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Convolution and Partial Sums of Certain Multivalent Analytic Functions Involving Srivastava–Tomovski Generalization of the Mittag–Leffler Function

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Abstract: We derive several properties such as convolution and partial sums of multivalent analytic functions associated with an operator involving Srivastava–Tomovski generalization of the Mittag–Leffler function.

Keywords: analytic function; Hadamard product (convolution); partial sum; Srivastava–Tomovski generalization of Mittag–Leffler function; subordination

1. Introduction

The Mittag–Leffler function $E_\alpha(z)$ [1] and its generalization $E_{\alpha,\beta}(z)$ [2] are defined by the following series:

$$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + 1)} \quad (z, \alpha \in \mathbb{C};\ \mathrm{Re}(\alpha) > 0) \tag{1}$$

and

$$E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + \beta)} \quad (z, \alpha, \beta \in \mathbb{C};\ \mathrm{Re}(\alpha) > 0), \tag{2}$$

respectively. It is known that these functions are extensions of exponential, hyperbolic, and trigonometric functions, since

$$E_1(z) = E_{1,1}(z) = e^z,$$
$$E_2(z^2) = E_{2,1}(z^2) = \cosh z$$

and

$$E_2(-z^2) = E_{2,1}(-z^2) = \cos z.$$

The functions $E_\alpha(z)$ and $E_{\alpha,\beta}(z)$ arise naturally in the resolvent of fractional integro-differential and fractional differential equations which are involved in random walks, super-diffusive transport problems, the kinetic equation, Lévy flights, and in the study of complex systems. In particular, the Mittag–Leffler function is an explicit formula for the solution the Riemann–Liouville fractional integrals that was developed by Hille and Tamarkin.

In [3], Srivastava and Tomovski defined a generalized Mittag–Leffler function $E^{\gamma,k}_{\alpha,\beta}(z)$ as follows:

$$E^{\gamma,k}_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{nk} z^n}{\Gamma(\alpha n + \beta) n!}, \tag{3}$$

$$(\alpha, \beta, \gamma, k, z \in \mathbb{C}; \quad \mathrm{Re}(\alpha) > \max\{0, \mathrm{Re}(k) - 1\}; \quad \mathrm{Re}(k) > 0),$$

where $(x)_n$ is the Pochhammer symbol

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = x(x+1)\cdots(x+n-1) \quad (n \in \mathbb{N}; \quad x \in \mathbb{C})$$

and $(x)_0 = 1$. They proved that the function $E^{\gamma,k}_{\alpha,\beta}(z)$ given by (3) is an entire function in the complex plane. Recently, Attiya [4] proved that, if $\mathrm{Re}(\alpha) \geq 0$ with $\mathrm{Re}(k) = 1$ and $\beta \neq 0$, the power series in (3) converges absolutely and analytically in $\mathbb{U} = \{z : |z| < 1\}$ for all $\gamma \in \mathbb{C}$. We call the function $E^{\gamma,k}_{\alpha,\beta}(z)$ the Srivastava–Tomovski generalization of the Mittag–Leffler function.

Let $\mathcal{A}(p)$ be the class of functions of the form

$$f(z) = z^p + \sum_{n=2}^{\infty} a_{n+p-1} z^{n+p-1} \quad (p \in \mathbb{N}) \tag{4}$$

which are analytic in $\mathbb{U}$. For $p = 1$, we write $\mathcal{A} := \mathcal{A}(1)$. The Hadamard product (or convolution) of two functions

$$f_j(z) = z^p + \sum_{n=2}^{\infty} a_{n+p-1,j} z^{n+p-1} \in \mathcal{A}(p) \quad (j = 1, 2)$$

is given by

$$(f_1 * f_2)(z) = z^p + \sum_{n=2}^{\infty} a_{n+p-1,1} a_{n+p-1,2} z^{n+p-1} = (f_2 * f_1)(z).$$

Let $\mathcal{P}$ denote the class of functions φ with $\varphi(0) = 1$. Suppose that f and g are analytic in $\mathbb{U}$. If there exists a Schwarz function w such that $f(z) = g(w(z))$ for $z \in \mathbb{U}$, then we say that the function f is subordinate to g and write $f(z) \prec g(z)$ for $z \in \mathbb{U}$. Furthermore, if g is univalent in $\mathbb{U}$, then the following equivalence holds true:

$$f(z) \prec g(z) \quad (z \in \mathbb{U}) \Leftrightarrow f(0) = g(0) \quad \text{and} \quad f(\mathbb{U}) \subset g(\mathbb{U}).$$

Throughout this paper, we assume that

$$\alpha, \beta, \gamma, k \in \mathbb{C}; \quad \mathrm{Re}(\alpha) > \max\{0, \mathrm{Re}(k) - 1\} \quad \text{and} \quad \mathrm{Re}(k) > 0.$$

We define the function $Q^{\gamma,k}_{\alpha,\beta}(z) \in \mathcal{A}(p)$ associated with the Srivastava–Tomovski generalization of the Mittag–Leffler function by

$$Q^{\gamma,k}_{\alpha,\beta}(z) = \frac{\Gamma(\alpha+\beta)}{(\gamma)_k} z^{p-1} \left(E^{\gamma,k}_{\alpha,\beta}(z) - \frac{1}{\Gamma(\beta)} \right) \quad (z \in \mathbb{U}). \tag{5}$$

For $f \in \mathcal{A}(p)$, we introduce a new operator $H^{\gamma,k}_{\alpha,\beta} : \mathcal{A}(p) \to \mathcal{A}(p)$ by

$$\begin{aligned} H^{\gamma,k}_{\alpha,\beta} f(z) &= Q^{\gamma,k}_{\alpha,\beta}(z) * f(z) \\ &= z^p + \sum_{n=2}^{\infty} \frac{\Gamma(\gamma+nk)\Gamma(\alpha+\beta)}{\Gamma(\gamma+k)\Gamma(\alpha n+\beta) n!} a_{n+p-1} z^{n+p-1}. \end{aligned} \tag{6}$$

Note that $H^{1,1}_{0,\beta} f(z) = f(z)$. From (6), we easily have the following identity:

$$z \left(H^{\gamma,k}_{\alpha,\beta} f(z) \right)' = \left(\frac{\gamma}{k} + 1 \right) H^{\gamma+1,k}_{\alpha,\beta} f(z) - \left(\frac{\gamma}{k} + 1 - p \right) H^{\gamma,k}_{\alpha,\beta} f(z). \tag{7}$$

It is noteworthy to mention that the Fox–Wright hypergeometric function ${}_q\Psi_s$ is more general than many of the extensions of the Mittag–Leffler function.

Now, we introduce a new subclass of $\mathcal{A}(p)$ by using the operator $H^{\gamma,k}_{\alpha,\beta}$.

Definition 1. *A function $f \in \mathcal{A}(p)$ is said to be in $\Omega_{\alpha,\beta}^{\gamma,k}(\lambda;\varphi)$ if it satisfies the first-order differential subordination:*

$$(1-\lambda)z^{-p}H_{\alpha,\beta}^{\gamma,k}f(z)+\frac{\lambda}{p}z^{-p+1}\left(H_{\alpha,\beta}^{\gamma,k}f(z)\right)'\prec\varphi(z), \tag{8}$$

where $\lambda \in \mathbb{C}$ and $\varphi \in \mathcal{P}$.

Lemma 1. *([5]). Let $g(z)=1+\sum_{n=m}^{\infty}b_nz^n$ $(m \in \mathbb{N})$ be analytic in $\mathbb{U}$. If $\mathrm{Re}(g(z))>0$ $(z \in \mathbb{U})$, then*

$$\mathrm{Re}\,(g(z)) \geq \frac{1-|z|^m}{1+|z|^m} \quad (z \in \mathbb{U}).$$

The study of the Mittag–Leffler function is an interesting topic in Geometric Function Theory. Many properties of the Mittag–Leffler function and the generalized Mittag–Leffler function can be found, e.g., in [6–22]. In this paper we shall make a further contribution to the subject by showing some interesting properties such as convolution and partial sums for functions in the class $\Omega_{\alpha,\beta}^{\gamma,k}(\lambda;\varphi)$.

2. Properties of the Class $\Omega_{\alpha,\beta}^{\gamma,k}(\lambda;\varphi)$

Theorem 1. *Let $\lambda \geq 0$ and*

$$f_j(z)=z^p+\sum_{n=2}^{\infty}a_{n+p-1,j}z^{n+p-1}\in\Omega_{\alpha,\beta}^{\gamma,k}(\lambda;\varphi_j) \quad (j=1,2), \tag{9}$$

where

$$\varphi_j(z)=\frac{1+A_jz}{1+B_jz} \quad \text{and} \quad -1\leq B_j<A_j\leq 1. \tag{10}$$

If $f \in \mathcal{A}(p)$ is defined by

$$H_{\alpha,\beta}^{\gamma,k}f(z)=\left(H_{\alpha,\beta}^{\gamma,k}f_1(z)\right)*\left(H_{\alpha,\beta}^{\gamma,k}f_2(z)\right), \tag{11}$$

then $f \in \Omega_{\alpha,\beta}^{\gamma,k}(\lambda;\varphi)$, where

$$\varphi(z)=\rho+(1-\rho)\frac{1+z}{1-z} \tag{12}$$

and ρ is given by

$$\rho=\begin{cases} 1-\frac{4(A_1-B_1)(A_2-B_2)}{(1-B_1)(1-B_2)}\left(1-\frac{p}{\lambda}\int_0^1\frac{t^{\frac{p}{\lambda}-1}}{1+t}dt\right) & (\lambda>0), \\ 1-\frac{2(A_1-B_1)(A_2-B_2)}{(1-B_1)(1-B_2)} & (\lambda=0). \end{cases} \tag{13}$$

The bound ρ is sharp when $B_1=B_2=-1$.

Proof. We consider the case when $\lambda>0$. Since $f_j \in \Omega_{\alpha,\beta}^{\gamma,k}(\lambda;\varphi_j)$, it follows that

$$\begin{aligned} p_j(z)&=(1-\lambda)z^{-p}H_{\alpha,\beta}^{\gamma,k}f_j(z)+\frac{\lambda}{p}z^{-p+1}\left(H_{\alpha,\beta}^{\gamma,k}f_j(z)\right)' \\ &\prec\frac{1+A_jz}{1+B_jz} \quad (j=1,2) \end{aligned} \tag{14}$$

and

$$H^{\gamma,k}_{\alpha,\beta}f_j(z) = \frac{p}{\lambda}z^{-\frac{p(1-\lambda)}{\lambda}}\int_0^z t^{\frac{p}{\lambda}-1}p_j(t)dt$$
$$= \frac{p}{\lambda}z^p\int_0^1 t^{\frac{p}{\lambda}-1}p_j(tz)dt \quad (j=1,2). \tag{15}$$

Now, if $f \in \mathcal{A}(p)$ is defined by (11), we find from (14) that

$$H^{\gamma,k}_{\alpha,\beta}f(z) = \left(H^{\gamma,k}_{\alpha,\beta}f_1(z)\right) * \left(H^{\gamma,k}_{\alpha,\beta}f_2(z)\right)$$
$$= \left(\frac{p}{\lambda}z^p\int_0^1 t^{\frac{p}{\lambda}-1}p_1(tz)dt\right) * \left(\frac{p}{\lambda}z^p\int_0^1 t^{\frac{p}{\lambda}-1}p_2(tz)dt\right)$$
$$= \frac{p}{\lambda}z^p\int_0^1 t^{\frac{p}{\lambda}-1}p_0(tz)dt, \tag{16}$$

where

$$p_0(z) = \frac{p}{\lambda}\int_0^1 t^{\frac{p}{\lambda}-1}(p_1 * p_2)(tz)dt. \tag{17}$$

Further, by using (14) and the Herglotz theorem, we see that

$$\mathrm{Re}\left\{\left(\frac{p_1(z)-\rho_1}{1-\rho_1}\right) * \left(\frac{1}{2} + \frac{p_2(z)-\rho_2}{2(1-\rho_2)}\right)\right\} > 0 \quad (z \in \mathbb{U}),$$

which leads to

$$\mathrm{Re}\{(p_1 * p_2)(z)\} > \rho_0 = 1 - 2(1-\rho_1)(1-\rho_2) \quad (z \in \mathbb{U}),$$

where

$$0 \leq \rho_j = \frac{1-A_j}{1-B_j} < 1 \quad (j=1,2).$$

Moreover, according to Lemma, we have

$$\mathrm{Re}\{(p_1 * p_2)(z)\} \geq \rho_0 + (1-\rho_0)\frac{1-|z|}{1+|z|} \quad (z \in \mathbb{U}). \tag{18}$$

Thus, it follows from (16) to (18) that

$$\mathrm{Re}\left\{(1-\lambda)z^{-p}H^{\gamma,k}_{\alpha,\beta}f(z) + \frac{\lambda}{p}z^{-p+1}\left(H^{\gamma,k}_{\alpha,\beta}f(z)\right)'\right\} = \mathrm{Re}\{p_0(z)\}$$
$$= \frac{p}{\lambda}\int_0^1 t^{\frac{p}{\lambda}-1}\mathrm{Re}\{(p_1 * p_2)(tz)\}dt$$
$$\geq \frac{p}{\lambda}\int_0^1 t^{\frac{p}{\lambda}-1}\left(\rho_0 + (1-\rho_0)\frac{1-|z|t}{1+|z|t}\right)dt$$
$$> \rho_0 + \frac{p(1-\rho_0)}{\lambda}\int_0^1 t^{\frac{p}{\lambda}-1}\frac{1-t}{1+t}dt$$
$$= 1 - 4(1-\rho_1)(1-\rho_2)\left(1 - \frac{p}{\lambda}\int_0^1 \frac{t^{\frac{p}{\lambda}-1}}{1+t}dt\right)$$
$$= \rho,$$

which proves that $f \in \Omega^{\gamma,k}_{\alpha,\beta}(\lambda;\varphi)$ for the function φ given by (12).

In order to show that the bound ρ is sharp, we take the functions $f_j \in \mathcal{A}(p)$ $(j=1,2)$ defined by

$$H^{\gamma,k}_{\alpha,\beta}f_j(z) = \frac{p}{\lambda}z^{-\frac{p(1-\lambda)}{\lambda}}\int_0^z t^{\frac{p}{\lambda}-1}\left(\frac{1+A_jt}{1-t}\right)dt \quad (j=1,2), \tag{19}$$

for which we have

$$p_j(z) = (1-\lambda)z^{-p}H^{\gamma,k}_{\alpha,\beta}f_j(z) + \frac{\lambda}{p}z^{-p+1}\left(H^{\gamma,k}_{\alpha,\beta}f_j(z)\right)' \\ = \frac{1+A_jz}{1-z} \quad (j=1,2)$$

and

$$(p_1 * p_2)(z) = \frac{1+A_1z}{1-z} * \frac{1+A_2z}{1-z} \\ = 1-(1+A_1)(1+A_2) + \frac{(1+A_1)(1+A_2)}{1-z}.$$

Hence, for the function f given by (11), we have

$$(1-\lambda)z^{-p}H^{\gamma,k}_{\alpha,\beta}f(z) + \frac{\lambda}{p}z^{-p+1}\left(H^{\gamma,k}_{\alpha,\beta}f(z)\right)' \\ = \frac{p}{\lambda}\int_0^1 t^{\frac{p}{\lambda}-1}\left(1-(1+A_1)(1+A_2) + \frac{(1+A_1)(1+A_2)}{1-tz}\right)dt \\ \to \rho \quad (as \quad z \to -1),$$

which shows that the number ρ is the best possible when $B_1 = B_2 = -1$.

For the case when $\lambda = 0$, the proof of Theorem 1 is simple, and we choose to omit the details involved. Now the proof of Theorem 1 is completed. □

Theorem 2. *Let α, β, γ, k, and λ be positive real numbers. Let $f(z) = z^p + \sum_{n=2}^{\infty} a_{n+p-1}z^{n+p-1} \in \mathcal{A}(p)$, $s_1(z) = z^p$, and $s_m(z) = z^p + \sum_{n=2}^{m} a_{n+p-1}z^{n+p-1}$ $(m \geq 2)$. Suppose that*

$$\sum_{n=2}^{\infty} c_n|a_{n+p-1}| \leq 1, \tag{20}$$

where

$$c_n = \frac{1-B}{A-B}\cdot\frac{\Gamma(\gamma+nk)\Gamma(\alpha+\beta)}{\Gamma(\beta+n\alpha)\Gamma(\gamma+k)n!}\left(1+\frac{\lambda}{p}(n-1)\right) \tag{21}$$

and $-1 \leq B < A \leq 1$.

(i) If $-1 \leq B \leq 0$, then $f \in \Omega^{\gamma,k}_{\alpha,\beta}\left(\lambda; \frac{1+Az}{1+Bz}\right)$.

(ii) If $\{c_n\}_1^{\infty}$ is nondecreasing, then

$$\mathrm{Re}\left\{\frac{f(z)}{s_m(z)}\right\} > 1 - \frac{1}{c_{m+1}} \tag{22}$$

and

$$\mathrm{Re}\left\{\frac{s_m(z)}{f(z)}\right\} > \frac{c_{m+1}}{1+c_{m+1}} \tag{23}$$

for $z \in \mathbb{U}$. The estimates in (22) and (23) are sharp for each $m \in \mathbb{N}$.

Proof. From the assumptions of Theorem 2, we have $c_n > 0$ $(n \in \mathbb{N})$. Let

$$J(z) = (1-\lambda)z^{-p}H^{\gamma,k}_{\alpha,\beta}f(z) + \frac{\lambda}{p}z^{-p+1}\left(H^{\gamma,k}_{\alpha,\beta}f(z)\right)' \\ = 1 + \sum_{n=2}^{\infty}\frac{\Gamma(\gamma+nk)\Gamma(\alpha+\beta)}{\Gamma(\beta+n\alpha)\Gamma(\gamma+k)n!}\left(1+\frac{\lambda}{p}(n-1)\right)a_{n+p-1}z^{n-1}. \tag{24}$$

(i) For $-1 \leq B \leq 0$ and $z \in \mathbb{U}$, it follows from (20), (21), and (24), that

$$
\begin{aligned}
&\left|\frac{J(z)-1}{A-BJ(z)}\right| \\
&= \left|\frac{\sum_{n=2}^{\infty} \frac{\Gamma(\gamma+nk)\Gamma(\alpha+\beta)}{\Gamma(\beta+n\alpha)\Gamma(\gamma+k)n!}\left(1+\frac{\lambda}{p}(n-1)\right) a_{n+p-1} z^{n-1}}{A-B-B\sum_{n=2}^{\infty} \frac{\Gamma(\gamma+nk)\Gamma(\alpha+\beta)}{\Gamma(\beta+n\alpha)\Gamma(\gamma+k)n!}\left(1+\frac{\lambda}{p}(n-1)\right) a_{n+p-1} z^{n-1}}\right| \\
&\leq \frac{\sum_{n=2}^{\infty} c_n |a_{n+p-1}|}{1-B+B\sum_{n=2}^{\infty} c_n |a_{n+p-1}|} \leq 1,
\end{aligned}
$$

which implies that

$$(1-\lambda)z^{-p} H_{\alpha,\beta}^{\gamma,k} f(z) + \frac{\lambda}{p} z^{-p+1}\left(H_{\alpha,\beta}^{\gamma,k} f(z)\right)' \prec \frac{1+Az}{1+Bz}.$$

Hence, $f \in \Omega_{\alpha,\beta}^{\gamma,k}\left(\lambda; \frac{1+Az}{1+Bz}\right)$.

(ii) Under the hypothesis in part (ii) of Theorem 2, we can see from (21) that $c_{n+1} > c_n > 1$ $(n \in \mathbb{N})$. Therefore, we have

$$\sum_{n=2}^{m} |a_{n+p-1}| + c_{m+1} \sum_{n=m+1}^{\infty} |a_{n+p-1}| \leq \sum_{n=2}^{\infty} c_n |a_{n+p-1}| \leq 1. \tag{25}$$

Upon setting

$$p_1(z) = c_{m+1}\left\{\frac{f(z)}{s_m(z)} - \left(1-\frac{1}{c_{m+1}}\right)\right\} = 1 + \frac{c_{m+1}\sum_{n=m+1}^{\infty} a_{n+p-1} z^{n-1}}{1+\sum_{n=2}^{\infty} a_{n+p-1} z^{n-1}},$$

and applying (25), we find that

$$\left|\frac{p_1(z)-1}{p_1(z)+1}\right| \leq \frac{c_{m+1}\sum_{n=m+1}^{\infty} |a_{n+p-1}|}{2-2\sum_{n=2}^{m} |a_{n+p-1}| - c_{m+1}\sum_{n=m+1}^{\infty} |a_{n+p-1}|} \leq 1 \quad (z \in \mathbb{U}),$$

which readily yields (22).

If we take

$$f(z) = z^p - \frac{z^{m+p}}{c_{m+1}}, \tag{26}$$

then

$$\frac{f(z)}{s_m(z)} = 1 - \frac{z^m}{c_{m+1}} \to 1 - \frac{1}{c_{m+1}} \quad \text{and} \quad z \to 1^-,$$

which shows that the bound in (22) is the best possible for each $m \in \mathbb{N}$.

Similarly, if we put

$$p_2(z) = (1+c_{m+1})\left(\frac{s_m(z)}{f(z)} - \frac{c_{m+1}}{1+c_{m+1}}\right),$$

then we can deduce that

$$
\begin{aligned}
\left|\frac{p_2(z)-1}{p_2(z)+1}\right| &\leq \frac{(1+c_{m+1})\sum_{n=m+1}^{\infty} |a_{n+p-1}|}{2-2\sum_{n=2}^{m} |a_{n+p-1}| - (c_{m+1}-1)\sum_{n=m+1}^{\infty} |a_{n+p-1}|} \\
&\leq 1 \quad (z \in \mathbb{U}),
\end{aligned}
$$

which yields (23).

The bound in (23) is sharp for each $m \in \mathbb{N}$, with the extremal function f given by (26). The proof of Theorem 2 is thus completed. □

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