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Tunable Topological Acoustic Tamm States in Comblike Structures Based on Band Inversion around Flat Bands

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Citation: Khattou, S.; Rezzouk, Y.; Amrani, M.; El Ghafiani, M.; El Boudouti, E.H.; Talbi, A.; Djafari-Rouhani, B. Tunable Topological Acoustic Tamm States in Comblike Structures Based on Band Inversion around Flat Bands. *Crystals* **2022**, *12*, 1685. https://doi.org/10.3390/cryst12121685

Academic Editors: Jin-Chen Hsu and Jia-Hong Sun

Received: 30 September 2022
Accepted: 15 November 2022
Published: 22 November 2022

S1. Derivation of the transfer matrix elements and the pressure field

Here we give the details of the derivation of the transfer matrix elements as well as the corresponding pressure field. The pressure fields in the cells (0), (1) and the stubs (s) that separate them (Figure S1) are given respectively as

$$p_0(x) = A_0 \exp(jk(x+d_1)) + B_0 \exp(-jk(x+d_1)); \qquad -d_1 \le x \le 0 \tag{S.1}$$

$$p_1(x) = A_1 \exp(jkx) + B_1 \exp(-jkx); \qquad 0 \le x \le d_1 \tag{S.2}$$

$$p_s(x) = C \sin(k(x-d_1')); \qquad 0 \le x \le d_1' \tag{S.3}$$

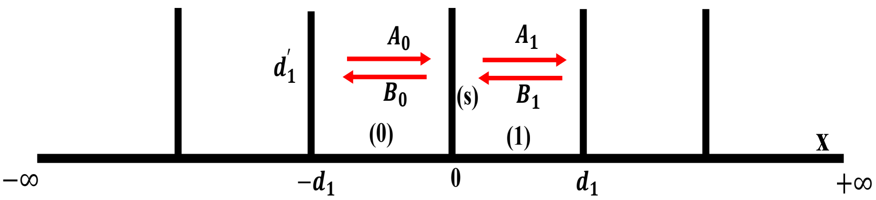

Figure S1. Sketch of an infinite periodic comb-like PnC made of stubs of lengths d_1' separated by periods of length d_1. A_i and B_i represent the amplitudes of the pressure fields propagating in positive and negative directions.

Using the boundary conditions at $x = 0$, we are able to relate the amplitudes of the pressure fields in the two successive cells (Figure S1) utilizing the 2×2 transfer matrix:

$$\begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \begin{pmatrix} A_0 \\ B_0 \end{pmatrix}, \tag{S.4}$$

where

$$t_{11} = \left[1 - j\frac{\cot(kd_1')}{2}\right] \exp(jkd_1), \tag{S.5a}$$

$$t_{12} = \left[-j\frac{\cot(kd_1')}{2}\right] \exp(-jkd_1), \tag{S.5b}$$

$$t_{21} = \left[j \frac{\cot(kd'_1)}{2} \right] \exp(jkd_1), \tag{S.5c}$$

and

$$t_{22} = \left[1 + j \frac{\cot(kd'_1)}{2} \right] \exp(-jkd_1), \tag{S.5d}$$

where $k = \frac{\omega}{v}$ is the wavevector and v is the velocity of sound in the tubes.

The eigenvector of the transfer matrix of the unit cell under consideration is given by

$$\begin{pmatrix} t_{12} \\ \exp(jk_B d_1) - t_{11} \end{pmatrix},$$

where k_B is the Bloch wave-vector given by the dispersion relation

$$\cos(k_B d_1) = \frac{t_{11} + t_{22}}{2}. \tag{S.6}$$

With these notations and using the Bloch theorem, the expressions of the pressure fields (Eqs. (S.1), (S.2) and (S.3)), become

$$p_0(x) = t_{12} \exp(jk(x + d_1)) + [\exp(jk_B d_1) - t_{11}] \exp(-jk(x + d_1)); \quad -d_1 \leq x \leq 0 \tag{S.7}$$

$$p_1(x) = [t_{12} \exp(jkx) + [\exp(jk_B d_1) - t_{11}] \exp(-jkx)] \exp(jk_B d_1); \quad 0 \leq x \leq d_1 \tag{S.8}$$

$$p_s(x) = -\frac{[t_{12} \exp(jkd_1) + [\exp(jk_B d_1) - t_{11}] \exp(-jkd_1)]}{\cos(kd'_1)} \cos(k(x - d'_1)); \quad 0 \leq x \leq d'_1 \tag{S.9}$$

S2. Relation between Friedel phase and variation of DOS for an asymmetric lossless comb-like PnC

Before discussing the relation between the Friedel phase and the total DOS, we will recall briefly the expressions of the Green's functions of the constituents from which the finite comb-like PnC is composed, namely

1. The inverse of the Green's function $g_i^{-1}(M_i M_i)$ of a finite tube of length d_i in the space of its interface $M_i = \{0, d_i\}$. This quantity can be written as a (2×2) matrix [1,2]

$$g_i^{-1}(M_i M_i) = \begin{pmatrix} -\frac{FC_i}{S_i} & \frac{F}{S_i} \\ \frac{F}{S_i} & -\frac{FC_i}{S_i} \end{pmatrix}, \tag{S.10}$$

   where $C_i = \cos(kd_i)$, $S_i = \sin(kd_i)$ (i=1,2) and $F = \frac{\omega}{Z}$.
2. The inverse of the Green's function of the stub of length d'_i in the space of interface $M_{i'} = \{0, 0\}$ is given by [1,2]

$$g_{i'}^{-1}(00) = -\frac{FC'_i}{S'_i}, \tag{S.11}$$

   where $C'_i = \cos(kd'_i)$, $S'_i = \sin(kd'_i)$ (i=1,2).

3. The inverse of the Green's function $g_{i''}^{-1}(M_{i''}M_{i''})$ of the tube at the surface in the space of interfaces $M_{i''} = \{0, \frac{d_i}{2}\}$ is similar to Eq.(S.10) but for the waveguide of length $\frac{d_i}{2}$. Its expression is given by

$$g_{i''}^{-1}(M_iM_i) = \begin{pmatrix} -\frac{FC_i''}{S_i''} & \frac{F}{S_i''} \\ \frac{F}{S_i''} & -\frac{FC_i''}{S_i''} \end{pmatrix}, \tag{S.12}$$

where $C_i'' = \cos(k\frac{d_i}{2})$, $S_i'' = \sin(k\frac{d_i}{2})$ (i=1,2).

4. The inverse of the Green's function of a semi-infinite tube is given by [1,2]

$$g_s^{-1}(i,i) = -jF. \tag{S.13}$$

The inverse of the Green's function of an asymmetric finite PnC composed by connecting two PnCs, each one made of three stubs (Figure 1 (c)) in the space of interface $M_0 = \{-3,-2,-1,0,1,2,3\}$, is obtained by the superposition of the inverse of the Green's function elements given in Eqs. (S.10), (S.11) and (S.12), namely

$$g^{-1}(M_0, M_0) = \begin{pmatrix} a_1 & b_1 & 0 & 0 & 0 & 0 & 0 \\ b_1 & c_1 & b_1 & 0 & 0 & 0 & 0 \\ 0 & b_1 & d_1 & e_1 & 0 & 0 & 0 \\ 0 & 0 & e_1 & f & e_2 & 0 & 0 \\ 0 & 0 & 0 & e_2 & d_2 & b_2 & 0 \\ 0 & 0 & 0 & 0 & b_2 & c_2 & b_2 \\ 0 & 0 & 0 & 0 & 0 & b_2 & a_2 \end{pmatrix} \tag{S.14}$$

where the expressions of a_i, b_i, c_i, d_i, e_i (i=1,2) and f are given by

$$a_i = -F(\frac{C_i}{S_i} + \frac{C_i'}{S_i'}), \tag{S.15a}$$

$$b_i = \frac{F}{S_i}, \tag{S.15b}$$

$$c_i = -F(\frac{2C_i}{S_i} + \frac{C_i'}{S_i'}), \tag{S.15c}$$

$$d_i = -F(\frac{C_i}{S_i} + \frac{C_i'}{S_i'} + \frac{C_i''}{S_i''}), \tag{S.15d}$$

$$e_i = \frac{F}{S_i''}, \tag{S.15e}$$

and

$$f = -F(\frac{C_1''}{S_1''} + \frac{C_1''}{S_2''}). \tag{S.15f}$$

In order to calculate the transmission and reflection coefficients through the finite PnC (Figure 1 (c)), we need to invert the Green's function in Eq. (S.14) and reinvert the truncated Green's function in the space of interface $M = \{-3, 3\}$ which reduces to a 2×2 matrix composed of the elements A, B and C. Then, the PnC should be inserted between two semi-infinite tubes, its final Green's function becomes

$$g^{-1}(M,M) = \begin{pmatrix} A - jF & B \\ B & C - jF \end{pmatrix}, \tag{S.16}$$

where $-jF$ is the inverse Green's function of the semi-infinite tubes surrounding the finite PnC (Eq. (S.13)). A and B are real quantities, their expressions are calculated numerically.

The expressions of the transmission and reflection coefficients through the finite PnC in Figure 1 (c) are given, respectively, as follows

$$t = -2jFg(-3,3) = 2jFBdet[g(M,M)], \tag{S.17}$$

$$r = -1 - 2jFg(-3,-3) = -1 - 2jF(C - jF)det[g(M,M)], \tag{S.18}$$

and

$$r' = -1 - 2jFg(3,3) = -1 - 2jF(A - jF)det[g(M,M)], \tag{S.19}$$

where r (r') are the reflection coefficients from left to right (right to left) side of the finite PnC (Figure 1 (c)). The expression of $det[g(M,M)]$ is given by

$$det[g(M,M)] = \frac{1}{AC - B^2 - F^2 - jF(A + C)}. \tag{S.20}$$

In order to give a comparison of the DOS with the Friedel phase, we consider the variation of the DOS between the finite PnC in Figure 1 (c) and a reference system formed out of the two semi-infinite waveguides and the finite PnCs. The variation of the DOS (noted $\Delta n(\omega)$) can be obtained by the following expression:

$$\Delta n(\omega) = \frac{1}{\pi} \frac{d}{d\omega} Arg\{det[g(M,M)]\}. \tag{S.21}$$

The Friedel phase θ_F is obtained by the phase of the determinant of the scattering matrix S. For an asymmetric system, S is defined by

$$S = \begin{pmatrix} r & t \\ t & r' \end{pmatrix}, \tag{S.22}$$

where t, r and r' are given by Eqs. (S.17)-(S.19).

For a lossless system, the determinant of the scattering matrix S can be deduced from the expressions of t, r and r', namely

$$detS = rr' - t^2 = \left\{(AC - B^2 + F^2)^2 + F^2(A - C)^2 + 4F^2B^2\right\}det[g(M,M)]^2. \tag{S.23}$$

Equation (S.23) shows that the argument of the first term vanishes as the quantities A, B, C and F are real quantities, and therefore,

$$\theta_F = Arg\{detS\} = 2Arg\{det[g(M,M)]\}. \tag{S.24}$$

From Eqs. (S.21) and (S.24) one can deduce that the Friedel phase θ_F is related to the DOS of the system by the following expression

$$\frac{d\theta_F}{d\omega} = 2\pi\Delta n(\omega). \tag{S.25}$$

S3. Influence of temperature on the interface state

The frequency of the interface state is related to the speed of sound v which is temperatue dependent. The relationship between the speed of sound and temperature in standard atmospheric pressure in air is expressed as [3]

$$v(t^{\circ}C) = \sqrt{\frac{\gamma R}{M}(273+t)} \tag{S.26}$$

where t is the temperature in Celsius ($^{\circ}C$), $\gamma = 1.402$ is the adiabatic index, $R = 8.31$ J/mol.K is the gas constant and $M = 29 \times 10^{-3}$ Kg/mol is the gas molar mass.

We plot in Figure S2 the interface mode frequency as a function of temperature for a finite system with geometrical parameters: $d_1 = 2$ cm, $d_2 = 13$ cm and $d_1' = d_2' = 10$ cm. One can see that the interface state is sensitive to the temperature and its frequency increases slightly when the temperature changes from $0^{\circ}C$ to $40^{\circ}C$.

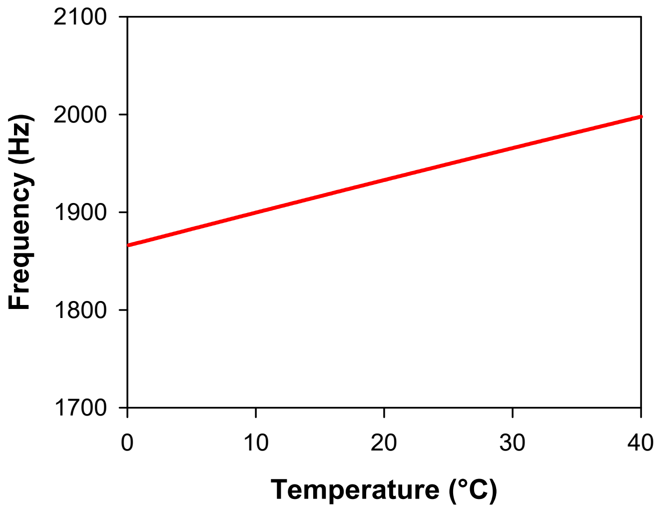

Figure S2. The interface mode frequency as a function of temperature. The geometrical parameters are chosen such as: $d_1 = 2$ cm, $d_2 = 13$ cm and $d_1' = d_2' = 10$ cm.

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