Supplementary Materials

Landfill Emissions of Methane Inferred from Unmanned Aerial Vehicle and Mobile Ground Measurements: Gaussian Plume Formulation

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The Transfer Matrix

The inverse modeling method described here is based on the dispersion of Gaussian plumes by a wind of horizontal speed, U, with horizontal and vertical turbulent spreads, σ_y and σ_z, respectively. The details of the Gaussian dispersion formulation are from USEPA (2004), unless otherwise noted.

A transfer matrix, P_{ij}, is defined such that each matrix element denotes the dispersion of a plume from point source j with emission rate, Q_j, to a receptor i with net concentration, C_i.

$$C_i = P_{ij}Q_j + B \tag{S1}$$

$$P_{ij} = \frac{exp\left(-\frac{x_{ij}^2}{2\sigma_y^2}\right)}{2\pi U\sigma_y\sigma_z}\sum_{m=0}^{1}\left\{exp\left[-\frac{\left(z_i - h_{sj} - 2mz_{mix}\right)^2}{2\sigma_z^2}\right] + exp\left[-\frac{\left(z_i + h_{sj} + 2mz_{mix}\right)^2}{2\sigma_z^2}\right]\right\}, \tag{S2}$$

where B is the background concentration, x_{ij} is the horizontal downwind distance of receptor i from emission source j, z_i is the vertical height of receptor i, h_{sj} is the effective height of source j, and z_{mix} is the mixing height. The dispersion parameters are computed based on two cases: (1) a stable boundary layer (SBL) and (2) a convective boundary layer (CBL). For simplicity, updrafts and downdrafts in the CBL are not accounted for, so that the plume centerline height is independent of vertical velocity. Plume contributions resulting from downward dispersion of lofted material near or above the mixing height in the CBL are likewise ignored. In USEPA (2004), these contributions are referred to as the indirect and penetrated sources, as compared to the direct plume accounted for in Equation (S2). Because only fugitive emission sources are considered (i.e., no plume rise from combustion sources), h_{sj} is simply the source elevation.

Friction Velocity and Monin–Obukhov Length

The friction velocity, u_*, in the SBL may be calculated based on the reference height, z_{ref} (= 10 m, that is anemometer level); reference temperature, T_{ref} (set to the ambient air temperature at the surface); reference wind speed, u_{ref} (the anemometer level wind); roughness length, z_0; cloud cover, n; and acceleration due to gravity, g, as follows:

$$u_* = \frac{C_D u_{ref}}{2}\left\{-1 + \left[1 + \left(\frac{2u_0}{C_D^{1/2}u_{ref}}\right)^2\right]^{1/2}\right\} \tag{S3}$$

$$u_0 = \left(\frac{5g\theta_* z_{ref}}{T_{ref}}\right)^{1/2} \tag{S4}$$

$$\theta_* = 0.09(1 - 0.5n^2) \quad \text{(S5)}$$

$$C_D = 0.4/ln(z_{ref}/z_0)\,. \quad \text{(S6)}$$

In Equation (S5), θ_* represents a temperature scale. Equation (S3) is valid only when $u_{ref} \geq 2u_0$. Otherwise, the friction velocity is computed from the following:

$$u_* = \frac{C_D u_{ref}}{4}\left(\frac{u_{ref}}{u_0}\right). \quad \text{(S7)}$$

The Monin–Obukhov length, L, in the SBL can then be computed from the friction velocity:

$$L = \frac{T_{ref}}{0.4g\theta_*}u_*^2\,. \quad \text{(S8)}$$

In the CBL, the surface sensible heat flux, H, must first be computed to derive the friction velocity and Monin–Obukhov length. The sensible heat flux is computed from the Bowen ratio, B_0 (ratio of sensible to latent heat fluxes at the surface):

$$H = \frac{0.9R_n}{(1 + 1/B_0)}\,. \quad \text{(S9)}$$

In Equation (S9), R_n is the net radiation as a function of the clear sky insolation, R_0; albedo, r; cloud cover, n; and ambient air temperature at the reference height, T_{ref}:

$$R_n = \frac{(1 - r)R_0(1 - 0.75n^{3.4}) + c_1 T_{ref}^6 - \sigma_{SB} T_{ref}^4 + c_2 n}{1.12}\,, \quad \text{(S10)}$$

where σ_{SB} is the Stefan–Boltzmann constant, $c_1 = 5.31 \times 10^{-13}$ W m^{-2} K^{-6}, and $c_2 = 60$ W m^{-2}. R_0 (W m^{-2}) and r are computed as a function of solar elevation angle, φ:

$$R_0 = 990 \sin\varphi - 30 \quad \text{(S11)}$$

$$r = r' + (1 - r')exp\{-0.1\varphi - 0.5(1 - r')^2\}\,. \quad \text{(S12)}$$

In Equation (S12), r' is the albedo at a solar elevation angle of 90 degrees.

The friction velocity and Monin–Obukhov length in the CBL can now be derived by iteratively solving a set of simultaneous equations, including Equation (S8), together with the following:

$$u_* = \frac{0.4u_{ref}}{ln(z_{ref}/z_0) - \Psi_c(z_{ref}/L) + \Psi_c(z_0/L)} \quad \text{(S13)}$$

$$\Psi_c(\omega) = 2\,ln\left(\frac{1+\mu}{2}\right) + ln\left(\frac{1+\mu^2}{2}\right) - 2\,tan^{-1}\mu + \pi/2 \quad \text{(S14)}$$

$$\mu = (1 - 16\omega)^{1/4}\,. \quad \text{(S15)}$$

Boundary Layer Structure

The SBL mixing height (m) at mid-latitudes can be approximated by the empirical relation:

$$z_{mix} = 2300\,{u_*}^{3/2}\,. \quad \text{(S16)}$$

The USEPA (2004) recommends that, if measurements of the CBL mixing height are available, z_{mix} should be set to the larger of the measured value and the estimate yielded by Equation (S16). However, because this study was performed during the transition from the SBL to the CBL, the CBL mixing height was computed directly from Equation (S16).

In contrast to the mixing height, the vertical profile of the wind speed is calculated differently for the two boundary layer types. In the CBL, this profile is given by the following:

$$U = U(7z_0)\left[\frac{z}{7z_0}\right] \qquad \text{for } z < 7z_0 \tag{S17}$$

$$U = 2.5u_*\left[ln\left(\frac{z}{z_0}\right) - \Psi_c\left(\frac{z}{L}\right) + \Psi_c\left(\frac{z_0}{L}\right)\right] \qquad \text{for } 7z_0 \le z \le z_{mix} \tag{S18}$$

$$U = U(z_{mix}) \qquad \text{for } z > z_{mix}\,. \tag{S19}$$

In the SBL, the function Ψ_c in Equations (S17)–(S19) is replaced by the function Ψ_s, which is given by the following equation:

$$\Psi_s(\omega) = -17[1 - exp(-0.29\omega)]\,. \tag{S20}$$

To derive the thermal structure of the boundary layer, it is convenient to refer to the Brunt–Väisälä frequency, *N*, based on the potential temperature, θ, according to the convention:

$$N^2 = \frac{g}{\theta}\frac{\partial\theta}{\partial z}\,. \tag{S21}$$

The vertical gradient of potential temperature is integrated to yield vertical profiles of θ and N^2.

In the SBL, the potential temperature gradient is calculated as follows:

$$\frac{\partial\theta}{\partial z} = 1.25\theta_*\left[1 + \frac{10}{L}\right] \qquad \text{for } z \le 2\ m \tag{S22}$$

$$\frac{\partial\theta}{\partial z} = 2.5\frac{\theta_*}{z}\left[1 + \frac{5z}{L}\right] \qquad \text{for } 2\ m < z \le 100\ m \tag{S23}$$

$$\frac{\partial\theta}{\partial z} = \frac{\partial\theta}{\partial z}|_{z=100} exp\left[-\frac{(z-100)}{0.44max(z_{mix},100)}\right] \qquad \text{for } z > 100\ m\,. \tag{S24}$$

In the CBL, the potential temperature gradient is set to zero below z = z_{mix}. Above z = z_{mix} + 500, the potential temperature gradient is set to 0.005 K m^{-1}. Linear interpolation is then used to compute the potential temperature gradient for $z_{mix} < z < z_{mix} + 500$.

Vertical Turbulence

In the SBL, vertical turbulence, σ_w, is due to mechanical mixing, represented by σ_{wm}, which is computed as the Pythagorean sum of the boundary layer, σ_{mb}, and the residual layer, σ_{mr}, components:

$$\sigma_w = \sigma_{wm} \tag{S25}$$

$$\sigma_{wm}^2 = \sigma_{mb}^2 + \sigma_{mr}^2\,. \tag{S26}$$

The expression for the boundary layer component is as follows:

$$\sigma_{mb} = 1.3u_*\left(1 - \frac{z}{z_{zmix}}\right)^{1/2} \qquad \text{for } z < z_{mix} \tag{S27}$$

$$\sigma_{mb} = 0 \qquad \text{for } z \ge z_{mix}\,. \tag{S28}$$

The residual layer component, on the other hand, is computed as follows:

$$\sigma_{mr} = 0.02U(z_{mix})\left(\frac{z}{z_{zmix}}\right) \qquad \text{for } z < z_{mix} \tag{S29}$$

$$\sigma_{mr} = 0.02U(z_{mix}) \qquad \text{for } z \geq z_{mix}\ . \tag{S30}$$

In the CBL, vertical turbulence is the Pythagorean sum of a mechanical mixing component, σ_{wm}, as described above, and a convective mixing component, σ_{wc}, so that obtain the following:

$$\sigma_w^2 = \sigma_{wm}^2 + \sigma_{wc}^2\ . \tag{S31}$$

The expression for the convective mixing component is as follows:

$$\sigma_{wc}^2 = 1.6w_*^2\left(\frac{z}{z_{zmix}}\right)^{2/3} \qquad \text{for } z \leq 0.1z_{mix} \tag{S32}$$

$$\sigma_{wc}^2 = 0.35w_*^2 \qquad \text{for } 0.1z_{mix} < z \leq 0.1z_{mix} \tag{S33}$$

$$\sigma_{wc}^2 = 0.35w_*^2 exp\left[-\frac{6(z - z_{mix})}{z_{mix}}\right] \qquad \text{for } z > 0.1z_{mix}\ . \tag{S34}$$

In Equations (S32)–(S34), the quantity, w_*, is the convective velocity scale, which is defined as follows:

$$w_* = \left(\frac{gHz_{mix}}{\rho c_p T_{ref}}\right)^{1/3}\ . \tag{S35}$$

In Equation (S35), ρ is the air density and c_p is the specific heat of air at constant pressure.

Lateral Turbulence

In the SBL, lateral turbulence, σ_v, is due to mechanical mixing only, represented by σ_{vm}, which varies with height.

$$\sigma_v = \sigma_{vm} \tag{S36}$$

$$\sigma_{vm0}^2 = 3.6u_*^2 \tag{S37}$$

$$\sigma_{vm}^2 = \sigma_{vm0}^2 + \left(\frac{z}{z_{zmix}}\right)[\sigma_{vm}^2(z_{mix}) - \sigma_{vm0}^2] \qquad \text{for } z < z_{mix} \tag{S38}$$

$$\sigma_{vm}^2 = min(\sigma_{vm0}^2, 0.25\, m^2 s^{-2}) \qquad \text{for } z \geq z_{mix}\ . \tag{S39}$$

In the CBL, lateral turbulence is due to both mechanical mixing as described above and convective mixing, σ_{vc}, such that we obtain the following:

$$\sigma_v^2 = \sigma_{vm}^2 + \sigma_{vc}^2 \tag{S40}$$

$$\sigma_{vc}^2 = \sigma_{vc0}^2 \qquad \text{for } z \leq z_{mix} \tag{S41}$$

$$\sigma_{vc}^2 = \sigma_{vc0}^2 + (\sigma_{vcr}^2 - \sigma_{vm0}^2)\left[\frac{z - z_{mix}}{0.2z_{mix}}\right] \qquad \text{for } z_{mix} < z < 1.2z_{mix} \tag{S42}$$

$$\sigma_{vc}^2 = \sigma_{vcr}^2 \qquad \text{for } z \geq 1.2 z_{mix}\,. \tag{S43}$$

In Equations (S41)–(S43), the following definitions hold:

$$\sigma_{vc0}^2 = 0.35 w_*^2 \tag{S44}$$

$$\sigma_{vcr}^2 = min(\sigma_{vc0}^2, 0.25\, m^2 s^{-2})\,. \tag{S45}$$

Complex Terrain Effects

To account for the effects of complex terrain, a two-layer structure is assumed such that the lower layer remains horizontal, while the upper layer rises over the terrain. The two layers are distinguished at a dividing streamline height, H_c, which is determined by the implicit equation:

$$\frac{U^2\{H_c\}}{2} = \int_{H_c}^{h_c} N^2 (h_c - z) dz\,, \tag{S46}$$

where h_c is the terrain influence height, which is defined as the minimum of the highest actual terrain and the local terrain-following height, z_t. The fraction of the plume mass below H_c is denoted by φ_p and computed as follows:

$$\varphi_p = \frac{\int_0^{H_c} C_G(x_r, y_r, z_r)\, dz}{\int_0^{\infty} C_G(x_r, y_r, z_r)\, dz}, \tag{S47}$$

where C_G is the concentration at the receptor coordinates (x_r, y_r, z_r) in the absence of the terrain.

The total plume concentration, C_T, is the superposition of a horizontal plume state and a terrain-following state:

$$C_T(x_r, y_r, z_r) = f \times C_G(x_r, y_r, z_r) + (1 - f) C_G(x_r, y_r, z_r - z_t) \tag{S48}$$

where we have the following:

$$f = 0.5\left(1 + \varphi_p\right). \tag{S49}$$

In USEPA (2004), Equation (S46) is applied only to the SBL, while φ_p is set to zero in the CBL. To avoid discontinuity in the transition from the SBL to the CBL, however, we apply Equation (S47) to both cases. This approach yields better inverse modeling results for the CBL case.

Vertical Inhomogeneity

The assumption of a steady-state plume requires that a single value of a meteorological parameter be used to describe dispersion within a given layer. Inhomogeneity in the layer must be averaged to yield effective homogeneous values, denoted by a tilde. For example, the wind speed in Equation (S2) should be interpreted as the effective wind speed, $\tilde{U}$. Effective parameters are such that, if x is the downwind horizontal distance of a receptor from the source, we obtain the following:

$$\tilde{\alpha} = \frac{1}{h_t - h_b} \int_{h_b}^{h_t} \alpha(z)\, dz \tag{S50}$$

$$h_b = H_p(x_r, y_r) \qquad \text{for } H_p(x_r, y_r) \leq z_r \tag{S51}$$

$$h_b = max\left(H_p(x_r, y_r) - 2.15\sigma_z(x), z_r\right) \quad \text{for } H_p(x_r, y_r) > z_r \tag{S52}$$

$$h_t = min\left(H_p(x_r, y_r) + 2.15\sigma_z(x), z_r\right) \quad \text{for } H_p(x_r, y_r) \le z_r \tag{S53}$$

$$h_t = H_p(x_r, y_r) \quad \text{for } H_p(x_r, y_r) > z_r. \tag{S54}$$

where $H_p(x_r, y_r)$ denotes the plume centroid (center of mass) height downwind of the source and is distinct from the centerline height. In the CBL, H_p is calculated as a function of source elevation, h_s, and downwind distance, x, as follows:

$$H_p = h_s + \frac{\langle \sigma_w \rangle}{\langle U \rangle}\left\{\frac{z_{mix}}{2} - h_s\right\}\left(\frac{x}{z_{mix}}\right). \tag{S55}$$

In Equation (S55), $\langle \sigma_w \rangle$ and $\langle U \rangle$ refer to averages of the vertical turbulence and wind speed over the entire boundary layer. In the SBL, the centerline and centroid heights coincide, i.e., $H_p = h_s$.

Equations (S50) through (S54) are overridden if $h_t > z_{mix}$, in which case $\tilde{\alpha} = \alpha(z_{mix})$.

Turbulent Spread

For the SBL, the turbulent spread parameters in Equation (S2) can now be written as follows:

$$\sigma_y = \frac{\widetilde{\sigma_v}\left(x/\widetilde{U}\right)}{\left\{1 + 78\frac{\widetilde{\sigma_v}\left(x/\widetilde{U}\right)}{z_{mix}}\right\}^{0.3}} \tag{S56}$$

$$\sigma_z = (1-\gamma)\sigma_{zgs} + \gamma\sigma_{zes} \tag{S57}$$

$$\gamma = min\left(\frac{h_s}{z_{mix}}, 1\right) \tag{S58}$$

$$\sigma_{zes} = \frac{\widetilde{\sigma_w}\left(x/\widetilde{U}\right)}{\left\{1 + \frac{\widetilde{\sigma_w}\left(x/\widetilde{U}\right)}{2}\left(\frac{1}{0.36h_s} + \frac{N}{0.27\widetilde{\sigma_w}}\right)\right\}^{1/2}} \tag{S59}$$

$$\sigma_{zgs} = \sqrt{\frac{2}{\pi}}\left(\frac{u_* x}{\widetilde{U}}\right)\left(1 + 0.7\frac{x}{L}\right)^{-1/3}. \tag{S60}$$

In Equations (S57) through (S60), σ_{zgs} and σ_{zes} refer to near-surface and elevated contributions to the vertical turbulent spread.

In the CBL, the horizontal spread is also given by Equation (S56). The vertical spread, however, is a Pythagorean sum of near-surface and elevated contributions, σ_{zgc} and σ_{zec}:

$$\sigma_z^2 = \sigma_{zec}^2 + \sigma_{zgc}^2 \tag{S61}$$

$$\sigma_{zec} = min\left(0.6 + \frac{4H_p}{z_{mix}}, 1.0\right)\widetilde{\sigma_w}\left(x/\widetilde{U}\right) \tag{S62}$$

$$\sigma_{zgc} = \frac{0.5}{|L|} max\left\{1 - 10\left(\frac{H_p}{z_{mix}}\right), 0\right\}\left(\frac{u_* x}{\widetilde{U}}\right)^2. \tag{S63}$$

Reference

1. U.S. Environmental Protection Agency (USEPA). AERMOD: Description of Model Formulation; EPA-600/R-05/047; Office of Air Quality Planning and Standards: Research Triangle Park, NC, 2004.