following colors: (1) **Normal**, (2) **Pre-alert**, (3) **Alert**, (4) **Emergency**, and (5) **no data available**. Ministerio para la Transición Ecológica y el Reto Demográfico, 2023.

As a result of this situation, the State has approved of drought plans intending to minimize the environmental, economic, and social impacts when shortages occur in the regions. These plans define four levels or thresholds for the progressive severity of droughts:

1. Normal.
2. Pre-alert.
3. Alert.
4. Emergency.

The main objective of these plans is to anticipate problems by knowing when a reservoir moves from one level to another, thereby enabling action protocols to be implemented, which include the following:

1. Monitoring and controlling measures at all levels.
2. Management measures that are considered from the alert level and higher (temporary agreements, specific changes in some concessions, the drafting of a royal decree on a drought, etc.).
3. With respect to the alert or emergency levels, restriction measures can be established, such as reducing the irrigation allocation and prioritizing human consumption.

We analyzed the case of the Riudecanyes community of irrigators at the Riudecanyes reservoir in Tarragona, Catalonia. The Riudecanyes reservoir was built in 1950 due to a severe drought season. Today, the Riudecanyes community still owns the reservoir, which receives water from the Riudecanyes stream and the Siurana reservoir's hydrographic system (see Figure 4). The problem appeared in 2022, which was characterized by a great drought period with reservoirs at a minimum levels. During the summer season, farmers in the country (Camp de Tarragona) need water for irrigation. Following the recommendations for the emergency level, in this paper, we propose an alternative way to distribute water to meet the needs of the farmers and to take care of this resource.

The first intuitive choice to look for a fair distribution might be cooperative games, where individuals join efforts to share gains or costs, thus looking for a better division in each situation. In such a scenario, the amount to be divided is big enough to satisfy the players (or smaller in the case that they share costs). Due to the nature of the analyzed problem, our approach must be different, as the amount to transfer does not guarantee agents' claims (every agent will receive less than their claim). To overcome this issue, we propose to study this case from the point of view of the conflicting claims problems theory [2]. A claims problem involves a set of agents demanding a share of some (perfectly divisible) endowment. It is a conflicting claims problem if the endowment can only partially honor some claims. We consider the supply of water from the reservoir as the endowment, and the claims are the amounts required by the owners of the plots based on their past demands (when there was no drought). Now, due to scarcity, the available water is not enough to satisfy the agents' claims. In the literature, several numbers of ways of modeling conflicts as games have been proposed. There are different types of solutions for games as starting points in defining the rules to solve claims problems. One of the approaches to study claims problems is the game theoretic approach; it consists of modeling the considered situation as a game, then applying a solution concept that is developed in the theory of games to solve the game. The context will affect the model that we adopt to solve the problem; in particular, it will affect whether the behavioral assumptions underlying it are strategic or cooperative. In our case, due to the situation and the data we have, we concerned ourselves with properties and axiomatic evidence instead of solutions, since they suited our case more.

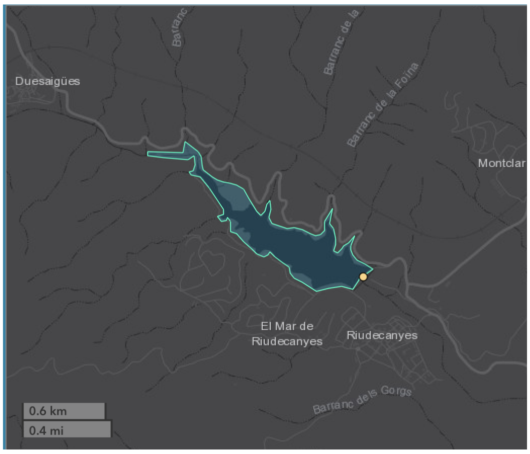

Figure 4. Riudecanyes reservoir. Boletín Hidrológico Peninsular, 2023.

Related Literature

Acording to the FAO, "If we don't change our habits now, global water demand could increase by 50 percent by 2030". Goal number six of the sustainable development goals of the United Nations is to ensure access to water and sanitation for all.

Water scarcity is a global concern. For this reason, many authors have studied this problem. Ref. [3] focused on studying water demand in periods of scarcity. They analyzed tools that help improve irrigation methods, such as distribution uniformity, as well as supplemental and deficit irrigation strategies. Finally, emerging technologies have been advocated to manage water use and management. Ref. [4] focused on the challenges regarding water management by studying the various proposals to improve productivity in water distribution. For example, it analyzes and compares some recommendations such as reducing the production of water-intensive products, managing the supply and demand in the water, energy, and agriculture sectors, using more efficient irrigation systems and household appliances, reducing leakages in water infrastructures, and making investments in water infrastructures and technologies, among others. The authors proposes using well-established sectoral-oriented models and creating a link between them. Ref. [5] reviewed the water deficit and proposed a method that optimized irrigation for cultivating indigo woad root with water-saving techniques in China. Ref. [6] evaluated the irrigation strategies in a water-scarse area in the central United States. The authors focused on how irrigation strategies impact economic value. The strategies are fixed (the same process used for every year), and potential (perfect adjustment to each year's weather) strategies and productivity were compared between them. To achieve the objective, the authors applied a crop simulation model. Ref. [7] studied the case of Kenya, which is known as a water-scarce country due to its low renewable freshwater supply (<1000 m^3/capita/year). Alternatives were sought to address these problems, and various solutions were proposed, such as the regulation of water-use rights; the regulation and management of the water supply and sewerage services; water sanitation (all of which are contemplated in the strategic plan through the construction of large and medium dams to store water, as well as the investment in groundwater storage); and water management. Ref. [8] examined the problems and possible solutions to persistent drought periods in Iraq. Ref. [9] studied the water allocation in Ghba Subbasin, Ethiopia. The authors estimated the demands of the sectors that use this resource the most and proposed a distribution based on this planning. These estimation methods may help reduce uncertainty in water allocation, optimize the utilization of water resource projects, and so on.

Therefore, these studies are relevant to provide more knowledge about the water sector, the effects of climate change, the problems, and the possible solutions to water scarcity. For this reason, we propose another approach, "the conflicting claims problems theory", to give an alternative solution for water distribution between farmers in the Riudecanyes area in Catalonia.

Upon analyzing the literature, we have found that many authors have applied this theory to various economic areas. For instance, in the negotiations with respect to CO_2emissions, Refs. [10,11] proposed a means to distribute CO_2 quotas to reduce pollution in the world. Regarding education, Ref. [12] put forward an efficient allocation of university funds among the departments through conflicting claims theory. Ref. [13] suggested the European Regional Development Fund (ERDF) distribution as a conflicting claim problem to achieve economic convergence among the concerned regions. Other authors also studied this subject , such as [14–16] who proposed a formal framework for rural development budget allocation using proper division techniques. Another example is the fishing sector [17–19], wherein fishing quotas have been proposed to find solutions to fish shortages. In addition, in the health sector, during the 2008 crisis, the Spanish government decided to cut public spending. Ref. [20] proposed to distribute the available budget for health care in Catalonia by applying distribution rules and inequality indexes.

This theory has been applied to study water problems. For example, Ref. [21] presented a scenario in which multiple agents were located along a river and produced waste that needed to be removed to restore the river to its natural state. The cost of cleaning was shared among all agents involved. The author applied an axiomatic analysis, as well as different rules of claims problems to propose a distribution; the results showed that a rule matched the Shapley value. Ref. [22] studied the case of the Nile basin, since the existing water demands generate disputes about sharing this resource. The authors applied the allocation rules and proposed an innovative way to account for the water contribution of the riparian states; they also proposed a mechanism for weighing the water deficit that they have allocated. In addition, Ref. [23] analyzed the Euphrates River Basin (ERB) with three littoral states from a conflicting claims perspective. Likewise, within the literature on the distribution of water resources, there are authors who study this problem from the perspective of cooperative game theory, considering that a greater distribution can be achieved through joint efforts. For example, Ref. [24] argued that cooperative strategies increased the allocation of water resources in countries with high utility and enhanced the overall utility of water throughout the basin. To explore this, they proposed a fuzzy coalition model for water resource allocation with spatial–temporal constraints to verify if a more beneficial distribution could be reached. Ref. [25] examined the case of water allocation in the Lancang-Mekong River Basin among five countries: China, Thailand, Laos, Cambodia, and Vietnam. The authors applied three cooperative game methods (the Shapley value, the Gately point, and Nash–Harsanyi solution) to find possible allocation solutions that maximized the benefits obtained by the countries.

In our scenario, we considered using claims problems solutions to propose an initial water allocation to achieve better resource use and to provide farmers with the necessary amount for their crops. In doing so, we considered the water shortage problem as a claims problem. We applied some solutions, called rules, and compared their proposals based on an inequality. For this, we use the Gini index of inequality. We also use some socially accepted axioms to obtain the best solution. As a consequence, we ascertained that the constrained equal awards rule (CEA) is the one that works best from both perspectives regarding axiomatic analysis and inequality.

The remainder of the paper is organized as follows. Section 2 formally presents the definition of the conflicting claims problem and some of the main solutions considered in the literature. Section 3 presents the case of the Riudecanyes reservoir as a conflicting claims problem and the application of solutions. Section 4 analyzes and compares the proposed allocations among those involved, and some final comments in Section 5 conclude the paper. Finally, all the tables with the data can be found in the Appendix A.

2. Conflicting Claims Problems

A *claims problem* appears whenever several economic and/or social actors, *the agents*, claim a part of some (perfectly divisible) *endowment*. If the endowment cannot fully honor all the claims, it is a *conflicting claims problem*. The typical example is that of a *bankruptcy problem*, wherein a firm does not have enough assets to pay all its debts, and the firm's assets (the endowment) must be distributed among its creditors. Another example would be the division of an estate among several heirs, particularly when the estate cannot meet all the deceased's commitments.

Although some references to this situation appear in the ancient literature (2000-year-old Babylonian Talmud), the modern literature begins with the seminal paper by [2], which also originated in a Talmud rights arbitration problem.

Next, we formally define the problem and some well-known rules from the literature.

We study problems where the *endowment* $E \in \mathbb{R}_+$ must be divided among a group of *agents*, where $N = \{1, 2, \ldots, n\}$. Each agent $i \in N$ is identified by her *claim* $c_i \in \mathbb{R}_+$ on the endowment E. We will denote this with $c = (c_1, c_2, \ldots, c_n)$, which is the vector of claims. The *aggregate claim* C is given by $C = \sum_{i=1}^{n} c_i$. A *claims problem* appears whenever the aggregate claim is greater than the available endowment; that is, $C > E$.

The pair (E, c) represents the claims problem, and we denote this with $\mathcal{B}$ to represent the set of all claims problems.

A *rule* (solution) is a single-valued function $\varphi : \mathcal{B} \to \mathbb{R}^N_+$ such that, for each agent $i \in N$, the following conditions are satisfied:

(i) $0 \leq \varphi_i(E, c) \leq c_i$ (*non-negativity* and *claim-boundedness*).

(ii) $\sum_{i=1}^{n} \varphi_i(E, c) = E$ (*efficiency*).

That is to say, no agent receives either a negative amount or an amount exceeding her claim, and the endowment E is completely distributed among the agents.

Many solution concepts about claims problems have been defined in the literature (see for instance [26,27]). Next, we define some commonly used claims rules.

The **proportional** rule (P) establishes that the endowment will be divided proportionally to the claim of the agents.

For each $(E, c) \in \mathcal{B}$ and each agent $i \in N$, $P_i(E, c) \equiv \lambda c_i$, $\lambda = \dfrac{E}{C}$.

The **constrained equal awards** rule (CEA) ([28]) balances the amount that each agent receives such that no agent obtains more than their claim.

For each $(E, c) \in \mathcal{B}$ and each agent $i \in N$, $CEA_i(E, c) \equiv \min\{c_i, \lambda\}$, λ is chosen so that $\sum_{i=1}^{n} \min\{c_i, \lambda\} = E$.

The **constrained equal losses** rule (CEL) ([28,29]) studies the problem from the point of view of losses (what the agents do not receive regarding their claims) and suggests equalizing losses, thus establishing that no agent receives a negative amount.

For each $(E, c) \in \mathcal{B}$ and each agent $i \in N$, $CEL_i(E, c) \equiv \max\{0, c_i - \lambda\}$, and λ is chosen so that $\sum_{i=1}^{n} \max\{0, c_i - \lambda\} = E$.

The **Talmud** rule (T) [29] is a combination of the CEA and the CEL rules, which takes into account half of the aggregate claim C as a reference. If the aggregate claim C is lower than the available resource E, then the CEA rule is applied over the half claims. Otherwise, each agent receives half of their claim, and the CEL rule is used to distribute the remaining endowment for the remaining claims (the other half).

For each $(E, c) \in \mathcal{B}$,

$$T(E,c) \equiv \begin{cases} CEA(E, \frac{1}{2}c) & \text{if} \quad E \leq \frac{1}{2}C. \\ \\ \frac{1}{2}c + CEL(E - \frac{1}{2}C, \frac{1}{2}c) & \text{if} \quad E > \frac{1}{2}C. \end{cases} \tag{1}$$

The $\boldsymbol{\alpha^{\min}}$egalitarian rule ($\boldsymbol{\alpha^{\min}}$) [30] ensures a minimum amount to each agent. All agents first receive an amount that coincides with the lowest claim, if this is possible. Then, the remaining endowment is distributed proportionally to the reduced claims (the initial claims minus the amount already received). If the endowment does not allow each agent to receive at least the lowest claim, then all agents receive the same amount $\frac{E}{n}$. Formally, for each $(E, c) \in \mathcal{B}$,

$$\alpha^{\min}(E,c) \equiv \begin{cases} (\frac{E}{n})\mathbf{1} & \text{if } E \leq nk, \\ \\ k^1 + P(E - nk, c - k^1) & \text{otherwise,} \end{cases} \tag{2}$$

where n is the number of agents, $k = \min\{c_1, \ldots, c_n\}$, $k^1 = (k, \ldots, k)_{1 \times n}$, and $\mathbf{1} = (1, \ldots, 1)_{1 \times n}$.

The next example shows how the previous rules work.

Example 1. *Let* $(E, c) = (600, (100, 200, 300, 400))$.

The $P(E, c) = (60, 120, 180, 240)$*; here, the endowment is divided proportionally to each agent's claim.*

The $CEA(E, c) = (100, 166.66, 166.66, 166.66)$*; here the first agent is fully compensated, and the remaining endowment is divided equally among the rest of the agents.*

The $CEL(E, c) = (0, 100, 200, 300)$ *divides the losses* ($L = 1000 - 600$) *equally among the agents such that each agent receives the subtraction of her claim and the equal portion of the losses.*

For the $T(E, c) = (50, 100, 175, 275)$*, each agent receives half of his claim, and the rest of the endowment for the other half of the claims is distributed by the CEL.*

The $\alpha^{min}(E, c) = (100, 133.33, 166.66, 200)$ *allocates a minimal right to all agents that is equal to* $c_1 = 100$*; then, the claims are revised down by the minimal right and the proportional rule is applied to allocate the remaining endowment.*

The Socially Accepted Properties: Axiomatic Analysis

In this section, we apply an axiomatic approach to analyze the previously considered claims rules. The minimal requirements set consists of the following properties: *the equal treatment of equals*, *anonymity*, *order preservation*, and *resource monotonicity*.

Note that these properties ensure that no discrimination between agents (farmers), as only the claim matters. Farmers with larger claims would not receive a smaller allocation than those with less needs. In Table 1, we observe that all these properties are satisfied by all the proposed rules. In addition, we used the *reasonable lower bounds on awards*, which implies ensuring a minimum amount for each farmer so that no area can be left with nothing. Therefore, we conclude that, by applying this property, we obtain a fair distribution.

The equal treatment of equals states that all agents with equal claims should receive an equal allocation. For each $(E, c) \in \mathcal{B}$, and each $i, j \in N$, such that $c_i = c_j$, then $\varphi_i(E, c) = \varphi_j(E, c)$.

Anonymity means that the identification of the agents should be ignored in the process of allocation. The only factor recognized as the allocation base is the agents' claims. For each $(E,c) \in \mathcal{B}$, such that $\pi \in \Pi^N$, and for each $i \in N$, then $\varphi_{\pi(i)}(E,c') = \varphi_i(E,c)$, where $c' \equiv (c_{\pi(j)})_{j \in N}$.

Order preservation [29] states that if the agent i's claim is smaller than agent j's claim, the agent i cannot receive an allocation greater than agent j's allocation. For each $(E,c) \in \mathcal{B}$ and each $i,j \in N$ such that $c_i \geq c_j$, then $\varphi_i(E,c) \geq \varphi_j(E,c)$, and $c_i - \varphi_i(E,c) \geq c_j - \varphi_j(E,c)$.

Reasonable lower bounds on awards ensures that each individual receives at least the minimum of her claim, as well as the endowment divided by the number of individuals. For each $(E,c) \in B$ and each $i \in N$, then $\varphi_i(E,c) \geq \frac{\min\{c_i,E\}}{n}$.

Composition down [31] requires that, if after distributing the endowment, this decreases, two options are available: First, cancel the initial allocation and apply the rule for the revised endowment. Second, consider the agents' initial awards as their claims and apply the rule to allocate the revised endowment in this situation. Both ways should lead to the same award vector. For each $(E,c) \in B$, each $i \in N$, and each $0 \leq E' \leq E$, $\varphi_i(E',c) = \varphi_i(E',\varphi(E,c))$.

Claims truncation invariance [32,33] states that agents cannot demand more than the available endowment. If an agent's claim is larger than the endowment, the part of the claim that exceeds the endowment is ignored. For each $(E,c) \in B$, $\varphi(E,c) = \varphi(E,\min\{c_i,E\})$.

Claim monotonicity [26] states that, if the agents' claims increase, they should receive at least as much as they did initially. For each $(E,c) \in \mathcal{B}$, each $i \in N$, and each $c'_i > c_i$, $\varphi_i(E,(c'_i,c_{-i})) \geq \varphi_i(E,c)$.

With the following table, we summarize the results by showing the axioms satisfied by the analyzed rules. We note that the CEA is the only rule that satisfies all these basic properties. The application of all these properties determines how the hours of irrigation water should be allocated, which could be considered a crucial aspect for the distribution to be accepted by the farmers.

Table 1. Properties and rules: The table shows which properties are satisfied by the rules considered. Each column corresponds to a rule, whereas each row corresponds to the proposed axiom. The results in the table can be found in [34].

Properties/Rules	P	CEA	CEL	T	$\alpha^{\min}$
Equal treatment of equals	Yes	Yes	Yes	Yes	Yes
Anonymity	Yes	Yes	Yes	Yes	Yes
Order preservation	Yes	Yes	Yes	Yes	Yes
Resource monotonicity	Yes	Yes	Yes	Yes	Yes
Reasonable lower bounds on awards	No	Yes	No	Yes	Yes
Composition down	Yes	Yes	Yes	No	Yes
Claims truncation invariance	No	Yes	No	Yes	No
Claim monotonicity	Yes	Yes	Yes	Yes	Yes

3. The Distribution of Riudecanyes Reservoir Water as a Conflicting Claims Problem

In this section, we present our scenario for the case of the Riudecanyes reservoir. 2022 was a period of significant drought, meaning that the reservoir was at a minimum level. Normally, during the summer, the farmers in Camp de Tarragona need water from the Riudecanyes reservoir to irrigate their crops. As we can see in Figure 5, where we have data from three hydrological years, the hydrological reserves for 2022 were lower than those of previous years.

Therefore, in 2022, the distribution of irrigation water was different from other summers, because it was marked by water scarcity. The Riudecanyes Reservoir Irrigation Community, which distributes this water, did not have enough water to guarantee irrigation in subsequent years. Additionally, it must be taken into account that there must always be a certain minimum amount of water in the reservoir. It must be noted that the water requested by the farmers is measured in irrigation hours. Therefore, the Camp de Tarragona farmers claimed a number of irrigation hours that was similar to those they received in previous years according to the irrigation rights of each member. The Riudecanyes Reservoir Irrigation Community was not able to allocate these hours, because it had fewer irrigation hours to distribute. In the summer of 2022, the Riudecanyes Reservoir had a maximum capacity of 4.5 Hm^3[2]. In a normal summer, that is to say, without drought, about 7.5 Hm^3 are needed to fully meet the demand for irrigation water.

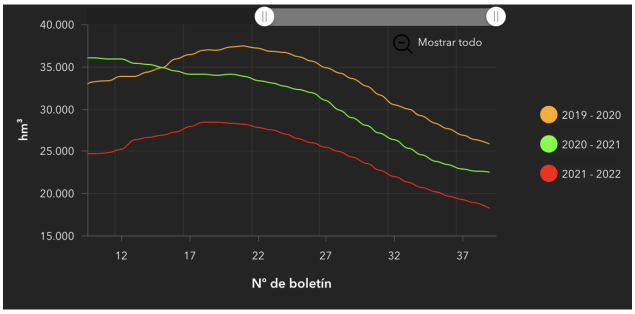

Figure 5. Hydrological reserves in Riudecanyes reservoir. Boletín Hidrológico Peninsular, 2023.

Each member of the community owns a certain number of water rights. All members have the right to use the available water equitably and proportionally to the rights that they own. Typically, each community member is allocated 2 h of water a week for each right during 12 weeks per year. If a member needs more water, additional water can be requested. Otherwise, if any member does not need all their allocated water, they can allow another member to use the available water (if they need it).

As mentioned above, the usual demand is more than 7.5 Hm^3 of water. In 2022, 4.5 Hm^3 were available, 1.5 of which had to remain in the reservoir. In other words, of the 7.5 Hm^3 required, only approximately 3 Hm^3 were available to meet the demand for a non-drought year. In a summer without anomalies, there is an irrigation period of twelve weeks, thus granting 2 h of irrigation rights per week. In 2022, due to the drought, the summer irrigation period was only ten weeks, and just one hour per week was allocated to each irrigation right. However, the requirements of the irrigation community members were the same as in the previous year, because the needs of the crops had not changed. If there were any changes, they would probably be for the worse, as the lack of rain would cause a greater need for irrigation.

Next, we defined our scenario as a claims problem. The endowment E is the current water available in the reservoir, which is E = 76,494 irrigation hours. The claimants are the farmers who ask for water. There are 1357 farmers in this community. Each farmer claimed the demand for irrigation hours from the previous year (2021), and the the sum of all demands was $\sum_{i=1}^{1,357} c_i = 168{,}288$, which is clearly more than what it is available.

4. Results

In this section, we present the results of our analysis. Table 2 shows the statistics of the amounts awarded according to different rules. Since there are 1357 farmers, there was a large amount of data, so we summarized the maximum, average, and minimum values assigned by each rule to each farmer[3].

Let us look at the results on the smaller claim. On the one hand, the CEA and the $\boldsymbol{\alpha}^{\mathbf{min}}$ rules were the only ones that assigned the entire claim to this agent, which totaled 8 h. On the other hand, the Talmud and the proportional rules afforded 4 h, thus fulfilling half of the claim. Finally, the CEL rule assigned 0 h to the agent with the smallest claim. Therefore, if we defend the idea that it is necessary to at least satisfy those who demand less water because they have less land, the rules that fulfill this are the CEA and the $\boldsymbol{\alpha}^{\mathbf{min}}$.

Upon analyzing the results of the highest claim, the rules that awarded the most allocations were the proportional rule and the $\boldsymbol{\alpha}^{\mathbf{min}}$, which allotted 3272.70 h and 3006.55 h, respectively. For example, the CEL granted 7110.88 h to agents with this type of claim, but remember that it was conducted from the perspective of the loss, that is to say, whoever has more claims will lose more.

Here we have two possibilities: consider the rules that favor farmers with a smaller demand and those with a larger demand. Moreover, according to the basic properties that we have considered, the CEA was the one that performed best with respect to these requirements.

In the next section, we analyze the statistical results (see Table 3). We focus on the coefficient of variation and the Gini index to justify the rule that shows better stability and less inequality in the distribution of irrigation hours.

Table 2. Statistics of the amounts awarded according to different rules.

	Claims	*CEA*	*CEL*	*Proportional*	$\alpha^{\min}$	*Talmud*
Minimum	8	8	0	4	8	4
1Q	50	50	0	23	26	25
Median	74	66	0	34	36	37
3Q	116	66	27	53	53	58
Maximum	7200.00	65.73	7110.88	3272.70	3006.55	764.94
Average	124.01	56.37	56.37	56.37	56.37	56.37
Std.Dev.	308.81	14.25	303.98	140.37	128.75	79.37
CV	2.49	0.25	5.39	2.49	2.28	1.41
Skewness	11.507	−1.396	12.106	11.507	11.507	6.350
Gini Index		0.12	0.90	0.51	0.47	0.46

Table 3. Statistics of percentages awarded according to different rules.

	CEA	*CEL*	**Prop**	$\alpha^{\min}$	*Talmud*
Minimum	0.009	0.000	0.455	0.418	0.106
1Q	0.567	0.000	0.455	0.457	0.500
Median	0.888	0.000	0.455	0.480	0.500
3Q	1.000	0.232	0.455	0.510	0.500
Maximum	1.000	0.988	0.455	1.000	0.500
Average	0.762	0.147	0.455	0.494	0.499
Std.Dev.	0.275	0.241	0.000	0.059	0.019
CV	0.361	1.633	0.000	0.120	0.039
Skewness	−0.852	1.591	0.000	2.496	−16.071

4.1. *Coefficient of Variation*

The **coefficient of variation (CV)** is the ratio of a sample's standard deviation μ to its mean $\bar{X}$.

$$CV = \frac{\mu}{\bar{X}}. \tag{3}$$

The coefficient of variation (CV) shows the dispersion of the distributions of the different rules. If we compare the values, the CEL, the proportional rule, and the $\boldsymbol{\alpha}^{\mathbf{min}}$ offered the highest values, that is to say, the highest dispersion. In contrast, the Talmud and the CEA rules provided a lower CV, which can be interpreted as a low dispersion.

4.2. *Inequality Measure: Gini Index*

According to the World Bank, the Gini index measures the extent to which the distribution of income among individuals or households within an economy deviates from a perfectly equal distribution. It is the most popular and most-used index to measure inequality in different research areas. A Lorenz curve plots the cumulative percentages of total income received against the cumulative number of beneficiaries, starting with the poorest individual or household [35]. In our case, we wanted to observe the inequality presented by the rules in their distribution of irrigation hours.

The **Gini index** [36] of these variables is defined by the following:

$$Gi = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} |x_i - x_j|}{2n^2\bar{x}}, \tag{4}$$

where $\bar{x}$ is the average of the irrigation hours distribution, n is the total number of observations, and x_i and x_j are the values of an individual's water quantity.

The Gini index takes values in the interval of $[0,1]$. Thus, a Gini index of 0 represents perfect equality, while an index of 1 implies perfect inequality. We explored how unevenly distributed the rules presented in our study were. If we look at Figure 6, the CEA represented the lowest value according to the Gini index.

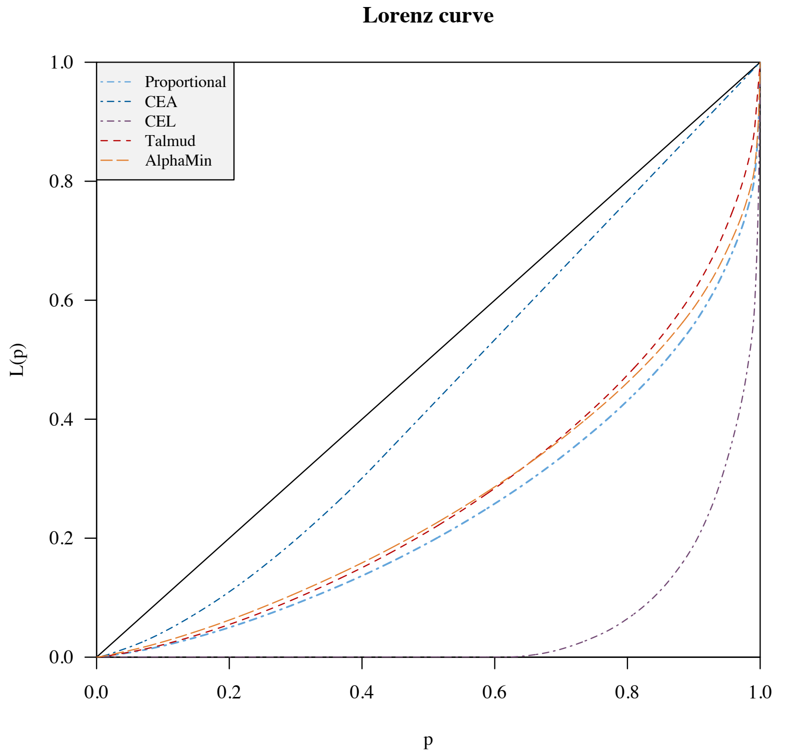

Figure 6. Lorenz curves of the amounts awarded according to different rules. Lorenz curves are represented by the functions L(p), where p, the cumulative portion of the population, is represented by the horizontal axis, and L, the cumulative portion of the total income received, is represented by the vertical axis.

The **Lorenz's dominance** is an interesting tool to compare the behavior of the solution concepts. In our case, we can use this tool to show if a rule favors smaller claimants over larger ones.

Let $\mathbb{R}^n$ be the set of positive n-dimensional vectors $x = (x_1, x_2, \ldots, x_n)$ such that the entries are ordered from small to large; i.e., $0 < x_1 \leq x_2 \leq \ldots \leq x_n$. Let x and y be in $\mathbb{R}^n$. Then, the x *Lorenz dominates* the y, which is denoted by $x \succ_L y$ if, for each $k = 1, 2, \ldots,$ the inequality comes out to $n-1$.

$$x_1 + x_2 + \cdots + x_k \geq y_1 + y_2 + \ldots + y_k \quad \text{and} \quad \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i. \tag{5}$$

If $x \succ_L y$, and $x \neq y$, then at least one of these $n-1$ inequalities is a strict inequality. We can extend this definition to claims rules.

Given two claims rules, φ and ψ, it is said that the φ *Lorenz dominates* ψ, and $\varphi \succ_L \psi$ if, for any $(E, c) \in \mathcal{B}$, $\varphi(E, c) \succ_L \psi(E, c)$.

Consequently, a Lorenz-dominated rule fulfills the claims. Ref. [27] obtained a Lorenz dominance comparison among several claims rules. Therefore, in our scenario, we have the following:

$$CEA \succ_L T \succ_L \alpha^{\min} \succ_L P \succ_L CEL. \quad (6)$$

Consequently, the CEA rule distributes the irrigation hours as equally as possible, thus maintaining the differences that existed before the water allocation. On the contrary, the CEL rule provides a less equal distribution of the irrigation hours. The reader may be given the same relationship between the inequalities of the distribution rules thanks to the Gini Index, as it numerically summarizes which Lorenz dominance displays graphically.

We applied a series of criteria—allocation rules, axiomatic perspective, the coefficient of variation, the Gini index, and Lorenz dominance—to make an allocation as equal as possible and to ensure that each farmer was satisfied with the allocated irrigation hours. Finally, we can say that the CEA meets all the criteria and meets all social axioms, with a Gini coefficient close to 0 (less inequality in the distribution), as well as with a low CV, thus showing less imbalance, and it is the only rule that distributes irrigation hours as equitably as possible.

5. Conclusions

In this paper, we analyzed the situation of the Riudecanyes Reservoir Irrigation Community, where, in 2022, there was a drought. As a consequence, the amount of water available in the reservoir decreased.

We proposed dividing the water by defining the situation as a conflicting claims problem. In this simulation, we specified the agents (*the farmers who own plots*) and the endowment (*the water available in the reservoir*). To complete the construction of the model, it was only necessary to define the *claim* of each farmer (claimant), which was the irrigation hours they used in the previous year when there was no drought.

Next, we applied the proportional rule, the constrained equal awards, the constrained equal losses, the Talmud, and the $\alpha^{\min}$ egalitarian rules.

As a result, the constrained equal awards rule was the one that proposed the (most) unequal allocation. By using the social axioms, Lorenz dominance, the coefficient of variation, and the Gini index, we observed that the CEA rule is the best option, because it proposes an unequal distribution of funds for the smaller claimants, satisfies all the social axioms provided in our scenario, and provides the lowest coefficient of variation, as well as the lowest inequality Gini index.

Finally, the results of this work are relevant as an alternative allocation method for irrigation water during drought periods. We consider the conflicting claims problem to be a tool for the efficient management and use of this resource. To conclude, and as a possible extension of this work, it would be interesting to obtain additional data, if possible, to be able to consider the same problem from the cooperative games approach in the case that the farmers cooperate among themselves and share their resources.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Here, we provide the tables with all the farmers' claims and the amounts awarded by all previously mentioned rules.

Table A1. Claims and amounts awarded by the selected rules (I).

Claim	*Freq*	*CEA*	*CEL*	*Prop*	α^{min}	*Talmud*	*Claim*	*Freq*	*CEA*	*CEL*	*Prop*	α^{min}	*Talmud*
8	1	8.0	0.0	3.6	8.0	4.0	45	7	45.0	0.0	20.5	23.4	22.5
10	1	10.0	0.0	4.5	8.8	5.0	46	18	46.0	0.0	20.9	23.8	23.0
11	3	11.0	0.0	5.0	9.3	5.5	47	6	47.0	0.0	21.4	24.3	23.5
12	3	12.0	0.0	5.5	9.7	6.0	48	13	48.0	0.0	21.8	24.7	24.0
13	1	13.0	0.0	5.9	10.1	6.5	49	9	49.0	0.0	22.3	25.1	24.5
14	5	14.0	0.0	6.4	10.5	7.0	50	27	50.0	0.0	22.7	25.5	25.0
15	1	15.0	0.0	6.8	10.9	7.5	51	10	51.0	0.0	23.2	25.9	25.5
16	2	16.0	0.0	7.3	11.3	8.0	52	16	52.0	0.0	23.6	26.3	26.0
17	2	17.0	0.0	7.7	11.8	8.5	53	6	53.0	0.0	24.1	26.8	26.5
18	1	18.0	0.0	8.2	12.2	9.0	54	20	54.0	0.0	24.5	27.2	27.0
19	2	19.0	0.0	8.6	12.6	9.5	55	15	55.0	0.0	25.0	27.6	27.5
20	4	20.0	0.0	9.1	13.0	10.0	56	25	56.0	0.0	25.5	28.0	28.0
21	3	21.0	0.0	9.5	13.4	10.5	57	9	57.0	0.0	25.9	28.4	28.5
22	33	22.0	0.0	10.0	13.8	11.0	58	24	58.0	0.0	26.4	28.8	29.0
23	7	23.0	0.0	10.5	14.3	11.5	59	7	59.0	0.0	26.8	29.3	29.5
24	9	24.0	0.0	10.9	14.7	12.0	60	19	60.0	0.0	27.3	29.7	30.0
25	5	25.0	0.0	11.4	15.1	12.5	61	9	61.0	0.0	27.7	30.1	30.5
26	12	26.0	0.0	11.8	15.5	13.0	62	16	62.0	0.0	28.2	30.5	31.0
27	4	27.0	0.0	12.3	15.9	13.5	63	11	63.0	0.0	28.6	30.9	31.5
28	6	28.0	0.0	12.7	16.3	14.0	64	14	64.0	0.0	29.1	31.3	32.0
29	6	29.0	0.0	13.2	16.8	14.5	65	5	65.0	0.0	29.5	31.8	32.5
30	10	30.0	0.0	13.6	17.2	15.0	66	18	65.7	0.0	30.0	32.2	33.0
31	4	31.0	0.0	14.1	17.6	15.5	67	5	65.7	0.0	30.5	32.6	33.5
32	13	32.0	0.0	14.5	18.0	16.0	68	20	65.7	0.0	30.9	33.0	34.0
33	9	33.0	0.0	15.0	18.4	16.5	69	4	65.7	0.0	31.4	33.4	34.5
34	12	34.0	0.0	15.5	18.8	17.0	70	16	65.7	0.0	31.8	33.8	35.0
35	6	35.0	0.0	15.9	19.3	17.5	71	10	65.7	0.0	32.3	34.3	35.5
36	15	36.0	0.0	16.4	19.7	18.0	72	19	65.7	0.0	32.7	34.7	36.0
37	8	37.0	0.0	16.8	20.1	18.5	73	7	65.7	0.0	33.2	35.1	36.5
38	19	38.0	0.0	17.3	20.5	19.0	74	14	65.7	0.0	33.6	35.5	37.0
39	13	39.0	0.0	17.7	20.9	19.5	75	5	65.7	0.0	34.1	35.9	37.5
40	14	40.0	0.0	18.2	21.3	20.0	76	7	65.7	0.0	34.5	36.4	38.0
41	8	41.0	0.0	18.6	21.8	20.5	77	8	65.7	0.0	35.0	36.8	38.5
42	14	42.0	0.0	19.1	22.2	21.0	78	24	65.7	0.0	35.5	37.2	39.0
43	5	43.0	0.0	19.5	22.6	21.5	79	4	65.7	0.0	35.9	37.6	39.5
44	19	44.0	0.0	20.0	23.0	22.0	80	13	65.7	0.0	36.4	38.0	40.0

Table A2. Claims and amounts awarded by the selected rules (II).

Claim	*Freq*	*CEA*	*CEL*	*Prop*	α^{min}	*Talmud*	*Claim*	*Freq*	*CEA*	*CEL*	*Prop*	α^{min}	*Talmud*
81	5	65.7	0.0	36.8	38.4	40.5	117	2	65.7	27.9	53.2	53.4	58.5
82	15	65.7	0.0	37.3	38.9	41.0	118	9	65.7	28.9	53.6	53.9	59.0
83	4	65.7	0.0	37.7	39.3	41.5	120	8	65.7	30.9	54.5	54.7	60.0
84	20	65.7	0.0	38.2	39.7	42.0	121	2	65.7	31.9	55.0	55.1	60.5
85	2	65.7	0.0	38.6	40.1	42.5	122	6	65.7	32.9	55.5	55.5	61.0
86	13	65.7	0.0	39.1	40.5	43.0	123	1	65.7	33.9	55.9	55.9	61.5
87	7	65.7	0.0	39.5	40.9	43.5	124	6	65.7	34.9	56.4	56.4	62.0
88	15	65.7	0.0	40.0	41.4	44.0	125	2	65.7	35.9	56.8	56.8	62.5
89	6	65.7	0.0	40.5	41.8	44.5	126	5	65.7	36.9	57.3	57.2	63.0
90	13	65.7	0.9	40.9	42.2	45.0	127	4	65.7	37.9	57.7	57.6	63.5
91	4	65.7	1.9	41.4	42.6	45.5	128	3	65.7	38.9	58.2	58.0	64.0
92	12	65.7	2.9	41.8	43.0	46.0	129	1	65.7	39.9	58.6	58.4	64.5

Table A2. *Cont.*

Claim	*Freq*	*CEA*	*CEL*	*Prop*	α^{min}	*Talmud*	*Claim*	*Freq*	*CEA*	*CEL*	*Prop*	α^{min}	*Talmud*
93	3	65.7	3.9	42.3	43.4	46.5	130	5	65.7	40.9	59.1	58.9	65.0
94	8	65.7	4.9	42.7	43.9	47.0	131	1	65.7	41.9	59.5	59.3	65.5
95	2	65.7	5.9	43.2	44.3	47.5	132	14	65.7	42.9	60.0	59.7	66.0
96	16	65.7	6.9	43.6	44.7	48.0	133	1	65.7	43.9	60.5	60.1	66.5
97	3	65.7	7.9	44.1	45.1	48.5	134	5	65.7	44.9	60.9	60.5	67.0
98	12	65.7	8.9	44.5	45.5	49.0	136	6	65.7	46.9	61.8	61.4	68.0
99	4	65.7	9.9	45.0	45.9	49.5	137	3	65.7	47.9	62.3	61.8	68.5
100	9	65.7	10.9	45.5	46.4	50.0	138	4	65.7	48.9	62.7	62.2	69.0
101	2	65.7	11.9	45.9	46.8	50.5	140	5	65.7	50.9	63.6	63.0	70.0
102	12	65.7	12.9	46.4	47.2	51.0	141	2	65.7	51.9	64.1	63.5	70.5
103	1	65.7	13.9	46.8	47.6	51.5	142	4	65.7	52.9	64.5	63.9	71.0
104	10	65.7	14.9	47.3	48.0	52.0	143	1	65.7	53.9	65.0	64.3	71.5
105	3	65.7	15.9	47.7	48.4	52.5	144	5	65.7	54.9	65.5	64.7	72.0
106	6	65.7	16.9	48.2	48.9	53.0	145	2	65.7	55.9	65.9	65.1	72.5
107	3	65.7	17.9	48.6	49.3	53.5	146	3	65.7	56.9	66.4	65.5	73.0
108	12	65.7	18.9	49.1	49.7	54.0	147	4	65.7	57.9	66.8	66.0	73.5
109	2	65.7	19.9	49.5	50.1	54.5	148	2	65.7	58.9	67.3	66.4	74.0
110	13	65.7	20.9	50.0	50.5	55.0	150	3	65.7	60.9	68.2	67.2	75.0
111	2	65.7	21.9	50.5	50.9	55.5	152	7	65.7	62.9	69.1	68.0	76.0
112	11	65.7	22.9	50.9	51.4	56.0	154	3	65.7	64.9	70.0	68.9	77.0
113	3	65.7	23.9	51.4	51.8	56.5	155	2	65.7	65.9	70.5	69.3	77.5
114	13	65.7	24.9	51.8	52.2	57.0	156	3	65.7	66.9	70.9	69.7	78.0
115	3	65.7	25.9	52.3	52.6	57.5	157	1	65.7	67.9	71.4	70.1	78.5
116	12	65.7	26.9	52.7	53.0	58.0	158	4	65.7	68.9	71.8	70.5	79.0

Table A3. Claims and amounts awarded by the selected rules (III).

Claim	*Freq*	*CEA*	*CEL*	*Prop*	α^{min}	*Talmud*	*Claim*	*Freq*	*CEA*	*CEL*	*Prop*	α^{min}	*Talmud*
160	2	65.7	70.9	72.7	71.4	80.0	221	1	65.7	131.9	100.5	96.8	110.5
162	3	65.7	72.9	73.6	72.2	81.0	224	1	65.7	134.9	101.8	98.1	112.0
164	2	65.7	74.9	74.5	73.0	82.0	225	3	65.7	135.9	102.3	98.5	112.5
165	1	65.7	75.9	75.0	73.5	82.5	227	2	65.7	137.9	103.2	99.3	113.5
166	3	65.7	76.9	75.5	73.9	83.0	228	1	65.7	138.9	103.6	99.7	114.0
167	1	65.7	77.9	75.9	74.3	83.5	230	1	65.7	140.9	104.5	100.6	115.0
168	6	65.7	78.9	76.4	74.7	84.0	232	1	65.7	142.9	105.5	101.4	116.0
169	1	65.7	79.9	76.8	75.1	84.5	234	2	65.7	144.9	106.4	102.2	117.0
170	2	65.7	80.9	77.3	75.5	85.0	238	1	65.7	148.9	108.2	103.9	119.0
171	2	65.7	81.9	77.7	76.0	85.5	240	7	65.7	150.9	109.1	104.7	120.0
172	3	65.7	82.9	78.2	76.4	86.0	241	2	65.7	151.9	109.5	105.1	120.5
173	1	65.7	83.9	78.6	76.8	86.5	242	1	65.7	152.9	110.0	105.6	121.0
174	4	65.7	84.9	79.1	77.2	87.0	243	1	65.7	153.9	110.5	106.0	121.5
176	3	65.7	86.9	80.0	78.0	88.0	256	1	65.7	166.9	116.4	111.4	128.0
178	4	65.7	88.9	80.9	78.9	89.0	258	1	65.7	168.9	117.3	112.2	129.0
179	1	65.7	89.9	81.4	79.3	89.5	260	1	65.7	170.9	118.2	113.1	130.0
180	2	65.7	90.9	81.8	79.7	90.0	262	2	65.7	172.9	119.1	113.9	131.0
181	1	65.7	91.9	82.3	80.1	90.5	264	2	65.7	174.9	120.0	114.7	132.0
182	1	65.7	92.9	82.7	80.5	91.0	270	2	65.7	180.9	122.7	117.2	135.0
184	1	65.7	94.9	83.6	81.4	92.0	274	2	65.7	184.9	124.5	118.9	137.0
186	3	65.7	96.9	84.5	82.2	93.0	278	1	65.7	188.9	126.4	120.6	139.0
188	5	65.7	98.9	85.5	83.0	94.0	282	1	65.7	192.9	128.2	122.2	141.0
190	3	65.7	100.9	86.4	83.9	95.0	283	1	65.7	193.9	128.6	122.7	141.5
192	1	65.7	102.9	87.3	84.7	96.0	284	1	65.7	194.9	129.1	123.1	142.0
194	1	65.7	104.9	88.2	85.5	97.0	285	1	65.7	195.9	129.5	123.5	142.5
196	3	65.7	106.9	89.1	86.4	98.0	286	1	65.7	196.9	130.0	123.9	143.0
198	2	65.7	108.9	90.0	87.2	99.0	287	1	65.7	197.9	130.5	124.3	143.5
201	1	65.7	111.9	91.4	88.5	100.5	290	1	65.7	200.9	131.8	125.6	145.0
202	1	65.7	112.9	91.8	88.9	101.0	291	1	65.7	201.9	132.3	126.0	145.5
206	1	65.7	116.9	93.6	90.6	103.0	292	1	65.7	202.9	132.7	126.4	146.0
208	1	65.7	118.9	94.5	91.4	104.0	294	1	65.7	204.9	133.6	127.2	147.0
210	3	65.7	120.9	95.5	92.2	105.0	296	1	65.7	206.9	134.5	128.1	148.0
211	1	65.7	121.9	95.9	92.6	105.5	300	2	65.7	210.9	136.4	129.7	150.0
216	3	65.7	126.9	98.2	94.7	108.0	304	2	65.7	214.9	138.2	131.4	152.0
218	2	65.7	128.9	99.1	95.6	109.0	306	1	65.7	216.9	139.1	132.2	153.0
220	1	65.7	130.9	100.0	96.4	110.0	308	3	65.7	218.9	140.0	133.1	154.0

Table A4. Claims and amounts awarded by the selected rules (IV).

Claim	Freq	CEA	CEL	Prop	α^{min}	Talmud	Claim	Freq	CEA	CEL	Prop	α^{min}	Talmud
310	3	65.7	220.9	140.9	133.9	155.0	556	1	65.7	466.9	252.7	236.5	278.0
314	1	65.7	224.9	142.7	135.6	157.0	574	1	65.7	484.9	260.9	244.0	287.0
317	1	65.7	227.9	144.1	136.8	158.5	594	1	65.7	504.9	270.0	252.3	297.0
320	2	65.7	230.9	145.5	138.1	160.0	620	1	65.7	530.9	281.8	263.2	310.0
328	3	65.7	238.9	149.1	141.4	164.0	682	1	65.7	592.9	310.0	289.0	341.0
331	2	65.7	241.9	150.5	142.7	165.5	716	1	65.7	626.9	325.5	303.2	358.0
332	1	65.7	242.9	150.9	143.1	166.0	816	1	65.7	726.9	370.9	344.9	408.0
334	1	65.7	244.9	151.8	143.9	167.0	954	1	65.7	864.9	433.6	402.4	477.0
344	2	65.7	254.9	156.4	148.1	172.0	1200	1	65.7	1110.9	545.5	505.0	600.0
346	1	65.7	256.9	157.3	148.9	173.0	1394	1	65.7	1304.9	633.6	585.9	697.0
351	1	65.7	261.9	159.5	151.0	175.5	1406	1	65.7	1316.9	639.1	590.9	703.0
354	1	65.7	264.9	160.9	152.3	177.0	1680	1	65.7	1590.9	763.6	705.1	764.9
362	2	65.7	272.9	164.5	155.6	181.0	1848	2	65.7	1758.9	840.0	775.1	764.9
366	1	65.7	276.9	166.4	157.3	183.0	2016	1	65.7	1926.9	916.4	845.2	764.9
370	1	65.7	280.9	168.2	158.9	185.0	2594	1	65.7	2504.9	1179.1	1086.2	764.9
374	1	65.7	284.9	170.0	160.6	187.0	3440	1	65.7	3350.9	1563.6	1438.9	764.9
375	1	65.7	285.9	170.5	161.0	187.5	4029	1	65.7	3939.9	1831.4	1684.5	764.9
378	1	65.7	288.9	171.8	162.3	189.0	4414	1	65.7	4324.9	2006.3	1845.0	764.9
395	2	65.7	305.9	179.5	169.4	197.5	7200	1	65.7	7110.9	3272.7	3006.6	764.9
401	1	65.7	311.9	182.3	171.9	200.5							
406	1	65.7	316.9	184.5	173.9	203.0							
414	1	65.7	324.9	188.2	177.3	207.0							
416	1	65.7	326.9	189.1	178.1	208.0							
428	1	65.7	338.9	194.5	183.1	214.0							
438	1	65.7	348.9	199.1	187.3	219.0							
444	1	65.7	354.9	201.8	189.8	222.0							
468	1	65.7	378.9	212.7	199.8	234.0							
480	1	65.7	390.9	218.2	204.8	240.0							
482	1	65.7	392.9	219.1	205.6	241.0							
490	1	65.7	400.9	222.7	209.0	245.0							
496	1	65.7	406.9	225.5	211.5	248.0							
500	2	65.7	410.9	227.3	213.1	250.0							
502	1	65.7	412.9	228.2	214.0	251.0							
506	1	65.7	416.9	230.0	215.6	253.0							
532	1	65.7	442.9	241.8	226.5	266.0							
534	1	65.7	444.9	242.7	227.3	267.0							

Notes

1 Ministerio para la Transición Ecológica y el Reto Demográfico.

2 The cubic hectometer is a unit of volume equivalent to one million cubic meters.

3 For more detailed data, please refer to the claim corresponding to each farmer and the solutions to the proposed rules in Tables A1–A4 in the Appendix A.

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