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Some q-Rung Dual Hesitant Fuzzy Heronian Mean Operators with Their Application to Multiple Attribute Group Decision-Making

Yuan Xu [1], Xiaopu Shang [1,*], Jun Wang [1], Wen Wu [1] and Huiqun Huang [2]

[1] School of Economics and Management, Beijing Jiaotong University, Beijing 100044, China; 17120627@bjtu.edu.cn (Y.X.); 14113149@bjtu.edu.cn (J.W.); wuwen@bjtu.edu.cn (W.W.)

[2] School of Software Engineering, Beijing Jiaotong University, Beijing 100044, China; 15301005@bjtu.edu.cn

* Correspondence: sxp@bjtu.edu.cn; Tel.: +86-10-5168-3854

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Abstract: The q-rung orthopair fuzzy sets (q-ROFSs), originated by Yager, are good tools to describe fuzziness in human cognitive processes. The basic elements of q-ROFSs are q-rung orthopair fuzzy numbers (q-ROFNs), which are constructed by membership and nonmembership degrees. As realistic decision-making is very complicated, decision makers (DMs) may be hesitant among several values when determining membership and nonmembership degrees. By incorporating dual hesitant fuzzy sets (DHFSs) into q-ROFSs, we propose a new technique to deal with uncertainty, called q-rung dual hesitant fuzzy sets (q-RDHFSs). Subsequently, we propose a family of q-rung dual hesitant fuzzy Heronian mean operators for q-RDHFSs. Further, the newly developed aggregation operators are utilized in multiple attribute group decision-making (MAGDM). We used the proposed method to solve a most suitable supplier selection problem to demonstrate its effectiveness and usefulness. The merits and advantages of the proposed method are highlighted via comparison with existing MAGDM methods. The main contribution of this paper is that a new method for MAGDM is proposed.

Keywords: q-rung orthopair fuzzy set; q-rung dual hesitant fuzzy; q-rung dual hesitant fuzzy Heronian mean; multiple attribute group decision-making

1. Introduction

With the rapid economic and technological development, competition among enterprises has become increasingly fierce. For manufacturing companies, choosing an appropriate supplier is of high importance. Generally speaking, companies need to collect relevant information for all suppliers and use some technologies to determine the most suitable one. In essence, supplier selection is a multiple attribute decision-making problem. Due to the complexity of modern decision-making problems, it is impossible for a single decision maker (DM) to grasp all the information of all decision objectives. Thus, many real decision-making problems often require group decision-making, i.e., multiple attribute group decision-making (MAGDM). Decision-making problems are constrained by a variety of internal and external factors. For example, as decision-making problems become increasingly complex, it is almost impossible to describe attribute values using crisp values. Decision-making problems often have enormous complexity and uncertainty. So, many scholars focus on how to deal with and describe uncertain phenomena. In 1986, Atanassov [1] proposed the concept of an intuitionistic fuzzy set (IFS) for coping with fuzziness and uncertainty. IFS is more powerful and useful than Zadeh's fuzzy set (FS) [2], as FS only has a membership degree, which makes it impossible to comprehensively describe imprecision. Since the appearance of IFS, it has been widely applied to medical diagnoses [3,4], pattern recognition [5,6], cluster analysis [7,8], and especially, MAGDM [9–12]. However, there are quite a few circumstances with which IFSs cannot cope. For instance, in some cases, the sum of the membership and nonmembership degrees provided by DMs is greater than that of their square sum being less than or equal to one. To effectively address these cases, the concept of the Pythagorean fuzzy set (PFS) was introduced by Yager [13]. Obviously, the PFS is a generalized form of an IFS and can describe a wider information range. Owing to the effectiveness and powerfulness of PFSs, MAGDM with Pythagorean fuzzy information have become a research topic of great interest. Studies on PFSs can be roughly divided into three categories. The first category includes extensions of classical decision-making methods to MAGDM with Pythagorean fuzzy information, the most representative of which are the Pythagorean fuzzy decision-making methods proposed by Zhang and Xu [14] and Khan et al. [15] based on TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution), and the one developed by Ren et al. [16] on the basis on TODIM (an acronym in Portuguese for Interactive Multi-Criteria Decision Making). The second category contains MAGDM methods with Pythagorean fuzzy information based on aggregation operators. Aggregation operators play a significantly important role in MAGDM. Solving MAGDM in different scenarios requires different aggregation operators. For example, to fairly treat membership and nonmembership degrees of PFSs, Ma et al. [17] raised symmetry operations of PFSs and proposed a battery of Pythagorean fuzzy symmetric aggregation operators. Xing et al. [18] put forward Pythagorean fuzzy Choquet integral aggregation operators based on Frank t-norm and t-conorm. To capture the interrelationship between aggregated Pythagorean fuzzy numbers (PFNs), Wei and Lu [19] put forward Pythagorean fuzzy Maclaurin symmetric mean operators. To fully absorb the advantages of Bonferroni mean and generalized Bonferroni mean in capturing the relationship among variables, Liang et al. [20] and Zhang et al. [21] introduced the Pythagorean fuzzy Bonferroni mean and generalized Pythagorean fuzzy Bonferroni mean operators, respectively. Due to the complexity of decision-making issues and the lack of sufficient experience, DMs often make unreasonable assessments. These unreasonable evaluation values have a serious negative impact on the final decision results. Thus, Li et al. [22] proposed the Pythagorean fuzzy power Muirhead mean operators to eliminate such bad impacts. Analogously, to fully utilize the advantages of Pythagorean fuzzy interaction operational rules in dealing with the interaction between membership and nonmembership degrees, Xu et al. [23] proposed the Pythagorean fuzzy interaction Muirhead mean operators. The third category is the investigation of combining PFSs with linguistic term sets. In actual MAGDM problems, the evaluations made by DMs need to be expressed from both qualitative and quantitative perspectives. Thus, Teng et al. [24], Du et al. [25], and Xian et al. [26] investigated MAGDM with Pythagorean fuzzy linguistic sets and interval-valued Pythagorean fuzzy linguistic sets, respectively. Considering uncertain linguistic terms provides DMs with a more convenient method to express their assessments. Geng et al. [27], Liu et al. [28], and Liu et al. [29] proposed the concept of Pythagorean fuzzy uncertain linguistic sets and studied their applications in MAGDM. In addition, some scientists also investigated MAGDM issues with Pythagorean 2-tuple linguistic information [30–32].

Although in the majority of cases IFSs and PFSs can successfully describe the attribute values in MAGDM, there are quite a few situations in which IFSs and PFSs are insufficient. According to the constraints of IFSs and PFSs, when the square sum of membership and nonmembership degrees exceed one, then the attribute value cannot be represented by both IFSs and PFSs. To deal with such a case, more recently, Yager [33] introduced the concept of the q-rung orthopair fuzzy set (q-ROFS), which can be viewed as an extension of IFS and PFS. From the definition of q-ROFSs, it is not difficult to see that q-ROFSs give DMs great freedom and a wider space within which to evaluate alternatives. Therefore, the decision-making opinions of DMs are greatly preserved, resulting in less information distortion. Analogous to PFSs, quite a few aggregation operators for q-ROFSs have been proposed [34–37]. To deal with both DMs' quantitative and qualitative evaluations in MAGDM, Li et al. [38] proposed q-rung orthopair linguistic sets as well as their aggregation operators. Moreover, Li et al. [39] introduced q-rung picture fuzzy linguistic sets by taking DMs' neutrality degree into consideration.

Due to the extreme complexity of realistic decision-making problems, the abovementioned decision-making methods with q-ROFSs are still insufficient. In reality, it is very common to encounter the following issues: (1) The complexity of decision-making problems causes DMs to be highly hesitant. In quite a few real-life decision-making scenarios, DMs may feel hesitant among a group of values when determining the attribute values in q-ROFSs. By taking such hesitancy into consideration, Torra [40] originated the concept of the hesitant fuzzy set (HFS), in which the membership degree is denoted by several discrete values instead of a single value. Afterwards, Zhu et al. [41] pointed out the drawback of HFS is that it only contains membership degrees. Subsequently, they proposed the concept of the dual hesitant fuzzy set (DHFS), which has both membership and nonmembership degrees. Recently, Wei and Lu [42] extended DHFS to PFS and proposed the concept of dual hesitant Pythagorean fuzzy set (DHPFS) (It is noted that Khan et al. [43] and Liang and Xu [44] also proposed the so-called hesitant Pythagorean fuzzy set, however, their definitions are the same as Wei and Lu's [42] DHPFS). Analogously, DMs may feel that it is difficult to determine membership and nonmembership degrees by single values, as they prefer to use several values to represent them in q-ROFSs. Therefore, this paper proposes the concept of the q-rung dual hesitant fuzzy set (q-RDHFS), which is constructed by a set of q-rung membership degrees and q-rung nonmembership degrees. Compared with DHFS and DHPFS, the proposed q-RDHFS allows the sum and square sum of membership and nonmembership degrees to be greater than one, providing decision makers more freedom to express their assessments. Compared with q-ROFS, the proposed q-RDHFS can effectively deal with DMs' hesitancy when determining membership and nonmembership degrees, consequently resulting in less information loss. Thus, the q-RDHFS exhibits more usefulness, power, and flexibility over DHFS, DHPFS, and q-ROFS. In Section 2, we introduce the concept of q-RDHFS in detail. (2) In most MAGDM, there is a strong correlation between attributes. Thus, in the process of information integration, it is not only necessary to aggregate the attribute values themselves but also to collect the correlation between them. Heronian mean (HM) [45] is the most common information aggregation method that can reflect the correlation between variables. Thus, we extended HM to q-RDHFSs to integrate q-rung dual hesitant fuzzy information. Then, we applied the proposed operators to solve MAGDM problems.

The main significance of this paper is that it expands the theory of q-ROFSs and DHFSs and proposes a new, powerful tool for describing uncertain phenomena, called q-RDHFSs. Compared with many existing fuzzy set theories, the newly proposed q-RDHFSs show great flexibility and effectiveness and can very effectively express the decision-making opinions of DMs in a very hesitant state. We also investigated their applications in MAGDM. The remainder of the paper is organized as follows. Section 2 briefly recalls some basic concepts. Section 3 presents some q-rung dual hesitant fuzzy Heronian mean operators. Section 4 introduces a novel approach to MAGDM. Section 5 provides a numerical example to demonstrate the validity and superiority of the proposed method. Finally, Section 6 summarizes the paper.

2. Basic Concepts

2.1. *q-Rung Orthopair Fuzzy Set*

Definition 1 [33]. *Let X be an ordinary fixed set. A q-ROFS A defined on X is given by*

$$A = \{\langle x, u_A(x), v_A(x)\rangle \,|x \in X\} \tag{1}$$

where $u_A(x)$ *and* $v_A(x)$ *represent the membership and nonmembership degrees, respectively, satisfying* $u_A(x) \in [0,1]$, $v_A(x) \in [0,1]$ *and* $0 \le u_A(x)^q + v_A(x)^q \le 1$, $(q \ge 1)$. *The indeterminacy degree is defined as* $\pi_A(x) = \left(u_A(x)^q + v_A(x)^q - u_A(x)^q v_A(x)^q\right)^{1/q}$. *For convenience,* $(u_A(x), v_A(x))$ *is called a q-rung orthopair fuzzy number (q-ROFN) by Liu and Wang [34], which can be denoted by* $A = (u_A, v_A)$.

From Definition 1, it is not difficult to find out that q-ROFS can describe a wider information range than IFSs and PFSs. To illustrate the difference among intuitionistic fuzzy numbers (IFNs), PFNs, and q-ROFNs, we present their space of acceptable membership degrees in Figure 1.

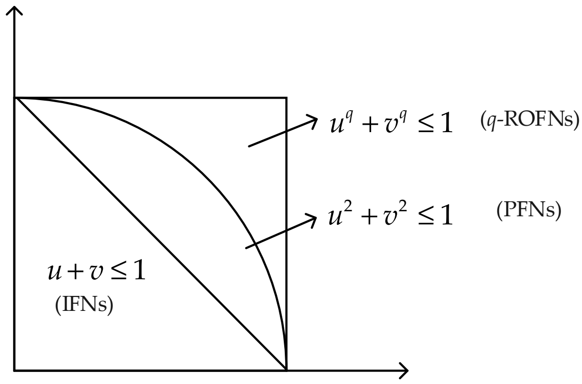

Figure 1. Comparison of grades of IFNs, PFNs, and q-ROFNs.

Figure 1 clearly shows that as the index of u and v increases, the range of information that the fuzzy numbers can describe also grows. Therefore, the q-ROFNs can expand the information that the attributes can describe and widen the space for experts to evaluate alternatives.

Definition 2 [34]. *Let $\widetilde{a}_1 = (u_1, v_1)$, $\widetilde{a}_2 = (u_2, v_2)$ be two q-ROFNs and λ be a positive real number. Then,*

1. $\widetilde{a}_1 \oplus \widetilde{a}_2 = \left(\left(u_1^q + u_2^q - u_1^q u_2^q \right)^{1/q}, v_1 v_2 \right)$.
2. $\widetilde{a}_1 \otimes \widetilde{a}_2 = \left(u_1 u_2, \left(v_1^q + v_2^q - v_1^q v_2^q \right)^{1/q} \right)$.
3. $\lambda \widetilde{a}_1 = \left(\left(1 - \left(1 - u_1^q \right)^{\lambda} \right)^{1/q}, v_1^{\lambda} \right)$.
4. $\widetilde{a}_1^{\lambda} = \left(u_1^{\lambda}, \left(1 - \left(1 - v_1^q \right)^{\lambda} \right)^{1/q} \right)$.

Definition 3 [34]. *Let $\widetilde{a} = (u_a, v_a)$ be a q-ROFN. Then, the score of $\widetilde{a}$ is defined as $S(\widetilde{a}) = u_a^q - v_a^q$ and the accuracy of $\widetilde{a}$ is defined as $H(\widetilde{a}) = u_a^q + v_a^q$. For any two q-ROFNs, $\widetilde{a}_1 = (u_1, v_1)$ and $\widetilde{a}_2 = (u_2, v_2)$*. Then,

1. If $S(\widetilde{a}_1) > S(\widetilde{a}_2)$, then $\widetilde{a}_1 > \widetilde{a}_2$;
2. If $S(\widetilde{a}_1) = S(\widetilde{a}_2)$, then
   if $H(\widetilde{a}_1) > H(\widetilde{a}_2)$, then $\widetilde{a}_1 > \widetilde{a}_2$;
   if $H(\widetilde{a}_1) = H(\widetilde{a}_2)$, then $\widetilde{a}_1 = \widetilde{a}_2$.

2.2. q-Rung Dual Hesitant Fuzzy Set

In this subsection, we introduce q-RDHFS, which is a new extension of q-ROFS and DHFS. Clearly, the proposed q-RDHFS is constructed of a set of membership degrees and several nonmembership degrees.

Definition 4. *Let X be an ordinary fixed set. A q-RDHFS A defined on X is given by*

$$A = \{ \langle x, h_A(x), g_A(x) \rangle | x \in X \}, \tag{2}$$

in which $h_A(x)$ and $g_A(x)$ are two sets of values in [0, 1] denoting the possible membership and nonmembership degrees of the element $x \in X$ to the set A, respectively, with the conditions

$$\gamma^q + \eta^q \leq 1 \ (q \geq 1),$$

where $\gamma \in h_A(x)$, $\eta \in g_A(x)$ for all $x \in X$. For convenience, the pair $d(x) = (h_A(x), g_A(x))$ is called a q-RDHFE denoted by $d = (h, g)$ with the conditions $\gamma \in h, \eta \in g$, $0 \leq \gamma, \eta \leq 1$, $0 \leq \gamma^q + \eta^q \leq 1$. Evidently, when q = 2, then q-RDHFS is reduced to Wei and Lu's [42] DHPFS, and when q = 1, then q-RDHFS is reduced to Zhu et al.'s [41] DHFS.

To compare any two q-RDHFEs, in the following, we propose a comparison law for q-RDHFEs.

Definition 5. *Let $d = (h, g)$ be a q-RDHFE, $S(d) = \left(\frac{1}{\#h}\sum\limits_{\gamma\in h}\gamma\right)^q - \left(\frac{1}{\#g}\sum\limits_{\eta\in g}\eta\right)^q$ be the score function of d, and $H(d) = \left(\frac{1}{\#h}\sum\limits_{\gamma\in h}\gamma\right)^q + \left(\frac{1}{\#g}\sum\limits_{\eta\in g}\eta^q\right)^q$ the accuracy function of d, where #h and #g are the numbers of the elements in h and g, respectively. Then, let $d_i = (h_i, g_i)(i = 1, 2)$ be any two q-RDHFEs. Thus, we have the following comparison laws:*

1. *If $S(d_1) > S(d_2)$, then d_1 is superior to d_2, denoted by $d_1 > d_2$;*
2. *If $S(d_1) > S(d_2)$, then*
   if $H(d_1) = H(d_2)$, then d_1 is equivalent to d_2, denoted by $d_1 = d_2$;
   if $H(d_1) > H(d_2)$, then d_1 is superior to d_2, denoted by $d_1 > d_2$.
   In the following, we define some operations of the q-RDHFEs.

Definition 6. *Let $d = (h, g)$, $d_1 = (h_1, g_1)$, and $d_2 = (h_2, g_2)$ be any three of q-RDHFEs and λ be a positive real number. Then,*

1. $d_1 \oplus d_2 = \cup_{\gamma_1\in h_1,\gamma_2\in h_2,\eta_1\in g_1,\eta_2\in g_2}\left\{\left\{\left(\gamma_1^q + \gamma_2^q - \gamma_1^q\gamma_2^q\right)^{\frac{1}{q}}\right\}, \{\eta_1\eta_2\}\right\}$;
2. $d_1 \otimes d_2 = \cup_{\gamma_1\in h_1,\gamma_2\in h_2,\eta_1\in g_1,\eta_2\in g_2}\left\{\{\gamma_1\gamma_2\}, \left\{\left(\eta_1^q + \eta_2^q - \eta_1^q\eta_2^q\right)^{\frac{1}{q}}\right\}\right\}$;
3. $\lambda d = \cup_{\gamma\in h,\eta\in g}\left\{\left\{\left(1 - (1-\gamma^q)^\lambda\right)^{\frac{1}{q}}\right\}, \{\eta^\lambda\}\right\}, \lambda > 0$;
4. $d^\lambda = \cup_{\gamma\in h,\eta\in g}\left\{\{\gamma^\lambda\}, \left\{\left(1 - (1-\eta^q)^\lambda\right)^{\frac{1}{q}}\right\}\right\}, \lambda > 0.$

2.3. Heronian Mean

The HM was first proposed by Sykora [45] for crisp numbers. It can process the interrelationship between arguments.

Definition 7 [45]. *Let $x_i(i = 1, 2, ..., n)$ be a group of real numbers, and $s, t > 0$. Then, the HM is defined as*

$$HM^{s,t}(x_1, x_2, ..., x_n) = \left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}x_i^s x_j^t\right)^{\frac{1}{s+t}}. \tag{3}$$

Recently, Yu [46] introduced the concept of geometric Heronian mean (GHM).

Definition 8 [46]. *Let $x_i(i = 1, 2, ..., n)$ be a group of numbers, and $s, t > 0$. Then, the GHM is defined as*

$$GHM^{p,q}(x_1, x_2, ..., x_n) = \frac{1}{p+q}\left(\prod_{i=1}^{n}\prod_{j=i}^{n}(px_i + qx_j)^{\frac{2}{n(n+1)}}\right). \tag{4}$$

3. The q-Rung Dual Hesitant Fuzzy Heronian Mean Operators

In this subsection, we extend the HM and GHM to q-RDHFSs and propose some new q-rung dual hesitant fuzzy Heronian mean aggregation operators.

3.1. The q-Rung Dual Hesitant Fuzzy Heronian Mean Operator

Definition 9. *Let $s, t \geq 0$ and $d_j = (h_j, g_j)$ $(j = 1, 2, ..., n)$ be a q-RDHFE. If*

$$q - RDHFHM^{s,t}(d_1, d_2, ..., d_n) = \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} d_i^s d_j^t \right)^{\frac{1}{s+t}}, \tag{5}$$

then $q - RDHFHM^{s,t}$ is called the q-rung dual hesitant fuzzy Heronian mean (q-RDHFHM) operator.

Based on the operational laws of the q-RDHFEs shown in Definition 6, we can get Theorem 1.

Theorem 1. *Let $s, t \geq 0$ and $d_j = (h_j, g_j)$ $(j = 1, 2, ..., n)$ be a q-RDHFE. Then, the aggregated value by the q-RDHFHM is also a q-RDHFE, and*

$$\begin{gathered} q - RDHFHM^{s,t}(d_1, d_2, ..., d_n) = \cup_{\gamma_i \in h_i, \gamma_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{ \left\{ \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(\gamma_i^s \gamma_j^t \right)^q \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{q(s+t)}} \right\}, \right. \\ \left. \left\{ \left(1 - \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(1 - \eta_i^q \right)^s \left(1 - \eta_j^q \right)^t \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{s+t}} \right)^{\frac{1}{q}} \right\} \right\} \end{gathered} \tag{6}$$

Proof. From Definition 6, we have

$$d_i^s = \cup_{\gamma_i \in h_i, \eta_i \in g_i} \left\{ \{\gamma_i^s\}, \left\{ \left(1 - \left(1 - \eta_i^q \right)^s \right)^{\frac{1}{q}} \right\} \right\},\ d_j^t = \cup_{\gamma_j \in h_j, \eta_j \in g_j} \left\{ \left\{ \gamma_j^t \right\}, \left\{ \left(1 - \left(1 - \eta_j^q \right)^t \right)^{\frac{1}{q}} \right\} \right\}.$$

Therefore,

$$d_i^s d_j^t = \cup_{\gamma_i \in h_i, \gamma_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{ \left\{ \gamma_i^s \gamma_j^t \right\}, \left\{ \left(1 - \left(1 - \eta_i^q \right)^s \left(1 - \eta_j^q \right)^t \right)^{\frac{1}{q}} \right\} \right\},$$

$$\sum_{j=i}^{n} d_i^s d_j^t = \cup_{\gamma_i \in h_i, \gamma_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{ \left\{ \left(1 - \prod_{j=i}^{n} \left(1 - \left(\gamma_i^s \gamma_j^t \right)^q \right) \right)^{\frac{1}{q}} \right\}, \left\{ \prod_{j=i}^{n} \left(1 - \left(1 - \eta_i^q \right)^s \left(1 - \eta_j^q \right)^t \right)^{\frac{1}{q}} \right\} \right\},$$

and

$$\sum_{i=1}^{n} \sum_{j=i}^{n} d_i^s d_j^t = \cup_{\gamma_i \in h_i, \gamma_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{ \left\{ \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(\gamma_i^s \gamma_j^t \right)^q \right) \right)^{\frac{1}{q}} \right\}, \left\{ \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(1 - \eta_i^q \right)^s \left(1 - \eta_j^q \right)^t \right)^{\frac{1}{q}} \right\} \right\},$$

$$\begin{gathered} \frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} d_i^s d_j^t = \cup_{\gamma_i \in h_i, \gamma_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{ \left\{ \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(\gamma_i^s \gamma_j^t \right)^q \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{q}} \right\}, \right. \\ \left. \left\{ \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(1 - \eta_i^q \right)^s \left(1 - \eta_j^q \right)^t \right)^{\frac{2}{qn(n+1)}} \right\} \right\}. \end{gathered}$$

Thus,

$$\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}d_i^s d_j^t\right)^{\frac{1}{s+t}} = \cup_{\gamma_i\in h_i,\gamma_j\in h_j,\eta_i\in g_i,\eta_j\in g_j}\left\{\left\{\left(1-\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-\left(\gamma_i^s\gamma_j^t\right)^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q(s+t)}}\right\},\right.$$

$$\left.\left\{\left(1-\left(1-\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-\left(1-\eta_i^q\right)^s\left(1-\eta_j^q\right)^t\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{s+t}}\right)^{\frac{1}{q}}\right\}\right\}.$$

The q-RDHFHM operator has the following properties. □

Theorem 2. *(Monotonicity) Let d_j and d'_j be two collections of q-RDHFEs. If $d_j \geq d'_j$ for all $j = 1, 2, \ldots, n$, then*

$$q-RDHFHM^{s,t}(d_1, d_2, ..., d_n) \geq q-RDHFHM^{s,t}\left(d'_1, d'_2, ..., d'_n\right) \tag{7}$$

Proof. Since $d_i \geq d'_i$ and $d_j \geq d'_j$ for $i = 1, 2, ..., n$ and $j = i, i+1, ..., n$, we have

$$d_i^s d_j^t \geq d'^s_i d'^t_j.$$

Then,

$$\sum_{i=1}^{n}\sum_{j=i}^{n}d_i^s d_j^t \geq \sum_{i=1}^{n}\sum_{j=i}^{n}d'^s_i d'^t_j,$$

and

$$\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}d_i^s d_j^t \geq \frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}d'^s_i d'^t_j.$$

So,

$$\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}d_i^s d_j^t\right)^{\frac{1}{s+t}} \geq \left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}d'^s_i d'^t_j\right)^{\frac{1}{s+t}}.$$

i.e.,

$$q-RDHFHM^{s,t}(d_1, d_2, ..., d_n) \geq q-RDHFHM^{s,t}\left(d'_1, d'_2, ..., d'_n\right).$$

□

Theorem 3. *(Idempotency) Let $d_j = \left(h_j, g_j\right)$, $j = 1, 2, ..., n$ be a collection of q-RDHFEs. If all the q-RDHFEs are equal, i.e., $d_j = d = (h, g)$, then*

$$q-RDHFHM^{s,t}(d_1, d_2, ..., d_n) = d. \tag{8}$$

Proof. Since $d_j = d$ for all i, we have

$$q-RDHFHM^{s,t}(d_1, d_2, ..., d_n) = \left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}d_i^s d_j^t\right)^{\frac{1}{s+t}} = \left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}d^{s+t}\right)^{\frac{1}{s+t}} = \left(d^{s+t}\right)^{\frac{1}{s+t}} = d.$$

□

Theorem 4. *(Boundedness) Let $d_j = \left(h_j, g_j\right)$, $j = 1, 2, \ldots, n$ be a collection of q-RDHFEs. If $d^+ = \max_j d_j$ and $d^- = \min_j d_j$, then*

$$d^+ \geq q-RDHFHM^{s,t}(d_1, d_2, ..., d_n) \geq d^-. \tag{9}$$

Proof. According to the Theorems 2 and 3, we can get

$$q-RDHFHM^{s,t}(d_1,d_2,\ldots,d_n) \le q-RDHFHM^{s,t}\left(d^+,d^+,\ldots,d^+\right),$$

and

$$q-RDHFHM^{s,t}(d_1,d_2,\ldots,d_n) \ge q-RDHFHM^{s,t}\left(d^+,d^+,\ldots,d^+\right).$$

Thus, we can get

$$d^+ \ge q-RDHFHM^{s,t}(d_1,d_2,\ldots,d_n) \ge d^-.$$

The advantages of q-RDHFHM are that it not only reflects the hesitation of DMs in the decision-making process and captures the correlation between attribute values, but it also shows great generality and flexibility. In the following, we can discuss some special cases of the q-RDHFHM operator. □

1. If $t \to 0$, then the q-RDHFHM reduces to a q-rung dual hesitant fuzzy generalized linear descending weighted mean operator, and we can obtain

$$\begin{aligned}
q-RDHFHM^{s,0}(d_1,d_2,\ldots,d_n) = \lim_{t\to 0}\Bigg\{ &\cup_{\gamma_i\in h_i,\gamma_j\in h_j,\eta_i\in g_i,\eta_j\in g_j}\left\{\left\{\left(1-\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-\left(\gamma_i^s\gamma_j^t\right)^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q(s+t)}}\right\},\right.\\
&\left.\left\{\left(1-\left(1-\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-\left(1-\eta_i^q\right)^s\left(1-\eta_j^q\right)^t\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{s+t}}\right)^{\frac{1}{q}}\right\}\right\}\Bigg\}\\
= \cup_{\gamma_i\in h_i,\eta_i\in g_i}&\left\{\left\{\left(1-\left(\prod_{i=1}^{n}\left(1-(\gamma_i^s)^q\right)^{n+1-i}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{qs}}\right\},\left\{\left(1-\left(1-\left(\prod_{i=1}^{n}\left(1-\left(1-\eta_i^q\right)^s\right)^{n+1-i}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{s}}\right)^{\frac{1}{q}}\right\}\right\}
\end{aligned} \tag{10}$$

Evidently, it is equivalent to weight the information $(d_1,d_2,\ldots,d_n)$ with the weight values $(n,\ n-1,\ \ldots,\ 1)$.

2. If $s \to 0$, then the q-RDHFHM reduces to a q-rung dual hesitant fuzzy generalized liner ascending weighted mean operator, and we can obtain

$$\begin{aligned}
q-RDHFHM^{0,t}(d_1,d_2,\ldots,d_n) = \lim_{s\to 0}\Bigg\{ &\cup_{\gamma_i\in h_i,\gamma_j\in h_j,\eta_i\in g_i,\eta_j\in g_j}\left\{\left\{\left(1-\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-\left(\gamma_i^s\gamma_j^t\right)^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q(s+t)}}\right\},\right.\\
&\left.\left\{\left(1-\left(1-\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-\left(1-\eta_i^q\right)^s\left(1-\eta_j^q\right)^t\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{s+t}}\right)^{\frac{1}{q}}\right\}\right\}\Bigg\}\\
= \cup_{\gamma_j\in h_j,\eta_j\in g_j}&\left\{\left\{\left(1-\left(\prod_{i=1}^{n}\left(1-\left(\gamma_j^t\right)^q\right)^i\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{qt}}\right\},\left\{\left(1-\left(1-\left(\prod_{i=1}^{n}\left(1-\left(1-\eta_j^q\right)^t\right)^i\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{t}}\right)^{\frac{1}{q}}\right\}\right\}
\end{aligned} \tag{11}$$

Obviously, it is equivalent to weight the information $(d_1,d_2,\ldots,d_n)$ with weight values $(1,\ 2,\ \ldots,\ n)$.

3. If $s = t = \frac{1}{2}$, then the q-RDHFHM reduces to a q-rung dual hesitant fuzzy basic Heronian mean operator, and we can obtain

$$\begin{aligned}
&q-RDHFHM^{\frac{1}{2},\frac{1}{2}}(d_1,d_2,\ldots,d_n) =\\
&\cup_{\gamma_i\in h_i,\gamma_j\in h_j,\eta_i\in g_i,\eta_j\in g_j}\left\{\left\{\left(1-\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-\left(\sqrt{\gamma_i\gamma_j}\right)^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q}}\right\},\left\{\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-\sqrt{\left(1-\eta_i^q\right)\left(1-\eta_j^q\right)}\right)^{\frac{2}{nq(n+1)}}\right\}\right\}
\end{aligned} \tag{12}$$

4. If $s = t = 1$, then the q-RDHFHM reduces to a q-rung dual hesitant fuzzy line Heronian mean operator. It follows that

$$q-RDHFHM^{1,1}(d_1,d_2,...,d_n) = \cup_{\gamma_i\in h_i,\gamma_j\in h_j,\eta_i\in g_i,\eta_j\in g_j} \begin{Bmatrix} \left\{\left(1-\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-(\gamma_i\gamma_j)^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{2q}}\right\}, \\ \left\{\left(1-\left(1-\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-\left(1-\eta_i^q\right)\left(1-\eta_j^q\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{2}}\right)^{\frac{1}{q}}\right\} \end{Bmatrix} \tag{13}$$

5. If $q = 2$, then the q-RDHFHM reduces to a dual hesitant Pythagorean fuzzy Heronian mean operator. So, we can obtain

$$q-RDHFHM^{s,t}(d_1,d_2,...,d_n) = \cup_{\gamma_i\in h_i,\gamma_j\in h_j,\eta_i\in g_i,\eta_j\in g_j} \begin{Bmatrix} \left\{\left(1-\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-\left(\gamma_i^s\gamma_j^t\right)^2\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{2(s+t)}}\right\}, \\ \left\{\left(1-\left(1-\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-(1-\eta_i^2)^s\left(1-\eta_j^2\right)^t\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{s+t}}\right)^{\frac{1}{2}}\right\} \end{Bmatrix} \tag{14}$$

6. If $q = 1$, then the q-RDHFHM reduces to the dual hesitant fuzzy Heronian mean operator proposed by Yu et al. [47]. It follows that

$$q-RDHFHM^{s,t}(d_1,d_2,...,d_n) = \cup_{\gamma_i\in h_i,\gamma_j\in h_j,\eta_i\in g_i,\eta_j\in g_j} \begin{Bmatrix} \left\{\left(1-\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-\left(\gamma_i^s\gamma_j^t\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{s+t}}\right\}, \\ \left\{\left(1-\left(1-\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-(1-\eta_i)^s(1-\eta_j)^t\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{s+t}}\right)\right\} \end{Bmatrix} \tag{15}$$

3.2. The q-Rung Dual Hesitant Fuzzy Weighted Heronian Mean (q-RDHFWHM) Operator

Definition 10. *Let $s,t \geq 0$ and $d_j = (h_j, g_j)$ $(j = 1, 2, ..., n)$ be a collection q-RDHFEs. The q-RDHFWHM operator is defined as*

$$q-RDHFWHM^{s,t}(d_1,d_2,...,d_n) = \left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}(nw_id_i)^s\left(nw_jd_j\right)^t\right)^{\frac{1}{s+t}}, \tag{16}$$

where $w = (w_1, w_2, ..., w_n)^T$ is the weight vector of $(d_1, d_2, ..., d_n)$, satisfying $w_j \in [0, 1]$, $\sum_{j=1}^{n} w_j = 1$.

According to the operations for q-RDHFEs, the following theorem can be obtained.

Theorem 5. *Let $s,t \geq 0$ and $d_j = (h_j, g_j)$ $(j = 1, 2, ..., n)$ be a collection q-RDHFEs. The aggregated value by the q-RDHFWHM is also a q-RDHFE and*

$$\begin{gathered} q-RDHFWHM^{s,t}(d_1,d_2,...,d_n) = \\ \cup_{\gamma_i\in h_i,\gamma_j\in h_j,\eta_i\in g_i,\eta_j\in g_j}\left\{\left\{\left(1-\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-\left(\left(1-\left(1-\gamma_i^q\right)^{nw_i}\right)^{\frac{1}{s}}\right)\left(\left(1-\left(1-\gamma_j^q\right)^{nw_j}\right)^{\frac{1}{t}}\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{q(s+t)}}\right\},\right. \\ \left.\left\{\left(1-\left(1-\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-\left(1-\eta_i^{qnw_i}\right)^s\left(1-\eta_j^{qnw_j}\right)^t\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{s+t}}\right)^{\frac{1}{q}}\right\}\right\} \end{gathered} \tag{17}$$

The proof of Theorem 5 is similar to that of Theorem 1.

Theorem 6. *Suppose* $w = (1/n, 1/n, ..., 1/n)^T$. *Then,*

$$q - RDHFWHM^{s,t}(d_1, d_2, ..., d_n) = q - RDHFHM^{s,t}(d_1, d_2, ..., d_n). \quad (18)$$

Proof. Since $w = (1/n, 1/n, ..., 1/n)^T$, then according to Equation (20),

$$\begin{aligned} q - RDHFWHM^{s,t}(d_1, d_2, ..., d_n) = & \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} (nw_i d_i)^s (nw_j d_j)^t \right)^{\frac{1}{s+t}} \\ & = \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left(n\frac{1}{n} d_i \right)^s \left(n\frac{1}{n} d_j \right)^t \right)^{\frac{1}{s+t}} \\ & = \left(\frac{2}{n(n+2)} \sum_{i=1}^{n} \sum_{j=i}^{n} d_i^s d_j^t \right)^{\frac{1}{s+t}} = q - RDHFWHM^{s,t}(d_1, d_2, ..., d_n) \end{aligned}$$

Moreover, it is easy to prove that the q-RDHFWHM operator has the properties of monotonicity and boundedness. □

3.3. The q-Rung Dual Hesitant Fuzzy Geometric Heronian Mean Operator

In this subsection, we shall extend the GHM to aggregate q-rung dual hesitant fuzzy information.

Definition 11. *Let* $s, t \geq 0$ *and* $d_j = (h_j, g_j)$ $(j = 1, 2, ..., n)$ *be a q-RDHFE. Then, the q-RDHFGHM operator is defined as*

$$q - RDHFGHM^{s,t}(d_1, d_2, ..., d_n) = \frac{1}{s+t} \left(\prod_{i=1}^{n} \prod_{j=i}^{n} (sd_i + td_j)^{\frac{2}{n(n+1)}} \right). \quad (19)$$

Based on the operational laws of q-RDHFEs, the following theorem can be obtained.

Theorem 7. *Let* $s, t \geq 0$ *and* $d_j = (h_j, g_j)$ $(j = 1, 2, ..., n)$ *be a q-RDHFE. The aggregated value by the q-RDHFGHM is also q-RDHFE and*

$$\begin{aligned} & q - RDHFGHM^{s,t}(d_1, d_2, ..., d_n) = \\ & \cup_{\gamma_i \in h_i, \gamma_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{ \begin{array}{l} \left\{ \left(1 - \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(1 - \gamma_i^q \right)^s \left(1 - \gamma_j^q \right)^t \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{s+t}} \right)^{\frac{1}{q}} \right\}, \\ \left\{ \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(\eta_i^s \eta_j^t \right)^q \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{q(s+t)}} \right\} \end{array} \right\} \end{aligned} \quad (20)$$

Proof. According to Definition 6, we can get

$$sd_i = \cup_{\gamma_i \in h_i, \eta_i \in g_i} \left\{ \left\{ \left(1 - \left(1 - \gamma_i^q \right)^s \right)^{\frac{1}{q}} \right\}, \left\{ \eta_i^s \right\} \right\} \text{ and}$$
$$td_j = \cup_{\gamma_j \in h_j, \eta_j \in g_j} \left\{ \left\{ \left(1 - \left(1 - \gamma_j^q \right)^t \right)^{\frac{1}{q}} \right\}, \left\{ \eta_j^t \right\} \right\}.$$

Then,

$$sd_i + td_j = \cup_{\gamma_i \in h_i, \gamma_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{ \left\{ \left(1 - \left(1 - \gamma_i^q\right)^s \left(1 - \gamma_j^q\right)^t \right)^{\frac{1}{q}} \right\}, \left\{ \eta_i^s \eta_j^t \right\} \right\},$$

and

$$\left(sd_i + td_j\right)^{\frac{2}{n(n+1)}} = \cup_{\gamma_i \in h_i, \gamma_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{ \left\{ \left(1 - \left(1 - \gamma_i^q\right)^s \left(1 - \gamma_j^q\right)^t \right)^{\frac{2}{n(n+1)}} \right\}, \left\{ \left(1 - \left(1 - \left(\eta_i^s \eta_j^t\right)^q \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{q}} \right\} \right\}.$$

Thus,

$$\prod_{j=i}^{n} \left(sd_i + td_j\right)^{\frac{2}{n(n+1)}} = \cup_{\gamma_i \in h_i, \gamma_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{ \left\{ \prod_{j=i}^{n} \left(1 - \left(1 - \gamma_i^q\right)^s \left(1 - \gamma_j^q\right)^t \right)^{\frac{2}{n(n+1)}} \right\}, \left\{ \left(1 - \prod_{j=i}^{n} \left(1 - \left(\eta_i^s \eta_j^t\right)^q \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{q}} \right\} \right\},$$

and

$$\begin{aligned} \prod_{i=1}^{n} \prod_{j=i}^{n} \left(sd_i + td_j\right)^{\frac{2}{n(n+1)}} = \cup_{\gamma_i \in h_i, \gamma_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} & \left\{ \left\{ \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(1 - \gamma_i^q\right)^s \left(1 - \gamma_j^q\right)^t \right)^{\frac{2}{n(n+1)}} \right\}, \right. \\ & \left. \left\{ \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(\eta_i^s \eta_j^t\right)^q \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{q}} \right\} \right\} \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{1}{s+t} \left(\prod_{i=1}^{n} \prod_{j=i}^{n} \left(sd_i + td_j\right)^{\frac{2}{n(n+1)}} \right) = \cup_{\gamma_i \in h_i, \gamma_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} & \left\{ \left\{ \left(1 - \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(1 - \gamma_i^q\right)^s \left(1 - \gamma_j^q\right)^t \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{s+t}} \right)^{\frac{1}{q}} \right\}, \right. \\ & \left. \left\{ \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(\eta_i^s \eta_j^t\right)^q \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{q(s+t)}} \right\} \right\}. \end{aligned}$$

In the following, we present some desirable properties of the q-RDHFGHM operator. □

Theorem 8. *(Monotonicity) Let $d_j = (d_1, d_2, ..., d_n)$ and $d'_j = (d'_1, d'_2, ..., d'_n)$ be two collections of q-RDHFEs. If $d_j \geq d'_j$ for all $j = 1, 2, ..., n$, then*

$$q - RDHFGHM^{s,t}(d_1, d_2, ..., d_n) \geq q - RDHFGHM^{s,t}\left(d'_1, d'_2, ..., d'_n\right). \quad (21)$$

The proof of the Theorem 8 is similar to that of Theorem 2, which is omitted here.

Theorem 9. *(Idempotency) Let $d_j = \left(h_j, g_j\right)$, $j = 1, 2, ..., n$ be a collection of q-RDHFEs. If all the q-RDHFEs are equal, i.e., $d_j = d = (h, g)$, then*

$$q - RDHFGHM^{s,t}(d_1, d_2, ..., d_n) = d. \quad (22)$$

Proof. Since $d_j = d$ for all i, we have

$$\begin{aligned} q - RDHFGHM^{s,t}(d_1, d_2, ..., d_n) &= \frac{1}{s+t} \left(\prod_{i=1}^{n} \prod_{j=i}^{n} \left(sd_i + td_j\right)^{\frac{2}{n(n+1)}} \right) = \frac{1}{s+t} \left(\prod_{i=1}^{n} \prod_{j=i}^{n} (sd + td)^{\frac{2}{n(n+1)}} \right) \\ &= \frac{1}{s+t} \left(\prod_{i=1}^{n} \prod_{j=i}^{n} ((s+t)d)^{\frac{2}{n(n+1)}} \right) = \frac{1}{s+t}((s+t)d) = d \end{aligned}$$

□

Theorem 10. *(Boundedness) Let* $d_j = \left(h_j, g_j\right)$, $j = 1, 2, \ldots, n$ *be a collection of q-RDHFEs. If* $d^+ = \max\limits_j d_j$ *and* $d^- = \min\limits_j d_j$, *then*

$$d^+ \geq q - RDHFGHM^{s,t}(d_1, d_2, \ldots, d_n) \geq d^-. \tag{23}$$

Analogous to the q-RDHFHM operator, the proposed q-RDHFGHM operator also exhibits high generality and flexibility. In the following, we shall discuss some special cases of the q-RDHFGHM operator.

1. If $t \to 0$, then the q-RDHFGHM reduces to a q-rung dual hesitant fuzzy generalized geometric linear descending weighted mean operator, and we can obtain

$$\begin{aligned} & q - RDHFGHM^{s,0}(d_1, d_2, \ldots, d_n) \\ & = \lim_{t \to 0} \left\{ \cup_{\gamma_i \in h_i, \gamma_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{ \left\{ \left(1 - \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(1 - \gamma_i^q \right)^s \left(1 - r_j^q \right)^t \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{s+t}} \right)^{\frac{1}{q}} \right\}, \right. \right. \\ & \qquad \left. \left. \left\{ \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(\mu_i^s \eta_j^t \right)^q \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{q(s+t)}} \right\} \right\} \right. \\ & = \cup_{\gamma_i \in h_i, \gamma_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{ \left\{ \left(1 - \left(1 - \left(\prod_{i=1}^{n} \left(1 - \left(1 - \gamma_i^q \right)^s \right)^{n+1-i} \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{s}} \right)^{\frac{1}{q}} \right\}, \right. \\ & \qquad \left. \left\{ \left(1 - \left(\prod_{i=1}^{n} \left(1 - \left(\eta_i^s \right)^q \right)^{n+1-i} \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{qs}} \right\} \right\} \end{aligned} \tag{24}$$

2. If $s \to 0$, the q-RDHFGHM reduces to a q-rung dual hesitant fuzzy generalized geometric liner ascending weighted mean operator, and we can obtain

$$\begin{aligned} & q - RDHFGHM^{0,t}(d_1, d_2, \ldots, d_n) \\ & = \lim_{s \to 0} \left\{ \cup_{\gamma_i \in h_i, \gamma_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{ \left\{ \left(1 - \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(1 - \gamma_i^q \right)^s \left(1 - \gamma_j^q \right)^t \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{s+t}} \right)^{\frac{1}{q}} \right\}, \right. \right. \\ & \qquad \left. \left. \left\{ \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(\eta_i^s \eta_j^t \right)^q \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{q(s+t)}} \right\} \right\} \right. \\ & = \cup_{\gamma_i \in h_i, \gamma_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{ \left\{ \left(1 - \left(1 - \left(\prod_{i=1}^{n} \left(1 - \left(1 - \gamma_j^q \right)^t \right)^{i} \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{t}} \right)^{\frac{1}{q}} \right\}, \right. \\ & \qquad \left. \left\{ \left(1 - \left(\prod_{i=1}^{n} \left(1 - \left(\eta_j^t \right)^q \right)^{i} \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{qt}} \right\} \right\} \end{aligned} \tag{25}$$

3. If $s = t = \frac{1}{2}$, the q-RDHFGHM reduces to a q-rung dual hesitant fuzzy basic geometric Heronian mean operator, and we can obtain

$$\begin{aligned} & q - RDHFGHM^{\frac{1}{2},\frac{1}{2}}(d_1, d_2, \ldots, d_n) \\ & = \cup_{\gamma_i \in h_i, \gamma_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{ \left\{ \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \sqrt{\left(1 - \gamma_i^q \right) \left(1 - \gamma_j^q \right)} \right)^{\frac{2}{nq(n+1)}} \right\}, \left\{ \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(\sqrt{\eta_i \eta_j} \right)^q \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{q}} \right\} \right\} \end{aligned} \tag{26}$$

4. If $s = t = 1$, the q-RDHFGHM reduces to a q-rung dual hesitant fuzzy line Heronian mean operator, and it follows that

$$q - RDHFGHM^{1,1}(d_1, d_2, ..., d_n) = \cup_{\gamma_i \in h_i, \gamma_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{ \left\{ \left(1 - \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(1 - \gamma_i^q \right) \left(1 - \gamma_j^q \right) \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{2}} \right)^{\frac{1}{q}} \right\}, \left\{ \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - (\eta_i \eta_j)^q \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{2q}} \right\} \right\} \tag{27}$$

5. If $q = 2$, then the q-RDHFGHM reduces to the dual hesitant Pythagorean fuzzy Heronian mean operator, and can we can obtain

$$q - RDHFGHM^{s,t}(d_1, d_2, ..., d_n) = \cup_{\gamma_i \in h_i, \gamma_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{ \left\{ \left(1 - \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - (1 - \gamma_i^2)^s \left(1 - \gamma_j^2 \right)^t \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{s+t}} \right)^{\frac{1}{2}} \right\}, \left\{ \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(\eta_i^s \eta_j^t \right)^2 \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{2(s+t)}} \right\} \right\} \tag{28}$$

6. If $q = 1$, then the q-RDHFGHM reduces to the dual hesitant fuzzy Heronian mean operator proposed by Yu et al. [47], and it follows that

$$q - RDHFHM^{s,t}(d_1, d_2, ..., d_n) = \cup_{\gamma_i \in h_i, \gamma_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{ \left\{ \left(1 - \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - (1 - \gamma_i)^s \left(1 - \gamma_j \right)^t \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{s+t}} \right) \right\}, \left\{ \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(\eta_i^s \eta_j^t \right) \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{s+t}} \right\} \right\} \tag{29}$$

Similarly, the q-RDHFGHM does not consider the importance of the input arguments, which means the weights of the aggregated q-RDHFGHM are not taken into consideration. However, in real decision-making problems, the weight vector of the aggregated values plays an important role in the final ranking orders. Therefore, we propose the q-rung dual hesitant fuzzy weighted geometric Heronian mean (q-RDHFWGHM) operator, which can take the weights of the aggregated q-RDHFEs into account.

3.4. The q-Rung Dual Hesitant Fuzzy Weighted Geometric Heronian Mean Operator

Definition 12. *Let $s, t \geq 0$ and $d_j = (h_j, g_j)$ $(j = 1, 2, ..., n)$ be a collection of q-RDHFEs:*

$$q - RDHFWGHM^{s,t}(d_1, d_2, ..., d_n) = \frac{1}{s+t} \left(\prod_{i=1}^{n} \prod_{j=i}^{n} \left(s(d_i)^{nw_i} + t(d_j)^{nw_j} \right)^{\frac{2}{n(n+1)}} \right), \tag{30}$$

where $w = (w_1, w_2, ..., w_n)^T$ is the weight vector of $(d_1, d_2, ..., d_n)$, satisfying $w_j \in [0, 1]$, $\sum_{j=1}^{n} w_j = 1$.

Based on the operational laws of q-RDHFEs, the following theorem can be obtained.

Theorem 11. *Let $s,t \geq 0$ and $d_j = \left(h_j, g_j\right)$ $(j = 1, 2, ..., n)$ be a collection of q-RDHFEs. The aggregated value by the q-RDHFWGHM is also a q-RDHFE and*

$$q - RDHFWGHM^{s,t}(d_1, d_2, ..., d_n) = \cup_{\gamma_i \in h_i, \gamma_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{ \left\{ \left(1 - \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(1 - \gamma_i^{qnw_i}\right)^s \left(1 - \gamma_j^{qnw_j}\right)^t \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{s+t}} \right)^{\frac{1}{q}} \right\}, \left\{ \left(1 - \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(1 - \gamma_i^{qnw_i}\right)^s \left(1 - \gamma_j^{qnw_j}\right)^t \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{s+t}} \right)^{\frac{1}{q}} \right\} \right\} \quad (31)$$

The proof of Theorem 11 is similar to Theorem 5, which is omitted here.

Theorem 12. *Suppose $w = (1/n, 1/n, ..., 1/n)^T$. Then,*

$$q - RDHFWGHM^{s,t}(d_1, d_2, ..., d_n) = q - RDHFGHM^{s,t}(d_1, d_2, ..., d_n). \quad (32)$$

The proof of Theorem 12 is similar to Theorem 6, which is omitted here.

Similarly, it is easy to prove that the q-RDHFWGHM has the properties of monotonicity and boundedness.

4. A Novel Approach to MAGDM with q-Rung Dual Hesitant Fuzzy Information

4.1. Description of a Typical MAGDM Problem with q-Rung Dual Hesitant Fuzzy Information

A typical MAGDM problem with q-rung dual hesitant fuzzy information can be described as follows: Let $A = \{A_1, A_2, ..., A_m\}$ and the set of attributes and $G = \{G_1, G_2, ..., G_n\}$ be a set of attributes. Let $w = (w_1, w_2, ..., w_n)^T$ be the weight vector of attributes, where $w_j \geq 0$, $j = 1, 2, ..., n$ and $\sum_{j=1}^{n} w_j = 1$. Suppose that $D = \left(d_{ij}\right)_{m \times n} = \left(h_{ij}, g_{ij}\right)_{m \times n}$ is the q-rung dual hesitant fuzzy decision matrix, where h_{ij} and g_{ij} indicate, respectively, the positive and negative degrees assessed by the decision maker that the alternative A_i satisfies the attribute G_j.

4.2. An Algorithm for q-Rung Dual Hesitant Fuzzy MAGDM Problems

In the following subsection, we present a novel algorithm for MAGDM based on the proposed operators.

Step 1. Standardize the original decision matrix according the following equation:

$$d_{ij} = \begin{cases} (h_{ij}, g_{ij}) & G_j \in I_1 \\ (g_{ij}, h_{ij}) & G_j \in I_2 \end{cases}, \quad (33)$$

where I_1 represents benefit attributes and I_2 represents cost attributes.

Step 2. For alternative $A_i (i = 1, 2, ..., m)$, utilize the q-RDHFWHM operator

$$d_i = q - RDHFWHM^{s,t}(d_{i1}, d_{i2}, \cdots, d_{in}), \quad (34)$$

or the q-RDHFWGHM operator

$$d_i = q - RDHFWGHM^{s,t}(d_{i1}, d_{i2}, \cdots, d_{in}), \quad (35)$$

to aggregate all the attributes values.

Step 3. Compute the score functions of all the alternatives and rank them.

Step 4. Rank the corresponding alternatives according to the rank of overall values and select the best alternative.

5. Numerical Example

In this section, to demonstrate the validity of the proposed method, we provide a numerical example adopted from [48]. A company wants to select a supplier, and after primary evaluation, four possible suppliers (A_1, A_2, A_3, and A_4) remain on the candidates list. To select the best supplier, a set of experts are invited to assess the four suppliers regarding four attributes: (1) relationship closeness (G_1); (2) product quality (G_2); (3) price competitiveness (G_3); and (4) delivery performance (G_4). The weight vector of the attributes is $w = (0.17, 0.32, 0.38, 0.13)^T$. The DMs are required to utilize DHFEs to express their preference information. The dual hesitant fuzzy decision matrix is shown in Table 1.

Table 1. The dual hesitant fuzzy decision matrix.

	G_1	G_2	G_3	G_4
A_1	{{0.3, 0.4}, {0.6}}	{{0.7, 0.9}, {0.1}}	{{0.4}, {0.2,0.3}}	{{0.5, 0.6}, {0.2}}
A_2	{{0.2, 0.3}, {0.5}}	{{0.6, 0.7}, {0.2}}	{{0.7, 0.8}, {0.2}}	{{0.6}, {0.1, 0.2, 0.3}}
A_3	{{0.4}, {0.2,0.3}}	{{0.2,0.3,0.4}, {0.6}}	{{0.7,0.8}, {0.1}}	{{0.7}, {0.2,0.3}}
A_4	{{0.6,0.7}, {0.3}}	{{0.5}, {0.4}}	{{0.3,0.4}, {0.5}}	{{0.4, 0.6}, {0.1,0.2}}

5.1. The Decision-Making Process

Step 1. As all the attributes are of the benefit type, the original decision matrix does not need to be normalized.

Step 2. Utilize the q-RDHFWHM operator to aggregate attributes values, so that the overall assessments are obtained (assume $s = t = 1$ and $q = 3$). Due to the relatively large numbers, the overall assessments are omitted.

Step 3. Calculate the scores of the overall assessments of alternatives to obtain $s(d_1) = 0.2235$, $s(d_2) = 0.2631$, $s(d_3) = 0.2097$, and $s(d_4) = 0.0780$.

Step 4. Rank the overall assessments so that we can obtain $A_2 \succ A_1 \succ A_3 \succ A_4$. Therefore, the best alternative is A_2.

In Step 2, if we utilize the q-RDHFWGHM operator to aggregate decision makers' assessments, we can obtain $s(d_1) = 0.1187$, $s(d_2) = 0.1819$, $s(d_3) = 0.0862$, and $s(d_4) = 0.0566$. Therefore, the ranking order is $A_2 \succ A_1 \succ A_3 \succ A_4$ and the best alternative is also A_2.

5.2. The Influence of the Parameters on the Results

Evidently, it is noted that the parameters s, t, and q play very important roles in the results. In the following subsection, we investigate the effect of parameters on the score functions and ranking results. To better illustrate the effect of the parameters s and t on the ranking results, we investigate the effects from the following three aspects: (1) We assign several fixed values to s and t and calculate the scores of the overall assessments. Further, we derive the ranking results of the alternatives. (2) Let $s \in (0, 10]$ and $t \in (0, 10]$, we investigate the influence of s and t on the ranking results. (3) Let s or t be a fixed value and investigate the influence of another parameter on the ranking results. Details can be found in Tables 1 and 2 and Figures 2–13.

Table 2. Scores and ranking results by using the q-rung dual hesitant fuzzy weighted Heronian mean (q-RDHFWHM) operator (q = 3).

Parameters	Score Function $s(d_i)(i = 1, 2, 3, 4)$	Ranking Results
$s = t = 1/2$	$s(d_1) = 0.1617$ $s(d_2) = 0.2086$ $s(d_3) = 0.1532$ $s(d_4) = 0.0685$	$A_2 \succ A_1 \succ A_3 \succ A_4$
$s = t = 1$	$s(d_1) = 0.2235$ $s(d_2) = 0.2631$ $s(d_3) = 0.2093$ $s(d_4) = 0.0780$	$A_2 \succ A_1 \succ A_3 \succ A_4$
$s = t = 2$	$s(d_1) = 0.3235$ $s(d_2) = 0.3375$ $s(d_3) = 0.2989$ $s(d_4) = 0.0942$	$A_2 \succ A_1 \succ A_3 \succ A_4$
$s = t = 5$	$s(d_1) = 0.4494$ $s(d_2) = 0.4363$ $s(d_3) = 0.4174$ $s(d_4) = 0.1198$	$A_1 \succ A_2 \succ A_3 \succ A_4$
$s = 1, t = 2$	$s(d_1) = 0.2769$ $s(d_2) = 0.3124$ $s(d_3) = 0.2626$ $s(d_4) = 0.0836$	$A_2 \succ A_1 \succ A_3 \succ A_4$
$s = 2, t = 1$	$s(d_1) = 0.3001$ $s(d_2) = 0.3079$ $s(d_3) = 0.2677$ $s(d_4) = 0.0930$	$A_2 \succ A_1 \succ A_3 \succ A_4$
$s = 1, t = 5$	$s(d_1) = 0.4024$ $s(d_2) = 0.4118$ $s(d_3) = 0.3719$ $s(d_4) = 0.1042$	$A_2 \succ A_1 \succ A_3 \succ A_4$
$s = 5, t = 1$	$s(d_1) = 0.4463$ $s(d_2) = 0.3952$ $s(d_3) = 0.3825$ $s(d_4) = 0.1214$	$A_1 \succ A_2 \succ A_3 \succ A_4$

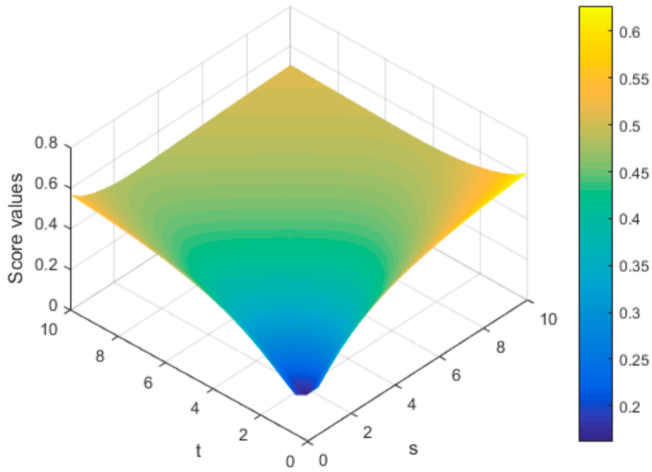

Figure 2. Scores of alternative A_1 when s, $t \in (0, 10)$ based on the q-RDHFWHM operator (q = 3).

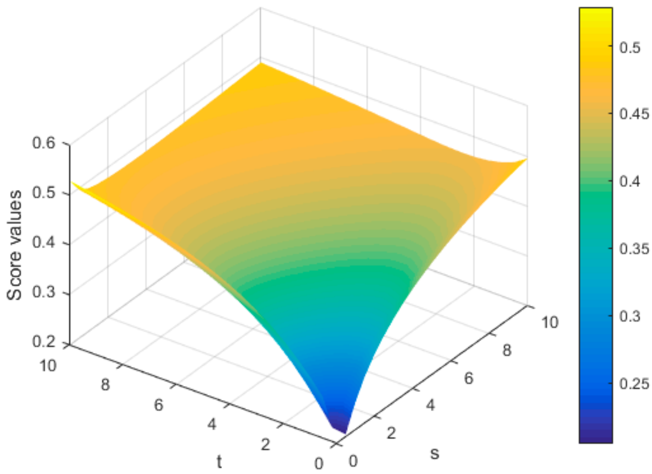

Figure 3. Scores of alternative A_2 when s, $t \in (0, 10)$ based on the q-RDHFWHM operator (q = 3).

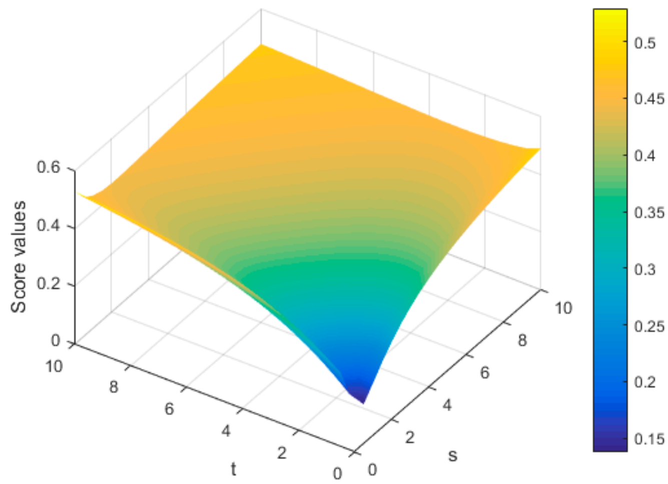

Figure 4. Scores of alternative A_3 when s, $t \in (0, 10)$ based on the q-RDHFWHM operator (q = 3).

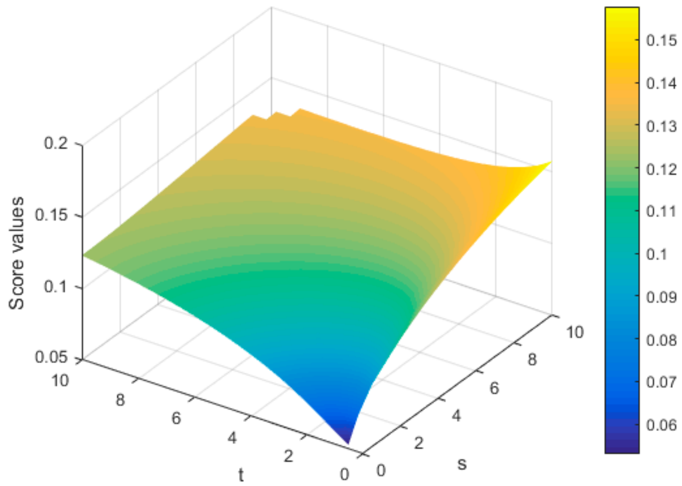

Figure 5. Scores of alternative A_4 when s, $t \in (0, 10)$ based on the q-RDHFWHM operator (q = 3).

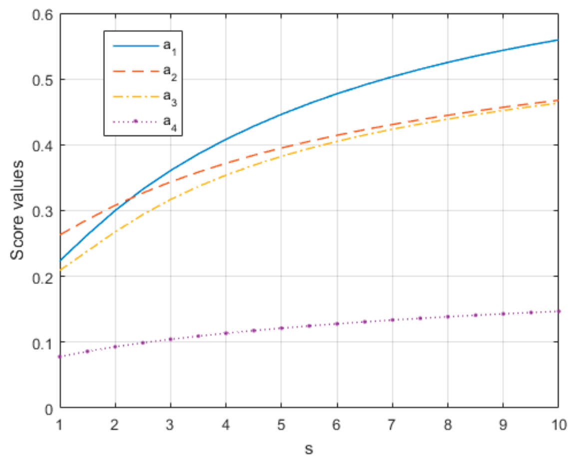

Figure 6. Scores of alternatives $A_i(i = 1, 2, 3, 4)$ when t = 1 and $s \in (1, 10)$ based on the q-RDHFWHM operator (q = 3).

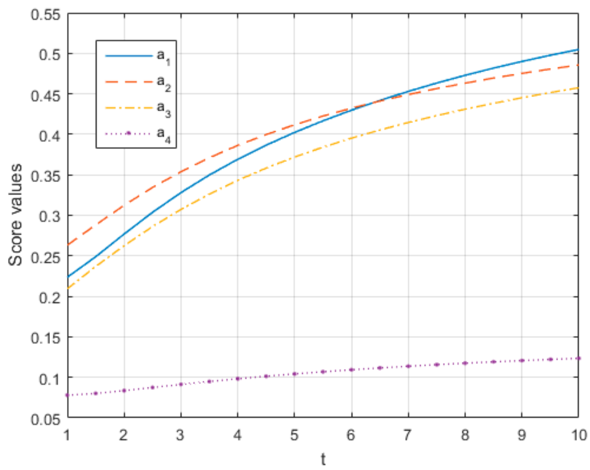

Figure 7. Scores of alternative $A_i(i = 1, 2, 3, 4)$ when s = 1 and $t \in (1, 10)$ based on the q-RDHFWHM operator (q = 3).

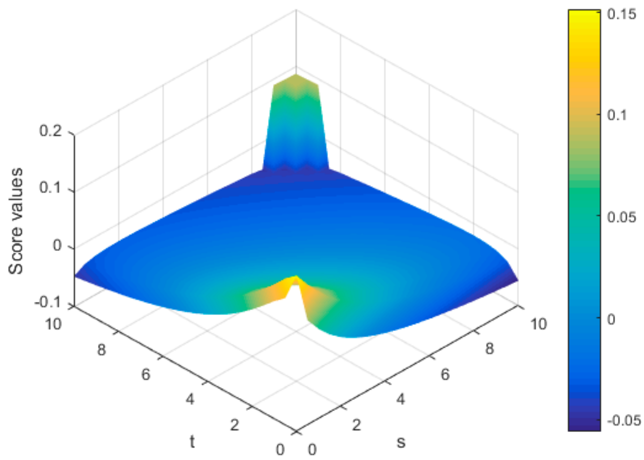

Figure 8. Scores of alternative A_1 when s, $t \in (0, 10)$ based on the q-rung dual hesitant fuzzy weighted geometric Heronian mean (q-RDHFWGHM) operator (q = 3).

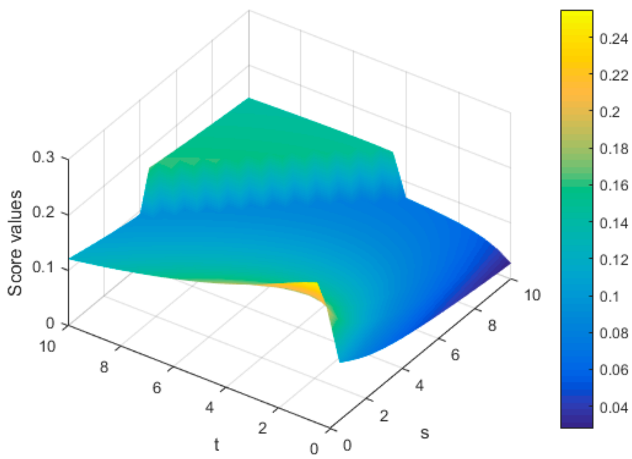

Figure 9. Scores of alternative A_2 when s, $t \in (0, 10)$ based on the q-RDHFWGHM operator (q = 3).

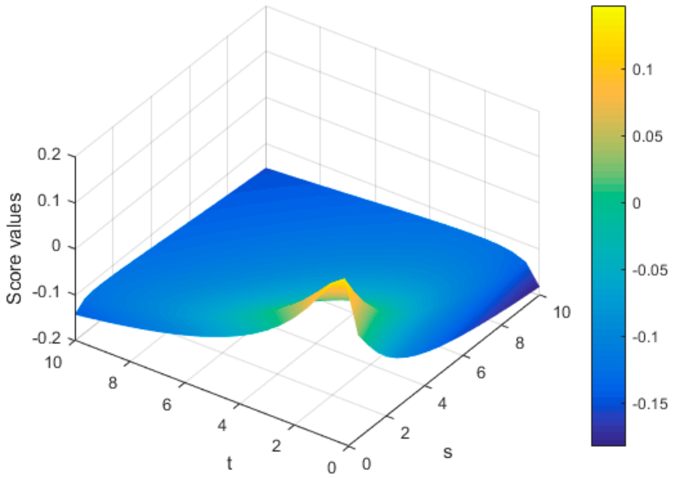

Figure 10. Scores of alternative A_3 when s, $t \in (0, 10)$ based on the q-RDHFWGHM operator (q = 3).

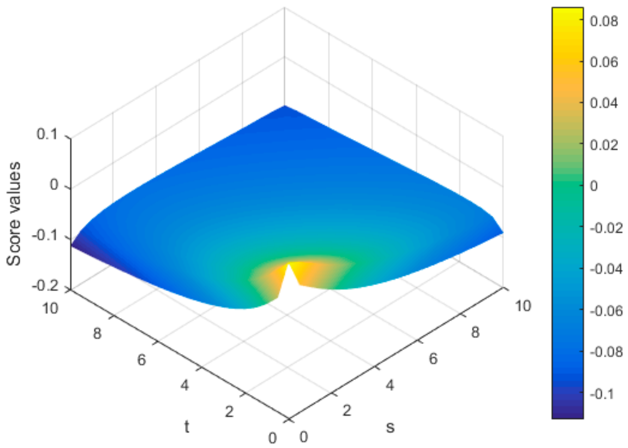

Figure 11. Scores of alternative A_4 when $s, t \in (0, 10)$ based on the q-RDHFWGHM operator (q = 3).

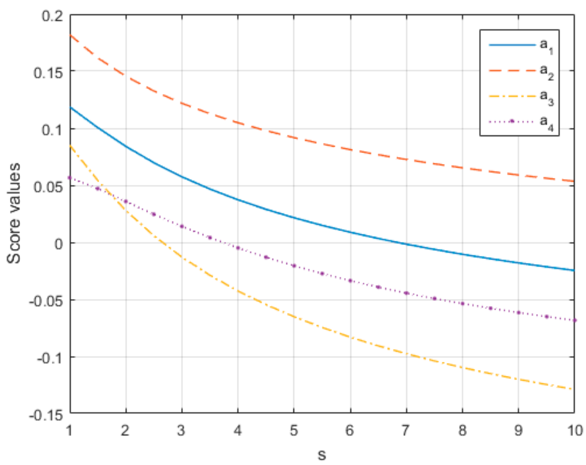

Figure 12. Scores of alternative $A_i (i = 1, 2, 3, 4)$ when $t = 1$ and $s \in (1, 10)$ based on the q-RDHFWGHM operator (q = 3).

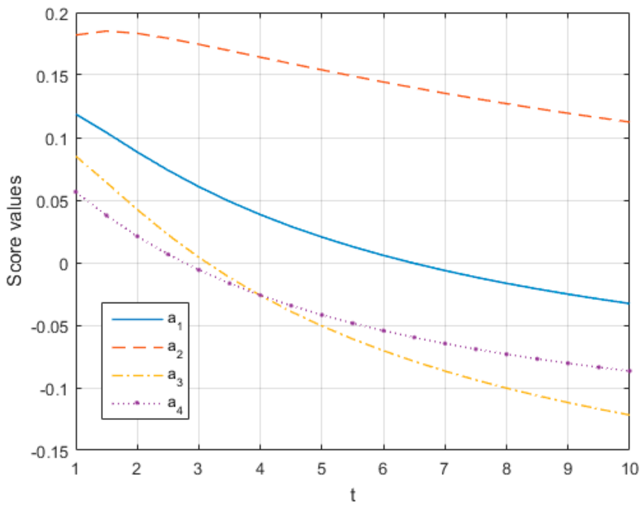

Figure 13. Scores of alternative $A_i(i = 1, 2, 3, 4)$ when $s = 1$ and $t \in (1, 10)$ based on the q-RDHFWGHM operator (q = 3).

From Table 2 and Figures 2–5, we can know that the scores and ranking results may be different for the different parameters s and t based on the q-RDHFWHM operator. However, the best alternative is A_2 or A_1. In addition, from Figures 6 and 7, we find that if we let t or s be a fixed value, then when s or t increases, the scores based on the q-RDHFWHM operator become greater and greater. Similarly, from Table 3 and Figures 8–11, we can obtain different scores and ranking results when s and t represent different values based on the q-RDHFWGHM operator. No matter what the values of s and t are, the best alternative is always A_2. However, what is opposite to the q-RDHFWHM operator is that if we let s or t be a fixed value, then when s or t increases, the scores based on the q-RDHFWGHM operator become smaller and smaller. The results shown in Tables 2 and 3 and Figures 2–13 demonstrate the flexibility of the aggregation processes by utilizing the q-RDHFWHM and q-RDHFWGHM operators. In real decision-making problems, DMs should choose the appropriate s and t according to their preference.

Table 3. Scores and ranking results by using the q-RDHFWGHM operator (q = 3).

Parameters	Score Function $s(d_i)(i = 1, 2, 3, 4)$	Ranking Results
$s = t = 1/2$	$s(d_1) = 0.1516$ $s(d_2) = 0.1972$ $s(d_3) = 0.1387$ $s(d_4) = 0.0861$	$A_2 \succ A_1 \succ A_3 \succ A_4$
$s = t = 1$	$s(d_1) = 0.1187$ $s(d_2) = 0.1819$ $s(d_3) = 0.0851$ $s(d_4) = 0.0566$	$A_2 \succ A_1 \succ A_3 \succ A_4$
$s = t = 2$	$s(d_1) = 0.0683$ $s(d_2) = 0.1544$ $s(d_3) = 0.0055$ $s(d_4) = 0.0132$	$A_2 \succ A_1 \succ A_4 \succ A_3$
$s = t = 5$	$s(d_1) = -0.0029$ $s(d_2) = 0.1054$ $s(d_3) = -0.0949$ $s(d_4) = -0.0521$	$A_2 \succ A_1 \succ A_4 \succ A_3$
$s = 1, t = 2$	$s(d_1) = 0.0884$ $s(d_2) = 0.1833$ $s(d_3) = 0.0423$ $s(d_4) = 0.0210$	$A_2 \succ A_1 \succ A_3 \succ A_4$
$s = 2, t = 1$	$s(d_1) = 0.0841$ $s(d_2) = 0.1456$ $s(d_3) = 0.0279$ $s(d_4) = 0.0358$	$A_2 \succ A_1 \succ A_4 \succ A_3$
$s = 1, t = 5$	$s(d_1) = 0.0204$ $s(d_2) = 0.1540$ $s(d_3) = -0.0507$ $s(d_4) = -0.0418$	$A_2 \succ A_1 \succ A_4 \succ A_3$
$s = 5, t = 1$	$s(d_1) = 0.0215$ $s(d_2) = 0.0917$ $s(d_3) = -0.0653$ $s(d_4) = -0.0206$	$A_2 \succ A_1 \succ A_4 \succ A_3$

In the following, we discuss the effects of the parameter q on the score function and ranking results based on q-RDHFWHM and q-RDHFWGHM operators. Details can be found in Figures 14 and 15.

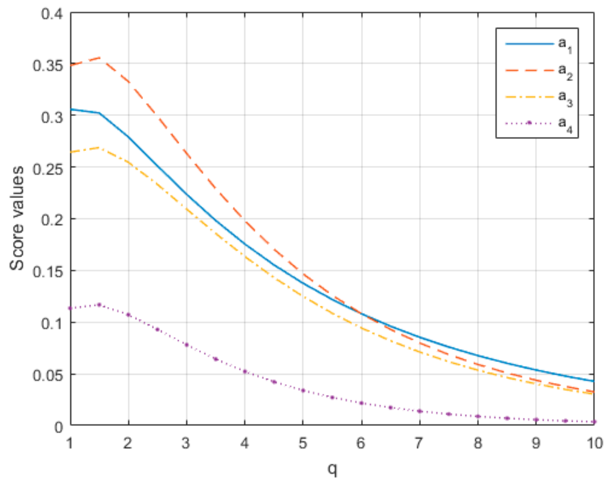

Figure 14. Scores of alternative $A_i (i = 1, 2, 3, 4)$ when s = t = 1 and $q \in (1, 10)$ based on the q-RDHFWHM operator.

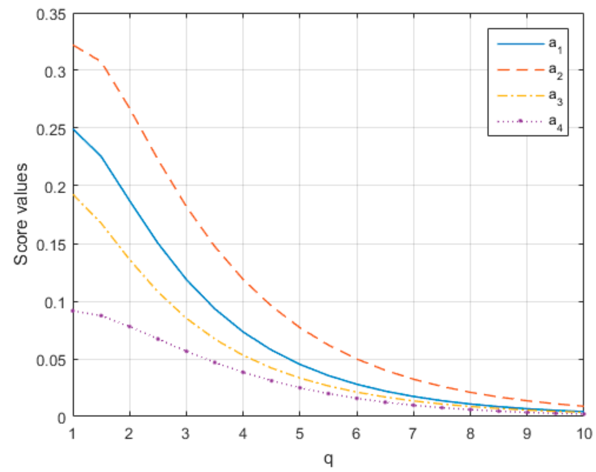

Figure 15. Scores of alternative $A_i (i = 1, 2, 3, 4)$ when s = t = 1 and $q \in (1,10)$ based on the q-RDHFWGHM operator.

As seen in Figures 14 and 15, the scores and ranking results can be different for the different parameter q based on the q-RDHFWHM and q-RDHFWGHM operators. However, the best alternative is always A_2 or A_4 based on the q-RDHFWHM, whereas the best alternative is always A_2 based on the q-RDHFWGHM operators. In addition, when q increases, both the scores obtained by the q-RDHFWHM and q-RDHFWGHM operators have the tendency to decrease.

5.3. Compared with Exiting MAGDM Methods

To demonstrate the advantages and superiorities of the proposed method, we compared our method with that proposed by Wang et al. [48], which was based on the dual hesitant fuzzy weighted averaging (DHFWA) operator proposed by Yu et al. [47], which was based on the dual hesitant fuzzy weighted Heronian mean (DHFWHM) operator proposed by Tu et al. [49], which was based on the dual hesitant fuzzy weighted Bonferroni mean (DHFWBM) operator that proposed by Wei and Lu [42], which was based on the dual hesitant Pythagorean fuzzy Hamacher weighted averaging (DHPFHWA) operator. We utilized these methods to solve the above example, and the score functions and ranking methods can be found in Table 4.

Table 4. Score functions and ranking results by using different methods.

Methods	Score Function $s(d_i)(i = 1,2,3,4)$	Ranking Results
Wang et al.' [48] method based on the DHFWA operator	$s(d_1) = 0.3915$ $s(d_2) = 0.4147$ $s(d_3) = 0.3573$ $s(d_4) = 0.1198$	$A_2 \succ A_1 \succ A_3 \succ A_4$
Yu et al.' s [47] method based on the DHFWHM operator ($s = t = 2$)	$s(d_1) = -0.3813$ $s(d_2) = -0.3916$ $s(d_3) = -0.3960$ $s(d_4) = -0.6147$	$A_1 \succ A_2 \succ A_3 \succ A_4$
Tu et al.'s [49] method based on the DHFWBM operator	$s(d_1) = 0.3152$ $s(d_2) = 0.3004$ $s(d_3) = 0.2978$ $s(d_4) = 0.0258$	$A_1 \succ A_2 \succ A_3 \succ A_4$
Wei and Lu's [42] method based on the DHPFHWA operator	$s(d_1) = 0.2369$ $s(d_2) = 0.2196$ $s(d_3) = 0.1284$ $s(d_4) = 0.0026$	$A_1 \succ A_2 \succ A_3 \succ A_4$
The proposed method in this paper	$s(d_1) = 0.2235$ $s(d_2) = 0.2631$ $s(d_3) = 0.2097$ $s(d_4) = 0.0780$	$A_2 \succ A_1 \succ A_3 \succ A_4$

First of all, Wang et al.'s [48], Yu et al.'s [47], and Tu et al.'s [49] methods are based on DHFSs. Wei and Lu's 429] method is based on DHPFSs. As mentioned above, DHFS and DHPFS are two special cases of q-RDHFS. When $q = 1$, then q-RDHFS is reduced to DHFS, and when $q = 2$, q-RDHFS is reduced to DHPFS. Evidently, q-RDHFS is more general and can describe a greater information range and process more information in the process of MAGDM. For instance, if an attribute value provided by DMs is {{0.1, 0.2, 0.5, 0.8}, {0.1, 0.2, 0.7}}, then obviously, the pair {{0.1, 0.2, 0.5, 0.8}, {0.1, 0.2, 0.7}} is not valid for DHFSs and DHPFSs. Thus, our method is more general, powerful, and can process more information in MAGDM.

Wang et al.'s [48] and Wei and Lu's [42] methods are based on the simple weighted averaging operator. The drawback of the two methods is that they do not consider the interrelationship between arguments. In other words, they assume all attributes are independent, which is not correct to some extent. In the abovementioned example, when choosing the most appropriate supplier, we need to consider not only the attribute values of each supplier but also the correlation between these attributes. Thus, Wang et al.'s [48] and Wei and Lu's [42] methods are not suitable for dealing with this problem. As our method has the ability to capture variable correlations, it is more reasonable than Wang et al.'s [48] and Wei and Lu's [42] methods for addressing this problem.

Tu et al.'s [49] method is based on Bonferroni mean (BM), and Yu et al.'s [47] and our methods are based on HM. The prominent characteristic of BM and HM is that both can consider the interrelationship between arguments. Therefore, all the three can process the interrelationship among attribute values. However, Yu et al.'s [47] method and ours are better than Tu et al.'s [49] method. In addition, as Yu et al.'s [47] is a special case of our method (when $q = 1$), our method is more general, scientific, and applicable than Yu et al.'s [47] method.

In real decision-making problems, we may encounter situations in which DMs are hesitant between several possible values when determining the membership and nonmembership degrees. Additionally, the sum and square sum of membership and nonmembership degrees may be more than one. Moreover, as attributes are related, the interrelationship between attribute values should be considered. In this paper, we present a novel approach to MAGDM problems based on q-RDHFS, which is a powerful tool for expressing and denoting DMs' assessments. It can deal with DMs' hesitancy and its lax constraints give DMs more freedom to express their preference information. In addition, our method is based on HM so that the interrelationship between attributes can be processed. Therefore, our method has some advantages and superiorities compared with existing methods.

6. Conclusions

Supplier selection is very important for manufacturing companies. Choosing a suitable supplier can greatly enhance the competitiveness and vitality of the company. In modern society, the selection of an appropriate supplier often requires a comprehensive assessment of all suppliers from multiple perspectives. Thus, supplier selection is one of the most common types of MAGDM problems in daily life. The main contributions of this paper are threefold. Firstly, we proposed the concept of q-RDHFS by combining DHFS with q-ROFS. The q-RDHFS can not only deal with DMs' hesitancy when determining the membership and nonmembership degrees but also gives DMs' more freedom to express their assessments. Secondly, we proposed the q-RDHFHM, q-RDHFWHM, q-RDHFGHM, and q-RDHFWGHM operators to effectively aggregate q-RDHFEs. Thirdly, we developed a novel method for MAGDM with q-rung dual hesitant fuzzy information. Considering the supplier selection problem is essentially a MAGDM issue, we also applied the proposed method to a real MAGDM problem to show its performance. Additionally, through comparative analysis the superiorities and advantages of the newly proposed method over existing methods are illustrated. Compared with the existing methods, the proposed method is more general and powerful. In addition, it has three parameters—q, s, and t—making the process of information aggregation more flexible. In real decision-making problems, DMs can choose the appropriate values of the parameters according to their preference. It is worth pointing out that as the newly proposed method is based on the HM operator, it mainly focuses on the interrelationship between any two q-RDHFEs. In future works, we should investigate more aggregation operators for fusing q-RDHFEs, such as the q-rung dual hesitant fuzzy Maclaurin symmetric mean, the q-rung dual hesitant fuzzy Hamy mean, and the q-rung dual hesitant fuzzy Muirhead mean operators, which have the ability of capturing the interrelationship among multiple q-RDHFEs.

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